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Observational consequences of chaotic inflation with nonminimal coupling to gravity

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Abstract. Recently there was an extensive discussion of Higgs inflation in the theory with the potential $\frac{\lambda}{4}(\phi^2 - v^2)^2$ and nonminimal coupling to gravity $\frac{\xi}{2}\phi^2 R$, for $\xi \gg 1$ and $v \ll 1$. We extend this investigation to the theories $\frac{m^2}{2}\phi^2$ and $\frac{\lambda}{4}(\phi^2 - v^2)^2$ with arbitrary values of ξ and v and describe implementation of these models in supergravity. We analyze observational consequences of these models and find a surprising coincidence of the inflationary predictions of the model $\frac{\lambda}{4}(\phi^2 - v^2)^2$ with $\xi < 0$ in the limit $|\xi| v^2 \to 1$ with the predictions of the Higgs inflation scenario for $\xi \gg 1$.

Keywords: inflation, supersymmetry and cosmology, cosmological parameters from CMBR

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Contents

1 Introduction 1

2 Canonical Field in Einstein Frame 2

3 Supergravity perspective 4

4 Basic models and their extensions 5

5 Quadratic Potential 6

6 Quartic Potential 9

7 Conclusions 14

1 Introduction

The chaotic inflation scenario [1] is one of the simplest and most general versions of inflationary cosmology. It can be realized in many theories with sufficiently flat potentials, including the models with $V \sim \phi^n$. However, for a long time it seemed very difficult to implement chaotic inflation in superstring theory and in supergravity. This situation changed when a simple realization of the chaotic inflation scenario in supergravity was proposed in [2], and a class of chaotic inflation models was developed in the context of string theory [3]. In this class of models, one could have an inflaton potential $\sim \phi^{2/3}$, or $\phi^{4/5}$, or ϕ, predicting a much smaller value of r.

Recently a broad class of models of chaotic inflation in supergravity was proposed in [4]. In these models one can obtain chaotic inflation with an arbitrary inflaton potential $V(\phi)$. In these models one can also implement the curvaton scenario [5] with a controllable degree of non-gaussianity [6].

Moreover, this new class of supergravity models can describe an inflaton field which is nonminimally coupled to gravity, with coupling $\frac{\xi}{2}\phi^2 R$ [4, 7]. Recently there was a wave of interest to such models after the suggestion to implement chaotic inflation in the standard model of electroweak interactions, with the Higgs field nonminimally coupled to gravity playing the role of the inflaton field [8, 9], see also [10]. However, the traditional textbook formulation of supergravity did not provide any description of scalar fields nonminimally coupled to gravity. This could suggest that the models with nonminimal coupling to gravity are outdated, because they are incompatible with modern developments in particle physics based on supersymmetry. Fortunately, this problem was solved in [4, 7], where a consistent generalization of supergravity describing scalar fields nonminimally coupled to gravity was developed and a supersymmetric generalization of the inflationary model of Refs. [8, 9] was proposed.

These developments prompted us to return to an investigation of various versions of chaotic inflation with the inflaton field nonminimally coupled to gravity. In this paper we will consider two basic models: the simplest model with a quadratic potential $\frac{m^2}{2}\phi^2$, and the model with the standard spontaneous symmetry breaking potential $\frac{\lambda}{4}(\phi^2 - v^2)^2$. Our goal is to analyze these models at the classical level, describe the way to incorporate them in supergravity, and study the stability of the inflationary trajectory in these theories. We will also calculate the spectral index n_s and the tensor-to-scalar ratio r for various values of the parameter ξ describing the nonminimal coupling of the inflaton field to gravity in these models. We will show that by changing parameters of these models one can cover a significant part of the space of parameters (n_s, r) allowed by recent observational data. Finally, we will show that the inflationary predictions of the model $\frac{\lambda}{4}(\phi^2 - v^2)^2$ with $\xi < 0$ in the limit $|\xi|v^2 \to 1$ coincide with the predictions of the Higgs inflation scenario for $\xi \gg 1$ and $v \ll 1$.

2 Canonical Field in Einstein Frame

The Lagrangian of the inflaton field in the models discussed in the Introduction describes a scalar field ϕ with a canonical kinetic term but with a *non-minimal* coupling to gravity:

$$L = \frac{1}{2}\sqrt{-g_J}\left[(1+\xi\phi^2)R_J - g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V_J(\phi)\right]. \tag{2.1}$$

The index J is to emphasize that this is a Jordan frame: the gravity part of the Lagrangian is different from the Einstein Lagrangian $\frac{1}{2}\sqrt{-g}R[g]$ and has a local function $1+\xi\phi^2$ in front. It is possible to proceed with this form of the Lagrangian but we will be able to use the familiar equations of general relativity, the inflationary solutions and the standard slow-roll analysis if we switch to the Einstein frame and use variables that show *minimal* coupling between the scalar field and gravity.

To this end, we change variables from g_J and ϕ to a conformally related metric $g_E^{\mu\nu} = (1+\xi\phi^2)^{-1}g_J^{\mu\nu}$ and a new field φ related to the field ϕ as follows:

$$\left(\frac{d\varphi}{d\phi}\right)^2 = \frac{1+\xi\phi^2+6\xi^2\phi^2}{(1+\xi\phi^2)^2} =: \frac{1}{Z(\phi)}. \tag{2.2}$$

In new variables, the Lagrangian, up to a total derivative, is given by

$$L = \frac{1}{2}\sqrt{-g_E}\left[R_E - g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 2V_E(\phi(\varphi))\right]. \tag{2.3}$$

where the potential in the Einstein frame is

$$V_E(\phi) = \frac{V_J(\phi)}{(1+\xi\phi^2)^2}, \tag{2.4}$$

The canonically normalized Einstein-Hilbert term of the action shows that g_E is the metric in the Einstein frame. Therefore we can use all results of the standard slow-roll analysis.

In doing the slow-roll analysis we should note that it is φ and not ϕ that has a canonical kinetic term. Therefore the slow-roll parameters are

$$\epsilon = \frac{1}{2}\left(\frac{V_\varphi}{V}\right)^2, \qquad \eta = \frac{V_{\varphi\varphi}}{V}. \tag{2.5}$$

(From now on, whenever we use V without a subscript or the word potential we mean the Einstein frame potential V_E.) The same quantities can be formally defined for the field ϕ but are not of physical meaning. However, they are easier to compute and are related to the physical slow-roll parameters via

$$\epsilon = Z\epsilon_\phi, \qquad \eta = Z\eta_\phi + \mathrm{sgn}(V')Z'\sqrt{\frac{\epsilon_\phi}{2}}, \tag{2.6}$$

where prime means $d/d\phi$. Similarly, we can find the number of e-folds by

$$N \approx \left|\int \frac{d\varphi}{\sqrt{2\epsilon}}\right| = \left|\int \frac{d\phi}{Z(\phi)\sqrt{2\epsilon_\phi}}\right|. \tag{2.7}$$

This enables us to do almost all computations in terms of ϕ without having to solve Eq. (2.2) to find φ. But we should not forget the fact that ϕ has a non-canonical kinetic term $-\frac{1}{2}Z^{-1}g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. So when we qualitatively analyze the motion of the field, in analogy with a particle with Lagrangian $\frac{1}{2}m\dot{x}^2 - V(x)$, we need to associate a variable 'rolling mass' $Z^{-1}(\phi)$ to the field ϕ.[1] This doesn't change the direction of rolling but modifies the velocity of the field such that with heavier rolling mass (smaller Z) the motion becomes slower (smaller ϵ and η) as indicated by Eq. (2.6).

In the small field limit, $\phi = \varphi$, but at large values of the fields their relation to each other is more complicated. In particular, for $\xi > 0$ and $\phi \gg 1/\sqrt{\xi}$ (or, equivalently, $\varphi \gg 1$), one has

$$\phi = \frac{1}{\sqrt{\xi}}e^{\frac{\varphi}{\sqrt{6}}}\ . \tag{2.8}$$

Therefore if the potential $V(\phi)$ is a polynomial of ϕ, at $\phi \gg 1/\sqrt{\xi}$ (i.e. at $\varphi \gg 1$) it depends on the canonically normalized field exponentially.

For negative ξ, the potential becomes singular when ϕ approaches $1/\sqrt{|\xi|}$, beyond which the effective gravitational coupling constant $G_N = \frac{1}{8\pi(1+\xi\phi^2)}$ would change sign (antigravity regime). However, in terms of the canonically normalized field φ, the singularity is never reached, because it corresponds to infinitely large values of φ. That is why in this scenario one should not worry about the possibility to cross the boundary between gravity and antigravity, or even to approach this boundary [11, 12].

One can especially clearly see it if one considers a vicinity of the would-be singularity, where $1 - |\xi|\phi^2 \ll 1$. In this regime

$$\phi = \frac{1}{\sqrt{|\xi|}}\left(1 - \frac{1}{2}\,e^{-\sqrt{\frac{2}{3}}\,\varphi}\right)\ . \tag{2.9}$$

Therefore $1 - \sqrt{|\xi|}\phi = \frac{1}{2}\,e^{-\sqrt{\frac{2}{3}}\,\varphi}$, which means that the limit $\phi \to 1/\sqrt{|\xi|}$ corresponds to the limit $\varphi \to \infty$. These considerations will help us to understand the properties of the inflaton potential in the Einstein frame.

Before investigating cosmological implications of this class of models, we would like to make a short detour and discuss implementation of these models in the context of supergravity. The results obtained in the paper do not depend on the possibility to implement the inflationary models with such potentials in supergravity, but supergravity allows to look at such models and evaluate them from an entirely different perspective. The readers interested only in observational consequences of the models with nonminimal coupling to gravity may skip the next section and go straight to section 5.

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[1] This 'rolling mass' should not be confused with the usual mass V'' of the field.

3 Supergravity perspective

The new class of chaotic inflation models in supergravity [4] describes two fields, S and Φ, with the superpotential

$$W = S\, f(\Phi), \tag{3.1}$$

where $f(\Phi)$ is a real holomorphic function such that $\bar{f}(\Phi) = f(\Phi)$. The Kähler potential in the simplest version of this model is given by

$$\mathcal{K} = -3\log\Big[1 - \frac{1}{3}(\Phi\bar{\Phi} + S\bar{S}) + \frac{\chi}{4}(\Phi^2 + \bar{\Phi}^2) + \zeta(S\bar{S})^2/3\Big]. \tag{3.2}$$

The term $\zeta(S\bar{S})^2/3$ is added for stabilization of the inflationary trajectory, to ensure that the fields S and $\mathrm{Im}\,\Phi$ vanish during inflation: $S = \mathrm{Im}\,\Phi = 0$. The field $\phi = \sqrt{2}\mathrm{Re}\,\Phi$ plays the role of the inflaton field in this scenario. According to [4, 7], the evolution of this field is described by the theory of the scalar field ϕ nonminimally interacting with gravity, with $\xi = -\frac{1}{6} + \frac{\chi}{4}$. The Jordan frame potential of this field is

$$V_J = f^2(\mathrm{Re}\,\Phi)\ . \tag{3.3}$$

It is convenient to present the result in terms of the real fields s, α, ϕ and β, related to the complex fields through

$$S = \frac{1}{\sqrt{2}}(s + i\alpha)\,, \qquad \Phi = \frac{1}{\sqrt{2}}(\phi + i\beta)\ . \tag{3.4}$$

The potential of the inflaton field ϕ is then

$$V_J(\phi) = f^2(\phi/\sqrt{2})\ , \tag{3.5}$$

and the Einstein frame potential is given by Eq. (2.4).

Thus, in this context one has a functional freedom of choice of the inflaton potential, determined by the function $f(\Phi)$. However, this scenario works only if one can actually ignore the cosmological evolution of the fields S and $\mathrm{Im}\,\Phi$, by fixing them at their values $S = \mathrm{Im}\,\Phi = 0$ during inflation. Therefore we must check whether the trajectory $S = \mathrm{Im}\,\Phi = 0$ corresponds to the minimum of the potential in the directions S and $\mathrm{Im}\,\Phi$. In our scenario, the potential has the same curvature in the directions s and α, so one should check whether $m_\beta^2, m_s^2 > 0$. We will impose a stronger condition, $m_\beta^2, m_s^2 \gtrsim H^2$, to avoid production of inflationary perturbations of any fields except for the inflaton field ϕ.

These conditions can be analyzed along the lines of [4], where it was shown that for the theories with $\xi = 0$ and arbitrary function $f(\Phi)$ one can always stabilize the inflationary trajectory $S = \mathrm{Im}\,\Phi = 0$ by a proper choice of the parameter ζ.

An investigation of this issue for $\xi \neq 0$ is similar, but more complicated, because m_β^2 and m_s^2 depend on ξ:

$$m_\beta^2 = \frac{\frac{4}{3}(1+3\xi)f^2 + (1+\xi\phi^2)((\partial_\Phi f)^2 - f\,\partial_\Phi^2 f)}{(1+\xi\phi^2)(1+\xi\phi^2+6\xi^2\phi^2)}, \tag{3.6}$$

$$m_s^2 = \frac{A(\phi)}{(1+\xi\phi^2)^2(1+\xi\phi^2+6\xi^2\phi^2)}. \tag{3.7}$$

where

$$\begin{aligned} A(\phi) = {} & \frac{2}{3}(-1-\xi\phi^2+6\xi^2\phi^2+6\zeta(1+2\xi\phi^2+6\xi^3\phi^4 \\ & + \xi^2\phi^2(6+\phi^2)))f^2 - 4\sqrt{2}\xi\phi(1+\xi\phi^2)f\,\partial_\Phi f \\ & + (1+\xi\phi^2)^2(\partial_\Phi f)^2 \end{aligned} \tag{3.8}$$

In this paper we studied two basic models. The first one corresponds to $f(\Phi) = m\Phi$, which leads to the quadratic inflaton potential $\frac{m^2}{2}\phi^2$. The second choice is $f(\Phi) = \sqrt{\lambda}(\Phi^2 - v^2/2)$, which leads to the inflaton potential $\frac{\lambda}{4}(\phi^2 - v^2)^2$. Investigation of stability in the supergravity versions of these models is straightforward but quite involved. Here we will skip the details and briefly summarize our main results.

First of all, we verified that the inflationary trajectory is stable with respect to generation of the field $\beta \sim \operatorname{Im}\Phi$. However, the situation with the field S is more complicated. Just as in the models studied in [7], stabilization requires the existence of the stabilizing term $\zeta(S\bar{S})^2/3$ in the Kähler potential. For $\xi > 0$, stabilization can be achieved by taking $0 < \zeta \lesssim O(1)$. Similar conclusion is valid for $\xi < 0$ as well, with one caveat.

As we discussed in the previous section, the point $|\xi|\phi^2 = 1$ for $\xi < 0$ corresponds to a singularity, where the effective gravitational constant becomes infinite and changes its sign. When $|\xi|\phi^2$ increases towards 1, which corresponds to the vicinity of this dangerous point, or to the infinitely large values of the canonically normalized inflaton field φ, stabilization of the field S requires $\zeta \gg 1$. This is not particularly attractive, but appearance of large parameters in the Kähler potential is not a novelty in this scenario. Indeed, the Higgs inflation scenario with $\xi \gg 1$ [9] also requires introduction of a very large coefficient χ in the Kähler potential (3.2) [7].

4 Basic models and their extensions

In this paper we will concentrate on observational consequences of two basic models, with potentials $\frac{1}{2}m^2\phi^2$ and $\frac{\lambda}{4}(\phi^2 - v^2)^2$, and extend their analysis for the case with $\xi \neq 0$. To put our results in a proper perspective, we will briefly remember what is known about observational consequences of some closely related models.

1) $V \sim \phi^{2\alpha}$, $\xi = 0$. For this model one has $1 - n_s = (1+\alpha)N^{-1}$, $r = 8\alpha/N$, where N is the number of e-folds. In this case $1 - n_s = 1/N + r/8$ [3]. The results are shown by two blue straight lines in Fig. 1. Not surprisingly, these lines go through the circles corresponding to the theories with potentials proportional to ϕ^2 and ϕ^4; these lines continue going down when α decreases.

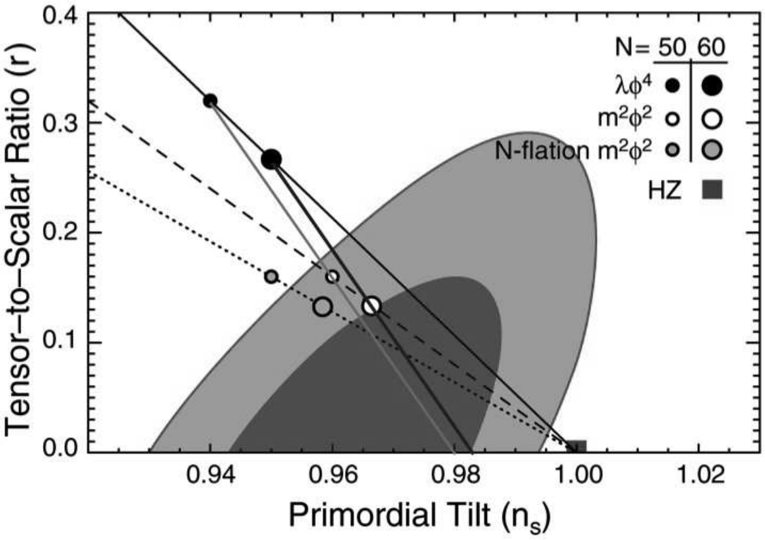

Figure 1. Predictions of the chaotic inflation model with the potential $V \sim \phi^{2\alpha}$ for different α in the range of $0 < \alpha < 2$ are shown by two blue lines corresponding to $N = 60$ and $N = 50$. Our results are shown on top of the WMAP results.

2) $V \sim (\phi^2 - v^2)^2$, $\xi = 0$. The results depend on the value of v, and also on the initial conditions for the field ϕ. If inflation occurs at $\phi > v$, and v is not too large, the results coincide with those in the model with the potential ϕ^4, see Fig. 2. For $v > O(10)$, the last stages of inflation occur not far from the minimum of the potential, where it looks quadratic. Therefore in this regime n_s and r coincide with their value for the quadratic potential. This property remains valid for $v > O(10)$ even if the field falls from $\phi < v$. However, inflation is also possible for $\phi < v$ and $1 \lesssim v \lesssim O(10)$, in which case n_s and r can be much smaller, as shown on the lower branch of the blue line in Fig. 2, below the white circle.

The green lines in Fig. 2 show n_s and r for the theory with the potential ϕ^4 with $\xi > 0$. This potential coincides with our potential $\sim (\phi^2 - v^2)^2$ in the limit $v = 0$ [14].

The investigation which we are going to perform should generalize the previously obtained results for the theory $m^2\phi^2/2$ with nonminimal coupling ξ, and for the theory $\frac{\lambda}{4}(\phi^2 - v^2)^2$ with arbitrary values of v and ξ.

5 Quadratic Potential

We begin with the investigation of the quadratic potential $V_J = \frac{1}{2}m^2\phi^2$ in the Jordan frame, which leads to the Einstein frame potential

$$V_E = \frac{m^2\phi^2/2}{(1+\xi\phi^2)^2} \ . \tag{5.1}$$

This equation immediately reveals an unusual behavior of the potential. At small ϕ, the potential behaves as the original quadratic potential. However, for $\xi > 0$ it has a maximum at $\xi\phi^2 = 1$, see the blue line in Fig. 3. If the field begins its evolution beyond this maximum,