Scale - Free model for governing universe dynamics

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We investigate the effects of scale-free model on cosmology, providing, in this way, a statistical background in the framework of general relativity. In order to discuss properties and time evolution of some relevant universe dynamical parameters (cosmographic parameters), such as $H(t)$ (Hubble parameter), $q(t)$ (deceleration parameter), $j(t)$ (jerk parameter) and $s(t)$ (snap parameter), which are well re-defined in the framework of scale-free model, we analyze a comparison between WMAP data. Hence the basic purpose of the work is to consider this statistical interpretation of mass distribution of universe, in order to have a mass density ρ dynamics, not inferred from Friedmann equations, via scale factor $a(t)$. This model, indeed, has been used also to explain a possible origin and a viable explanation of cosmological constant, which assumes a statistical interpretation without the presence of extended theories of gravity; hence the problem of dark energy could be revisited in the context of a classical probability distribution of mass, which is, in particular, for the scale-free model, $P(k) \sim k^{-\gamma}$, with $2 < \gamma < 3$. The ΛCDM model becomes, with these considerations, a consequence of the particular statistics together with the use of general relativity.

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I. INTRODUCTION

We found, in literature, many evidences of a lot of unsolved problems, which are still open questions in the picture of Einstein field equations. One of the most important issue of general relativity (GR) is a complete comprehension and interpretation of the cosmological constant in accordance with data. In the framework of Friedmann equations we write equations of motion of ρ, and of pressure p, in terms of scale parameter $a(t)$; on the other hand we do not consider any kind of mass dynamics, induced as an emergent effect of a statical mass distribution too; this could mean that we lose information about matter dynamics and distribution, in sense that we consider it as the static part of a dynamic equation, where the scale factor $a(t)$ is the true dynamical element. In order to consider a more direct matter dynamics we focus on ρ of universe and we supposed the validity of the scale-free model [7] into the framework of GR; i.e. to say that could exist a statistical model which describes the motion of matter and defines implicitly, in this way, the true dynamics of the scale factor. This interpretation will allow us to describe a dynamics of universe, starting only from the basic assumptions that every part of universe (in the framework of homogeneity and of isotropy) is characterized by a distribution of gravitons, induced by the simple presence of mass. Every graviton is linked to each other by a network or by a family of networks and its existence is provided by the implicit presence of nodes, characterized by a space-time vector x^μ, in which is possible to have a graviton. This model appears to be useful for matter distribution of universe, making us able, also, to consider the universe like a disordered system.

In particular, it is possible to describe the existence of heterogeneous spectrum of local interaction patterns topologically linked to the presence of nodes. Following the features done by other authors [6], we try to investigate the consequences of degree heterogeneity on dynamic properties of networks isolating a region of universe of correlated networks with a completely random organization, defined by the distribution $P(k) \sim k^{-\gamma}$, where γ has to be considered as an integer number. This ansatz shows apparently that dynamics would be limited to a particular region but it is generalized, simply, assuming, as in the beginning, a homogeneous and isotropic universe, which allows us to extend a particular result to the whole system (the universe evolution). Moreover the possibility to consider an elevate number of networks, N, is the basic idea to consider firstly, because, although $N < \infty$, it is true, at the same time, that N is a very large number.

In order to describe dynamics, we need all the quantities [16] describing it. These are the cosmographic parameters $H(t)$, $q(t)$, $j(t)$ and $s(t)$ [1, 9, 17, 18, 20, 21]; respectively the Hubble parameter, the deceleration parameter, the jerk parameter and finally the snap parameter; these are defined in principle for all t, or redshifts

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z. Assuming the power law degree distribution of the form $P(k) \sim k^{-\gamma}$, which is the basic idea of the model, and supposing no interactions among networks, that is equivalent to say that the type of interaction described by us, i.e. mean-field type, we are also able to infer how the scale-free model also deals with transitions and, so, it could be also fruitful to explain, for a fixed number of gravitons into account, for what values of the involved quantities the transition era could start, that could be a real topic to test the statistical model proposed [5].

It is also possible to characterize the model considering the value of parameter γ, which is the most influent parameter of the model. For example some evidences describe a probability distribution with $2 < \gamma < 3$, which for real systems appears to be a correct choice [6, 7]. Of course, indeed, depending to the value of γ, we find many different behaviors; in the present paper we do not matter initially about the exact value of γ in order to describe a particular behavior[1].

For our purposes, following the description of [6, 7], the first quantity introduced to describe the matter distribution is $\rho_k(t)$, which represents the average density of particles in nodes of degree k and for all the variables, it holds

$$\dot{\rho}_k(t) = -\rho_k + k\rho z_1 + \Lambda[\rho_k], \tag{1}$$

which is a first example of Langevin equation for our model; it suggests that the first two terms on the right side of equation are due to diffusion in uncorrelated random graphs and the third could be considered as a reaction kernel that depends only on ρ_k. From this equation we derive the dynamics of a particular choice of k. In order to show the evolution of density $\rho(t)$, the density which is present in Friedmann equations, we must require that

$$\rho(t) = \sum_k P(k)\rho_k(t). \tag{2}$$

In the continous limit, the above sum becomes an integrals over the k space, which has to be troncated by using $k^*(t) \sim z_1/\rho(t)$. Easily, following [6, 7]

$$\int P(k)\rho_k^2 dk \propto \begin{cases} \rho^{\gamma-1} + ck_c^{2-\gamma}\rho & \text{if } k_c > k^*, \\ k_c^{3-\gamma}\rho^2 & \text{if } k_c < k^*. \end{cases} \tag{3}$$

We consider that near a critical point $\mu_c \Longleftrightarrow k_c$, density is small, thus we expect that the mean-field regime holds when $k^* > k_c$ and $N < \infty$. The corresponding Langevin equation is

$$\dot{\rho}(t) \simeq a\rho - b\rho^2 + \sqrt{(a\rho + 2b\rho^2)/N}\eta(t) \tag{4}$$

where we considered a, b, c three integration constants and, in particular

$$k_c \propto N^{1/\omega}. \tag{5}$$

[1] See for details [7, 19].

If we assume universe as a whole system we must require the possible presence of noise term; we describe an universe without noise if

$$\rho \gg N^{(\gamma-\omega-3)/(2\omega)}, \tag{6}$$

with the ω parameter which has to be in relation with γ because, if eq. (6) holds, we must require

$$\gamma < 3\omega + 3, \tag{7}$$

or equivalently at least $\omega > -(1/3)$; because [6, 7] ω appears to be the maximum degree of the network size N, it must satisfy $\omega > 1$ and so our interval is always in agreement with this request. The basic idea becomes, so, to start with the scale-free probability, concerning a discrete interactions among gravitons and so, after defining the density time evolution (eq. (4)), considering the cosmographic evolution of parameters of the universe. All the considerations we will do, have to respect eqs. (6) and (7). It will be possible to link the universe description also to critical parameter, as said before, in order to describe the transition era, and it will be clear in the next sections.

The article is divided as follows: In the next section we describe, briefly, all the results involved by the use of GR in the framework of scale-free model for generic redshift z; in particular, we derive the functional form of the dynamical parameter $a(t)$, first, which will be the basis for the construction of the cosmographic parameters, later; their own expressions will be found very easily for generic redshifts without approximation in this section. In the third section we compare the results, obtained in the previous one, in the observable limit $z \ll 1$ with comparison with WMAP data, trying to link also the expressions found with the definition of Ω_Λ, which represents the density of cosmological constant Λ, i.e. *dark energy*, and we try to infer the role of Λ, just in the statistical way considered. The last section is devoted to some qualitative considerations, conclusion and perspectives for future works on this direction.

II. THE MODEL DYNAMICS

In this section we infer from scale-free model a specific dynamics for universe, without a particular limit on z, or of time t, of the dynamical quantities involved.

Such a dynamics, defined from the statistical background is easily valuable, thanks to the first Friedmann equation, which is an expression that makes a correspondence between $H(t)$ and ρ as follows

$$H^2 = \alpha\rho + \beta, \tag{8}$$

where we have considered, in principle, to take $k = 0$ in the above expression; and using the positions $\alpha = \frac{8\pi G}{3}$ and $\beta = \frac{\Lambda c^2}{3}$, we can imagine to evaluate ρ and its derivative $\dot{\rho}$ by putting them into eq. (4), having,

so, a differential equation for $H(t)$, in the case $\eta(t) = \delta(t)$, which translates the matter statistical distribution into an astrophysical differential equation for $H(t)$, which reads

$$\dot{H} = -\left(\frac{p\beta}{2} + \frac{b\beta^2}{2\alpha}\right)\frac{1}{H} + \\ + \left(\frac{p}{2} + \frac{q\beta}{\alpha}\right)H - \frac{b}{2\alpha}H^3 + \sqrt{\frac{f(H)}{N}}\delta(t). \quad (9)$$

It could give us the correspondent evolution of universe dynamics in terms of $H(t)$ and $a(t)$; in fact, the solution for $H(t)$ is writable, in terms of $a(t)$, because of definition $H(t) = \frac{d}{dt}\log a(t)$. All the discussion is completely global, in sense that, at this stage, we are not considering any kind of approximation on t, or on z, as already mentioned before; that is the same to say that a complete comprehension of dynamics is, in this paragraph, involved.

For the sake of simplicity we rewrite eq. (9) at $t \neq 0$, making the following positions

$$a \equiv \frac{p\beta}{2} + \frac{b\beta^2}{2\alpha},$$

$$b \equiv \frac{p}{2} + \frac{q\beta}{\alpha}, \quad (10)$$

$$c \equiv \frac{b}{2\alpha}.$$

and so we get the simple solution for $H(t)$

$$H\left(t - t^*\right) = A\sqrt{b + \sqrt{d}\tanh\left\{\sqrt{d}\left(t - t^*\right)\right\}}, \quad (11)$$

where $A = \frac{1}{\sqrt{2c}}$. In the previous expression we consider the position $d = b^2 - 4ac$ too and we notice that the integration constant could be considered as an arbitrary time t^*. Depending from d sign we could have different evolutions of $H(t)$; this sign, indeed, cannot be less than zero: A similar choice, i.e. $d < 0$, shows us a definite time interval in which $H(t)$ (and so $a(t)$) loses its meaning. Hence, for describing a variable $a(t)$, for all the time, we must have $d > 0$. On the other hand, this is evident, in order to have a solution, which has no periodicity, into account. This could, also, be tested, by considering directly its expression, $d = b^2 - 4ac$ and it gets

$$d = \frac{p^2}{4} + pq\beta\left(\frac{3}{8\pi G} - 2\right), \quad (12)$$

which is always positive for all the possible values assumed by p and q, if we require that $pq > 0$. Our choice is the simplest one: Because of p, q are "phenomenological" quantities, defining the type of reaction and process, we could assume, without losing information, $p > 0$ and $q > 0$. Indeed, it is possible in principle, that $p < 0$ and $q < 0$, but we will deny this, comparing also with WMAP data. Our efforts, on the other side, describe a model, whose validity is strongly dependent from p, q values, which has to be chosen by data.

From integrating eq. (9), it is possible to get, also, a precise value for $H(0)$. The theoretical expression, that we find, is

$$\frac{1}{\alpha^2}4H^4N = p\left(\frac{H^2 - \beta}{\alpha}\right) + 2q\left(\frac{H^2 - \beta}{\alpha}\right)^2, \quad (13)$$

which is the solution of equation $H(0) = \sqrt{\frac{f(H(0))}{N}}$, where $f\left(H\left(t = 0^+\right)\right) = f\left(H(0)\right)$, together with $H\left(t = 0^-\right) = 0$ condition, which is obvious, if we consider the positive direction of time. Eq. (13) gives two positive solutions, i.e.

$$H(0)_1 = \frac{\sqrt{\frac{p\alpha}{4N-2q} - \frac{4q\beta}{4N-2q} - \frac{\sqrt{p^2\alpha^2 - 16Np\alpha\beta + 32Nq\beta^2}}{4N-2q}}}{\sqrt{2}}, \quad (14)$$

$$H(0)_2 = \frac{\sqrt{\frac{p\alpha}{4N-2q} - \frac{2q\beta}{2N-q} + \frac{\sqrt{p^2\alpha^2 - 16Np\alpha\beta + 32Nq\beta^2}}{2(2N-q)}}}{\sqrt{2}}. \quad (15)$$

Both solutions are, in the thermodynamic limit, equivalent and they would read $H_0 = 0$; in our case, for $N < \infty$, only eq. (15) is relevant, if happens that $p\alpha > 2q\beta$. This condition will represent the correct interval for p and q choice.

Moreover, eq. (15) is also useful because tells us the value of t^* in the expression (11); it reads

$$t^* = \tanh^{-1}\left(\frac{b - 2cH_0^2}{\sqrt{d}}\right). \quad (16)$$

After evaluating the $H(t)$ dynamics we naturally achieve to find the expression for scale factor $a(t)$. We get

$$a(t) = a(0)e^{\frac{G\left(t - t^*\right)}{\sqrt{2cd}}}, \quad (17)$$

where the function $G\left(t - t^*\right)$ is expressed by

$$G\left(t - t^*\right) - \tilde{G} = \\ = -\sqrt{b - \sqrt{d}}\tanh^{-1}\left\{\frac{\sqrt{b + \sqrt{d}\tanh\left\{\sqrt{d}\left(t - t^*\right)\right\}}}{\sqrt{b - \sqrt{d}}}\right\} + \\ + \sqrt{b + \sqrt{d}}\tanh^{-1}\left\{\frac{\sqrt{b + \sqrt{d}\tanh\left\{\sqrt{d}\left(t - t^*\right)\right\}}}{\sqrt{b + \sqrt{d}}}\right\} \quad (18)$$

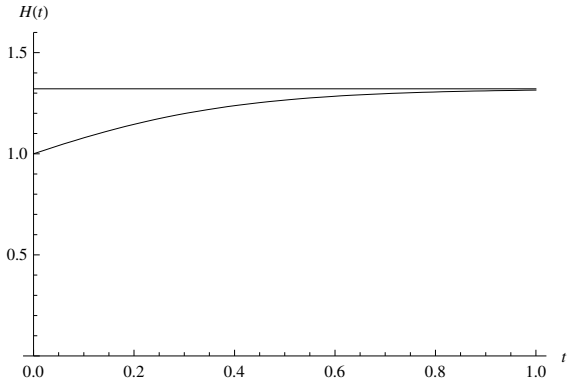

FIG. 1: In this graphic is plotted $H(t)$ with $k = 0$. The set of parameters is $a = 1$, $b = 3$ and $c = 1$, with normalization $H(0) = 1$.

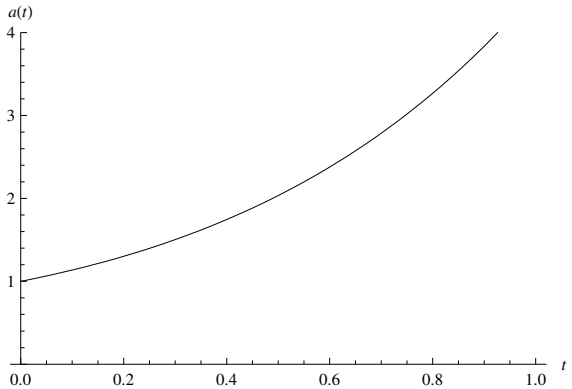

FIG. 2: In this graphic is plotted $a(t)$ with $k = 0$. The set of parameters is $a = 1$, $b = 3$ and $c = 1$, with normalization $a(0) = 1$.

with the integration constant $\tilde{G}$ of the form

$$\tilde{G} = \sqrt{b-\sqrt{d}}\tanh^{-1}\left\{\frac{\sqrt{b}}{\sqrt{b-\sqrt{d}}}\right\} - \sqrt{b+\sqrt{d}}\tanh^{-1}\left\{\frac{\sqrt{b}}{\sqrt{b+\sqrt{d}}}\right\}. \quad (19)$$

Following all the prescriptions we wrote, and considering the dynamical scale factor, in order to describe, for all the redshift z, the dynamics of universe, immediately, the expression for the deceleration parameter, $q(t)$ could be found; it reads

$$q(t) = -1 - \frac{\sqrt{c/2}d\sqrt{b+\sqrt{d}\tanh\left[\sqrt{d}\,(t-t^*)\right]}}{\left(b\cosh\left[\sqrt{d}\,(t-t^*)\right]+\sqrt{d}\sinh\left[\sqrt{d}\,(t-t^*)\right]\right)^2}. \quad (20)$$

It appears, of course, negative, which suggests that the model is compatible with accelerating universe model. The other expressions for $j(t)$ and $s(t)$, which discriminates the type of universe dynamics, are described in appendix for their complexity.

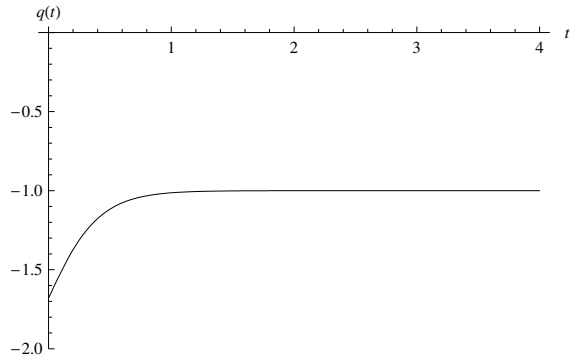

FIG. 3: In this graphic is plotted $q(t)$ with $k = 0$. The set of parameters is $a = 1$, $b = 3$ and $c = 1$.

III. THE OBSERVABLE LIMIT

Recently the possibility to get, from data, all the elements for universe dynamics, has been reported by several works (see for example [1–4, 20, 21]). A form of so-called cosmography, which corresponds to the possibility to infer, by some useful parameters, the universe evolution, is involved. We know, in fact, that we are living in the so-called period of *Precision Cosmology*, in which observations are extremely precise and allows us to test better than past, a given evolutionary models. The expressions we need are the so-called deceleration $q(t)$, jerk $j(t)$, snap $s(t)$ [1, 20, 21]. The cosmographic parameters are defined by derivatives of the scale factor for our epoch; they, instead of their simplicity, allow to fit the data also in terms of redshift, that is an implicit measure of time. It is remarkable to remember that the model, we are going to consider, here and in the following, is the standard model [1, 8, 9] which is not useful when z is not little, i.e. $z \ll 1$, that is, indeed, the case of validity of the Taylor expansion of scale parameter.
In fact, for our time, the cosmographic parameters are defined by the expression

$$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \\ +\frac{1}{6}j_0H_0^3(t-t_0)^3 + \frac{1}{24}s_0H_0^4(t-t_0)^4 + \ldots, \quad (21)$$

and so they reads

$$H(t) = \frac{\dot{a}}{a}, \quad (22)$$

$$q(t) = -\frac{\ddot{a}a}{\dot{a}^2}, \quad (23)$$

$$j(t) = \frac{a^{(3)}a^2}{\dot{a}^3}, \quad (24)$$

$$s(t) = -\frac{a^{(4)}a^3}{\dot{a}^4}. \tag{25}$$

We denote with a subscript 0 these parameters, in sense that they are evaluated at $t = t_0$; on the meaning of these parameters see for example [13]. In this paper we will take into account WMAP observations and so we will assume that the value of the Hubble constant is $H_0 \simeq 70 \pm 2$ km/sec/Mpc at our time $t = t_0$ [9, 10].

Following these advice, it is clear that, the cosmographic parameters may also be tested by WMAP data [10] and, above all, for the observed limit $z \ll 1$. It is also possible to express them, in terms of the dark energy density Ω_Λ, which could stress also the role of a cosmological constant involved into Einstein field equations, and so, give a description of ΛCDM model in terms of statistical background.

In order to understand such description, we can follow the prescription of [11]. Hence, we will use the Chevallier - Polarski - Linder (CPL) parameterization for the equation of state setting [14], which reads $w = w_0 + w_a - \frac{w_a}{1+z}$, with $a(z) = \frac{a_0}{1+z}$. Easily, noting the correspondence $t \longleftrightarrow z$, i.e. $dt = -\frac{d\log(1+z)}{H(z)}$. Moreover, for the scenario, we are considering, in the ΛCDM case, i.e.

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1, \tag{26}$$

where Ω_m is normalized[2] matter density, Ω_k spatial curvature density and Ω_Λ cosmological constant density, the list of above parameters has to be set $(w_0, w_a) = (-1, 0)$, and, then, after some straightforward calculations, it is useful to note $q_0 = \frac{1}{2} - \frac{3}{2}\Omega_\Lambda$, j_0 and $s_0 = 1 - \frac{9}{2}\Omega_m$.

Now we have all the ingredients to understand a complete comparison with scale-free quantities and cosmological parameters. In the next section, so, will be possible to write down an expression for Λ, by comparing with the above expression for Ω_Λ in terms of q_0.

A. The comparison with WMAP data

After comparing with the experimental value [23, 24] defined by H_0, q_0, s_0, l_0, in a easy way, we link measurable parameters, in terms of scale-free parameters

$$q_0 = -1 - \frac{d}{2bH_0}, \tag{27}$$

$$j_0 = 1 + 3\left(\frac{d}{2bH_0}\right) - \left(\frac{d}{2bH_0}\right)^2, \tag{28}$$

$$s_0 = 1 + \left(\frac{d}{2bH_0}\right)\left[6 - \left(\frac{d}{2bH_0}\right) + 3\left(\frac{d}{2bH_0}\right)^2 - \frac{4cd}{b}\right], \tag{29}$$

and inverting, we get expression for the scale-free parameters in function of deceleration, jerk and Hubble parameters as follows

$$c = \frac{1 - s_0 - (1+q_0)\left[7 + q_0 + 3(1+q_0)^2\right]}{8H_0(1+q_0)^2}, \tag{30}$$

$$b = H_0\frac{1 - s_0 - (1+q_0)\left[7 + q_0 + 3(1+q_0)^2\right]}{4(1+q_0)^2}, \tag{31}$$

$$d = -2H_0^2\frac{1 - s_0 - (1+q_0)\left[7 + q_0 + 3(1+q_0)^2\right]}{4(1+q_0)}. \tag{32}$$

All the previous results deal with a dynamics, induced by the use of the scale-free model, and treat the link among scale-free parameters and the observed parameters.
It is necessary to note that all the expressions obtained are in the case $k = 0$, which is the observed one [9]. Inducing a curvature term $k \neq 0$ is possible to achieve an expression for $a(t)$, which is not analytically defined. It appears as a strong complication of the model; indeed, it seems so also because observation provide a curvature density $\Omega_k \sim 0$, but we could easily consider a graphic of it, in order to fix ideas. It is important for us to

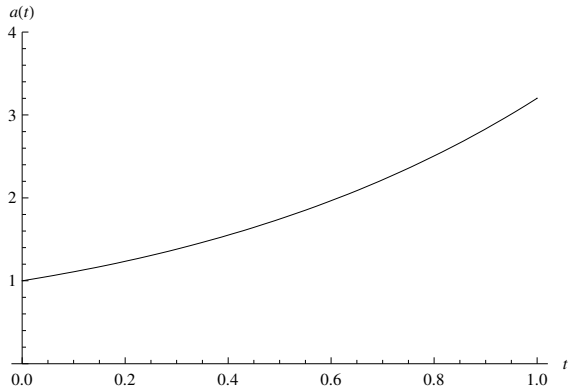

FIG. 4: In this graphic is plotted $a(t)$ with $k \neq 0$. The set of parameters is $a = 1$, $b = 3$ and $c = 1$, with normalization $a(0) = 1$.

underline that all the graphics are evaluated by a specific choice of parameters a, b, c reported in the graphic labels, not chosen by particular reasons but only for graphic simplicity.

Now we can write down all the parameters in terms of H_0, as follows

$$\begin{aligned} c &\sim 0.35H_0^{-1}\,, \\ b &\sim -2H_0\,, \\ d &\sim 2H_0^2\,. \end{aligned} \tag{33}$$

[2] Normalized by $\rho_c \equiv \frac{3H_0^2}{8\pi G}$.

and eqs. (?? and 33) allow us to write down cosmographic parameters in terms of Ω_Λ and, in particular, we find the definition of Λ in our model as follows

$$\Lambda \sim \frac{7\rho_c}{\cosh^2\left(\sqrt{2}H_0 t_0\right)} + 1. \tag{34}$$

Remembering the definition of scale-free parameters, postulated at the beginning, in particular of critical transition parameter, we find for it

$$k_c \sim 1.429\, H_0^3. \tag{35}$$

In other words the effect of a dark energy are modeled by the presence of a cut off k_c and for our epoch, in which $k > k_c$, all the previous results suggest how to consider the correct meaning of Λ: It could be considered as an effect of modeling universe with a $P(k) \sim k^{-\gamma}$, or, in other words, as a statistical result.

IV. CONCLUSION AND PERSPECTIVES

We investigated in this paper that the scale-free model, achieving a statistical interpretation of the "background" of space time, by the presence of a large number of gravitons, allowed us, thanks to networks interactions, to define directly ρ dynamics, without using, the scale factor $a(t)$ dynamics, in the picture of mean field approximation.

Moreover we showed, that, if that dynamics is involved into the framework of GR, we are able to define all the cosmographic parameters, in terms of time evolution and, in principle, also of redshift z. Best results were $q(t) < 0$ which represents the acceleration parameter of universe and tells us the universe expansion and also the cosmological constant Λ, which became dependent from statistical and cosmological parameters. This could be thought as an explanation of the peculiar reason for what a cosmological constant has to be put into Einstein field equations, in order to respect the correct statistical matter distribution and matter evolution.

We have, also, discussed the possibility to evaluate the parameters of scale-free theory at our epoch in terms of measured cosmographic parameters and vice-versa.

Starting from these results, we can discuss, indeed, also the possibility of defining a dark matter dynamical equations thanks to the expression of density motion, defined in the introduction and, of course, we can also focus a possible reformulation of Λ in the framework of a microscopic theory, involved by the scale-free model directly.

The choice of $P(k) \sim k^{-\gamma}$ appears to be comparable with the density of a high-energy astrophysical object and could suggest that scale-free model could be a possible adaptable statistical model for the cases of Fermi processes [9].

The scale-free model appears to be a framework model for gravitation so, in principle, extensions of GR are adaptable with scale-free model.

Moreover, all the previous results, concerning finite networks, could be extended in the case of infinite networks by using diverging cut-off k_c. It is possible to show, that depending from $\gamma - 1$ and q values, different results are involved, and this will concern to future efforts.

Other fields of future interest are to be found in the possibility to join the statistical behavior, described in this work, not necessary in the case of GR [15]. It will be clear in future work, that modifications occur if a different field theory is involved; idea which is possible, because the model appears to be independent from the field background. Concluding, of course, the statistics in this sense is a first kind of example in which the possibility to consider a different approach to the problem of Λ. This is possible, also passing through a quantum picture as showed, for example, in [22]. The deep difference of this kind of work is the classical way in which all the calculations have been evaluated.

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APPENDIX A: COSMOGRAPHIC PARAMETERS

The jerk parameter $j(t)$ is defined as follows

$$\frac{j(t)}{f(t)} = \beta_1 + \beta_2 \cosh 4\sqrt{d}t + 4\cosh 2\sqrt{d}t\,(\beta_3 + g_4) + \\ +\beta_5 \sinh(4\sqrt{d}t) + g_6 + 2\sqrt{d}\sinh(2\sqrt{d}t)\,(\beta_7 + g_8)\,,$$

where $f(t) = \frac{\cosh^{-4}\left(\sqrt{d}t\right)}{8\sqrt{2}}\left[b + \sqrt{d}\tanh\left(\sqrt{d}t\right)\right]^3$, and $g_n \equiv \frac{\beta_n H(t)}{A}$. The snap parameter $s(t)$ is

$$\frac{s(t)}{h(t)} = g_{11} - g_{12} + g_{13} + g_{14} + \cosh\left(6\sqrt{d}t\right)(g_{15} + g_{16} + g_{17}) + \\ \sinh\left(2\sqrt{d}t\right)(\beta_9 + g_{18} - g_{19} - g_{20} + g_{21}) + \\ +\sinh\left(4\sqrt{d}t\right)(\beta_{10} + g_{22} - g_{23} - g_{24}) + \\ +\sinh\left(6\sqrt{d}t\right)(g_{25} + g_{26}) + \\ +2\cosh\left(4\sqrt{d}t\right)(\beta_{27} + g_{28} + g_{29} - g_{30} - g_{31}) + \\ -\cosh\left(2\sqrt{d}t\right)(\beta_{32} - g_{33} + g_{34} + g_{35} + g_{36}).$$

The complete list of constants involved into calculations is

$$\beta_1 = \sqrt{2}(3b^3 - 3bd + 4cd^2), \beta_2 = \sqrt{2}b(b^2 + 3d),$$

$$\beta_3 = \sqrt{2}(b^3 - 2cd^2), \beta_4 = 3\sqrt{c}bd, \beta_5 = \sqrt{2d}(3b^2 + d),$$

$$\beta_6 = 12b\sqrt{c}d, \beta_7 = -\sqrt{2}(-3b^2 + d + 4bcd), \beta_8 = 6\sqrt{c}d,$$

$$\beta_9 = -8\sqrt{2c}d^{3/2}(-9b^2 + 3d + 10bcd), \qquad \text{(A1)}$$

$$\beta_{10} = 4\sqrt{2c}d^{3/2}(9b^2 + 3d + 8bcd), \beta_{11} = 10b^4, \beta_{12} = -12b^2d,$$

$$\beta_{13} = 2d^2, \beta_{14} = 56bcd^2, \beta_{15} = b^4, \beta_{16} = 6b^2d, \beta_{17} = d^2,$$

$$\beta_{18} = 20b^3\sqrt{d}, \beta_{19} = -12bd^{3/2}, \beta_{20} = -64b^2cd^{3/2},$$

$$\beta_{21} = 56cd^{5/2}, \beta_{22} = 16b^3\sqrt{d}, \beta_{23} = -32b^2cd^{3/2},$$

$$\beta_{24} = -32cd^{5/2}, \beta_{25} = 4b^3\sqrt{d}, \beta_{26} = 4bd^{3/2},$$

$$\beta_{27} = 2\sqrt{2c}d(3b^3 + b(9 + 4bc)d + 4cd^2), \beta_{28} = 3b^4,$$

$$\beta_{29} = 6b^2d, \beta_{30} = -d^2, \beta_{31} = -32bcd^2,$$

$$\beta_{32} = 16\sqrt{2c}d(-3b^3 + 2b^2cd + 3cd^2), \beta_{33} = -15b^4,$$

$$\beta_{34} = 6b^2d, \beta_{35} = d^2, \beta_{36} = 8bcd^2.$$

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