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125 GeV Higgs Boson and the Type-II Seesaw Model

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Abstract: We study the vacuum stability and unitarity conditions for a 125 GeV Standard Model (SM)-like Higgs boson mass in the type-II seesaw model. We find that, as long as the seesaw scale is introduced below the SM vacuum instability bound, there exists a large parameter space predicting a 125 GeV Higgs mass, irrespective of the exact value of the seesaw scale, satisfying both stability and unitarity conditions up to the Planck scale. We also study the model predictions for the Higgs partial decay widths in the diphoton and Z+photon channels with respect to their SM expectations and find that the decay rates for these two processes are correlated. We further show that for any given enhancement in the Higgs-to-diphoton rate over its SM expectation, there exists an *upper* bound on the type-II seesaw scale, and hence, on the masses of the associated doubly- and singly-charged Higgs bosons in the allowed parameter space. For instance, if more than 10% enhancement persists in the Higgs-to-diphoton channel, the upper limit on the type-II seesaw scale is about 450 GeV which is completely within the reach of the 14 TeV LHC. We believe this to be an encouraging result for the experimental searches of the singly- and doubly-charged Higgs bosons which, in combination with improved sensitivity in the Higgs-to-diphoton and Higgs-to-Z+photon channels, could probe the *entire* allowed parameter space of the minimal type-II seesaw model, and establish/eliminate it as a single viable extension of the SM.

Keywords: Higgs Physics, Beyond Standard Model

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1 Introduction

A new neutral boson with mass around 125 GeV has recently been observed with more than 5σ significance by both ATLAS [1] and CMS [2] experiments at the LHC, which is also consistent with the earlier observations of an excess in the 115 - 140 GeV range made by the Tevatron [3] experiment. While this new particle mostly resembles the highly sought after Standard Model (SM) Higgs boson (h), there still exist some deviations from the SM expectations of its signal strength in some decay modes [4, 5], most notably in one of the highest mass resolution channels, namely, $h \to \gamma\gamma$ [6, 7]. Although more data and further detailed analysis are required to confirm whether these deviations are just statistical fluctuations or indeed hints of some New Physics beyond the SM, it might be worthwhile examining some of the beyond SM scenarios which could possibly lead to these deviations.

Even if the newly discovered particle turns out to be *the* SM Higgs boson, its current favored mass range $M_h = 125 \pm 1$ GeV [4, 5] will make the SM Higgs self-coupling negative in its Renormalization Group (RG) running at some energy scale below the natural Planck scale $M_P = 1.2 \times 10^{19}$ GeV, thus leading to an instability of the effective Higgs potential, if no New Physics is assumed at an intermediate scale (for a review, see e.g., [8]). Recently, a next-to-next-to-leading order (NNLO) analysis of the SM Higgs potential derived the following lower bound on the Higgs boson mass from the condition of absolute vacuum stability up to the Planck scale [9]:

$$M_h(\mathrm{GeV}) > 129.4 + 1.4\left[\frac{M_t(\mathrm{GeV}) - 173.1}{0.7}\right] - 0.5\left[\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right] \pm 1.0(\mathrm{th}). \quad (1.1)$$

According to this result, the vacuum stability of the SM up to the Planck scale is excluded at 98% CL for $M_h < 126$ GeV. Putting it another way, $M_h = 125 \pm 1$ GeV leads to the vacuum instability scale $\Lambda_I = 10^9$ - 10^{12} GeV (depending on the exact values of the top quark mass and the strong coupling constant) at which the Higgs self-coupling becomes negative, and hence, necessitates the existence of some new physics beyond the SM at or below Λ_I in order to make the electroweak vacuum absolutely stable (or sufficiently long-lived compared to the age of the Universe) all the way up to the Planck scale [1]. This provides us one of the main motivations to consider a simple and testable extension of the SM that can alleviate the vacuum stability problem at high scales while being consistent with *all* the experimental results at low scales.

On the other hand, the discovery of neutrino oscillations, and hence, non-zero neutrino masses from solar, atmospheric, reactor and accelerator neutrino experiments [16] has given us a definitive evidence of some beyond SM physics. A precise understanding of the smallness of neutrino masses, along with their observed large mixing is a potential gateway to New Physics [17]. It is certainly interesting to see if a simple extension of the SM which accounts for the non-zero neutrino masses and mixing can simultaneously solve the electroweak vacuum stability problem. Note that in general, the vacuum structure of the Higgs potential in any SM extension can be modified due to the addition of one or more of the following interaction terms to the SM Lagrangian: (i) Yukawa interactions associated with the generation of neutrino mass, (ii) additional Higgs interactions associated with the new degrees of freedom, and (iii) extended gauge interactions.

The simplest theoretical way to obtain non-zero neutrino masses is by breaking the global $(B - L)$ symmetry of the SM. This can be parametrized within the SM through an effective dimension-5 operator due to Weinberg [18]. There exist three tree-level realizations [19] of this Weinberg operator using only renormalizable interactions, and are generically known as the *seesaw mechanism*. In the simplest case, the so-called type-I seesaw [20–24], the SM particle content is supplemented by SM gauge-singlet right-handed (RH) Majorana neutrinos (ν_R), thus leading to the effective Weinberg operator of the type $y_{ij}(L_i^{\mathsf{T}}\Phi)(L_j^{\mathsf{T}}\Phi)/M$, where $L = (\nu_\ell, \ell)_L^{\mathsf{T}}$ (with $\ell = e, \mu, \tau$) is the $SU(2)_L$ lepton doublet, $\Phi = (\phi^+, \phi^0)^{\mathsf{T}}$ is the SM Higgs doublet, and M is the heavy neutrino mass scale. The presence of these massive RH neutrinos has an impact on the structure of the Higgs potential above the seesaw scale M, and hence on its vacuum stability, via loop corrections induced by the RH neutrino Yukawa couplings [12, 15, 25–29]. The requirement that the electroweak vacuum has a lifetime longer than the age of the Universe implies an upper bound on the RH neutrino mass scale of $10^{13} - 10^{14}$ GeV (depending on the physical masses of the light neutrinos) for $M_h = 125$ GeV [12, 15, 25]. However, it was found that the vacuum stability lower bound on the SM Higgs mass *increases* for large Dirac neutrino Yukawa couplings, and for the Majorana neutrino mass in the LHC-accessible TeV range, a 125 GeV vacuum stability bound requires that the Dirac neutrino Yukawa coupling responsible for the light-heavy neutrino mixing should be small: $y < 0.1$ [29] thus limiting our ability to observe the heavy neutrino signals at the LHC. For current collider limits on the heavy Majorana neutrino mass and its mixing with the light neutrinos, see [30, 31].

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[1] The issue of electroweak vacuum stability for a SM Higgs mass in the vicinity of 125 GeV is not yet fully settled since this mass range is close to the transition between absolute stability and metastability [9–15], and the uncertainties (both theoretical and experimental), mainly on the top mass, are too large at the moment. In addition, the effective quantum field-theoretic treatment and the RG evolution of the SM parameters may not be valid all the way up to the Planck scale due to some non-perturbative quantum gravity effects. A proper treatment of all these yet unresolved issues is beyond the scope of this paper.

Another realization of the seesaw mechanism, known as the type-II seesaw [24, 32–35], is by adding an $SU(2)_L$ triplet scalar field $\vec{\Delta} = (\Delta^{++}, \Delta^{+}, \Delta^{0})$ carrying hypercharge $Y = 2$ to the SM, thus leading to the effective Weinberg operator $(L^{\mathsf{T}}\vec{\sigma}L) \cdot (\Phi^{\mathsf{T}}\vec{\sigma}\Phi)/M$, where σ^i's are the usual 2×2 Pauli matrices. One of the most interesting features of the type-II seesaw is that the seesaw messenger field Δ, being a SM non-singlet, couples to the SM Higgs doublet via both cubic and quartic scalar couplings which has direct implications for the electroweak vacuum stability [36–42]. It turns out that a light SM Higgs boson in the (124 - 126) GeV range can easily be realized in type-II seesaw models. In fact, as shown in [36], the vacuum stability lower bound on the SM Higgs boson mass is well below 125 GeV (even lower than the LEP2 Higgs mass bound of 114.4 GeV) for a TeV-scale type-II seesaw. Another important phenomenological consequence of the type-II seesaw mechanism is that the $SU(2)_L$ triplet scalar fields can couple *directly* to the SM gauge bosons ($W^{\pm}$, Z, γ), and hence, are relatively easier to be detected at the LHC, if kinematically accessible [43, 44] as compared to the SM singlet heavy neutrinos in the type-I seesaw case which require a large mixing with the light neutrinos to have any observable effects at colliders. The current LHC bounds on the scalar triplet masses are given in [45, 46].

The third realization of the seesaw mechanism, known as the type-III seesaw [47], is by adding an $SU(2)_L$ fermion triplet $\vec{\Sigma}$, leading to the effective Weinberg operator $(L^{\mathsf{T}}\vec{\sigma}\Phi)^2/M$. The effect of these fermion triplets on the electroweak vacuum stability has been analyzed in [26, 48], and it was shown that similar to the type-II case, the vacuum stability bound becomes *lower* than the SM bound of 129 GeV for decreasing seesaw scale. These results have generated a lot of interest in type-II and III seesaw models [2] in the light of the recent LHC discovery of the 125 GeV Higgs-like boson. For a review of various seesaw models and their testability at colliders and other experiments, see e.g., [50].

In this paper, we will focus on the minimal type-II seesaw model with a single $SU(2)_L$ scalar triplet added to the SM and analyze the parameter space allowed by the stability of the electroweak vacuum and the perturbativity of the scalar couplings for a 125 GeV SM-like Higgs mass. In this model, there are seven scalar mass eigenstates: two doubly-charged ($H^{\pm\pm}$), two singly-charged ($H^{\pm}$), two CP-even (h, H^0) and a CP-odd (A^0) neutral scalar particles. However in a large part of the parameter space of this model, the mixing between the doublet and triplet scalar fields is usually small (unless the CP-even neutral scalar eigenstates are mass-degenerate), and hence, the lightest CP-even scalar field has essentially the same couplings to the SM fermions and massive vector bosons as the SM Higgs boson has [37, 51–54]. Therefore, we would expect its production rate as well as the branching ratios for its decay channels to be very similar to those of the SM Higgs boson, except for the loop-induced $h \to \gamma\gamma$ [44, 55, 56] and also $h \to Z\gamma$ [42, 56, 57] channels which receive additional contributions from $H^{\pm\pm}$ and $H^{\pm}$ running in the loop. This has motivated many recent studies within the context of the Higgs triplet model [39, 41, 42, 44, 53, 58–60] to explain the persistent 'excess' in the $\gamma\gamma$ signal strength of the Higgs-like particle discovered at the LHC.

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[2] The vacuum stability constraints for another variation of the seesaw, namely, linear and inverse seesaw models, have been recently studied in [49].

Our approach in this paper is mainly motivated by the vacuum stability problem for a 125 GeV Higgs boson in the SM. As argued earlier, we require to have some new physics below the SM vacuum instability scale $\Lambda_I = 10^9$ - 10^{12} GeV to solve this problem. We assume this new physics scale to be the type-II seesaw scale which is well-motivated for explaining the non-zero neutrino masses and mixing, and systematically analyze the modified RG running of the SM Higgs quartic coupling above this scale to obtain the allowed parameter space satisfying both vacuum stability and perturbativity conditions for all the couplings up to the Planck scale. We emphasize that there exists a large allowed parameter space for the type-II seesaw model which yields a 125 GeV SM-like Higgs boson pole mass, irrespective of the seesaw scale. We demonstrate this point by considering some typical values of the seesaw scale, both low (200 - 500 GeV) and high (10^9 - 10^{10} GeV). We have included the effects of the neutrino Yukawa couplings on the RG evolution of the scalar couplings, which cannot be neglected in certain cases, unlike what was previously assumed in the literature.

As a consequence of the low-scale seesaw case, we also study the predictions for the $h \to \gamma\gamma$ and $h \to Z\gamma$ branching ratios in the allowed parameter space of the type-II seesaw model with respect to the purely SM expectations. We find a correlation between the $h \to \gamma\gamma$ and $h \to Z\gamma$ decay rates which, in combination with its collider signals, could be used to test the low-scale type-II seesaw model at the LHC, when the signal strength sensitivities in the $\gamma\gamma$ and $Z\gamma$ channels improve in future. We buttress the motivation for collider searches of the low-scale type-II seesaw by obtaining an *upper* bound on the seesaw scale, and hence, on the associated doubly- and singly-charged Higgs boson masses, for a given enhancement in the $h \to \gamma\gamma$ decay rate over its SM expectation, under the assumption that no other new physics effects contribute to this enhancement. For example, with an enhancement of 10% or more, the corresponding upper limit on the type-II seesaw scale is about 450 GeV which is completely within the reach of the 14 TeV LHC. Thus, the LHC might be able to offer a definitive answer to the fate of the type-II seesaw model as a single viable extension of the SM up to the Planck scale.

The plan of this paper is as follows: In Section 2, we review the type-II seesaw model and its scalar sector. In Section 3, we discuss the RG evolution of the Higgs quartic coupling and other scalar couplings in the model, including the effect of the neutrino Yukawa couplings. In Section 4, we present our results for the allowed parameter space at a given seesaw scale predicting a SM-like Higgs mass of 125 GeV while satisfying the vacuum stability as well as the unitarity conditions up to the Planck scale. In Section 5, we study the predictions of the decay rates for the processes $h \to \gamma\gamma$ and $h \to Z\gamma$ with respect to the SM expectations in the allowed parameter space of the type-II seesaw model, and show the correlation between the two decay rates. We also derive upper bounds on the seesaw scale for a given enhancement in the $h \to \gamma\gamma$ signal strength. Our conclusions are given in Section 6. The matching conditions for the $\overline{\text{MS}}$ and pole masses of the top-quark and the Higgs boson are collected in Appendix A.

2 Review of the Type-II Seesaw Model

In this section, we briefly review the minimal type-II seesaw model (for a detailed discussion, see e.g., [61]) where, in addition to the SM fields, a triplet scalar field Δ is introduced, which transforms as $(\mathbf{3}, 2)$ under the $SU(2)_L \times U(1)_Y$ gauge group:

$$\Delta = \frac{\sigma^i}{\sqrt{2}}\Delta_i = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}, \tag{2.1}$$

with $\Delta_1 = (\delta^{++} + \delta^0)/\sqrt{2},\ \Delta_2 = i(\delta^{++} - \delta^0)/\sqrt{2},\ \Delta_3 = \delta^+$. The Lagrangian for this model is given by

$$\mathcal{L} = \mathcal{L}_Y + \mathcal{L}_{\text{kinetic}} - \mathcal{V}(\Phi, \Delta), \tag{2.2}$$

where the relevant kinetic and Yukawa interaction terms are respectively

$$\mathcal{L}_{\text{kinetic}} = \mathcal{L}^{\text{SM}}_{\text{kinetic}} + \text{Tr}\left[(D_\mu\Delta)^\dagger (D^\mu\Delta)\right], \tag{2.3}$$

$$\mathcal{L}_Y = \mathcal{L}^{\text{SM}}_Y - \frac{1}{\sqrt{2}}(Y_\Delta)_{ij} L_i^\intercal C i\sigma_2 \Delta L_j + \text{H.c.}\,. \tag{2.4}$$

Here C is the Dirac charge conjugation matrix with respect to the Lorentz group, and

$$D_\mu\Delta = \partial_\mu\Delta + i\frac{g}{2}[\sigma^a W^a_\mu, \Delta] + i\frac{g'}{2}B_\mu\Delta \qquad (a = 1, 2, 3) \tag{2.5}$$

is the covariant derivative of the scalar triplet field, with the GUT-normalization for the electroweak couplings $g = g_2$ and $g' = \sqrt{3/5}g_1$.

Following the notation of [63], we write the scalar potential in Eq. (2.2) as [3]

$$\begin{aligned}\mathcal{V}(\Phi, \Delta) = &-m_\Phi^2(\Phi^\dagger\Phi) + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + M_\Delta^2\text{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\text{Tr}(\Delta^\dagger\Delta)\right]^2 \\ &+ \frac{\lambda_2}{2}\left(\left[\text{Tr}(\Delta^\dagger\Delta)\right]^2 - \text{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right) + \lambda_4(\Phi^\dagger\Phi)\text{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger[\Delta^\dagger, \Delta]\Phi \\ &+ \left(\frac{\Lambda_6}{\sqrt{2}}\Phi^\intercal i\sigma_2\Delta^\dagger\Phi + \text{H.c.}\right).\end{aligned} \tag{2.6}$$

The coupling constants λ_i can be chosen to be real through a phase redefinition of the field Δ. Also, we have chosen $m_\Phi^2 > 0$ in order to ensure the spontaneous symmetry breaking of the $SU(2)_L \times U(1)_Y$ gauge group to $U(1)_Q$ by a nonzero vacuum expectation value (vev) for the neutral component of the SM Higgs doublet, $\langle \phi^0 \rangle = v/\sqrt{2}$ with $v \simeq 246.2$ GeV while M_Δ^2 can be of either sign. Note that the last term in Eq. (2.6) is the *only* source of lepton number violation at the Lagrangian level before the spontaneous symmetry breaking.

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[3] The general form of the potential given in [55] can be recovered with a simple redefinition of the couplings: $\lambda \to \lambda/2,\ (\lambda_1 + \lambda_2) \to 2\lambda_2,\ \lambda_2 \to -2\lambda_3,\ (\lambda_4 + \lambda_5) \to \lambda_1,\ \lambda_5 \to -\lambda_4/2$, and using the identity $(\Phi^\dagger\Phi)\text{Tr}(\Delta^\dagger\Delta) = \Phi^\dagger\{\Delta^\dagger, \Delta\}\Phi$ which is valid for any traceless 2×2 matrix Δ.

2.1 Neutrino Masses and Mixing

A non-zero vev for the Higgs doublet field Φ induces a tadpole term for the scalar triplet field Δ via the Λ_6 term in Eq. (2.6), thereby generating a nonzero vev for its neutral component, $\langle \delta^0 \rangle = v_\Delta/\sqrt{2}$, and breaking lepton number by two units. This results in the following Majorana mass matrix for the neutrinos:

$$(M_\nu)_{ij} = v_\Delta (Y_\Delta)_{ij} \tag{2.7}$$

The triplet vev contributes to the weakly interacting gauge boson masses at tree-level:

$$M_W^2 = \frac{g^2}{2}(v^2 + 2v_\Delta^2), \quad M_Z^2 = \frac{g^2}{2\cos^2\theta_W}(v^2 + 4v_\Delta^2), \tag{2.8}$$

thereby affecting the SM ρ-parameter:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1 + \frac{2v_\Delta^2}{v^2}}{1 + \frac{4v_\Delta^2}{v^2}}. \tag{2.9}$$

The electroweak precision data constraints require the ρ-parameter to be very close to its SM value of unity: $\rho = 1.0004^{+0.0003}_{-0.0004}$ [64]. This requires

$$\frac{v_\Delta}{v} < 0.02\,, \quad \text{or} \quad v_\Delta \lesssim 5 \text{ GeV}. \tag{2.10}$$

Hence, we will be working in the limit $v_\Delta \ll v$.

The electroweak symmetry breaking (EWSB) conditions are obtained after minimizing the scalar potential given by Eq. (2.6):

$$m_\Phi^2 = \frac{1}{2}\lambda v^2 - \Lambda_6 v_\Delta + \frac{\lambda_4 - \lambda_5}{2} v_\Delta^2, \tag{2.11}$$

$$M_\Delta^2 = \frac{1}{2}\frac{\Lambda_6 v^2}{v_\Delta} - \frac{1}{2}(\lambda_4 - \lambda_5)v^2 - \frac{1}{2}\lambda_1 v_\Delta^2. \tag{2.12}$$

In the limit $v_\Delta \ll v$, we obtain from Eq. (2.12)

$$v_\Delta = \frac{\Lambda_6 v^2}{2M_\Delta^2 + v^2(\lambda_4 - \lambda_5)}\,. \tag{2.13}$$

Note that for $M_\Delta \gg v$, Eq. (2.7) for the neutrino masses becomes

$$M_\nu \simeq \frac{\lambda_6 v^2}{2M_\Delta} Y_\Delta \tag{2.14}$$

with the dimensionless parameter $\lambda_6 \equiv \Lambda_6/M_\Delta$. Eq. (2.14) resembles a typical seesaw formula with $M_\nu \propto M_\Delta^{-1}$.

In order to satisfy the low-energy neutrino oscillation data [64], we fix the structure of the Yukawa coupling matrix Y_Δ as follows: From Eq. (2.7), we obtain

$$Y_\Delta = \frac{M_\nu}{v_\Delta} = \frac{1}{v_\Delta} U^\mathsf{T} M_\nu^{\rm diag} U \tag{2.15}$$

where $M_\nu^{\rm diag} = {\rm diag}(m_1, m_2, m_3)$ is the diagonal neutrino mass eigenvalue matrix. U is the standard Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix, usually parametrized in terms of the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, and one Dirac ($\delta$) and two Majorana ($\alpha_1, \alpha_2$) CP phases:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times {\rm diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) ,$$

where $c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}$. For illustration purposes, we assume the Majorana phases in the PMNS matrix to be zero, and also choose a normal hierarchy for the neutrino masses with $m_1 = 0$ so that $m_2 = \sqrt{\Delta m^2_{\rm sol}}$ and $m_3 = \sqrt{\Delta m^2_{\rm atm}}$. Using the central values of a recent global analysis of the 3-neutrino oscillation data [65]:

$$\begin{aligned} &\Delta m^2_{\rm sol} = 7.62 \times 10^{-5}\ {\rm eV}^2,\ \Delta m^2_{\rm atm} = 2.55 \times 10^{-3}\ {\rm eV}^2, \\ &\theta_{12} = 34.4^\circ,\ \theta_{23} = 40.8^\circ, \theta_{13} = 9.0^\circ, \delta = 0.8\pi\ , \end{aligned} \tag{2.16}$$

we obtain the following structure of the Yukawa coupling matrix:

$$Y_\Delta = \frac{10^{-2}\ {\rm eV}}{v_\Delta} \times \begin{pmatrix} 0.31 - 0.12i & -0.09 + 0.32i & -0.72 + 0.37i \\ -0.09 + 0.32i & 2.53 + 0.04i & 2.19 + 0.01i \\ -0.72 + 0.37i & 2.19 + 0.01i & 3.07 - 0.03i \end{pmatrix} . \tag{2.17}$$

From Eq. (2.17), we see that there are two extreme cases:
(i) Small Yukawa couplings which correspond to large $v_\Delta \lesssim \mathcal{O}({\rm GeV})$, where the upper bound comes from the ρ-parameter constraint given by Eq. (2.10).
(ii) Large Yukawa couplings $(Y_\Delta)_{ij} \sim \mathcal{O}(1)$ which corresponds to small $v_\Delta \gtrsim \mathcal{O}(10^{-2}\ {\rm eV})$. Note that v_Δ cannot be arbitrarily small because of the naturalness consideration for the Yukawa couplings in Eq. (2.17).

2.2 Scalar Masses and Mixing

Expanding the scalar fields ϕ^0 and δ^0 around their vevs, we obtain 10 real-valued field components:

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \phi + i\chi) \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \frac{1}{\sqrt{2}}(v_\Delta + \delta + i\eta) & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix} \tag{2.18}$$

which, upon minimization of the scalar potential $\mathcal{V}(\Phi, \Delta)$ in Eq. (2.6) with respect to the vevs, yields a 10×10 squared mass matrix for the scalars. There are seven physical massive eigenstates $H^{\pm\pm}, H^\pm, h, H^0, A^0$ and three massless Goldstone bosons $G^\pm, G^0$ which are eaten up to give mass to the SM gauge bosons $W^{\pm}, Z$. The physical mass eigenvalues for the scalar sector are given by

$$m_{H^{\pm\pm}}^2 = M_\Delta^2 + \frac{1}{2}(\lambda_4+\lambda_5)v^2 + \frac{1}{2}(\lambda_1+\lambda_2)v_\Delta^2, \tag{2.19}$$

$$m_{H^{\pm}}^2 = \left(M_\Delta^2 + \frac{1}{2}\lambda_4 v^2 + \frac{1}{2}\lambda_1 v_\Delta^2\right)\left(1+\frac{2v_\Delta^2}{v^2}\right), \tag{2.20}$$

$$m_{A^0}^2 = \left(M_\Delta^2 + \frac{1}{2}(\lambda_4-\lambda_5) v^2 + \frac{1}{2}\lambda_1 v_\Delta^2\right)\left(1+\frac{4v_\Delta^2}{v^2}\right), \tag{2.21}$$

$$m_h^2 = \frac{1}{2}\left(A+C-\sqrt{(A-C)^2+4B^2}\right), \tag{2.22}$$

$$m_{H^0}^2 = \frac{1}{2}\left(A+C+\sqrt{(A-C)^2+4B^2}\right), \tag{2.23}$$

$$\text{with}\ \ A=\lambda v^2, \quad B=-\frac{2v_\Delta}{v}\left(M_\Delta^2+\frac{1}{2}\lambda_1 v_\Delta^2\right), \quad C=M_\Delta^2+\frac{1}{2}(\lambda_4-\lambda_5)v^2+\frac{3}{2}\lambda_1 v_\Delta^2.$$

Note that among the two CP-even neutral Higgs bosons, $m_{H^0} > m_h$ is always satisfied.

The mixing between the doublet and triplet scalar fields in the charged, CP-even and CP-odd scalar sectors are respectively given by

$$\begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta' & \sin\beta' \\ -\sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} \phi^{\pm} \\ \delta^{\pm} \end{pmatrix}, \tag{2.24}$$

$$\begin{pmatrix} h \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi \\ \delta \end{pmatrix}, \tag{2.25}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi \\ \eta \end{pmatrix}, \tag{2.26}$$

where the mixing angles are given by

$$\tan\beta' = \frac{\sqrt{2}v_\Delta}{v}, \tag{2.27}$$

$$\tan\beta = \frac{2v_\Delta}{v} \equiv \sqrt{2}\tan\beta', \tag{2.28}$$

$$\tan 2\alpha = \frac{2B}{A-C} = \frac{4v_\Delta}{v}\frac{M_\Delta^2+\frac{1}{2}\lambda_1 v_\Delta^2}{M_\Delta^2+\frac{1}{2}(\lambda_4-\lambda_5-2\lambda)v^2+\frac{3}{2}\lambda_1 v_\Delta^2}. \tag{2.29}$$

Thus in the limit $v_\Delta \ll v$, the mixing between the doublet and triplet scalars is usually small (unless the CP-even scalars h and H^0 are close to being mass-degenerate). In this limit, the mass of the (dominantly doublet) lightest CP-even scalar is simply given by $m_h^2 = \lambda v^2$ (as in the SM) independent of the mass scale M_Δ, whereas the other (dominantly triplet) scalars have M_Δ-dependent mass. The mass scale M_Δ will be simply referred to as the “seesaw scale" for the rest of our paper.

2.3 Stability and Unitarity Conditions

Here we summarize the constraints on the scalar potential of the type-II seesaw model given by Eq. (2.6) in order to ensure the stability of the electroweak vacuum and the preservation of the tree-level unitarity in various scattering processes.

The necessary and sufficient conditions valid for all directions in field space to ensure that the scalar potential in Eq. (2.6) is bounded from below are given by [55]

$$\begin{aligned}
&\lambda \geq 0, \quad \lambda_1 \geq 0, \quad 2\lambda_1 + \lambda_2 \geq 0, \\
&\lambda_4 + \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \quad \lambda_4 + \lambda_5 + \sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} \geq 0, \\
&\lambda_4 - \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \quad \lambda_4 - \lambda_5 + \sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} \geq 0.
\end{aligned} \tag{2.30}$$

In addition, the tree-level unitarity of the S-matrix for elastic scattering imposes the following constraints [55]:

$$\begin{aligned}
&\lambda \leq \frac{8}{3}\pi, \quad \lambda_1 - \lambda_2 \leq 8\pi, \quad 4\lambda_1 + \lambda_2 \leq 8\pi, \quad 2\lambda_1 + 3\lambda_2 \leq 16\pi, \\
&|\lambda_5| \leq \frac{1}{2}\mathrm{min}\left[\sqrt{(\lambda \pm 8\pi)(\lambda_1 - \lambda_2 \pm 8\pi)}\right], \\
&|\lambda_4| \leq \frac{1}{\sqrt{2}}\sqrt{\left(\lambda - \frac{8}{3}\pi\right)(4\lambda_1 + \lambda_2 - 8\pi)}.
\end{aligned} \tag{2.31}$$

3 The Renormalization Group Equations

In this section, we present the RG equations (RGEs) for the scalar, gauge and Yukawa couplings relevant for our analysis. For the SM fermions, we will only keep the dominant top-quark Yukawa coupling terms. Depending on whether the renormalization scale μ is below or above the seesaw scale M_Δ, the RG running will be different, as follows:

3.1 For $\mu < M_\Delta$

Below the seesaw scale M_Δ, the heavy Higgs triplets can be integrated out to obtain a low-energy effective scalar potential for the SM Higgs doublet:

$$V_{\rm eff}(\Phi) = -m_\Phi^2(\Phi^\dagger\Phi) + \frac{1}{2}(\lambda - \lambda_6^2)(\Phi^\dagger\Phi)^2, \tag{3.1}$$

and hence, the effective SM Higgs quartic coupling is shifted down:

$$\lambda \to \lambda_{\rm SM} = \lambda - \lambda_6^2\,, \tag{3.2}$$

$$\text{where} \quad \lambda_6 = \frac{\Lambda_6}{M_\Delta} = \frac{2v_\Delta M_\Delta}{v^2}\left(1 + \frac{v^2}{2M_\Delta}(\lambda_4 - \lambda_5)\right) \tag{3.3}$$

from Eq. (2.13). Thus, for a low-scale seesaw with M_Δ comparable to v, we always have $\lambda_6^2 \ll \lambda$ below M_Δ, and hence, its effect on the SM Higgs quartic coupling can be ignored. However, for a high-scale seesaw with $M_\Delta \gg v$, the effect of λ_6 could be non-negligible.

The two-loop RG equation for the Higgs quartic coupling is given by

$$\frac{d\lambda}{d\ln\mu} = \frac{\beta_\lambda^{(1)}}{16\pi^2} + \frac{\beta_\lambda^{(2)}}{(16\pi^2)^2}, \tag{3.4}$$

with the β-functions [66–69]

$$\beta_\lambda^{(1)} = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right) + 12y_t^2\lambda - 12y_t^4, \tag{3.5}$$

$$\begin{aligned}\beta_\lambda^{(2)} = &-78\lambda^3 + 18\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda^2 - \left(\frac{73}{8}g_2^4 - \frac{117}{20}g_1^2 g_2^2 - \frac{1887}{200}g_1^4\right)\lambda - 3\lambda y_t^4 \\ &+\frac{305}{8}g_2^6 - \frac{289}{40}g_1^2 g_2^4 - \frac{1677}{200}g_1^4 g_2^2 - \frac{3411}{1000}g_1^6 - 64g_3^2 y_t^4 - \frac{16}{5}g_1^2 y_t^4 - \frac{9}{2}g_2^4 y_t^2 \\ &+10\lambda\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)y_t^2 - \frac{3}{5}g_1^2\left(\frac{57}{10}g_1^2 - 21g_2^2\right)y_t^2 - 72\lambda^2 y_t^2 + 60y_t^6.\end{aligned} \tag{3.6}$$

The boundary condition for $\lambda(\mu)$ at a given renormalization scale μ can be determined from the one-loop matching condition [70] for the SM Higgs boson pole mass M_h and its running mass $m_h(\mu) = \sqrt{\lambda(\mu)}v$: [4]

$$\lambda(\mu) = \frac{M_h^2}{v^2}\left[1 + \Delta_h(\mu)\right], \tag{3.7}$$

with the expression for $\Delta_h(\mu)$ explicitly given in Appendix A.

For the top-quark Yukawa coupling, we have the two-loop RG equation

$$\frac{dy_t}{d\ln\mu} = \left(\frac{\beta_t^{(1)}}{16\pi^2} + \frac{\beta_t^{(2)}}{(16\pi^2)^2}\right)y_t, \tag{3.8}$$

where [66]

$$\beta_t^{(1)} = \frac{9}{2}y_t^2 - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{3.9}$$

$$\begin{aligned}\beta_t^{(2)} = &-12y_t^4 + \left(\frac{393}{80}g_1^2 + \frac{225}{16}g_2^2 + 36g_3^2\right)y_t^2 + \frac{1187}{600}g_1^4 - \frac{9}{20}g_1^2 g_2^2 + \frac{19}{15}g_1^2 g_3^2 \\ &-\frac{23}{4}g_2^4 + 9g_2^2 g_3^2 - 108g_3^4 + \frac{3}{2}\lambda^2 - 6\lambda y_t^2.\end{aligned} \tag{3.10}$$

The boundary condition for $y_t(\mu)$ can be determined from the matching condition between the running top quark mass $m_t(\mu) = y_t(\mu)v/\sqrt{2}$ and its pole mass M_t, analogous to Eq. (3.7):

$$y_t(\mu) = \frac{\sqrt{2}M_t}{v}\left[1 + \Delta_t(\mu)\right], \tag{3.11}$$

where $\Delta_t(\mu)$ gets contributions from QCD [71–74] as well as electroweak corrections [75, 76]. The QCD corrections up to $\mathcal{O}(\alpha_3^2)$ and the electroweak corrections up to order $\mathcal{O}(\alpha)$ are explicitly given in Appendix A.

The two-loop RG equations for the SM gauge couplings are given by [66, 68]

$$\frac{dg_i}{d\ln\mu} = -\frac{g_i^3}{16\pi^2}b_i - \frac{g_i^3}{(16\pi^2)^2}\sum_{j=1}^{3}b_{ij}g_j^2 - \frac{g_i^3 y_t^2}{(16\pi^2)^2}a_i, \tag{3.12}$$

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[4] Henceforth, we will denote the pole masses by upper case, and the running masses by lower case, whenever applicable.