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A sufficient condition for de Sitter vacua in type IIB string theory

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Markus Rummel[a] **and Alexander Westphal**[b]

[a] *II. Institut für Theoretische Physik der Universität Hamburg, D-22761 Hamburg, Germany*

[b] *Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany*

E-mail: markus.rummel@desy.de, alexander.westphal@desy.de

Abstract: We derive a sufficient condition for realizing meta-stable de Sitter vacua with small positive cosmological constant within type IIB string theory flux compactifications with spontaneously broken supersymmetry. There are a number of 'lamp post' constructions of de Sitter vacua in type IIB string theory and supergravity. We show that one of them – the method of 'Kähler uplifting' by F-terms from an interplay between non-perturbative effects and the leading α'-correction – allows for a more general parametric understanding of the existence of de Sitter vacua. The result is a condition on the values of the flux induced superpotential and the topological data of the Calabi-Yau compactification, which guarantees the existence of a meta-stable de Sitter vacuum if met. Our analysis explicitly includes the stabilization of all moduli, i.e. the Kähler, dilaton and complex structure moduli, by the interplay of the leading perturbative and non-perturbative effects at parametrically large volume.

Keywords: moduli stabilization, string vacua, flux compactifications

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1 Introduction & Motivation

String theory is a candidate for a fundamental theory of nature, providing at the same time a UV-finite quantum theory of gravity and unification of all forces and fermionic matter. Mathematical consistency requires string theory to live in a ten dimensional space-time, and a description of our large four-dimensional physics thus necessitates compactification of the additional six dimensions of space.

The need for compactification confronts us with two formidable consequences: Firstly, even given the known internal consistency constraints of string theory, there are unimaginably large numbers of 6d manifolds available for compactification. Secondly, many compact manifolds allow for continuous deformations of their size and shape while preserving their defining properties (such as topology, vanishing curvature, etc) – these are the moduli, massless scalar fields in 4d. This moduli problem is exacerbated if we wish to arrange for low-energy supersymmetry in string theory, as compactifications particularly suitable for this job – Calabi-Yau manifolds – tend to come with hundreds of complex structure and Kähler moduli.

Therefore, a very basic requirement for string theory to make contact with low-energy physics is moduli stabilization – the process of rendering the moduli fields very massive. Moreover, as supersymmetry is very obviously broken – and so far has not been detected – ideally, moduli stabilization should tolerate or even generate supersymmetry breaking. And finally, the process should produce a so-called meta-stable de Sitter (dS) vacuum with tiny positive cosmological constant, so as to accommodate the observational evidence for the accelerated expansion of our universe by dark energy [1–3].

The task of moduli stabilization and supersymmetry breaking has recently met with considerable progress, which is connected to the discovery of an enormous number [4–8] of stable and meta-stable 4d vacua in string theory. The advent of this landscape [7] of isolated, moduli stabilizing minima marks considerable progress in the formidable task of constructing realistic 4d string vacua.

There are several methods of moduli stabilization. The first one uses supersymmetric compactifications of string theory on a Calabi-Yau manifold, and the strong gauge dynamics of gaugino condensation in the 'racetrack' mechanism to stabilize the dilaton and several of the bulk volume and complex structure moduli [9–11]. Recently, this method has been applied to supersymmetric compactifications of M-theory on G_2-manifolds, where the structure of the manifolds allows for the racetrack superpotential to generically depend on all the moduli of the compactification [12].

The second, more recent, method relies on the use of quantized closed string background fluxes in a given string compactification. These flux compactifications can stabilize the dilaton and the complex structure moduli of type IIB string theory compactified on a Calabi-Yau orientifold supersymmetrically [5]. The remaining volume moduli are then fixed supersymmetrically by non-perturbative effects, e.g. gaugino condensation on stacks of D7-branes [6]. The full effective action of such fluxed type IIB compactifications on Calabi-Yau orientifolds was derived in [13]. In type IIA string theory on a Calabi-Yau manifold all geometric moduli can be stabilized supersymmetrically by perturbative means using the larger set of fluxes available [14].

If the moduli are stabilized supersymmetrically, parametrically small and controlled supersymmetry breaking can happen, e.g, by means of inserting an anti-D3-brane into a warped throat of the Calabi-Yau [6], by D-terms originating in magnetic flux on a D7-brane [15], or dynamically generated F-terms of a matter sector [16]. This process is known as 'uplifting' and allows for dS vacua with extremely small vacuum energy by means of fine-tuning the $\mathcal{O}(100)$ independent background fluxes available in a typical Calabi-Yau compactification [4, 6]. The reliability of this last step of uplifting supersymmetric AdS vacua without unstabilized moduli into a dS vacuum is still under discussion. Some of the points in question e.g. concern the fact that the existence of D7-brane D-terms as well as F-terms from hidden matter sectors are very model dependent, rendering statistical sweeps over large sets of compactifications difficult. Supersymmetry breaking and uplifting by a warped-down anti-D3-brane also remains under ongoing discussion on whether its presence can be completely described in a probe approximation or causes dangerous non-normalizable perturbations to the compact geometry [17–22]. Very recently, the use of internal F_2 gauge flux on a CY threefold in heterotic string theory has been used to stabilize all geometric moduli except the dilaton and one Kähler modulus in a supersymmetric Minkowski vacuum [23, 24].

Alternatively, in non-Calabi-Yau flux compactifications of type IIB or IIA string theory, all geometric moduli can be stabilized perturbatively in a non-supersymmetric way using a combination of background fluxes, D-branes, orientifold planes, and negative curvature. Examples here are flux compactifications of type IIB with 3-form fluxes on a product of Riemann surfaces [25] and almost Calabi-Yau 4-folds in F-theory [26], type II compactifications with generalized fluxes on manifolds of $SU(3)$ (see, e.g., the reviews [27, 28]), as well as of type IIA with fluxes on a product of two 3d nil manifolds [29]. The ingredients used typically lead to scalar potential dominated by three perturbative terms with alternating signs, which depend as varying power laws on the dilaton and the geometric moduli. Such a '3-term structure' structure generically allows for tunable dS vacua [25, 29]. Supersymmetry is generically broken in these perturbative mechanisms of moduli stabilization at a high scale, which typically is the Kaluza-Klein (KK)-scale. The geometric and flux part of these type IIA compactifications were studied in more detail in [22, 30–37]. The conclusion there so far seems to be that in absence of the KK5-branes used in [29] (which play a similar role as 'explicit' supersymmetry breaking objects as the anti-D3-brane in [6]) there are no stable dS vacua. A complete analysis including the effects of the KK5-branes in the language of [22, 30–37] still remains open.

Finally, in type IIB flux compactifications on Calabi-Yau manifolds there are constructions of a 'hybrid' type, where fluxes fix the complex structure moduli and the dilaton supersymmetrically, but the volume moduli are stabilized non-supersymmetrically by an interplay of non-perturbative effects on D7-brane stacks and the leading perturbative correction at $\mathcal{O}(\alpha'^3)$ in type IIB [38], or by perturbative corrections to the Kähler potential alone. Examples for the latter consist of the Large-Volume-Scenario (LVS) [39], stabilization by perturbative corrections to the Kähler potential of the volume moduli alone [40–43] which are uplifted by D7-brane D-terms [44], and the method of 'Kähler uplifting' [45, 46].

For 'Kähler uplifted' dS vacua, an interplay between the leading perturbative correction at $\mathcal{O}(\alpha'^3)$ and a non-perturbative effect in the superpotential serves to generate a dS vacuum with supersymmetry spontaneously broken by an F-term generated in the volume moduli sector. For some recent reviews on flux compactifications and the associated questions of the landscape of string vacua and string cosmology ensuing from the meta-stable dS vacua, with a much more complete list of references, please see [28, 47, 48].

'Kähler uplifting' has the benefit of generating meta-stable dS vacua in terms of just background 3-form fluxes, D7-branes and the leading perturbative $\mathcal{O}(\alpha'^3)$-correction, data which are completely encoded in terms of the underlying F-theory compactification on a fluxed Calabi-Yau fourfold. In addition, supersymmetry is spontaneously broken at a scale of order of the inverse Calabi-Yau volume, measured in string units this is typically $\sim M_{\rm GUT}$ here, and still below the KK-scale), by an F-term generated in the volume moduli sector. No extra anti-branes, D-terms or F-term generating matter fields are needed or involved. The existing analysis of these models consists of including manifestly the dilaton and one complex structure modulus [45].

Therefore, in this paper we develop a method towards a rigorous analytical understanding of 'Kähler uplifting' driven by the leading $\mathcal{O}(\alpha'^3)$ correction to the Kähler potential of the volume moduli. Our derivation will be carried out in the presence of an arbitrary number $h^{2,1}$ of complex structure moduli. A large value of 3-cycles $h^{2,1} = \mathcal{O}(100)$ is a prerequisite to use the associated 3-form fluxes for the required fine-tuning of the cosmological constant.

Note the relationship between the supersymmetric KKLT-type AdS vacua [6] (prior to uplifting) with the flux superpotential tuned small, the SUSY-breaking LVS-type AdS vacua [39] (again, prior to uplifting), and the SUSY-breaking 'Kähler uplifted' AdS/dS vacua [45, 46] (inherently liftable to dS by the pure moduli sector itself) discussed here. These three classes of moduli stabilizing vacua are three branches of solutions in the same low-energy 4d $\mathcal{N} = 1$ supergravity arising from type IIB compactified on a Calabi-Yau orientifold with D7-branes.

In section 2, we will review the method of 'Kähler uplifting' and analytically derive the existence of the meta-stable dS vacuum for the volume modulus of a one-parameter Calabi-Yau compactification with $h^{1,1} = 1$ Kähler modulus, and then extend this to the case of several Kähler moduli $h^{1,1} > 1$ explicitly. The interplay of perturbative and non-perturbative effects implies for $h^{1,1} = 1$ that here a structure of *two* terms with alternating signs is sufficient to approximate the volume modulus scalar potential and its tunable dS vacuum. This contrasts with the '3-term structure' generically necessary in purely perturbatively stabilized situations [25, 29]. For $h^{1,1} > 1$ a '3-term structure' reappears for the additional $h^{1,1} - 1$ blow-up Kähler moduli of a 'swiss cheese' Calabi-Yau.

Finally, we will show that we can express the existence of the meta-stable dS vacuum for the volume modulus in terms of a *sufficient* condition on the microscopic parameters. These are consisting of the fluxes, the D7-brane configuration, and the Euler number of the Calabi-Yau governing the perturbative $\mathcal{O}(\alpha'^3)$-correction, which are all in turn determined by the underlying F-theory compactification on an elliptically fibred Calabi-Yau fourfold. Thus, the result amounts to a sufficient condition for the existence of meta-stable dS vacua in terms of purely F-theory geometric and topological data which can be satisfied for a sizable subclass of all 4d $\mathcal{N} = 1$ F-theory compactifications, instead of just single 'lamp post' models. We also check that our sufficient condition satisfies the necessary condition for meta-stable dS vacua in 4d $\mathcal{N} = 1$ supergravity given in [49] and the longevity of the metastable vacuum under tunneling.

Section 3 includes the dilaton into a full analytical treatment of the combined dS minimum. We show that supersymmetry breaking happens predominantly in the volume modulus direction, and explicitly determine the shift of the dilaton away from its flux-stabilized supersymmetric locus as suppressed by inverse powers of the volume of the Calabi-Yau.

Section 4 extends the analysis by including an arbitrary number of complex structure moduli with unspecified dependence in the Kähler and superpotential. We then show that the shift of the complex structure moduli and the dilaton in general is suppressed by inverse powers of the volume, and that the dilaton and all complex structure moduli generically are fixed at positive-definite masses. Finally, we estimate the backreaction of the shifted dilaton and complex structure moduli onto the volume modulus. The ensuing shift of the stabilized volume is generically found to be small and suppressed by inverse powers of the volume. This crucially extends the sufficient condition for the existence of dS vacua in type IIB F-theory compactifications to a large class of ‘swiss cheese’ style fluxed Calabi-Yau compactifications with arbitrary $h^{1,1} < h^{2,1}$.

In section 5, we apply our methods to a simple toy model where the Kähler and superpotential of complex structure moduli are approximated by the structure found in a torus compactification. We verify the general results of the previous sections, and show that the shifts of the moduli and the backreaction effects are either independent of the number of complex structure moduli $h^{2,1}$, or decreasing as an inverse power of $h^{2,1}$. We conclude in section 6.

While this paper was being finished, we became aware of [50], whose section 2 contains overlapping results with our section 2. The main results of section 2 and 3 here have first been presented in talk by one of the authors in [51]. Additionally, we find numerical disagreement concerning the values of x in section 2 permissible for a meta-stable dS vacuum of T compared to the results for the same quantity given in section 2 of [50] due to an approximation used between eq.s (16) and (17) *ibid.*

2 ‘Kähler uplifting’ – a meta-stable dS vacuum for the Kähler modulus

We will start with reviewing the structure of ’Kähler uplifted’ dS vacua in type IIB flux compactifications on an orientifolded CY threefold [45]. We will at first restrict ourselves to one-parameter models with $h^{1,1} = 1$ and $h^{2,1} > 1$ so that the Euler number $\chi = 2(h^{1,1} - h^{2,1}) < 0$ (which will be shown to be part of the the sufficient condition for the existence dS vacua). Later, we will extend the analysis given here to all so-called swiss-cheese Calabi-Yau threefolds with arbitrary $h^{1,1} > 1$ and $h^{2,1} > h^{1,1}$, giving a strong indication that the mechanism discussed here works for all threefolds with $\chi < 0$.

For type IIB compactifications on Calabi-Yau orientifolds with 3-form fluxes and D7-branes the effective 4d $\mathcal{N} = 1$ supergravity of the moduli sector is determined by [5, 13, 38, 52]

$$K = -2\ln\left(\hat{\mathcal{V}} + \alpha'^3\frac{\hat{\xi}}{2}\right) - \ln(S+\bar{S}) - \ln\left(-i\int_{CY_3}\bar{\Omega}\wedge\Omega\right) \ , \tag{2.1}$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}, \quad \text{with } W_0 = \frac{1}{2\pi}\int_{CY_3} G_{(3)}\wedge\Omega \ . \tag{2.2}$$

Note, that this 4d $\mathcal{N} = 1$ supergravity has three branches of vacua. Firstly, we may look for vacua where $|W_0| \ll 1$ is tuned small. Then supersymmetric solutions $D_I W = 0$ (with I running over all $h^{1,1}$ Kähler moduli, $h^{2,1}$ complex structure moduli, and the dilaton S) stabilizing all moduli, with 4-cycle volumes $\mathrm{Re}\, T_i >> 1$, are possible including the α'-correction discussed above [6]. On swiss-cheese style Calabi-Yau manifolds, a second branch of solutions are the SUSY-breaking AdS vacua of the Large-Volume-Scenario which work for arbitrary W_0 [39], and the third branch consists of the 'Kähler uplifted' solutions studied below, where typically $|W_0| - \mathcal{O}(1 \ldots 10)$ to get dS vacua.

For one-parameter models we have $\hat{\mathcal{V}} = \gamma(T + \bar{T})^{3/2}$ and we set $\alpha' := 1$. Here

$$\gamma = \sqrt{3}/(2\sqrt{\kappa}) \quad , \tag{2.3}$$

$$\hat{\xi} = -\frac{\zeta(3)}{4\sqrt{2}\,(2\pi)^3}\,\chi\,(S + \bar{S})^{3/2} \quad , \tag{2.4}$$

and κ denotes the self-intersection number of the single Kähler modulus T in terms of the Poincare-dual 2-cycle volume modulus v of the underlying $\mathcal{N} = 2$ theory prior to orientifolding. The volume of 1-parameter CY threefolds is then given by [13]

$$\hat{\mathcal{V}} = \frac{\kappa}{6} v^3 \equiv \gamma\,(T + \bar{T})^{3/2} \quad , \quad \mathrm{Re}\, T = \frac{1}{3}\,\partial_v \hat{\mathcal{V}} \quad . \tag{2.5}$$

The flux-superpotential W_0 is determined by the integral over the holomorphic 3-form Ω of the Calabi-Yau and the 3-form flux $G_{(3)}$ [52]. The Kähler potential K and superpotential W determine the F-term scalar potential to be

$$V = e^K \left(K^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right) \tag{2.6}$$

with $D_a W = W_a + K_a W$, and a runs over the dilaton S, the single Kähler modulus T and the $h^{2,1}$ complex structure moduli U_i. We will now stabilize the Kähler modulus

$$T = t + i\tau \quad , \tag{2.7}$$

(τ denotes its axion) using the interplay between the leading perturbative α' correction $\hat{\xi}$ to the Kähler potential [38] and non-perturbative corrections to the superpotential. For now, we assume the dilaton S and the complex structure moduli U_i to be stabilized already. Thus, we have to find local stable minima of the scalar potential descending from eq.s (2.1) assuming $D_S W = D_{U_i} W = 0$.

Following [38, 45, 46] we can write the resulting scalar potential in the following form

$$\begin{aligned} V(T) &= e^K \left(K^{T\bar{T}} D_T W \overline{D_T W} - 3|W|^2 \right) \\ &= e^K \left(K^{T\bar{T}} \left[W_T \overline{W_T} + (W_T \cdot \overline{W K_T} + c.c) \right] + 3\hat{\xi} \frac{\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2\hat{\mathcal{V}})^2} |W|^2 \right) \quad . \end{aligned} \tag{2.8}$$

Here $K^{T\bar{T}}$ denotes the $T\bar{T}$-component of the inverse of the Kähler metric $(K_{I\bar{J}})^{-1}$ where I, J run over all fields involved.

The non-trivial task is to find stationary points of $V(T)$ with respect to t. It is straightforward to show that the axionic direction has an actual minimum at $\tau = 0$. The Kähler potential does not depend on τ and the exponential in eq. (2.1) introduces trigonometric functions $\sin(a\tau)$ and $\cos(a\tau)$ into $V(T)$. Then it can be shown that $V_\tau = 0$ for $\tau = n\pi/a$ for $n \in \mathbb{Z}$. We restrict to the case $\tau = 0$ so that after insertion of W_T we obtain

$$V(t) = e^K \left(K^{T\bar{T}} \left[a^2 A^2 e^{-2at} + (-aAe^{-at}\overline{WK_T} + c.c) \right] + 3\hat{\xi} \frac{\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2\hat{\mathcal{V}})^2} |W|^2 \right) . \tag{2.9}$$

2.1 Approximating the scalar potential $V(T)$ in the large volume limit

In [45], it was shown that one can get de Sitter minima for T at parametrically large volume $\hat{\mathcal{V}} \simeq \mathcal{O}(100 \ldots 1000)$ and weak string coupling $g_S \simeq 0.1$. The stable minimum is realized at $\hat{\xi}/(2\hat{\mathcal{V}}) \simeq 0.01$ so small that neglecting higher orders in the α' expansion is well justified and string loop effects are double-suppressed due to the smallness of g_S and the extended no-scale structure [42]. This minimum can be constructed under the following conditions

- Put a stack of $N \simeq \mathcal{O}(30 \ldots 100)$ D7-branes on the single 4-cycle that undergoes gaugino condensation.[1] The parameter A is assumed to be $\mathcal{O}(1)$.
- Choose the flux induced superpotential $W_0 \simeq \mathcal{O}(-30)$ and the parameter $\hat{\xi} \simeq \mathcal{O}(10)$. Note that a W_0 of this rather large magnitude does not induce problematic back reactions, as in type IIB the fluxes are imaginary self-dual (ISD) and of (1,2) or (0,3) type which limitates the back reaction to the warp factor.

In this setup, one typically obtains a minimum at $T \simeq \mathcal{O}(40)$ so that the non-perturbative contribution to the superpotential Ae^{-aT} is small enough to also trust the Ansatz for the non-perturbative superpotential.

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[1] For example, the 2-parameter model $\mathbb{P}^4_{11169}$ was shown in [53] to have an F-theory lift containing an E_8 ADE-singularity for the condensing gauge group, giving a rank of 30. In general, the achievable rank of the gauge groups is limited for compact CY fourfolds, due to the compactness interfering with enforcing an ADE-singularity of arbitrarily high rank along a given divisor. Still, on compact F-theory fourfolds very large gauge groups with very large ranks can be generated, e.g. in [54] F-theory was compactified to 4d on a compact fourfold to yield a gauge group with 251 simple factors, the largest of which was $SO(7232)$.

We now want to give a parametric understanding of this scenario by approximating the scalar potential eq. (2.9) under the constraint of the typical values of the parameters $a, A, W_0, \hat{\xi}, \gamma$. We use the condition $\hat{\xi}/(2\hat{\mathcal{V}}) \simeq 0.01$ and the validity of the non-perturbative superpotential:

$$\hat{\mathcal{V}} \gg \hat{\xi}, \qquad |W_0| \gg Ae^{-at} \; . \tag{2.10}$$

Under these approximations, the Kähler Potential and its derivatives simplify in the following way:

$$\begin{aligned} K &= -2\ln\left(\hat{\mathcal{V}} + \frac{\hat{\xi}}{2}\right) \simeq -2\ln\left(\hat{\mathcal{V}}\right) \; , \\ K_T &= \frac{-3\gamma^{2/3}\sqrt[3]{\hat{\mathcal{V}}}}{\hat{\mathcal{V}} + \frac{\hat{\xi}}{2}} \simeq \frac{-3\gamma^{2/3}}{\hat{\mathcal{V}}^{2/3}} \; , \\ \left(K_{T\bar{T}}\right)^{-1} &= \gamma^{-4/3}\frac{\sqrt[3]{\hat{\mathcal{V}}}(4\hat{\mathcal{V}}^2 + \hat{\xi}\hat{\mathcal{V}} + 4\hat{\xi}^2)}{12(\hat{\mathcal{V}} - \hat{\xi})} \simeq \frac{\hat{\mathcal{V}}^{4/3}}{3\gamma^{4/3}} \; . \end{aligned} \tag{2.11}$$

Also the last term of eq. (2.9) simplifies under the approximation eq. (2.10). Implementing eq. (2.10), the scalar potential eq. (2.9) becomes

$$V(t) \simeq \frac{e^{-2at}(3aA^2 + a^2A^2t)}{6\gamma^2t^2} + \frac{aAe^{-at}W_0}{2\gamma^2t^2} + \frac{3W_0^2\hat{\xi}}{64\sqrt{2}\gamma^3t^{9/2}} \; . \tag{2.12}$$

We also neglect the term $\propto e^{-2at}$ since it is suppressed by one more power of e^{-at} compared to the second term in eq. (2.12) and obtain a '2-term structure' for the scalar potential

$$V(t) \simeq \frac{aAe^{-at}W_0}{2\gamma^2t^2} + \frac{3W_0^2\hat{\xi}}{64\sqrt{2}\gamma^3t^{9/2}} \; . \tag{2.13}$$

Note that the flux-superpotential is negative, $W_0 < 0$, so that the *two* terms have opposite sign and a minimum is in principle allowed. Eq. (2.13) is a drastic simplification of the rather complicated scalar potential eq. (2.9) that allows us to extract an analytic condition on the parameters to obtain a meta-stable de Sitter vacuum. Factorizing eq. (2.13), we can write it in terms of two characteristic variables $x = a \cdot t$ and C

$$V(x) \simeq \frac{-W_0a^3A}{2\gamma^2}\left(\frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}\right), \quad C = \frac{-27W_0\hat{\xi}a^{3/2}}{64\sqrt{2}\gamma A} \; . \tag{2.14}$$

The overall constant in eq. (2.14) does not influence the extrema of this potential. For completeness, we mention that the stationary point in the axionic direction $\tau = 0$ is always a minimum since the mass

$$V_{\tau\tau} = -\frac{a^3 A e^{-at} W_0}{2\gamma^2 t^2} > 0 \quad \text{if} \quad W_0 < 0 \ . \tag{2.15}$$

The mass matrix V_{ij} for $i, j \in \{t, \tau\}$ is diagonal since the mixed derivative $V_{t\tau}$ vanishes at $\tau = 0$.

Note, that it is the presence of the exponential factor in the negative term with the slower inverse power-law dependence on x, which renders this term as a 'negative middle term' in terms of the analysis of [29]. Here, however, this term shuts down exponentially fast for large enough x. This combined behavior of being a power-law at small x and an exponential at larger x is responsible for the fact, that a '2-term' combination with a single positive inverse power-law term is enough to obtain a tunable dS vacuum.

2.2 A sufficient condition for meta-stable de Sitter vacua

To calculate extrema of eq. (2.14) we need to calculate the first and second derivative with respect to x ($V' = \frac{\partial V}{\partial x}$)

$$V'(x) = \frac{-W_0 a^3 A}{2\gamma^2} \frac{1}{x^{11/2}} \left(C - x^{5/2}(x+2)e^{-x}\right) \ , \tag{2.16}$$

$$V''(x) = \frac{-W_0 a^3 A}{2\gamma^2} \frac{1}{x^{13/2}} \left(\frac{11}{2} C - x^{5/2}(x^2+4x+6)e^{-x}\right) \ . \tag{2.17}$$

Solving for an extremum $V'(x) = 0$ yields

$$x^{5/2}(x+2)e^{-x} = C \tag{2.18}$$

which cannot be solved explicitly in an analytic way. Plotting the approximate expression eq. (2.14) of $V(x)$ for different values of the constant C in figure 1 we observe the following behavior:

We see that with growing C we first obtain an AdS minimum. This minimum breaks supersymmetry since

$$F_T \simeq \frac{-3W_0}{2t\hat{\mathcal{V}}} \neq 0 \ . \tag{2.19}$$

Then at some point the minimum transits to dS, and for even larger C the potential eventually develops a runaway in the x direction. We can analytically calculate the window for C where we obtain a meta-stable de Sitter vacuum by identifying: