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Some Aspects regarding Renormalisation Theory of Critical Phenomena

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Abstract: Concerning renormalisation group theory applied to phase transitions, we examine the value of positive numerical and analytical evidence, the divergent short-wavelength behaviour of classical free fields and the absence of UV-divergences in quantum field theories with proper mathematical handling. We conclude as to the inadequacy of the conceptual and mathematical framework of renormalisation theory of critical phenomena.

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This paper treats the viability of the contemporary renormalisation theory of critical phenomena (RTCP). We begin with a brief review of the theory followed by an analysis of its various aspects.

The following draws essentially from [1]. RTCP is supposed to give a unified picture of universal features of phase transitions. These features concern essentially the critical exponents associated with a certain number of physical quantities such as susceptibilities, specific heat, correlation length and so on. Very close to transition temperature T_c some physical quantities diverge according to power laws $(T-T_c)^{-g}$; g is called the critical exponent of the quantity investigated. One observes that the values of the critical exponents are very similar (if not identical within experimental errors which are often quite large, however) among phase transitions of different materials, temperature ranges etc., hence the term "universal feature". The number of independent critical coefficients per phase transition situation is two (thermodynamic considerations relate all among them). The predictive power of the model is thus very limited.

RTCP was inspired by two theoretical strands, Ginzburg-Landau phenomenology (GL) and field-theoretic renormalisation theory (FTRT).

GL views a phase transition as a discontinuous change of some "order parameter" at a certain temperature T_c . GL expand the free energy in the order parameter amplitude and amplitude fluctuations according to (for real scalar order parameter)

(1) $$Z_g = tr \exp\left(-\beta\left[\int d^n x \left[\pi^2 + (\nabla\phi)^2 + m\phi^2\right]\right]\right)$$

and look for a minimum as a function of F . This gives the observable order parameter value; the most

popular example being spontaneous magnetization in magnetic transitions. Above the transition temperature, one assumes that (a,b,c)(T)>0, so F is always zero. Below T_c , b(T) can be < 0, so F ¹ 0 becomes allowed. The reason for expanding to quartic order only is the treatment in proximity to T_c where F is already very small.

RTCP uses the GL-partition function (GLPF) instead, given by

(2) $$Z = \int D\Phi(\vec{r}) \exp(-1/kT\left[a(T)\int d^n x(\nabla\Phi)^2 + b(T)\int d^n x\Phi^2 + c(T)\int d^n x\Phi^4\right)$$

DF (r) is the functional integral over all order-parameter configurations F (r).

The motivation is that, as correlation lengths increase (diverge) towards the transition temperature, the microscopic properties of a system are averaged over and the macroscopic concept of an order parameter applies. The state of the system is then completely determined by its order parameter configuration and the partition function is the Boltzmann sum over these configurations. The smallness of the order parameter (proximity to T_c) justifies expansion to forth order only. All thermodynamic properties can now be derived from Z. We can adapt the GLPF to different situations of phase transitions by choosing an order-parameter field with several components, real or complex. The model is thus very flexible.

Can RTCP be considered a physical theory at this stage? In fact, no. As we have seen, the only predictions possible are two numbers (critical exponents). As long as we retain arbitrary functions (a,b,c)(T), we will always arrive at adjusting Z such that we match the experimental results, but this is purely arbitrary and not physical. In fact RTCP claims that the critical exponents are independent of the original "bare" form of (a,b,c) through the renormalisation procedure. We can set them independent of temperature and RTCP now qualifies as a physical theory since all arbitrariness is eliminated. Unfortunately, Z, as claimed in RTCP, is not a well-defined mathematical quantity but exhibits UV- and IR-divergences. IR-divergences are supposed to arise from considering an infinite system volume, UV-divergences arise from extrapolating to the microscopic limit. All quantities in (equ. 2) are called "bare" quantities, not measurable. We are supposed to retrieve the physical ones after performing a "renormalisation" operation, removing divergences and deriving the "true" physical and measurable quantities. The theory will then be insensitive to the original "bare" coefficients used. RTCP still qualifies as a physical theory if one can give a prescription scheme for the unambiguous removal of the divergences mentioned. These schemes mainly come from quantum field theory and the confidence to handle divergent expressions stems from the successful treatment of quantum electrodynamics (QED).

The first question that one can pose is whether the GLPF reflects the physical situation. One remarks that it is really used out of its original context. The aim of Ginsburg and Landau, when introducing the fluctuation term (the "Ginsburg" term), was to treat the case of spatially varying superconductors in magnetic fields at constant temperature. This was a beautiful success, which did not need any ad-hoc assumption about the temperature variation of the coefficients. In particular, in equ. (1) we notice the presence of the fluctuation term which is crucial for the superconductor treatment but completely disappears from the picture for homogeneous systems. So, it is difficult to see what justifies its use for homogeneous systems in the RTCP-framework. Landau's treatment of the temperature-dependent case, with which RTCP is routinely compared [2], does need an assumption of the form of b(T), which is usually b(T) µ $(T-T_c)$. This gives a critical exponent b =1/2 (associates magnetisation and temperature

m(T) μ (T-T_c)b). If we had chosen b(T) μ (T-T_c)^3 instead (which is just one further order of expansion, assuming that the quadratic coefficient vanishes), we would have obtained b = 3/2. This sensitivity to the ad-hoc form, however justified, of the coefficients renders any comparison highly unreliable.

Still, as we have seen, RTCP does not suffer this drawback because we can set the coefficients constant and are supposed to derive the critical exponents in a unambiguous manner.

However, using continuous-space classical (not quantum) fields is impossible in statistical mechanics. Consider the free scalar field, whose partition function is given, for the quantum and classical case, respectively, by

(3a) $$Z_Q = tr\exp\left(-\beta\left[\int d^n x\left[\pi^2 + (\nabla\phi)^2 + m\phi^2\right]\right]\right)$$

(3b) $$Z_C = tr\exp\left(-\beta\left[\int d^n x\left[(\nabla\phi)^2 + m\phi^2\right]\right]\right)$$

The free energy can be calculated explictely in both cases to give

(4a) $$F_Q = -\frac{\partial Z_Q}{\partial\beta} = V\int_{k_m}^{k_M} d^n k\,\frac{A(k)}{e^{\beta\omega(k)}-1} = VT^4\int_0^{\infty} d^n x\,\frac{x}{e^x-1}$$

(4b) $$F_C = -\frac{\partial Z_C}{\partial\beta} = N\frac{T}{2}$$

k_m , k_M are low- and high-wave number cutoffs, V is system volume, n spatial dimension; the last term in (4a) corresponds to a massless field

In (4b), N is the number of points in phase space, defined by

(5) $$N = \frac{k_M^n}{k_m^n}$$

The thermodynamic limit (V® ¥) is given by $k_{m®}$ 0. Hence, for all calculations, we keep $k_{m¹}$ 0. We see immediately that the free energy is well defined for the UV-limit in the quantum case only, where we get a finite result. For the classical case, the energy of the free (!) field diverges (at fixed volume) with k_M^n .

The classical free field gives, contrary to the quantum case, a nonsensical result. This is exactly the situation Max Planck faced when he treated blackbody radiation classically. It was the inadequacy of classical field theory for thermodynamics (James Jeans' treatment) that pushed him to the quantum treatment. Here, the situation is the same: either we treat a lattice, or phase space "cells" (usually related to *quantum* uncertainties), to calculate classically or we adopt quantum operators.

In the light of the foregoing, we can divide the approaches into those treating classical lattice partition functions (without UV-limit) and quantum continuous-space partition functions (with UV-limit)

On lattice-GLPF as well as Ising-type models there are a lot of Monte-Carlo (and the like) calculations. These numerical investigations, however useful for particular purposes, are to be discarded as positive evidence for RTCP for several reasons: they all use small lattices (4x4 to 10x10 or 4x4x4 points) and cannot simulate the crucial property of diverging correlation length. Phase transitions are mathematically pathological and one cannot numerically distinguish between a "physical" instability and a numerical one. Calculation power is, apart from these general arguments, still too limited to simulate phase transitions (think of the - likewise non-abelian- situation in the proton, a three-body system "only").

For the quantum case, analytical approaches are used and quite the only tool are perturbative S-matrix calculations inherited from QED, using the quartic coupling parameter (usually called l) as the expansion parameter [1]. This is both mathematically and physically untenable. Mathematically, we get ill-defined quantities for $b<0$, which is the only interesting case for obtaining spontaneous symmetry breaking and a phase transition. The results obtained reflect this situation and workers usually pick out subseries, more or less arbitrarily (Borel sums are one example), to match the experimental results (which are few, one only has two critical exponents as observables to calculate). Clearly questionable but still enjoying some respectability, it is claimed to be part of the "renormalisation" procedure. This is, however, not possible because there is no need for renormalisation in QED if you treat the S-matrix carefully. This state of affairs has been discovered in the mid-1970s [3, 4] but attracted little attention probably because all you can measure with QED has already been measured and calculated the "wrong" way with renormalisation as a (very efficient) "crutch" that got you to the goal. Conceptually, however, it is important, especially in the phase-transition context, that QED S-matrix expansions are well-behaved with "bare" coefficients. This removes all justification of using mathematically untenable methods in RTCP.

Apart from the mathematical considerations regarding S-matrix calculations, there is a simpler, physical, one. A perturbation scheme is as good as its starting point. In QED, one looks at high-energy scattering events where particles are initially and finally free. There, it is perfectly justified to start out from free fields since you want to end up with free ones. In RTCP, using perturbation theory starting out from non-interacting fields is only viable if you expect free fields at the end. This then automatically excludes discontinuous behaviour such as phase transitions. Last not least, we cannot present any arguments derived from a prolonged experience with Yang-Mill fields, since the latter are still too poorly understood and we could not yet build up any physical intuition on them.

We have seen that the renormalisation theory of critical phenomena has no foundation left. The legitimacy of transposing the Ginsburg-Landau treatment to phase-transition phenomena as a whole has to be questioned and the use of classical fields has been shown to be inadequate for continuous-field thermodynamics. For quantum fields, the underlying renormalisation philosophy inherited from quantum electrodynamics has been discarded by the work of Epstein and Glaser a long time ago and there is no positive evidence in favor of the theory that could stand a mathematical or conceptional test.

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