arXiv:cond-mat/0002038 2 Feb 2000

SU-4240-714

The Statistical Mechanics of Membranes

Mark J. Bowick* and Alex Travesset†
[1]Physics Department, Syracuse University,
Syracuse, NY 13244-1130, USA

Abstract

The fluctuations of two-dimensional extended objects (*membranes*) is a rich and exciting field with many solid results and a wide range of open issues. We review the distinct universality classes of membranes, determined by the local order, and the associated phase diagrams. After a discussion of several physical examples of membranes we turn to the physics of *crystalline* (or *polymerized*) membranes in which the individual monomers are rigidly bound. We discuss the phase diagram with particular attention to the dependence on the degree of *self-avoidance* and *anisotropy*. In each case we review and discuss analytic, numerical and experimental predictions of critical exponents and other key observables. Particular emphasis is given to the results obtained from the renormalization group ε-expansion. The resulting renormalization group flows and fixed points are illustrated graphically. The full technical details necessary to perform actual calculations are presented in the Appendices. We then turn to a discussion of the role of topological defects whose liberation leads to the *hexatic* and *fluid* universality classes. We finish with conclusions and a discussion of promising open directions for the future.

*bowick@physics.syr.edu
†alex@suhep.phy.syr.edu

Contents

1 Introduction **4**

2 Physical examples of membranes **5**

3 The Renormalization Group **11**

4 Crystalline Membranes **14**
- 4.1 Phantom 16
  - 4.1.1 The crumpled phase 19
  - 4.1.2 The Crumpling Transition 19
  - 4.1.3 The Flat Phase 21
- 4.2 Self-avoiding 26
  - 4.2.1 The nature of the SAFP 28
  - 4.2.2 Attractive potentials 32
  - 4.2.3 The properties of the SAFP 33

5 Anisotropic Membranes **34**
- 5.1 Phantom 36
  - 5.1.1 The Phase diagram 36
  - 5.1.2 The Crumpled Anisotropic Phase 38
  - 5.1.3 The Flat Phase 38
- 5.2 The Tubular Phase 38

6 Defects in membranes: The Crystalline-Fluid transition and Fluid membranes **41**
- 6.1 Topological Defects 43
  - 6.1.1 Topological Defects in fluctuating geometries 44
  - 6.1.2 Melting and the hexatic phase 45
- 6.2 The Hexatic membrane 46

7 The Fluid Phase **49**

8 Conclusions **50**

A Useful integrals in dimensional regularization **53**

B Some practical identities for RG quantities **53**

C Discretized Model for tethered membranes 54

D The crumpling Transition 56

E The Flat Phase 58

F The Self-avoiding phase 60

G The mean field solution of the anisotropic case 64

1 Introduction

The statistical mechanics of one-dimensional structures (polymers) is fascinating and has proved to be fruitful from the fundamental and applied points of view [1, 2]. The key reasons for this success lie in the notion of *universality* and the relative simplicity of one-dimensional geometry. Many features of the long-wavelength behavior of polymers are independent of the detailed physical and chemical nature of the monomers that constitute the polymer building blocks and their bonding into macromolecules. These microscopic details simply wash out in the thermodynamic limit of large systems and allow predictions of critical exponents that should apply to a wide class of microscopically distinct polymeric systems. Polymers are also sufficiently simple that considerable analytic and numerical progress has been possible. Their statistical mechanics is essentially that of ensembles of various classes of *random walks* in some d-dimensional bulk or embedding space.

A natural extension of these systems is to intrinsic two-dimensional structures which we may call generically call *membranes*. The statistical mechanics of these *random surfaces* is far more complex than that of polymers because two-dimensional geometry is far richer than the very restricted geometry of lines. Even planar two-dimensional — monolayers — are complex, as evidenced by the KTNHY [3, 4, 5] theory of defect-mediated melting of monolayers with two distinct *continuous* phase transitions separating an intermediate hexatic phase, characterized by quasi-long-range bond orientational order, from both a low-temperature *crystalline* phase and a high-temperature *fluid* phase. But full-fledged membranes are subject also to shape fluctuations and their macroscopic behavior is determined by a subtle interplay between their particular microscopic order and the entropy of shape and elastic deformations. For membranes, unlike polymers, distinct types of microscopic order (crystalline, hexatic, fluid) will lead to distinct long-wavelength behavior and consequently a rich set of universality classes.

Flexible membranes are an important member of the enormous class of *soft* condensed matter systems [6, 7, 8, 9], those which respond easily to external forces. Their physical properties are to a considerable extent dominated by the entropy of thermal fluctuations.

In this review we will describe some of the presently understood behavior of crystalline (fixed-connectivity), hexatic and fluid membranes, including the relevance of self-avoidance, intrinsic anisotropy and topological defects. Emphasis will be given to the role of the renormalization group in elucidating

the critical behavior of membranes. The polymer pastures may be lovely but a dazzling world awaits those who wander into the membrane meadows.

The outline of the review is the following. In sec. 2 we describe a variety of important physical examples of membranes, with representatives from the key universality classes. In sec. 3 we introduce basic notions from the renormalization group and some formalism that we will use in the rest of the review. In sec. 4 we review the phase structure of crystalline membranes for both phantom and self-avoiding membranes, including a thorough discussion of the fixed-point structure, RG flows and critical exponents of each global phase. In sec. 5 we turn to the same issues for intrinsically anisotropic membranes, with the new feature of the *tubular* phase. In sec. 6 we address the consequences of allowing for membrane defects, leading to a discussion of the hexatic membrane universality class. We end with a brief discussion of fluid membranes in sec. 7 and conclusions.

2 Physical examples of membranes

There are many concrete realizations of membranes in nature, which greatly enhances the significance of their study. Crystalline membranes, sometimes termed *tethered* or *polymerized* membranes, are the natural generalization of linear polymer chains to intrinsically two-dimensional structures. They possess in-plane elastic moduli as well as bending rigidity and are characterized by broken translational invariance in the plane and fixed connectivity resulting from relatively strong bonding. Geometrically speaking they have a preferred two-dimensional metric. Let's look at some of the examples. One can polymerize suitable chiral oligomeric precursors to form molecular sheets [10]. This approach is based directly on the idea of creating an intrinsically two-dimensional polymer. Alternatively one can permanently cross-link fluid-like Langmuir-Blodgett films or amphiphilic bilayers by adding certain functional groups to the hydrocarbon tails and/or the polar heads [11, 12] as shown schematically in Fig. 1.

The cytoskeletons of cell membranes are beautiful and naturally occurring crystalline membranes that are essential to cell membrane stability and functionality. The simplest and most thoroughly studied example is the cytoskeleton of mammalian erythrocytes (red blood cells). The human body has roughly 5×10^{13} red blood cells. The red blood cell cytoskeleton is a fishnet-like network of triangular plaquettes formed primarily by the proteins

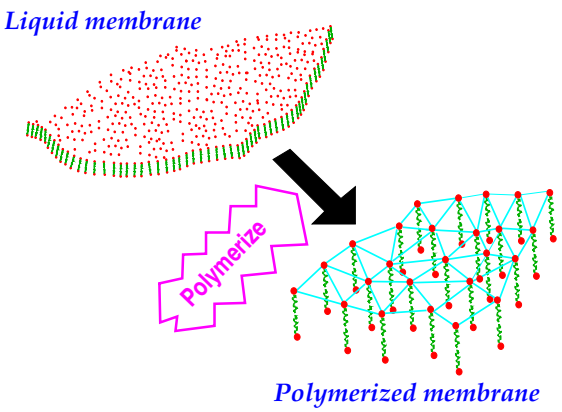
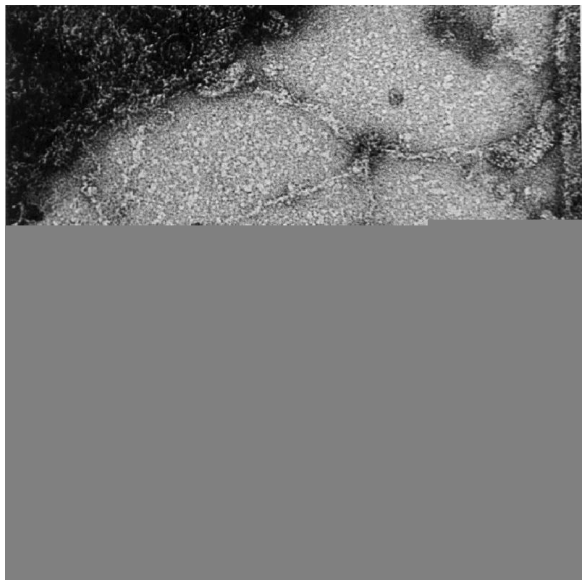


Figure 1: The polymerization of fluid-like membrane to a crystalline membrane.

spectrin and *actin*. The links of the mesh are spectrin tetramers (of length approximately 200 nm) and the nodes are short actin filaments (of length 37 nm and typically 13 actin monomers long) [13, 14], as seen in Fig.2 and Fig.3. There are roughly 70,000 triangular plaquettes in the mesh altogether and the cytoskeleton as a whole is bound by ankyrin and other proteins to the cytoplasmic side of the fluid phospholipid bilayer which constitutes the other key component of the red blood cell membrane.

There are also inorganic realizations of crystalline membranes. Graphitic oxide (GO) membranes are micron size sheets of solid carbon with thicknesses on the order of 10Å, formed by exfoliating carbon with a strong oxidizing agent. Their structure in an aqueous suspension has been examined by several groups [17, 18, 19]. Metal dichalcogenides such as MoS_2 have also been observed to form rag-like sheets [20]. Finally similar structures occur in the large sheet molecules, shown in Fig.4, believed to be an ingredient in glassy B_2O_3.

In contrast to crystalline membranes, fluid membranes are characterized by vanishing shear modulus and dynamical connectivity. They exhibit significant shape fluctuations controlled by an effective bending rigidity parameter.

A rich source of physical realizations of fluid membranes is found in *amphiphilic* systems [21, 22, 23]. Amphiphiles are molecules with a two-fold character – one part is hydrophobic and another part hydrophilic. The classic examples are lipid molecules, such as phospholipids, which have polar or ionic head groups (the hydrophilic component) and hydrocarbon tails (the