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Heavy Higgs at Tevatron and LHC in Universal Extra Dimension Models

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Abstract

Universal Extra Dimension (UED) models tend to favor a distinctively heavier Higgs mass than in the Standard Model (SM) and its supersymmetric extensions when the Kaluza-Klein (KK) scale is not much higher than the electroweak one, which we call the weak scale UED, in order to cancel the KK top contributions to the T-parameter. Such a heavy Higgs, whose production through the gluon fusion process is enhanced by the KK top loops, is fairly model independent prediction of the weak scale UED models regardless of the brane-localized mass structure at the ultraviolet cutoff scale. We study its cleanest possible signature, the Higgs decay into a Z boson pair and subsequently into four electrons and/or muons, in which all the four-momenta of the final states can be measured and both the Z boson masses can be checked. We have studied the Higgs mass $500\,\mathrm{GeV}$ (and also $700\,\mathrm{GeV}$ with $\sqrt{s} = 14\,\mathrm{TeV}$) and have found that we can observe significant resonance with the integrated luminosity $10\,\mathrm{fb}^{-1}$ for six dimensional UED models.

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1 Introduction

The Universal Extra Dimension (UED) scenario [1], in which all the Standard Model (SM) fields propagate in bulk of compactified extra dimension(s), is an attractive possibility whose simplest five dimensional realization on orbifold S^1/Z_2, the minimal UED model (mUED), may account for the existence of the dark matter as the Lightest Kaluza-Klein Particle (LKP) [2] and can give a loose gauge coupling unification at around 30 TeV [3]. See also Refs. [4, 5] for review on mUED. For the mUED, the latest analysis including the effects from second KK resonances gives the preferred Kaluza-Klein (KK) scale at around $M_{\text{KK}} \sim 1.3\,\text{TeV}$ [6]. As is mentioned in [2, 6], this result strongly depends on the brane-localized mass structure, which is assumed to be vanishing at the UV cutoff scale [7] in mUED.

One of the most important signature to establish the model would be the direct search of KK resonances at the CERN Large Hadron Collider (LHC). See Refs. [8]–[14] for mUED and Refs. [15]–[21] for 6D UED models. We note that some of them also pertain to the International Linear Collider (ILC), see also Refs. [22]–[27]. The LHC already puts a lower bound on the KK scale for mUED as $M_{\text{KK}} \gtrsim 500\,\text{GeV}$ at the 95% CL from M_{T2} analysis of cascade decay of first KK particles into the LKP [14, 28]. It is noted that bounds on mUED [29] and T^2/Z_4 UED [30] from $b \to s\gamma$ processes claim $M_{\text{KK}} \gtrsim 600\,\text{GeV}$ and 650 GeV, respectively. Again all the above bounds strongly depend on the KK mass splitting and flavor mixing patterns and hence on the boundary mass structure.[1] In particular, we cannot see a decay product unless there are enough mass splitting among the first KK modes so that it becomes sufficiently energetic. In this paper, we present a complementary signal that is insensitive to such detailed boundary structure.

In the SM, the electroweak data constrain the Higgs mass to be $M_H \lesssim 170\,\text{GeV}$ at the 95% CL, see e.g. [32, 33]. On the contrary, mUED prefer heavier Higgs when the KK scale is not much higher than the electroweak scale $v_{\text{EW}} \simeq 246\,\text{GeV}$, namely, the KK scale should be $M_{\text{KK}} \gtrsim 800\,\text{GeV}$ ($300\,\text{GeV} \lesssim M_{\text{KK}} \lesssim 400\,\text{GeV}$) at the 95% CL for $M_H = 115\ (700)\,\text{GeV}$ [34, 35, 36]. We note that this is fairly model independent feature of a general UED model since KK top modes always contribute positively to the T-parameter and such an effect requires a heavy Higgs in order to cancel these KK top contributions by the ordinary negative $\log M_H$ dependence. In this paper, we call such a natural UED model without big mass splitting among electroweak, Higgs and KK scales: $M_H \sim M_{\text{KK}} \sim \mathcal{O}(10^2)\,\text{GeV}$, the *weak scale UED model*. Concretely, we will pick up the cases: $M_H = 330, 500$ and 700 GeV. To summarize, existence of a heavy Higgs is a model independent prediction of the weak scale UED models in contrast to the relic abundance of the dark matter and the cascade decay signature of the KK particles that are dependent on the detailed boundary mass structure at the Ultra-Violet (UV) cutoff scale of the higher-dimensional gauge theory.

In five and six dimensional UED models, the Higgs production cross section via the gluon fusion process is enhanced by the KK top loops [37, 38, 39]. In this paper, we analyze its cleanest possible signature, the Higgs decay into a Z boson pair and subsequently into four electrons and/or muons, in which all the four momenta of the final states can be measured and both the Z boson masses can be checked.

In Section 2, we review the relevant part of all the known 5D and 6D UED models to the

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[1] A 95% CL bound on the KK scale $M_{\text{KK}} > 961\,\text{GeV}$ is put on a "UED" model, assuming existence of large additional extra dimensions compactified with radius of order eV^{-1}, in which SM fields cannot propagate, so that the LKP decays into KK-gravitons [31]. In this paper, we do not assume such additional large extra dimensions.

Higgs production process via the gluon fusion through the KK top loops. Considered models are the 5D mUED model on S^1/Z_2 [1], the Dirichlet Higgs (DH) model on an interval [40, 41], the 6D T^2-based models on T^2/Z_2 [1], $T^2/(Z_2 \times Z_2')$ [42], T^2/Z_4 [43, 44], RP^2 [45], and the 6D S^2-based models on Projective Sphere (PS) [46], S^2 (see Section 2.3.2), S^2/Z_2 [47].[2] In Section 3, we present the concrete computation of the process. The cross sections for the DH, T^2/Z_2, $T^2/(Z_2 \times Z_2')$, RP^2, and S^2 UED models are newly obtained. We also review the estimation of the UV cutoff scale in the T^2-based geometry [48] and extend it to the S^2-based one. In Section 4, we show our numerical results. The last section is for summary and discussions. In Appendix A, we present the relevant Feynman rules for our computation in the DH model. In Appendix B we explain our estimation of the UV cutoff scale for 6D UED models, based on the Renormalization Group Equation (RGE) analysis in the renormalizable KK picture. In Appendix C, we review the way to take into account the width in the amplitude and justify our approximation.

2 Review on known 5D and 6D UED models

In this section, we give a brief review on various UED models. Readers who are not interested in the details of these models may skip this section. In the first part of this section, we briefly review the 5D minimal UED model on S^1/Z_2 [1] and Dirichlet Higgs model on an interval [40, 41]. The remaining of the section is devoted to an overview of various types of 6D UED models.

2.1 5D UED models

2.1.1 Minimal UED model on S^1/Z_2

First we review the 5D UED model [1]. The matter contents of the model are the same as those of the SM, but they are living in the bulk of flat five-dimensional space, compactified on the orbifold S^1/Z_2. The action S is written down as

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \left[\mathcal{L}_{\text{bulk}} + \delta(y)\mathcal{L}_0 + \delta(y-\pi R)\mathcal{L}_{\pi R}\right]. \tag{1}$$

Usually when one say mUED model, it is implied that all the boundary masses are zero at the UV cutoff scale and are generated through radiative corrections [7]. Hereafter, when we call mUED model, we do not assume any boundary mass structure and concentrate on the signal that is independent of it. In particular, we do not include the constraints from the direct KK search [14, 28] and from the relic abundance of the LKP [6] that are dependent on the KK mass splitting pattern.

The Z_2 twist conditions on the bulk SM fields are put as

$$\begin{aligned}
\mathcal{B}_\mu(x,-y) &= \mathcal{B}_\mu(x,y), & \mathcal{B}_5(x,-y) &= -\mathcal{B}_5(x,y),\\
\mathcal{W}_\mu(x,-y) &= \mathcal{W}_\mu(x,y), & \mathcal{W}_5(x,-y) &= -\mathcal{W}_5(x,y),\\
\mathcal{G}_\mu(x,-y) &= \mathcal{G}_\mu(x,y), & \mathcal{G}_5(x,-y) &= -\mathcal{G}_5(x,y),
\end{aligned} \tag{2}$$

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[2] In [46] the terminology "real projective plane" is employed for the compactified space, the sphere with its antipodal points being identified. In order to distinguish [46] from [45], we call the former the Projective Sphere.

$$\begin{aligned} L(x,-y) &= \gamma^5 L(x,y), & E(x,-y) &= -\gamma^5 E(x,y), \\ Q(x,-y) &= \gamma^5 Q(x,y), & U(x,-y) &= -\gamma^5 U(x,y), \\ & & D(x,-y) &= -\gamma^5 D(x,y), \end{aligned} \tag{3}$$

and

$$\Phi(x,-y) = \Phi(x,y), \tag{4}$$

where x and y ($= x^5$) denote four and extra dimensional coordinates, respectively.[3] We can see that the wanted zero modes remain after the twist (2)–(4). There are fixed points of the Z_2 orbifolding at $y = 0, \pi R$. If the boundary Lagrangians at $y = 0, \pi R$ are equal at the UV cutoff scale, there remains an additional accidental symmetry under the reflection $\frac{\pi R}{2} - y \to \frac{\pi R}{2} + y$, called the KK parity, which ensures the stability of the LKP and makes it a dark matter candidate.

The gauge and Yukawa interactions for the KK-top quarks, which we need for later calculation, are:

$$\begin{aligned} \mathcal{L}_{\text{KK top}} = &-ig_{4s} \sum_{n=1}^{\infty} \begin{bmatrix} \overline{t_1} & \overline{t_2} \end{bmatrix}^{(n)} \gamma^\mu \mathcal{G}_\mu^{(0)} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}^{(n)} \\ &- \frac{m_t}{v_{\text{EW}}} H^{(0)} \sum_{n=1}^{\infty} \begin{bmatrix} \overline{t_1} & \overline{t_2} \end{bmatrix}^{(n)} \begin{bmatrix} \sin 2\alpha^{(n)} & -\gamma^5 \cos 2\alpha^{(n)} \\ \gamma^5 \cos 2\alpha^{(n)} & \sin 2\alpha^{(n)} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}^{(n)}, \end{aligned} \tag{5}$$

where $g_{4s} = g_s/\sqrt{2\pi R}$ is a dimensionless 4D $SU(3)_C$ coupling constant and $v_{\text{EW}} \simeq 246\,\text{GeV}$ is the 4D Higgs vacuum expectation value which appear after the KK expansion; $\mathcal{G}^{(0)}(H^{(0)})$ shows zero-mode gluon (zero-mode physical Higgs); $t_1^{(n)}$ and $t_2^{(n)}$ are mass eigenstates of n-th KK top quarks and each mixing angle $\alpha^{(n)}$ is determined to be $\cos 2\alpha^{(n)} = m_{(n)}/\sqrt{m_t^2 + m_{(n)}^2}$, $\sin 2\alpha^{(n)} = m_t/\sqrt{m_t^2 + m_{(n)}^2}$, with $m_{(n)} := n/R$. Each KK state is twofold degenerate and n-th KK top mass is

$$m_{t,(n)} = \sqrt{m_t^2 + m_{(n)}^2}. \tag{6}$$

KK tops give dominant contribution to the gluon fusion process due to their large Yukawa coupling to the Higgs. We note that γ^5 is put in Eq. (5) merely to arrange the sign of both the KK masses positive.

2.1.2 Dirichlet Higgs (DH) model

Dirichlet Higgs model is defined on an interval: $0 \leq y \leq \pi R$. The action S is as follows:

$$S = \int d^4x \int_0^{\pi R} dy \left[\mathcal{L}_{\text{bulk}} + \delta(y)\mathcal{L}_0 + \delta(y - \pi R)\mathcal{L}_{\pi R}\right], \tag{7}$$

where R is a radius of the extra spacial direction. The structure of the bulk Lagrangian, covariant derivatives and field strength of gauge bosons are the same as that of the mUED model. There is no difference between the matter contents of this model and those of the mUED model. As in the mUED model, we neglect the possible boundary interactions in this paper. The zero-mode sector of the UED on an interval becomes the same as that of the mUED on the orbifold S^1/Z_2 when we choose the boundary conditions for the SM degrees of freedom $\Psi^N = \mathcal{G}_\mu, \mathcal{W}_\mu, \mathcal{B}_\mu; L_L, Q_L; E_R, U_R, D_R$ to be Neumann (at $y = 0$ and πR):

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[3] We follow the metric and spinor conventions of [49].

$$\partial_5 \Psi^N(x, 0) = \partial_5 \Psi^N(x, \pi R) = 0 \tag{8}$$

and for other non-SM modes $\Psi^D = \mathcal{B}_5, \mathcal{W}_5; L_R, Q_R; E_L, U_L, D_L$ to be Dirichlet:

$$\Psi^D(x, 0) = \Psi^D(x, \pi R) = 0. \tag{9}$$

We note that mode functions with Dirichlet and Neumann boundary conditions are not orthogonal to each other, unlike the orbifolding on S^1/Z_2.[4] Kinetic terms turn out to be diagonal even though the expansion is not orthonormal. We can explicitly check that the non-orthogonality does not lead to extra mixing for spinors even after the EWSB because non-orthogonal terms drop out due to the 4D-chirality.

If we had put the Neumann condition on the Higgs Φ, we would get exactly the same zero-mode sector as in the mUED model on S^1/Z_2. In the Drichlet Higgs model on interval, the EWSB is caused by a non-zero Dirichlet boundary condition on the $SU(2)_W$-doublet Higgs field [40, 41]. We assume that the KK-parity is respected by the boundary conditions on the Higgs field too. The advantage of the Dirichlet EWSB is that we do not need to assume the negative mass-squared in the bulk Lagrangian nor the quartic coupling which is a higher dimensional operator in 5D. Throughout this paper, we consider the minimal case: $\mathcal{V}(\Phi) = 0$. We list the necessary Feynman rules in Appendix A.

2.2 6D UED models based on T^2

We consider a gauge theory on six-dimensional spacetime $M^4 \times T^2$, which is a direct product of the four-dimensional Minkowski spacetime M^4 and two-torus T^2: $0 \leq y \leq 2\pi R_y, 0 \leq z \leq 2\pi R_z$. We assume that the two radii of T^2 have the same value $R = R_y = R_z$ for simplicity.[5]

When we use 6D Weyl spinor for 6D UED model construction, there is a constraint on the choice of 6D chiralities. The origin of this constraint is the cancellation of 6D gravitational and $\mathrm{SU}(2)_L$ global anomalies that cannot be removed by use of the Green-Schwarz mechanism. This constraint requires the number of matter generation to be (multiple of) three [51]. A suitable choice of the 6D chirality for a single matter generation is as follows:

$$(Q_+, U_-, D_-; L_+, E_-, N_-), \tag{10}$$

where the ± suffixes represent 6D chirality of each field. Number of d.o.f. of 6D Weyl fermion is 4, the same as that of a 4D Dirac fermion. Therefore we can construct 6D UED models on T^2 following the orbifolding method of the 5D UED model. We have several options for the orbifolding to realize the SM chiral fermions in the zero mode sector of (10). Let us review them in turn. The range for KK summation is listed in Table 2.

type of orbifolding	identification	fixed points (y_i, z_i)
T^2/Z_2	$(y,z) \sim (-y,-z)$	$(0,0)$, $(\pi R, 0)$, $(0, \pi R)$, $(\pi R, \pi R)$
$T^2/(Z_2 \times Z_2')$	$(y,z) \sim (-y,z)$ and $(y,z) \sim (y,-z)$	$(0,0)$, $(\pi R, 0)$, $(0, \pi R)$, $(\pi R, \pi R)$
T^2/Z_4	$(y,z) \sim (-z,y)$	$(0,0)$, $(\pi R, \pi R)$

Table 1: Fixed points which stem from each identification.

2.2.1 Orbifold T^2/Z_2, $T^2/(Z_2 \times Z_2')$, and T^2/Z_4

We consider T^2/Z_2 [1], $T^2/(Z_2 \times Z_2')$ [42], and T^2/Z_4 [43, 44] orbifolds. Let us write down the action

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \int_{-\pi R}^{\pi R} dz \left[\mathcal{L}_{\text{bulk}}(x,y,z) + \sum_{\vec{y}\,\in\,\vec{y}_i} \delta(\vec{y}_i) \mathcal{L}_{\vec{y}_i}(x) \right], \tag{11}$$

where $\vec{y}_i = (y_i, z_i)$ are orbifold fixed points. We note that terms localized at the fixed points are induced at quantum level even if we assume that they are vanishing at tree level [7, 52, 53]. For the orbifold we consider in this paper, the projections are:

$$(y,z) \sim (-y,-z) \qquad \text{for } T^2/Z_2, \tag{12}$$

$$(y,z) \sim (-y,z) \text{ and } (y,z) \sim (y,-z) \qquad \text{for } T^2/(Z_2 \times Z_2'), \tag{13}$$

$$(y,z) \sim (-z,y) \qquad \text{for } T^2/Z_4. \tag{14}$$

See also Table. 1. In each case, we can choose a suitable boundary condition of 6D Weyl fermions, whose exact forms are not discussed in this paper, to generate 4D Weyl fermions at the zero modes.

The bulk Lagrangian, covariant derivatives and field strengths of gauge bosons are essentially the same as that of the 5D mUED model except for the structure of spinors. For 6D Weyl fermions, the kinetic and Yukawa terms are

$$\begin{aligned}\mathcal{L}_{\text{kinetic}} &= -\overline{Q_+}\Gamma^M \mathcal{D}_M Q_+ - \overline{U_-}\Gamma^M \mathcal{D}_M U_- - \overline{D_-}\Gamma^M \mathcal{D}_M D_- \\ &\quad - \overline{L_+}\Gamma^M \mathcal{D}_M L_+ - \overline{E_-}\Gamma^M \mathcal{D}_M E_- - \overline{N_-}\Gamma^M \mathcal{D}_M N_-, \qquad (15)\\ \mathcal{L}_{\text{Yukawa}} &= -\lambda_U \overline{U_-}\left(Q_+ \cdot \Phi\right) - \lambda_D \left(\overline{Q_+}\Phi\right) D_- - \lambda_E \left(\overline{L_+}\Phi\right) E_- + \text{h.c.}, \qquad (16)\end{aligned}$$

where contraction of $SU(2)$ indices are understood.[6] Resultant interactions relevant for our discussion are

$$\begin{aligned}\mathcal{L}_{\text{KK top}} &= -ig_{4s} \sum_{(m,n)}^{\infty} \begin{bmatrix} \overline{t_1} & \overline{t_2} \end{bmatrix}^{(m,n)} \gamma^\mu \mathcal{G}_\mu^{(0)} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}^{(m,n)} \\ &\quad - \frac{m_t}{v_{\text{EW}}} H^{(0)} \sum_{(m,n)}^{\infty} \begin{bmatrix} \overline{t_1} & \overline{t_2} \end{bmatrix}^{(m,n)} \begin{bmatrix} \sin 2\alpha^{(m,n)} & -\gamma^5 \cos 2\alpha^{(m,n)} \\ \gamma^5 \cos 2\alpha^{(m,n)} & \sin 2\alpha^{(m,n)} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}^{(m,n)}, \end{aligned} \tag{17}$$

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[4] In other words the KK mass-squared operator ∂_5^2 is not hermitian in this setup, though the kinetic term is still positive definite.

[5] In the T^2/Z_4 orbifold case, the condition $R_y = R_z$ is imposed by the consistency with the Z_4 discrete symmetry. See also [50] for a realization of CP violation from the complex structure of T^2/Z_4, which appears in 4D effective interactions after KK decomposition.

[6] We leave the neutrino sector untouched since it is irrelevant for the Higgs signal considered in this paper.

type of orbifolding	range of (m,n)
T^2/Z_2	$m+n \geq 1$, or $m=-n \geq 1$
$T^2/(Z_2 \times Z_2')$	$0 \leq m < \infty,\ 0 \leq n < \infty;\ (m,n) \neq (0,0)$
T^2/Z_4	$1 \leq m < \infty,\ 0 \leq n < \infty$

Table 2: The range of the parameter (m,n) except the zero mode case $(m,n)=(0,0)$ in each case of the orbifolding.

where $g_{4s} = g_s/(2\pi R)$ is the dimensionless 4D $SU(3)_C$ coupling constant and $v_{\rm EW} \simeq 246\,\text{GeV}$ is the 4D Higgs vev, $\mathcal{G}^{(0)}(H^{(0)})$ shows zero-mode gluon (zero-mode physical Higgs), and $t_1^{(m,n)}, t_2^{(m,n)}$ are mass eigenstates of (m,n)-th KK top quarks. Again we only consider the KK top quark loops since contributions from other flavors are suppressed by the small Yukawa coupling. Each mixing angle $\alpha^{(m,n)}$ is determined to be $\cos 2\alpha^{(m,n)} = m_{(m,n)}/\sqrt{m_t^2 + m_{(m,n)}^2}$, $\sin 2\alpha^{(m,n)} = m_t/\sqrt{m_t^2 + m_{(m,n)}^2}$. Each KK state is twofold degenerate and (m,n)-th KK top mass is

$$m_{t,(m,n)} = \sqrt{m_t^2 + m_{(m,n)}^2}, \tag{18}$$

with

$$m_{(m,n)} := \frac{\sqrt{m^2+n^2}}{R}. \tag{19}$$

It should be mentioned that the difference from the mUED case appears only in the form of KK mass and the number of d.o.f. in each KK level when we consider the gluon fusion process. We adopt m(n) as the y(z)-directional KK index, whose parameter region is determined by the way of the orbifolding. This information has a great influence on the enhancement of the Higgs production through the gluon fusion.

2.2.2 Real Projective Plane (RP^2)

We can construct a UED model on a non-orientable geometry: Real Projective Plane (RP^2) [45]. RP^2 is defined by two types of identifications: a π-rotation r and a glide g:

$$r : (y,z) \sim (-y,-z), \quad g : (y,z) \sim (y+\pi R, -z+\pi R). \tag{20}$$

The system is invariant under each manipulation in Eq. (20). Note that the shifts $y \sim y + 2\pi R$ and $z \sim z + 2\pi R$ can be obtained as different combinations of r and g, respectively. Note also that no fixed point exists globally in this background geometry.

Under r and g, Weyl fermions transforms as

$$r : \Psi_\pm(x;-y,-z) = p_r \Gamma_r \Psi_\pm(x;y,z), \quad \Gamma_r = i\Gamma^5\Gamma^6\Gamma^7, \tag{21}$$

$$g : \tilde{\Psi}_\pm(x;y+\pi R, -z+\pi R) = p_g \Gamma_g \Psi_\mp(x;y,z), \quad \Gamma_g = \Gamma^6\Gamma^7, \tag{22}$$

where Γ^7 is the 6D chirality operator and p_r, p_g (Z_2-parities) can take the value ± 1. The $\tilde{\Psi}_\pm$ is what we call the "mirror" fermion. Eq. (21) has the same form with that of the T^2/Z_2 orbifold condition for 6D fermion. An essential point of this model is that the condition (22) does not generate a 4D Weyl fermion in the zero mode sector. In other words, the 6D chirality of both sides of Eq. (22) are different from each other. This means that we have to introduce new fermions $\tilde{\Psi}_\pm$ which have opposite 6D chirality and the same SM quantum number compared to each corresponding field $\Psi_\mp$. Concretely, "mirror" fermions:

$$\mathcal{Q}_-, \mathcal{U}_+, \mathcal{D}_+; \text{Ł}_-, \mathcal{E}_+, \mathcal{N}_+, \tag{23}$$

are identified with $\{Q_+, U_-, D_-; L_+, E_-, N_-\}$, respectively. The choice of 6D chiralities in Eq. (23) obeys the condition for realizing the 6D anomaly cancellation which we have argued before.

The bulk Lagrangian is the same as that of the T^2-based models using orbifold except for the existence of the mirror fermions:

$$\begin{aligned}
\mathcal{L}_{\text{kinetic}} = \frac{1}{2}\Big[& -\overline{Q_+}\Gamma^M \mathcal{D}_M Q_+ - \overline{U_-}\Gamma^M \mathcal{D}_M U_- - \overline{D_-}\Gamma^M \mathcal{D}_M D_- \\
& -\overline{L_+}\Gamma^M \mathcal{D}_M L_+ - \overline{E_-}\Gamma^M \mathcal{D}_M E_- - \overline{N_-}\Gamma^M \mathcal{D}_M N_- \\
& -\overline{\mathcal{Q}_-}\Gamma^M \mathcal{D}_M \mathcal{Q}_- - \overline{\mathcal{U}_+}\Gamma^M \mathcal{D}_M \mathcal{U}_+ - \overline{\mathcal{D}_+}\Gamma^M \mathcal{D}_M \mathcal{D}_+ \\
& -\overline{\text{Ł}_-}\Gamma^M \mathcal{D}_M \text{Ł}_- - \overline{\mathcal{E}_+}\Gamma^M \mathcal{D}_M \mathcal{E}_+ - \overline{\mathcal{N}_+}\Gamma^M \mathcal{D}_M \mathcal{N}_+ \Big],
\end{aligned} \tag{24}$$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} = \frac{1}{2}\Big[& -\lambda_U \overline{U_-}\left(Q_+ \cdot \Phi\right) - \lambda_D \left(\overline{Q_+}\Phi\right) D_- - \lambda_E \left(\overline{L_+}\Phi\right) E_- \\
& -\lambda_U \overline{\mathcal{U}_+}\left(\mathcal{Q}_- \cdot \Phi\right) - \lambda_D \left(\overline{\mathcal{Q}_-}\Phi\right) \mathcal{D}_+ - \lambda_E \left(\overline{\text{Ł}_-}\Phi\right) \mathcal{E}_+ + \text{h.c.}\Big],
\end{aligned} \tag{25}$$

where we introduce the "1/2" factors for later convenience. The neutrino sector is again left untouched as it is irrelevant for our discussion. By use of Eq. (22), we can erase all the mirror fermions and obtain the ordinary form of Lagrangian same as T^2 cases. The form of π-rotation r given in Eq. (21) is the same as that of Z_2 orbifolding in Eq. (12). Therefore, the interactions of RP^2 model, needed to calculate the gluon fusion process, take the same form as that of T^2/Z_2 one given in Eq. (17).

2.3 6D UED models based on S^2

Let us review UED models based on the S^2 compactification. We span the extra dimension by the zenith and azimuthal angles θ and ϕ, respectively. The two-sphere S^2 has a positive curvature and to stabilize the radius R, we introduce an extra $U(1)_X$ gauge field which has a monopole-like classical configuration [54]

$$\left[\mathcal{X}^c_\phi(x^\mu, \theta, \phi)\right]^{\substack{N\\S}} = \frac{n}{2g_X}(\cos\theta \mp 1), \qquad \text{(other components)} = 0, \tag{26}$$

where the superscript c denotes the classical configuration, g_X is the 6D $U(1)_X$ gauge coupling, the integer n is the (negative) monopole charge, and the superscripts N and S indicate that the field is given in north (involving the $\theta = 0$ point) and south (involving the $\theta = \pi$ point) charts, respectively. The $U(1)_X$ transition function from the north to the south chart is given by

$$\left[\mathcal{X}_M(x^\mu, \theta, \phi)\right]^S = \left[\mathcal{X}_M(x^\mu, \theta, \phi)\right]^N + \frac{1}{g_X}\partial_M \alpha(x^\mu, \theta, \phi) \tag{27}$$

with $\alpha(x^\mu, \theta, \phi) = n\phi$. Because of the monopole-like configuration, the radius of S^2 is stabilized spontaneously at

$$R^2 = \left(\frac{n}{2g_X M_*^2}\right)^2, \tag{28}$$

where M_* is the 6D Planck scale.

We mention that any 6D field Ξ on S^2 is KK expanded by use of the spin-weighted spherical harmonics ${}_sY_{jm}(\theta, \phi)$ as follows:

$$\Xi(x, \theta, \phi)^{\substack{N\\S}} = \sum_{j=|s|}^{\infty} \sum_{m=-j}^{j} \xi^{(j,m)}(x) f_{\Xi}^{(j,m)}(\theta, \phi)^{\substack{N\\S}}, \quad f_{\Xi}^{(j,m)}(\theta, \phi)^{\substack{N\\S}} := \frac{{}_sY_{jm}(\theta, \phi) e^{\pm is\phi}}{R}, \tag{29}$$

where $\xi^{(j,m)}$ is the (j, m)-th expanded 4D field, $f_{\Xi}^{(j,m)}$ is the corresponding mode function and s is the spin weight of the field Ξ. The spin-weighted spherical harmonics ${}_sY_{jm}(\theta, \phi)$ matches the orthonormal condition as

$$\int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \, \overline{{}_sY_{jm}(\theta, \phi)} \, {}_sY_{j'm'}(\theta, \phi) = \delta_{jj'} \delta_{mm'}. \tag{30}$$

A spin weight of fermion is closely related to its $U(1)_X$ charge. When we assign $U(1)_X$ charges of 6D Weyl fermions $\Psi_\pm$ as $q_{\Psi_\pm}$, the corresponding spin weights of 4D Weyl fermions $\{\psi_{+\substack{L\\R}}, \psi_{-\substack{L\\R}}\}$ are given as follows in our convention:

$$s_{+\substack{L\\R}} = \frac{nq_{\Psi_+} \pm 1}{2}, \quad s_{-\substack{L\\R}} = \frac{nq_{\Psi_-} \mp 1}{2}. \tag{31}$$

Note that if a 6D Weyl fermion takes a spin weight $s = 0$, a $j = 0$ mode appears as a 4D Weyl fermion with vanishing KK mass. This means that we can get chiral SM fermions without orbifolding in the case of S^2. When we take the values:

$$(s_{+R}, s_{+L}, s_{-R}, s_{-L}) = (0, 1, 1, 0), \tag{32}$$

we can create the same situation as in the T^2-based models discussed before. The spin weight of the 4D-vector component of a 6D gauge boson is always $s = 0$ and then there is a zero mode that can be identified as the SM gauge boson. On the other hand, extra dimensional components of the 6D gauge boson are expanded by the $|s| = 1$ spin-weighted spherical harmonics and has no zero-mode.

In our configuration, any (j, m)-th KK mode has the KK mass:

$$m_{(j,m)} = \frac{\sqrt{j(j+1)}}{R}. \tag{33}$$

An important point is that the form of the above KK mass is independent of the index of m. This means that there are $2j + 1$ degenerate modes for each j. Note that the lightest KK mode has the mass $\sqrt{2}/R$.

As discussed above, the 4D-vector component of a 6D gauge boson has a zero mode. This is the case for the extra $U(1)_X$ gauge boson too. Phenomenologically the existence of an extra $U(1)$ interaction, under which SM fields are charged, is problematic [46]. In the following, let us see how to get rid of this massless $U(1)_X$ vector.

2.3.1 Projective Sphere (PS)

We can construct a UED model compactified on the Projective Sphere (PS), a sphere S^2 with its antipodal points being identified by $(\theta, \phi) \sim (\pi - \theta, \phi + \pi)$ [46]. In the UED model based on PS, the 6D action takes a different form from that of the 6D orbifold UED models. One of the remarkable points of this model is that there is no fixed point on the background geometry PS. As in the RP^2 model, we introduce "mirror" 6D Weyl fermions:

$$\mathcal{Q}_-, \mathcal{U}_+, \mathcal{D}_+; \text{Ł}_-, \mathcal{E}_+, \mathcal{N}_+ \tag{34}$$

which have opposite 6D chirality and opposite SM and $U(1)_X$ charges, compared to the fields $\{Q_+, U_-, D_-; L_+, E_-, N_-\}$. Because of the existence of mirror fermions the kinetic term takes the same form as in the RP^2 model (24) and the Yukawa interaction is modified to

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = \frac{1}{2}\Big[& - \lambda_U \overline{U_-} \left(Q_+ \cdot \Phi\right) - \lambda_D \left(\overline{Q_+} \Phi\right) D_- - \lambda_E \left(\overline{L_+} \Phi\right) E_- \\ & - \lambda_U^* \overline{\mathcal{U}_+} \left(\mathcal{Q}_- \cdot \Phi\right) - \lambda_D^* \left(\overline{\mathcal{Q}_-} \Phi\right) \mathcal{D}_+ - \lambda_E^* \left(\overline{\text{Ł}_-} \Phi\right) \mathcal{E}_+ + \text{h.c.}\Big]. \end{aligned} \tag{35}$$

Like the RP^2 case which we have discussed before, we introduce the "1/2" factors for a later convenience. The covariant derivatives in this model are given as

$$\begin{aligned} \mathcal{D}_M &= \partial_M + i g_s \mathcal{G}_M^a T_s^a + i g \mathcal{W}_M^a T^a + i g_Y \mathcal{B}_M Y \\ & \qquad\qquad\qquad\qquad\qquad\qquad (\text{for } \Phi), \qquad\qquad (36) \\ \mathcal{D}_M &= \partial_M + i g_s \mathcal{G}_M^a T_s^a + i g \mathcal{W}_M^a T^a + i g_Y \mathcal{B}_M Y + i g_X q_\Psi (\mathcal{X}_M^c + \mathcal{X}_M) + \Omega_M \\ & \qquad\qquad\qquad\qquad\qquad\qquad (\text{for } Q_+, U_-, D_-; L_+, E_-, N_-), \qquad (37) \\ \mathcal{D}_M &= \partial_M + i g_s \mathcal{G}_M^a [-T_s^a]^{\mathrm{T}} + i g \mathcal{W}_M^a [-T^a]^{\mathrm{T}} + i g_Y \mathcal{B}_M [-Y] + i g_X q_\Psi (\mathcal{X}_M^c + \mathcal{X}_M) + \Omega_M \\ & \qquad\qquad\qquad\qquad\qquad\qquad (\text{for } \mathcal{Q}_-, \mathcal{U}_+, \mathcal{D}_+; \text{Ł}_-, \mathcal{E}_+, \mathcal{N}_+), \qquad (38) \end{aligned}$$

where Ω_M is the spin connection. The covariant derivative of Higgs is the same as that in the S^2/Z_2 case, but there is a difference between those of fermions and these "mirror" fermions. We discuss these points shortly below.

As we mentioned before, projective sphere is a non-orientable manifold and has no fixed point. Let us consider the 6D P and CP transformations. Under the antipodal projection,

$$\left\{ \begin{array}{lcr} \mathcal{X}_\mu(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} & = & \mathcal{X}_\mu^C(x, \theta, \phi)\,{}^{S}_{N}, \\ \mathcal{X}_\theta(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} & = & -\mathcal{X}_\theta^C(x, \theta, \phi)\,{}^{S}_{N}, \\ \{\mathcal{X}_\phi^c, \mathcal{X}_\phi\}(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} & = & \{(\mathcal{X}_\phi^c)^C, \mathcal{X}_\phi^C\}(x, \theta, \phi)\,{}^{S}_{N}, \end{array} \right. \tag{39}$$

where the superscript C denotes the 6D C transformation. Recall that the superscript c denotes the classical configuration. These conditions leave the monopole-like configuration invariant under the antipodal identification and projects out the unwanted $U(1)_X$ 4D-vector zero mode. In contrast, identification of a SM gauge boson $\mathcal{A}_M^{(i)}$ should be done by another condition since we want the corresponding 4D-vector zero mode, where i shows the type of gauge group. We adopt the 6D P transformation and those identifications are written as

$$\left\{ \begin{array}{rcl} \mathcal{A}_\mu^{(i)}(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} & = & \mathcal{A}_\mu^{(i)}(x, \theta, \phi)\,{}^{S}_{N}, \\ \mathcal{A}_\theta^{(i)}(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} & = & -\mathcal{A}_\theta^{(i)}(x, \theta, \phi)\,{}^{S}_{N}, \\ \mathcal{A}_\phi^{(i)}(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} & = & \mathcal{A}_\phi^{(i)}(x, \theta, \phi)\,{}^{S}_{N}, \end{array} \right. \tag{40}$$

where it is evident that the zero mode of $\mathcal{A}_\mu^{(i)}$ survives. We also identify Higgs with the 6D P transformation to obtain its zero mode:

$$\Phi(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} = \Phi(x, \theta, \phi)\,{}^{S}_{N}\,. \tag{41}$$

Finally, we discuss the identification of 6D Weyl fermions. Since 6D Weyl fermions have $U(1)_X$ charge and interact with the $U(1)_X$ gauge boson, they should be identified by the 6D CP transformation. The specific form of the 6D CP transformation, for example in the case of U_-, is as follows:

$$\mathcal{U}_+(x, \pi - \theta, \phi + \pi)\,{}^{N}_{S} = PU_-^C(x, \theta, \phi)\,{}^{S}_{N}\,, \tag{42}$$

where the matter field U_- is identified to the mirror $\mathcal{U}_+$. We decide the forms of covariant derivatives (37) and (38) on the criterion of invariance of the action under the 6D CP transformation. Using the identification conditions (39)–(42), we can see that the mirror fermions drop out of the action after the identifications and eventually we obtain the usual type of UED model action. This can be interpreted that all the modes of mirror fermions $\{\mathcal{Q}_-, \mathcal{U}_+, \mathcal{D}_+; \mathcal{L}_-, \mathcal{E}_+, \mathcal{N}_+\}$ are erased and no mode of $\{Q_+, U_-, D_-; L_+, E_-, N_-\}$ is projected out. The interaction terms which we need for calculation in this model is the same as those in Eq. (17). Only difference is the number of degenerate top KK modes in each j-level.

2.3.2 S^2 UED with a Stueckelberg Field (S^2)

As a solution to the massless $U(1)_X$ problem, we can simply give a Stueckelberg mass [55, 56], see also [57, 58] for reviews, to the $U(1)_X$ field. We can make the unwanted $U(1)_X$ 4D-vector zero mode to be massive while preserving the classical monopole structure in Eq. (26). This way, we can formulate a UED model on S^2 with no field identification. Let us call this simple model the S^2 UED model. In the S^2 UED model, the matter contents, bulk Lagrangian, definition of field strengths and covariant derivatives and the configuration of the classical $U(1)_X$ field are the same as the PS model after removing the mirror fermions, except for the Stueckelberg field part. In contrast to the S^2/Z_2 orbifold below, there are no fixed point nor a localized Lagrangian anywhere on S^2, as in the case of PS UED. The d.o.f. of KK fermions has no difference between the S^2 UED and PS one (after the antipodal projection). There is no need for an additional computation; all we have to do is to borrow the PS result as a whole when we are only interested in the gluon fusion process.

2.3.3 S^2/Z_2 orbifold

Although the above S^2 UED model with a Stueckelberg $U(1)_X$ mass is already phenomenologically viable, we may further perform a Z_2 orbifolding on it [47].[7] On this orbifold, the point (θ, ϕ) is identified with $(\pi - \theta, -\phi)$. The 6D action S is as follows:

$$S = \int d^4x \int_0^\pi d\theta \int_0^{2\pi} d\phi \sqrt{-g} \left[\mathcal{L}_{\text{bulk}}(x, y, z) + \delta\left(\theta - \frac{\pi}{2}\right) \delta\left(\phi\right) \mathcal{L}_{(\pi/2,0)}(x) + \delta\left(\theta - \frac{\pi}{2}\right) \delta\left(\phi - \pi\right) \mathcal{L}_{(\pi/2,\pi)}(x)\right], \tag{43}$$

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[7] This extra Z_2 cannot project out the $U(1)_X$ gauge field. In [47], the $U(1)_X$ is assumed to be broken by an anomaly. Since we need a classical configuration of the $U(1)_X$, it would be theoretically preferable to break it by a tiny Stueckelberg mass.

where $\sqrt{-g} = R^2 \sin\theta$. This system has two fixed points of the Z_2 symmetry at $(\theta,\phi) = (\frac{\pi}{2},0),(\frac{\pi}{2},\pi)$ and we describe the localized terms with $\mathcal{L}_{(\pi/2,0)},\mathcal{L}_{(\pi/2,\pi)}$, respectively. Like the T^2-case, we do not discuss those parts in this paper.

We can easily construct mode functions of S^2/Z_2 $f^{(j,m)}_{s,t}(\theta,\phi)$ with spin weight s in both north and south charts following the general prescription [59] as follows:

$$f^{(j,m)}_{s,t}(\theta,\phi)\,{}^{N}_{S} = \begin{cases} \dfrac{1}{2R}\left[{}_sY_{jm}(\theta,\phi) + (-1)^{j-s}{}_sY_{j-m}(\theta,\phi)\right]e^{\pm is\phi} & \text{for } t=+1 \\ \dfrac{1}{2R}\left[{}_sY_{jm}(\theta,\phi) - (-1)^{j-s}{}_sY_{j-m}(\theta,\phi)\right]e^{\pm is\phi} & \text{for } t=-1 \end{cases}, \tag{44}$$

where $t=\pm1$ is the Z_2 parity. These mode functions have the property that $f^{(j,m)}_{s,t=\pm1}(\pi-\theta,-\phi)\,{}^{N}_{S} = \pm f^{(j,m)}_{s,t=\pm1}(\theta,\phi)\,{}^{S}_{N}$. To realize the Z_2 symmetry, we identify a field at (θ,ϕ) in north chart with the same field at $(\pi-\theta,-\phi)$ in south chart.

The range of the summation over m shrinks from $[-j,j]$ to $[0,j]$ after the Z_2 identification. Under the transformation of $(\theta,\phi)\to(\theta,\phi+\pi)$, mode functions behave as

$$f^{(j,m)}_{s=0,t=+1}(\theta,\phi+\pi)\,{}^{N}_{S} = (-1)^m f^{(j,m)}_{s=0,t=+1}(\theta,\phi)\,{}^{N}_{S}, \quad f^{(j,m)}_{s=\pm1,t=-1}(\theta,\phi+\pi)\,{}^{N}_{S} = -(-1)^m f^{(j,m)}_{s=\pm1,t=-1}(\theta,\phi)\,{}^{N}_{S}. \tag{45}$$

After some fields redefinition, we can find that each KK field has a KK parity $(-1)^m$, which is a remnant of the KK angular momentum conservation.

We focus on the $m=0$ modes of each j level. When we see the concrete forms of mode functions in $m=0$, which are

$$f^{(j,m=0)}_{s=0,t=+1}(\theta,\phi)\,{}^{N}_{S} = \frac{1}{2R}(1+(-1)^j)\cdot{}_0Y_{j0}(\theta,\phi), \tag{46}$$

$$f^{(j,m=0)}_{s=+1,t=-1}(\theta,\phi)\,{}^{N}_{S} = \frac{1}{2R}(1+(-1)^j)\cdot{}_1Y_{j0}(\theta,\phi)e^{\pm i\phi}, \tag{47}$$

$$f^{(j,m=0)}_{s=-1,t=-1}(\theta,\phi)\,{}^{N}_{S} = \frac{1}{2R}(1+(-1)^j)\cdot{}_{-1}Y_{j0}(\theta,\phi)e^{\mp i\phi}, \tag{48}$$

where we find that $m=0$ modes appear only in the case of even j. Then degeneracy of KK masses is

$$\begin{array}{ccl} j+1 & \text{for} & j:\text{even}, \\ j & \text{for} & j:\text{odd}, \end{array} \tag{49}$$

since m runs from 0 to j. Again, this mode counting is the only important point when computing the enhancement of the Higgs production via gluon fusion process. We do not discuss the form of interactions which we need for calculating the gluon fusion process because there is essentially no difference from the T^2 case (17).

3 Higgs production and decay into four leptons in UED models

In the SM, the cross section for the Leading Order (LO) one-loop Higgs production via the gluon fusion process and its subsequent decay into a Z boson pair: $gg\to H\to ZZ$ is given by [60]. The LO parton-level cross section shown in Fig. 1 is:

$$\hat\sigma^{\rm SM}_{gg\to H\to ZZ} = \frac{\alpha^2_{4s}}{256\pi^3}\left(\frac{m_Z}{v_{\rm EW}}\right)^4\left[1+\frac{\left(\hat s-2m_Z^2\right)^2}{8m_Z^4}\right]\frac{\hat s}{\left(\hat s-M_H^2\right)^2+\Delta^2}\sqrt{1-\frac{4m_Z^2}{\hat s}}\;|I(\hat s)|^2, \tag{50}$$