arXiv:cond-mat/0210142 v1 7 Oct 2002

Gauge Theory of Composite Fermions: Particle-Flux Separation in Quantum Hall Systems

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(October 9, 2002)

Fractionalization phenomenon of electrons in quantum Hall states is studied in terms of U(1) gauge theory. We focus on the Chern-Simons(CS) fermion description of the quantum Hall effect(QHE) at the filling factor $\nu = p/(2pq \pm 1)$, and show that the successful composite-fermions(CF) theory of Jain acquires a solid theoretical basis, which we call particle-flux separation(PFS). PFS is studied efficiently by a gauge theory and characterized as a deconfinement phenomenon in the corresponding gauge dynamics. The PFS takes place at low temperatures, $T \leq T_{\rm PFS}$, where each electron or CS fermion splinters off into two quasiparticles, a fermionic chargeon and a bosonic fluxon. The chargeon is nothing but Jain's CF, and the fluxon carries $2q$ units of CS fluxes. At sufficiently low temperatures $T \leq T_{\rm BC} (< T_{\rm PFS})$, fluxons Bose-condense uniformly and (partly) cancel the external magnetic field, producing the correlation holes. This partial cancellation validates the mean-field theory in Jain's CF approach. FQHE takes place at $T < T_{\rm BC}$ as a joint effect of (i) integer QHE of chargeons under the residual field ΔB and (ii) Bose condensation of fluxons. We calculate the transition temperature $T_{\rm PFS}$ and the CF mass. Repulsive interactions between electrons are essential to establish PFS. PFS is a counterpart of the charge-spin separation in the t-J model of high-T_c cuprates in which each electron dissociates into holon and spinon. Quasiexcitations and resistivity in the PFS state are also studied. The resistivity is just the sum of contributions of chargeons and fluxons, and we conjecture that ρ_{xx} acquires extra reduction factor below $T_{\rm PFS}$ from the expected T-linear behavior.

I. INTRODUCTION

Fractional quantum Hall effect(FQHE) is one of the most interesting phenomena in the current condensed matter physics. As a result of Coulomb interactions between electrons, several excentric quasiparticles may appear as low-energy excitation modes. Among them, composite fermions (CFs) proposed by Jain[1] play a very important role for a unified view of the FQHE at the states with the electron filling factor $\nu = p/(2pq \pm 1)$ (p, q are positive integers). Experiments support the CF picture not only in these FQH states, but also in the compressible states like $\nu = 1/2$. In theoretical studies of CF, the Chern-Simons(CS) gauge theory is often employed. However, there are no transparent and consistent theoretical explanations why the CF picture works so nicely. In particular, in the CS gauge theory of CF given so far, it is not clear how to treat the CS constraints consistently, although this problem is essential to determine the low-energy quasiparticles and discuss the (in)stability of CFs. Another problem is to calculate the CF mass, which is to be determined by the Coulomb interactions between electrons. *Nonperturbative* study on the CS gauge theory is necessary for solving these problems, though most of the studies use perturbative calculations with respect to the CS gauge field assuming a small gauge coupling.[2,3]

Recently, some papers on the low-energy properties of the QHE and/or the CS gauge theory appeared. Shankar and Murthy (SM)[4] tried to separate intra-Landau level (LL) excitations from inter-LL excitations by enlarging the Hilbert space and redefining field variables. However, as a result of these redefinitions, the resultant field operators satisfy very complicated nonlocal commutation relations, though SM just ignore these complexities. Therefore these field operators cannot be capable to describe quasiexcitations at low energies. In other words, enlargement of the Hilbert space and redefinition of the field operators are not sufficient to describe low-energy physics of FQHE and its closely related states.

In the SM approach, there appears a local gauge symmetry and they impose a constraint on physical states, which is similar to the Gauss' law constraint in quantum electrodynamics(QED). However, the way how this gauge symmetry is realized dynamically (e.g., in confienement phase or in deconfinement phase) is crucial for the physical properties of the system at low energies. For example, in QED, the field operators of electrons and photons in the *Heisenberg picture* satisfy the Gauss' law, but the *asymptotic fields* describing incoming and outgoing particles *do not*. This stems from the fact that the gauge symmetry in QED is realized in the deconfinement-Coulomb phase. As a result, electrons and photons appear as quasi-free particles and their interactions can be treated by the perturbative calculations with respect to the *small* gauge-coupling constant at low energies.

In Jain's idea of CF,[1] notion and dynamics of fictitious

(i.e., CS) magnetic fluxes are very important to understand the FQHE and $\nu = 1/2$ states intuitively. In the previous papers[5,6], we studied an electron system in a strong magnetic field at $\nu = 1/2$. In Ref.[5] we made the usual CS gauge transformation to electrons and studied the resulting CS gauge theory of self-interacting fermions. We point out the possibility of separation phenomenon between particle and flux degrees of freedom, which we called particle-flux separation (PFS). In Ref.[6], we associated a particle picture to flux degrees of freedom by writing the CS fermion operator as a product of two operators of new particles named chargeon and fluxon by enlarging the Hilbert space and imposing a local constraint. The chargeons are just the fermions that describe Jain's CFs, exhibitting IQHE at low temperatures, whereas the fluxons are bosons that describe CS flux degrees of freedom and associated correlated holes. We showed that PFS takes place at low temperatures ($T \leq T_{\rm PFS}$) where a CS fermion fractionalizes into a chargeon and a fluxon, and these "constituents" behave as quasi-free low-energy excitations there.

PFS is closely related with the charge-spin separation (CSS) in high-T_c cuprates, in which each electron dissociates into holon and spinon.[7–9] Both the PFS and CSS are understood as deconfinement phenomena of dynamical gauge fields that appear as a result of introducing constituents of an electron, the chargeon and fluxon in PFS and the holon and spinon in CSS.

In our recent letter[10] we generalized the gauge theory of PFS at $\nu = 1/2$ of Ref.[5,6] to the cases of $\nu = p/(2pq \pm 1)$ (p, q : positive integers). The case of $\nu = 1/2$ is viewed as the limit $p \to \infty$ at $q = 1$. In the PFS states, bosonic fluxons may Bose condense at $T \leq T_{\rm BC} (< T_{\rm PFS})$. The resulting uniform CS magnetic field (partly) cancels the external magnetic field, and chargeons move in this reduced field. This partial cancellation of magnetic field just validates Jain's idea that FQHE is nothing but the integer QHE of CFs, where chargeons are nothing but CFs. In Fig.1 we illustrate the basic idea of our chargeon-fluxon approach to FQHE.

In this paper, we shall present detailed and self-contained account of the gauge theory of composite fermions summerized in our letter.[10] We extend also the previous calculations of the transition temperature $T_{\rm PFS}$ and masses of chargeon and fluxon at $\nu = 1/2$ to the general fillings. Furthermore, we investigate the physical properties of the low-energy excitations in PFS states in the present picture.

The paper is organized as follows; In Sec.II, we introduce our model. Field operators of chargeons and fluxons are introduced to describe strongly-correlated electron systems in a magnetic field. Chargeons and fluxons are quantized as ordinary fermions and bosons, respectively. A local gauge symmetry emerges as a result of the chargeon-fluxon representation of electron, and a gauge field is introduced as an auxiliary field, which mediates the interaction between chargeon and fluxon in an electron. In Sec.III, an effective action or a Ginzburg-Landau(GL) theory of the dynamical gauge field is obtained by integrating out chargeon and fluxon variables by the hopping expansion. By referring to the established knowledge of lattice gauge theory, it is concluded that a confinement-deconfinement phase transition of the gauge dynamics takes place at the transition temperature $T_{\rm PFS} (> 0)$, and the PFS (deconfinement phase) is realized *below* $T_{\rm PFS}$. In Sec.IV, the ground states, quasiexcitations, and the EM transport properties in PFS states are discussed. The ground state is a direct product of the chargeon ground state and Bose condensate of fluxons, which is shown to be just the Laughlin states for $p = 1$. As a result of the local gauge invariance, we obtain a formula of resistivity similar to the Ioffe-Larkin formula in the holon-spinon theory of high-T_c cuprates; the resistivity tensor is just the summation of chargeon contribution and fluxon contribution.This formula gives rise to the results that agree with the experimantal observations. We conjecture that ρ_{xx} acquires an extra reduction factor for $T_{\rm BC} < T < T_{\rm PFS}$ from the expected T-linear behavior.

Sec.V is devoted to discussion. We explain flaws of SM theory in some details, e.g., insufficient treatment of noncommuting operaotrs. These flaws are removed in our theory, which is characterized as a detailed study of the dynamics of the relevant degrees of freedom in a framework of second-quantized field theory.

II. MODEL

A. Electrons on a lattice

Let us consider a two-dimensional system of electrons in a parpendicular constant magnetic field $B^{\rm ex}$. Instead of working in the continuum space, we put the system on a two-dimensional square lattice as a way of regularization, which is useful for nonperturbative study of gauge dynamics as demonstrated in lattice gauge theory by Wilson.[11] The lattice model below is regarded as an effective system of renormalization-group theory, and the main results obtained below, e.g., the existence of PFS, are not the artifacts of introduction of a lattice but survive in the continuum. For example, the critical temperature $T_{\rm PFS}$ is a physical quantity that is renormalization-group invariant.(We discuss more on this point at the end of Sec.III.) For definiteness, in numerical estimation, we take the lattice spacing a as $a \sim \ell$ where

$$\ell = \frac{1}{\sqrt{eB^{\rm ex}}} \tag{2.1}$$

is the magnetic length. In this paper we use the units $\hbar = 1$ and $c = 1$.

We start with the electron annihilation operator C_x on the site x (it has a definite spin component due to the Zeeman effect), satisfying the canonical anticommutaiton relations,

$$[C_x, C_y^\dagger]_+ = \delta_{xy},\ [C_x, C_y]_+ = 0. \tag{2.2}$$

The Hamiltonian is written as

$$H_C = -\frac{1}{2ma^2}\sum_x \sum_{j=1}^{2} \left(C_{x+j}^\dagger e^{-ie(A_{xj}^{\rm ex}+a_{xj})} C_x + {\rm H.c.} \right) + H_{\rm int}(C_x^\dagger C_x), \tag{2.3}$$

where m is the effective electron mass (i.e., the band mass). The first term describes the process of electron hopping from the site x to its nearest-neighbor $x+j$ (we use the direction index $j = 1, 2$ also as the lattice unit vector in the j-th direction) under the EM vector potential $A_{xj}^{\rm ex}$ for $B^{\rm ex}a^2 = \epsilon_{ij}\nabla_i A_{xj}^{\rm ex}$ ($\epsilon_{12} = 1$ and $\nabla_i F(x) \equiv F(x+i) - F(x)$). We included also the dynamical (fluctuating) EM vector potential, a_{xj}, in order to derive the formula of the EM current later. By making the field rescalings,

$$A_{xj}^{\rm ex} \to aA_j(x),\ a_{xj} \to aa_j(x),\ C_x \to aC(x), \tag{2.4}$$

and taking a naive continuum limit, $a \to 0$, the first term reduces to a well known continuum Hamiltonian $(2m)^{-1}\int d^2x |D_j C(x)|^2$, $D_j \equiv \partial_j - ieA_j^{\rm ex}(x) - iea_j(x)$ (up to a chemical potential term). The second term $H_{\rm int}$ represents repulsive interactions between electrons. Its explicit form is specified later.

Note that $A_{xj}^{\rm ex}$ is defined on the link $(x, x+j)$ and its exponentiated phase factor $U_{xj}^{\rm ex} \equiv \exp(-ieA_{xj}^{\rm ex})$ is a U(1) link variable of lattice gauge theory.[11] In terms of U_{xj} the local (i.e., site-dependent) U(1) EM gauge transformation is written as

$$C_x \to G_x^{\rm EM} C_x,$$
$$U_{xj}e^{-iea_{xj}} \to G_{x+j}^{\rm EM} U_{xj} e^{-iea_{xj}} G_x^{\rm EM\dagger},\ G_x \in U(1). \tag{2.5}$$

B. CS fermions

Let us express C_x in terms of CS fermion operator ψ_x as follows;

$$C_x = \exp[2iq\sum_y \theta_{xy}\psi_y^\dagger\psi_y]\psi_x, \tag{2.6}$$

where $\theta_{xy}[\equiv \tan^{-1}((x_2 - y_2)/(x_1 - y_1))]$ is the multi-valued azimuthal angle of the vector $\vec{x} - \vec{y}$ on the lattice.[12] From (2.6) ψ_x satisfy

$$[\psi_x, \psi_y^\dagger]_+ = \delta_{xy},\ [\psi_x, \psi_y]_+ = 0. \tag{2.7}$$

Eq.(2.6) indicates that each electron is viewed as a composite of $2q$ flux quanta ($q = 1, 2, 3, \cdots$) and a CS fermion.[12]

Then the Hamiltonian (2.3) is rewritten as

$$H_\psi = -\frac{1}{2ma^2}\sum_{x,j}\left(\psi_{x+j}^\dagger e^{i(A_{xj}^{\rm CS} - eA_{xj}^{\rm ex} - ea_{xj})}\psi_x + {\rm H.c.} \right) + H_{\rm int}(\psi_x^\dagger\psi_x), \tag{2.8}$$

where the CS gauge field is introduced as

$$A_{xj}^{\rm CS} = 2q\epsilon_{ij}\sum_y \nabla_i\theta_{xy}\cdot\psi_y^\dagger\psi_y. \tag{2.9}$$

Note $H_{\rm int}(C_x^\dagger C_x) = H_{\rm int}(\psi_x^\dagger\psi_x)$ due to the relation between the number operators, $C_x^\dagger C_x = \psi_x^\dagger\psi_x$. The filling factor ν is defined as usual by

$$\nu = \frac{2\pi\rho}{eB^{\rm ex}},\ \rho = \frac{n}{a^2},$$
$$n \equiv \langle C_x^\dagger C_x\rangle = \langle\psi_x^\dagger\psi_x\rangle, \tag{2.10}$$

where n is the average electron number *per site*. Eqs.(2.8) and (2.9) show that the CS fermions ψ_x not only minimally couple with the CS gauge field but also they themselves are the sources of $A_{xj}^{\rm CS}$, hence producing the CS magnetic fluxes. In fact, eq(2.9) leads to the relation,

$$B_x^{\rm CS} \equiv \epsilon_{ij}\nabla_i A_{xj}^{\rm CS} = 4\pi q\psi_x^\dagger\psi_x, \tag{2.11}$$

so each CS fermion accompanies with 2q units of CS flux.

One may conceive that Jain's CF idea can be realized if the phase factor and the CS fermion ψ_x in (2.6) "sepatate" dynamically. However, since both of these quantities are desctibed by the same variables ψ_x, such a separation is not straightforward. We need to prepare a set of independent variables to describe the phase (CS fluxes) and the fermion. This leads us to introduce the chargeon and fluxon variables, which is the subject of the next subsection.

C. Chargeons and fluxons

Let us rewrite the CS fermion operator ψ_x as a composite field of a canonical *fermion* operator η_x and a canonical *boson* operator ϕ_x,

$$\psi_x = \phi_x\eta_x. \tag{2.12}$$

We call particles described by η_x and ϕ_x chargeons and fluxons, respectively. Meaning of this terminology becomes clear shortly. They satisfy

$$[\phi_x, \phi_y^\dagger] = \delta_{xy},\ [\phi_x, \phi_y] = 0,$$
$$[\eta_x, \eta_y^\dagger]_+ = \delta_{xy},\ [\eta_x, \eta_y]_+ = 0,$$
$$[\phi_x, \eta_y] = [\phi_x, \eta_y^\dagger] = 0. \tag{2.13}$$

To keep the equivalence of C_x representation and $\phi_x - \eta_x$ representation, we impose the following local constraint on the physical states $|{\rm Phys}\rangle$;

$$\eta_x^\dagger\eta_x|{\rm Phys}\rangle = \phi_x^\dagger\phi_x|{\rm Phys}\rangle \ \text{ for each } x. \tag{2.14}$$

Then there is the following one-to-one correpondence between the states made of ψ_x and the physical states made of ϕ_x and η_x;

$$\begin{aligned}\psi_x|0\rangle_\psi = 0,\ \eta_x|V\rangle_\eta = \phi_x|V\rangle_\phi = 0,\\ |0\rangle_\psi = |V\rangle \equiv |V\rangle_\eta|V\rangle_\phi,\\ |1\rangle_\psi \equiv \psi_x^\dagger|0\rangle_\psi = \eta_x^\dagger\phi_x^\dagger|V\rangle.\end{aligned} \tag{2.15}$$

One may check the product $\phi_x\eta_x$ satisfies the canonical anticommutation relation (2.7) of ψ_x in the physical subspace defined above.

By substituting (2.12) to H_ψ of (2.8) the Hamiltonian is rewritten in terms of η_x and ϕ_x as

$$\begin{aligned}H_{\eta\phi} = -\frac{1}{2ma^2}\sum_{x,j}\Big(\eta_{x+j}^\dagger\phi_{x+j}^\dagger W_{x+j}M_{x+j}M_x^\dagger W_x^\dagger e^{-iea_{xj}}\phi_x\eta_x\\ +\text{H.c.}\Big) + H_{\rm int}(\eta_x^\dagger\eta_x\phi_x^\dagger\phi_x) - \sum_x(\mu_\eta\eta_x^\dagger\eta_x + \mu_\phi\phi_x^\dagger\phi_x)\\ -\sum_x\lambda_x(\eta_x^\dagger\eta_x - \phi_x^\dagger\phi_x),\end{aligned} \tag{2.16}$$

where λ_x is the Lagrange multiplier to enforce the constraint (2.14). W_x and M_x are U(1) phase factors defined as

$$W_x = \exp\Big[2iq\sum_y\theta_{xy}\Big(\phi_y^\dagger\phi_y - n\Big)\Big], \tag{2.17}$$

$$M_x = \exp\Big[i\sum_y\theta_{xy}(2q - \frac{1}{\nu})n\Big], \tag{2.18}$$

where we have replaced $\psi_y^\dagger\psi_y$ in $A_{xj}^{\rm CS}$ by $\phi_y^\dagger\phi_y$ using the relations $\psi_x^\dagger\psi_x = \phi_x^\dagger\phi_x = \eta_x^\dagger\eta_x$ that hold due to the constraints. The chemical potentials μ_η and μ_ϕ are introduced to enforce the conditions $\langle\eta_x^\dagger\eta_x\rangle = \langle\phi_x^\dagger\phi_x\rangle = n$.

The original electron operator C_x is expressed in terms of η_x and ϕ_x as

$$C_x = \exp[2iq\sum_y\theta_{xy}\phi_y^\dagger\phi_y]\phi_x\eta_x. \tag{2.19}$$

Here we have expressed the CS phase factor in terms of ϕ_x, i.e., we have assigned so that each ϕ_x carries CS $2q$ flux quanta, while η_x does not. Thus the fluxons accompany a CS field, the average value B_ϕ of which is given by using (2.11) as

$$B_\phi = \frac{\langle B_x^{\rm CS}\rangle}{e} = \frac{4\pi qn}{ea^2}, \tag{2.20}$$

where we divided $B_x^{\rm CS}$ by e so that B_ϕ has the dimension of a magnetic field.

By using the constraint (2.14), two expressions of the electron operators, (2.6) and (2.19), are equivalent. Eq.(2.19) shows that an electron is viewed as a composite of a chargeon η_x and a fluxon ϕ_x that bears $2q$-flux quanta. In other words, the *chargeon* η_x is a *composite* of an electron and $2q$-flux quanta in the direction opposite to $B^{\rm ex}$. This implies that the chargeon in the present formalism may be regarded as Jain's CF. However, we stress that, in order to justify the CF picture as a physically correct picture, the PFS must take place dynamically, as we explained in Sec.I.

We should also remark here that the CS fermion and the electron operators are invariant under the following "gauge transformation" of the chargeon and fluxon;

$$(\eta_x, \phi_x) \to (e^{i\alpha_x}\eta_x, e^{-i\alpha_x}\phi_x), \tag{2.21}$$

where α_x is an arbitrary function of x.As we shall see later, this local gauge symmetry plays a crucial role for the PFS transition and for the EM transport properties of the present system.

D. Gauge theory of chargeons and fluxons

We are mainly interested in the structure of the ground state and the low-energy excitations of the system $H_{\eta\phi}$ of (2.16). For this purpose, we employ the Lagrangian path-integral formalism, since this formalism is suitable to introduce a gauge field as an auxiliary field and to obtain an effective gauge theory. By using this gauge theory and examining its gauge dynamics, one can study the possibility of PFS in a natural and convincing manner.

The partition funciton Z at the temperature T is expressed by a path integral[13] as

$$\begin{aligned}Z \equiv {\rm Tr}\exp(-\beta H_{\phi\eta}) = \int[d\eta][d\phi][d\lambda]\ \exp\Big(\int_0^\beta d\tau L(\tau)\Big),\\ L = -\sum_x\eta_x^\dagger(\partial_\tau + i\lambda_x - \mu_\eta)\eta_x\\ -\sum_x\phi_x^\dagger(\partial_\tau - i\lambda_x - \mu_\phi)\phi_x\\ +\frac{1}{2m}\sum_{x,j}\Big(\eta_{x+j}^\dagger\phi_{x+j}^\dagger W_{x+j}M_{x+j}M_x^\dagger W_x^\dagger e^{-iea_{xj}}\phi_x\eta_x\\ +\text{H.c.}\Big) - H_{\rm int}(\eta_x^\dagger\eta_x\phi_x^\dagger\phi_x),\\ [d\eta][d\phi][d\lambda] \equiv \prod_\tau\prod_x d\eta_x(\tau)d\phi_x(\tau)d\lambda_x(\tau),\end{aligned} \tag{2.22}$$

where $\tau \in [0, \beta(\equiv (k_{\rm B}T)^{-1})]$ is the imaginary time. The fields in $L(\tau)$ are functions of τ; $\eta_x(\tau), \phi_x(\tau), \lambda_x(\tau)$. $\eta_x(\tau)$ is a Grassmann number, $\phi_x(\tau)$ is a complex number, and $\lambda_x(\tau)$ is a real number.

We decouple the third line of L by introducing a complex link auxiliary field[13] V_{xj} on the link $(x, x+j)$ by using the formula for arbitrary J_{xj},

$$\begin{aligned}\int dV_{xj}\exp\Big(\Delta\tau\Big[-\frac{|V_{xj}|^2}{2ma^2} + (V_{xj}^\dagger J_{xj} + \text{H.c.})\Big]\Big)\\ \propto \exp\Big(\Delta\tau\ 2ma^2|J_{xj}|^2\Big).\end{aligned} \tag{2.23}$$

Then we have

$$
\begin{aligned}
Z &= \int [d\eta][d\phi][d\lambda][dV]\ \exp\Big(\int_0^\beta d\tau L_V(\tau)\Big),\\
L_V &= -\sum_x \eta_x^\dagger(\partial_\tau + i\lambda_x - \mu_\eta)\eta_x\\
&\quad -\sum_x \phi_x^\dagger(\partial_\tau - i\lambda_x - \mu_\phi)\phi_x + \sum\Big(V_{xj}^\dagger J_{xj} + \text{H.c.}\Big)\\
&\quad -\frac{1}{2ma^2}\sum_{x,j}|V_{xj}|^2 - H_4 - H_{\rm int}(\eta_x^\dagger\eta_x\phi_x^\dagger\phi_x),\\
H_4 &\equiv \sum_{x,j}\Big(\frac{\gamma^2}{2m}\phi_{x+j}^\dagger\phi_{x+j}\phi_x^\dagger\phi_x - \frac{1}{2ma^2\gamma^2}\eta_{x+j}^\dagger\eta_{x+j}\eta_x^\dagger\eta_x\Big),\\
J_{xj} &\equiv \frac{1}{2ma^2}\Big(\gamma\phi_{x+j}W_x e^{iecA_{xj}}W_{x+j}^\dagger\phi_x^\dagger\\
&\quad +\frac{1}{\gamma}\eta_{x+j}^\dagger M_{x+j}e^{-ie(1-c)A_{xj}}M_x^\dagger\eta_x\Big). \qquad (2.24)
\end{aligned}
$$

Here γ is a real parameter, which measures the ratio of the chargeon and fluxon masses as we shall see in (3.32). We determine its value later in (2.37) of Sec.II E. γ c is an *arbitrary* real constant that appears in the EM charges Q_ϕ of ϕ_x and Q_η of η_x, which are read off from the couplings to a_{xj} in L_V as

$$Q_\phi = ce,\ Q_\eta = (1-c)e. \qquad (2.25)$$

In Sec.4C, we shall discuss some consequences of this arbitrariness in the EM transport properties; we shall see that the physical results are independent of the value of c.

To see the physical meaning of the auxiliary field V_{xj}, we use the following trivial identity;

$$
\begin{aligned}
&\int d^2V_{xj}\frac{\partial}{\partial V_{xj}^\dagger}\exp\Big(\Delta\tau\Big[-\frac{|V_{xj}|^2}{2ma^2} + (V_{xj}^\dagger J_{xj} + \text{H.c.})\Big]\Big)\\
&= \int d^2V_{xj}\Delta\tau\Big[-\frac{V_{xj}}{2ma^2} + J_{xj}\Big]\\
&\quad\times\exp\Big(\Delta\tau\Big[-\frac{|V_{xj}|^2}{2ma^2} + (V_{xj}^\dagger J_{xj} + \text{H.c.})\Big]\Big)\\
&= 0, \qquad (2.26)
\end{aligned}
$$

(the surface terms vanish) to obtain the relation (equations of motion),

$$\langle V_{xj}\rangle = 2ma^2\langle J_{xj}\rangle, \qquad (2.27)$$

which shows that V_{xj} describes the hopping amplitudes of η_x and ϕ_x. We note that the way of decoupling in (2.24) is slightly different from our previous papers.[6] The present expression is more suitable to discuss PFS since the current J_{xj} coupled to V_{xj} is just the sum of the η_x part and the ϕ_x part without mixing terms.

From (2.24), $A_{x0} \equiv \lambda_x$ and A_{xj} defined through $V_{xj} \equiv |V_{xj}|U_{xj}$, $U_{xj} \equiv \exp(iA_{xj})$ can be regarded as the time and transverse component of a gauge field $A_{x\mu}$ ($\mu = 0(\tau), 1, 2$), respectively. Actually, under the U(1) local gauge transformation (2.21) with τ-dependent $\alpha_x(\tau)$, they transform as

$$\lambda_x \to \lambda_x - \partial_\tau\alpha_x,\ \ A_{xj} \to A_{xj} + \nabla_j\alpha_x. \qquad (2.28)$$

The Lagrangian L_V of (2.24) is invariant under (2.28). Thus the system has a local gauge invariance, and one may regard the system as a lattice gauge theory.[11]

A lattice gauge theory has generally two possibilities to realize its gauge dynamics;

(i) Confinement phase:
Here the gauge-field fluctuations, ΔA_{xj}, are large and random i.e.,

$$\langle(\Delta A_{xj})^2\rangle = \infty,\ \langle\exp(iA_{xj})\rangle = 0, \qquad (2.29)$$

and only the gauge-invariant (i.e., charge-neutral) objects may appear as physical excitations.

(ii) Deconfinement phase (like Coulomb phase and Higgs phase):
Here ΔA_{xj} are small and can be treated in perturbation theory,

$$\langle(\Delta A_{xj})^2\rangle \sim 0,\ \langle\exp(iA_{xj})\rangle \sim 1, \qquad (2.30)$$

and the *gauge-variant* objects may appear as excitations. (For more detailed discussion on this point, see Sec.VI of the first reference of Ref.[9].)

In the confinement phase of the present system, the only CS fermions ψ_x or the electrons may appear as quasiparticles, while in the deconfinement phase, the chargeons and fluxons may appear as quasiparticles. The latter case corresponds to the PFS. Therefore, the PFS is characterized as a deconfinement phase of the gauge dynamics of the gauge field $(\lambda_x,\ A_{xj})$.[6] In fact, in the PFS states, one may set $V_{xj} = |V_{xj}|U_{xj}$ by some constant $V_{xj} \simeq V_0U_{xj} \simeq V_0$ as the first approximation. (We shall calculate V_0 later in Sec.III A.) Then the coupling $J_{xj}V_{xj}$ reduces to the sum of chargeon and fluxon hopping terms. The chargeons η_x just hop in a constant field described by $M_{x+j}M_x^\dagger$, i.e., in the reduced magnetic field;

$$\Delta B \equiv B^{\rm ex} - B_\phi = \frac{2\pi\rho}{e}\Big(\frac{1}{\nu} - 2q\Big) = \pm\frac{2\pi\rho}{ep}. \qquad (2.31)$$

The fluxons ϕ_x hop in a field $W_xW_{x+j}^\dagger$ generated by ϕ_x themselves. At low T, we expect that ϕ_x form a Bose condensation and one may set $\phi_x^\dagger\phi_x = n$ as the first approximation. Then $W_x \to 1$ and ϕ_x describes just free bosons. The central problem is to identify the condition when the deconfinement phase, hence the PFS, is realized in the present system (2.24).

Before studying this problem in detail, let us present some general considerations. First, we observe that the Lagrangian L_V contains *no* kinetic (i.e., Maxwell) terms

of gauge field like $g^{-2}\sum V^{\dagger}_{xj}V^{\dagger}_{x+j,i}V_{x+i,j}V_{xi}$ (which corresponds to the $(\partial_i A_j - \partial_j A_i)^2$ term in the continuum); that is, the gauge coupling constant g^2 is infinite. Therefore one may suspect that V_{xj} should fluctuate randomly leading to the confinement phase, so the PFS does no take place at all. This argument is valid if the system is a pure gauge theory, Lagrangian of which consists of only a Maxwell term and no couplings to charged matter fields.[11] However, our system has couplings to matter fields, chargeons and fluxons, $\phi^{\dagger}_{x+j}V_{xj}\phi_x$ and $\eta^{\dagger}_{x+j}V_{xj}\eta_x$. It is possible that these couplings may suppress gauge-field fluctuations at low-energies as a renormalization effect due to the high-momentum and/or high-energy modes of ϕ_x and η_x. If these factors are efficient enough, the gauge dynamics is to be realized in the deconfinement phase, hence the PFS takes place.

Similar question arose for the CSS, and some physicists concluded that the CSS never takes place in the $U(1)$ gauge theory of strongly-correlated models like the t-J model.[14] In our previous paper,[9], we clarified this misunderstanding by pointing out the renormalization effects by couplings to matter fields. Also we emphasized there that there exists a counter example against the above naive expectation. For example, the SU(3) gauge theory in $(3+1)$ dimensions *without* quark fields is always in the confinement phase regardless of the magnitude of gauge coupling constant g. However, onset of couplings to quarks, relativistic fermions, changes its phase structure drastically.[15] Especially, in quantum chromodynamics (QCD) with a sufficiently large number of light quarks (the number of quark flavor $N_f \geq 7$), the deconfinement phase is realized even if the coupling constant $g_{\rm QCD} \to \infty$. Furthermore, for $N_f \geq 17$, the effective coupling constant is renormalized as $g_{\rm eff} \to 0$ at lower energies even for $g_{\rm QCD} = \infty$. One may argue that a similar phenomenon should certainly occur in the present gauge system, because the chargeons, nonrelativistic fermions, behave as a set of many species of relativistic fermions. This is because nonrelativistic fermions have a Fermi surface (Fermi line in two-dimensions) instead of Fermi points of relativistic fermions, and it contains a large number of low-energy excitations. In the following Section, we shall study this possibility by deriving an effective action of the dynamical gauge field $A_{x\mu}$, finding that the PFS is possible at sufficiently low temperatures.

E. Repulsive interaction between electrons

To make an explicit and detailed study of the gauge dynamics, one needs to specify the interaction term $H_{\rm int}$ in (2.24). This interaction between electrons (or CS fermions) should play a crucial role in the CF picture and the PFS. Actually, if this term were missing, the system would be just an ensemble of independent electrons, each electron occupying degenerate LL's, and no FQHE would be observed. Generally speaking, appropriate interactions are certainly necessary in order that separation phenomena of degrees of freedom take place.

Let us focus on the effects of short-range part of Coulomb interaction by taking the following nearest-neighbor repulsion as $H_{\rm int}$;

$$H_{\rm int}(C^{\dagger}_x C_x) = H_{\rm int}(\psi^{\dagger}_x\psi_x) = g\sum \psi^{\dagger}_{x+j}\psi_{x+j}\psi^{\dagger}_x\psi_x\,, \tag{2.32}$$

where $g(>0)$ is the coupling constant of the Coulomb repulsion. Since we set $a \simeq \ell$, g is estimated as

$$g \simeq \frac{e^2}{\epsilon\ell}, \tag{2.33}$$

where ϵ is the dielectric constant of materials. The effects of the long-range part $H^{\rm LR}$ of Coulomb repulsion shall be discussed in Sec.IV A. By using the relations $\psi^{\dagger}_{x+j}\psi_{x+j}\psi^{\dagger}_x\psi_x = \eta^{\dagger}_{x+j}\eta_{x+j}\eta^{\dagger}_x\eta_x = \phi^{\dagger}_{x+j}\phi_{x+j}\phi^{\dagger}_x\phi_x$ by (2.15), $H_{\rm int}$ above is rewritten as

$$H_{\rm int} = \sum_{x,j}\Big(g_1\eta^{\dagger}_{x+j}\eta_{x+j}\eta^{\dagger}_x\eta_x + g_2\phi^{\dagger}_{x+j}\phi_{x+j}\phi^{\dagger}_x\phi_x\Big),$$
$$g_1 + g_2 = g. \tag{2.34}$$

The parameters g_1 and g_2 shall be determined shortly. Then we note that this $H_{\rm int}$ has the same form as H_4 in L_V of (2.24). Since we expect that chargeons and fluxons become quasiexcitations at low energies, we should adjust their environments so that they behave as freely as possible. This self-consistency condition leads us to require that $H_{\rm int}$ and H_4 in L_V of (2.24) should cancel out each other,

$$H_4 + H_{\rm int} = 0. \tag{2.35}$$

Then g_1 and g_2 are related with γ as

$$g_1 = \frac{1}{2ma^2\gamma^2},\ g_2 = -\frac{\gamma^2}{2ma^2}. \tag{2.36}$$

From $g_1 + g_2 = g$, γ satisfies

$$\frac{1}{2ma^2\gamma^2} - \frac{\gamma^2}{2ma^2} \simeq \frac{e^2}{\epsilon\ell}. \tag{2.37}$$

The relation (2.37) is suggestive, expressing the relation between two energy scales, one is the inter-LL gap for electrons (the cyclotron frequency),

$$\omega_B = \frac{eB^{\rm ex}}{m}, \tag{2.38}$$

and another is the short-range Coulomb energy $e^2/(\epsilon\ell)(\propto\sqrt{B^{\rm ex}})$ for chargeons and fluxons. In fact, (2.37) is rewritten with $a \simeq \ell$ as

$$\frac{1}{2ma^2}\Big(\frac{1}{\gamma^2} - \gamma^2\Big) \simeq \frac{\omega_B}{2}\Big(\frac{1}{\gamma^2} - \gamma^2\Big) \simeq \frac{e^2}{\epsilon\ell}. \tag{2.39}$$

Thus the dimensionless constant $\gamma^{-2}-\gamma^2$ is a reduction factor to reduce ω_B down to $e^2/(\epsilon\ell)$. In the experiment, the effective electron mass m, the dielectric constant ϵ, and the magnetic length ℓ are all given, so γ is determined from (2.37) by these parameters.[16,17]

III. EFFECTIVE GAUGE THEORY AND PFS

In this Section, we derive the effective lattice gauge theory of the gauge field $A_{x\mu}$ by the hopping expansion over ϕ_x and η_x, and study its phase structure. In the effective gauge theory, we shall ignore fluctuations of the absolute value $|V_{xj}|$ since they are massive, and focus on its U(1) phase part U_{xj}. We shall also see the fluctuations of $\lambda_x = A_{x0}$ are massive and can be ignored. Thus the effective action is a function of U_{xj}'s. As the lattice regularization is used in the present study, we can study the problem nonperturbatively. This is in sharp contrast to the most of other studies of the CS gauge theory.

A. Estimation of $V_0 \equiv \langle |V_{xj}| \rangle$

We start with writing the spatial component of the gauge field, V_{xj}, in the polar coodinate as

$$V_{xj} = |V_{xj}| U_{xj},\ U_{xj} = \exp(iA_{xj}) \in \mathrm{U}(1). \tag{3.1}$$

Let us first estimate the expectaiton value of the amplitude, $V_0 \equiv \langle |V_{xj}| \rangle$ by MFT, which is obtained by setting $V_{xj} = V_0$ and $\lambda_x = 0$ in the Lagrangian L_V of (2.24). This MFT is jusitified for studying the low-energy physics a posteriori, because fluctuations of both $|V_{xj}|$ and λ_x are shown to be massive and irrelevant to the low-energy physics. On the other hand, the phase part, A_{xj}, may be massless and should be treated carefully. Estimation of V_0 is further simplified by putting the fluxon variables as $\phi_x = \sqrt{n}$, since one expects Bose condensation of fluxons at low T. In the bosonic CS gauge theory of FQHE, Bose condensation actually takes place and the Bose-condensed ground state describes the Laughlin state as is shown in Ref.[18]. The Lagrangian L_{V_0} of the MFT then takes the following form;

$$\begin{aligned} L_{V_0} = &-\sum_x \eta_x^\dagger(\partial_\tau - \mu_\eta)\eta_x \\ &+ \frac{1}{2ma^2}\sum_{x,j}\Big[-V_0^2 + V_0\Big(\gamma n + \frac{1}{\gamma}\eta_{x+j}^\dagger \eta_x + \mathrm{H.c.}\Big)\Big]. \end{aligned} \tag{3.2}$$

A typical behavior of V_0 was plotted in Fig.1 of Ref.[5]. It decreases as T increases. We expect V_0 vanishes at certain temperature T_{V_0}. Near T_{V_0}, the MFT (3.2) is not reliable since Bose condensation would disappear. Instead, an expansion in term of small V_0 is possible.[5] We shall calculate T_{V_0} in (3.30) later.

The small fluctuations of $|V_{xj}|$ around V_0 are described by inserting $V_{xj} = V_0 + v_{xj}, v_{xj} \in R$ to V_0 in L_{V_0} and expand L_{V_0} up to $O(v_{xj}^2)$. The squared mass of v_{xj} is positive, hence these fluctuations are irrelevant at low energies.

B. Hopping expansion and effective gauge theory

In the gauge theory of nonrelativistic fermions, the time component of gauge field is often screened (getting massive) and becomes irrelevant to the low-energy physics. This is exploited, for example, in the study of CS gauge theory of CF at $\nu = 1/2$ by Halperin, Lee and Read.[3] Let us examine this problem.

From Sec.III A, we are allowed to set

$$V_{xj} = V_0 U_{xj},\ U_{xj} = \exp(iA_{xj}) \in \mathrm{U}(1) \tag{3.3}$$

below T_{V_0}. Thus we are to study the dynamics of the U(1) gauge field U_{xj} and λ_x. To this end, we obtain an effective action of U_{xj} and λ_x by using the hopping expansion over ϕ_x and η_x, which is an expansion in powers of V_0 or equivalently of U_{xj}. This expansion is a nonperturbative expansion w.r.t. A_{xj} that is legitimate for small expectation values of U_{xj}. It is especially useful to study the phase transition for which $\langle U_{xj} \rangle$ may be regarded as an order parameter with a continuous change.

To be explicit, we use the so-called temporal gauge. At $T = 0$, one can set $\lambda_x(\tau) = 0$. However, at finite T, one degree of freedom of $\lambda_x(\tau)$ should survive as an independent variable.[11] We make the eigen-frequency expansion,

$$\lambda_x(\tau) = \sum_{n\in Z} \lambda_{x,n} \exp(i\omega_n \tau),\ \omega_n = \frac{2\pi n}{\beta}, \tag{3.4}$$

and let the zero mode θ_x,

$$\theta_x \equiv \lambda_{x,0} = \int_0^\beta d\tau \lambda_x(\tau) \tag{3.5}$$

as the remaining integration variable. (This θ_x should not be confused with the azimuthal angle of x.) All the other modes are set zero; $\lambda_{x,n}\ (n \neq 0) = 0$.

In the hopping expansion, we need the on-site propagators of ϕ_x and η_x, $\langle \phi_x(\tau_1)\phi_y^\dagger(\tau_2)\rangle_0$, $\langle \eta_x(\tau_1)\eta_y^\dagger(\tau_2)\rangle_0$ determined from the following Lagrangian L_0 obtained from L_V by setting $V_{xj} = 0$;

$$\begin{aligned} L_0 = &-\sum_x \eta_x^\dagger(\partial_\tau + i\beta^{-1}\theta_x - \mu_\eta)\eta_x \\ &-\sum_x \phi_x^\dagger(\partial_\tau - i\beta^{-1}\theta_x - \mu_\phi)\phi_x. \end{aligned} \tag{3.6}$$

Calculations are straightforward, and we obtain

$$\begin{aligned} &\langle \eta_x(\tau_1)\eta_y^\dagger(\tau_2)\rangle_0 = \delta_{xy}\frac{e^{\mu_\eta(\tau_1-\tau_2)}}{1+e^{\beta\mu_\eta - i\theta_x}} \\ &\times\Big[e^{-i\theta_x(\tau_1-\tau_2)/\beta}\theta(\tau_1-\tau_2) \\ &-e^{\beta\mu_\eta - i\theta_x - i\theta_x(\tau_1-\tau_2)/\beta}\theta(\tau_2-\tau_1)\Big], \end{aligned} \tag{3.7}$$

$$\langle\phi_x(\tau_1)\phi_y^\dagger(\tau_2)\rangle_0 = \delta_{xy}\frac{e^{\mu_\phi(\tau_1-\tau_2)}}{1-e^{\beta\mu_\phi+i\theta_x}}$$
$$\times\Big[e^{i\theta_x(\tau_1-\tau_2)/\beta}\theta(\tau_1-\tau_2)$$
$$+e^{\beta\mu_\phi+i\theta_x+i\theta_x(\tau_1-\tau_2)/\beta}\theta(\tau_2-\tau_1)\Big], \tag{3.8}$$

where $\theta(\tau)$ is the step function. The chemical potentials are determined by

$$\langle\eta_x^\dagger\eta_x\rangle = \lim_{\epsilon\to+0}\langle\eta_x^\dagger(\tau+\epsilon)\eta_x(\tau)\rangle_0 = \frac{e^{\beta\mu_\eta}}{1+e^{\beta\mu_\eta}} = n,$$
$$\langle\phi_x^\dagger\phi_x\rangle = \lim_{\epsilon\to+0}\langle\phi_x^\dagger(\tau+\epsilon)\phi_x(\tau)\rangle_0 = \frac{e^{\beta\mu_\phi}}{1-e^{\beta\mu_\phi}} = n. \tag{3.9}$$

By expanding the integrand of Z of (2.24) in powers of V_0, and by using the propagators, (3.7) and (3.8), the effective action of the gauge field $A_{\rm eff}(\theta,U)$ is obtained, which is a kind of GL theory for the gauge field (θ,U);[20]

$$Z = \int[dU][d\theta]\exp\Big(A_{\rm eff}(\theta,U)\Big),$$
$$A_{\rm eff}(\theta,U) = A_0(\theta)+A_2(\theta,U)+{\rm O}(V_0^4). \tag{3.10}$$

In the leading order ${\rm O}(V_0^0)$ of the expansion, the effective action, $A_0(\theta)$ of θ_x, is obtained as

$$A_0(\theta) = \sum_x \ln F(\theta_x),$$
$$F(\theta_x) \equiv \frac{1+e^{\beta\mu_\eta-i\theta_x}}{1-e^{\beta\mu_\phi+i\theta_x}} = \frac{1+\dfrac{n}{1-n}e^{-i\theta_x}}{1-\dfrac{n}{1+n}e^{i\theta_x}}. \tag{3.11}$$

For the decoupled partition function at each x for A_0, we have

$$\int_0^{2\pi}\frac{d\theta_x}{2\pi}F(\theta_x) = 1+e^{\beta(\mu_\phi+\mu_\eta)} = \sum_{n_x=0}^{1}e^{\beta(\mu_\phi+\mu_\eta)n_x}, \tag{3.12}$$

as expected from the constraint. (Here $n_x = \eta_x^\dagger\eta_x = \phi_x^\dagger\phi_x$ are the particle numbers.) We note that Re $F(\theta_x)$ is an even funciton of θ_x and has the maximum at $\theta_x = 0$ (mod 2π), while Im $F(\theta_x)$ is an odd funciton of θ_x.

Similarly, the second-order calculation of the hopping expansion gives rise to

$$A_2(\theta,U) = -\beta\sum_{x,j}\frac{V_0^2}{2ma^2}+\sum_{x,j}\int_0^\beta d\tau_1\int_0^\beta d\tau_2$$
$$\times C(\theta_x,\theta_{x+j};\tau_1,\tau_2)U_{xj}^\dagger(\tau_1)U_{xj}(\tau_2),$$
$$C(\theta_x,\theta_{x+j};\tau_1,\tau_2) = V_0^2\Big[\Big(\frac{\gamma}{2ma^2}\Big)^2$$
$$\times\langle\phi_{x+j}W_xW_{x+j}^\dagger\phi_x^\dagger(\tau_1)\cdot\phi_xW_{x+j}W_x^\dagger\phi_{x+j}^\dagger(\tau_2)\rangle_0$$
$$+\Big(\frac{1}{2\gamma ma^2}\Big)^2\langle\eta_{x+j}^\dagger\eta_x(\tau_1)\eta_x^\dagger\eta_{x+j}(\tau_2)\rangle_0\Big]$$
$$\equiv \frac{V_0^2}{4m^2a^4}f(\theta_x,\theta_{x+j};\tau_1-\tau_2), \tag{3.13}$$

where $f(\theta_x,\theta_{x+j};\tau_1-\tau_2)$ is a certain complicated function of θ_x and θ_{x+j}. In particular, for $\tau_1-\tau_2\sim 0$, we have

$$f(\theta_x,\theta_{x+j};\tau_1-\tau_2)$$
$$\simeq\Big[\theta(\tau_2-\tau_1)e^{-i\theta_x}+\theta(\tau_1-\tau_2)e^{-i\theta_{x+j}}\Big]$$
$$\times\Big[\frac{\gamma^{-2}e^{\beta\mu_\eta}}{(1+e^{\beta\mu_\eta-i\theta_x})(1+e^{\beta\mu_\eta-i\theta_{x+j}})}$$
$$+\frac{\gamma^2e^{\beta\mu_\phi}}{(1-e^{\beta\mu_\phi+i\theta_x})(1-e^{\beta\mu_\phi+i\theta_{x+j}})}\Big]. \tag{3.14}$$

C. Integration over zero modes θ_x

Let us make a rough estimation of θ_x-integration in (3.10) by using (3.11), (3.13), and (3.14). We shall obtain a simple result that θ_x can be treated as small fluctuations around the minimum $\theta_x = 0$. To see this, we start by replacing U_{xj}-dependent part in $A_2(\theta,U)$ by its average. Explicitly, we start with the following general expression of Z up to ${\rm O}(U^2)$ after the θ-integration;

$$Z = \int[dU]\exp(A_2(U)),$$
$$A_2(U) = \int d\tau_1d\tau_2C_U(\tau_1-\tau_2)\sum_{x,j}U_{xj}^\dagger(\tau_1)U_{xj}(\tau_2), \tag{3.15}$$

where C_U depends on T and n, and it is shown to be positive for $\tau_1-\tau_2\sim 0$ by (3.14). From this result, configurations like $U_{xj}^\dagger(\tau_1)U_{xj}(\tau_2)\sim 1$ dominate the functional integral at least for $(\tau_1-\tau_2)\sim 0$. Then one can assume

$$\langle U_{xj}^\dagger(\tau_1)U_{xj}(\tau_2)\rangle \propto e^{-m_0|\tau_1-\tau_2|}\ \text{ or }\ \sim|\tau_1-\tau_2|^{-r},\ r>0. \tag{3.16}$$

By substituting the correlator (3.16) into (3.13) and by the decoupling, the τ-integrals in $A_2(U)$ takes the form,

$$\int d\tau_1d\tau_2f(\theta_x,\theta_{x+j};\tau_1-\tau_2)\langle U_{xj}^\dagger(\tau_1)U_{xj}(\tau_2)\rangle$$
$$= -b(\nabla_j\theta_x)^2, \tag{3.17}$$

with a positive constant b. From (3.11) and (3.17), the general form of the effective action for θ_x is fixed as

$$A_2(\theta) = -\sum_x\Big[b\sum_j(\nabla_j\theta_x)^2+\ln F(\theta_x)\Big]. \tag{3.18}$$

The first kinetic term generates correlations among θ_x and disfavors configurations of independent θ_x's. In such a field theory (having infinite degrees of freedom) the limit $b\to 0$ is singular, and one obtains qualitatively correct results for $b>0$ by expanding θ_x around $\theta_x=0$, the minimum of $\ln F(\theta_x)$, up to $O(\theta_x^2)$. In other words,

$\lambda_x = \theta_x/\beta = 0$ in the ground state and their excitations are massive. This implies that the effects of λ_x are screened by fluctuations (radiative corrections) of chargeons and fluxons, and the constraint (2.14) becomes irrelevant at low energies. This nonperturbative observation supports the "weak-coupling" perturbative calculation in gauge theories of nonrelativistic fermions which is often used in random-phase approximations, etc. An intuitive picture of the above screening phenomenon at long wavelength is supplied by block-spin-type renormalization-group (RG) transformations. At first, numbers of chargeons and fluxons at each lattice site are well-defined quantities. However, through block-spin RG transformations, the number of particles at each site of the transformed larger lattice becomes ambiguous. As a result, the local constraint (2.14) tends to irrelevant at long wavelengthes.

D. Confinement-deconfinement phase transition

The above result that the ground state of θ_x is $\theta_x = 0$ and its excitations are massive leads that the question whether the PFS takes place or not is determined just by the dynamics of the transverse gauge field U_{xj}. The PFS corresponds to the deconfinement phase of U_{xj}.

Thus, to evaluate A_2 of (3.13), we simply put $\theta_x = 0$ in $f(\theta_x, \theta_{x+j}; \tau_1 - \tau_2)$. Then by using (3.9), we have $f(0,0;\tau_1-\tau_2) = \gamma^{-2}n(1-n)+\gamma^2 n(1+n)$. So A_2 reduces to[21]

$$
\begin{aligned}
A_2(\theta_x = 0, V_0 U_{xj}) &= -\beta \sum_{x,j} \frac{V_0^2}{2ma^2} \\
&+ D_2 V_0^2 \sum_{xj} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 U^\dagger_{xj}(\tau_1) U_{xj}(\tau_2) \\
&= V_0^2 \sum_{x,j} \Big[-\frac{\beta}{2ma^2} + D_2 \beta^2 U^\dagger_{xj,0} U_{xj,0} \Big], \\
D_2 &\equiv \frac{1}{4m^2a^4} \Big(\frac{n(1-n)}{\gamma^2} + \gamma^2 n(1+n) \Big), \qquad (3.19)
\end{aligned}
$$

where we have introduced Fourier decomposition of $U_{xj}(\tau)$,

$$
\begin{aligned}
U_{xj}(\tau) &= \sum_{n \in Z} U_{xj,n}\, e^{i\omega_n \tau}, \ \omega_n \equiv \frac{2\pi n}{\beta}, \\
\sum_n U^\dagger_{xj,n} U_{xj,n+m} &= \delta_{m0}, \qquad (3.20)
\end{aligned}
$$

where the second line represents $U^\dagger_{xj}(\tau) U_{xj}(\tau) = 1$. The higher-order terms of $A_{\rm eff}$ in the hopping expansion can be calculated in a straightforward manner. There appear plaquette terms (i.e., magnetic terms) like

$$
A_{\rm plaq} = C_{\rm plaq} \sum_x U_{x2,0} U_{x+2,1,0} U^\dagger_{x+1,2,0} U^\dagger_{x1,0} + \text{H.c.} \quad (3.21)
$$

Their coefficient $C_{\rm plaq}$ is positive and also getting large at low T.

From (3.19) and (3.21), the static-modes of the transverse gauge field $U_{xj,0}$ are enhanced at large β and nonvanishing V_0, i.e., $\langle U_{xj,0} \rangle \equiv U_0 \sim 1$ (up to irrelevant pure-gauge freedoms). Explicitly, we estimate that $U_{xj,0}$ has nonvanishing expectation value U_0 when the coefficient of $|U_{xj,0}|^2$ in A_2 is larger than unity,

$$
D_2 V_0^2 \beta^2 > 1. \qquad (3.22)
$$

Since V_0 is a decreasing function of T, this happens at lower T's. On the other hand, the other oscillating modes $U_{xj,n\neq 0}$ are strongly suppressed by (3.20), i.e., $\sum_{n\neq 0} |U_{xj,n}|^2 \ll 1$. This implies that, at low T where the condition (3.22) holds, the gauge dynamics is in the deconfinement phase, where $\langle U_{xj}(\tau) \rangle \simeq U_0$ (up to gauge freedom) and the fluctuations ΔA_{xj} are small. To support this conclusion, we estimate the expectation value P of the Polyakov line, an order parameter of the confinement-deconfinement phase transition (CDPT) at finite T,[19] for finite U_0 as

$$
P \equiv \Big\langle \prod_\tau U_{xj}(\tau) \Big\rangle \sim (U_0)^{\frac{\beta}{\Delta\tau}} \neq 0. \qquad (3.23)
$$

Therefore, PFS takes place at low T such that (3.22) holds. The quasiexcitations there are the chargeons η_x, fluxons ϕ_x, and the gauge bosons A_{xj}. The transition temperature $T_{\rm PFS}$ can be estimated from the "GL theory" (3.19) by setting the coefficient of $|U_{xj,0}|^2$ unity;[19]

$$
\begin{aligned}
& D_2 V_0^2 \beta^2 |_{T=T_{\rm PFS}} \\
&= \frac{V_0^2(T_{\rm PFS})}{4m^2a^4 k_{\rm B}^2 T_{\rm PFS}^2} \Big(\frac{n(1-n)}{\gamma^2} + \gamma^2 n(1+n) \Big) \simeq 1. \qquad (3.24)
\end{aligned}
$$

The analysis using the method by Polyakov and Susskind[19] predicts that the phase transition at $T_{\rm PFS}$ is smooth as in CSS,[7,9] so our hopping expansion of $A_{\rm eff}$ in powers of $V_0 U_{xj}$ is justified a posteriori.[20] Concerning to the order of the CDPT of the present system, we remind that the usual lattce gauge theory on a 3D spatial lattice exhibits a second-order CDPT, while the gauge theory on a 2D spatial lattice exhibit a CDPT of the Kosterlitz-Thouless (KT) type.[19] However, the present effective lattice gauge theory (3.19) on a 2D lattice has a stronger correlations along τ-direction than the usual coupling $|\partial_\tau U_{xj}|^2$, so the CDPT may be of second order instead of the KT type. Further study is required to clarify this point.

For comparison, let us list up the temperature regions in which the deconfinement phases take place for various models;

Model	Region of deconfinement
QED, QCD($7 \leq N_f$)	$0 \leq T$
QCD($0 \leq N_f \leq 7$)	$0 < T_{\rm CD} < T$
t-J Model	$0 \leq T < T_{\rm CSS}$
FQH System	$0 \leq T < T_{\rm PFS}$

(3.25)

One may feel it strange that the deconfinement region is at high T for QED and QCD, while it is low T for the t-J model and the present FQH system. From Ref.[19], the condition of deconfinement is estimated as $\beta g_{\rm G}^2 < 1$ for a gauge theory with a gauge coupling constant $g_{\rm G}$. When $g_{\rm G}$ is a T-independent constant as in QED and QCD, the deconfinement occurs at high T. However, in the strongly-correlated electron systems, the gauge theory in question is an effective theory obtained by integrating out the "electron" degrees of freedom, and the resulting gauge coupling may be T-dependent. In fact, we estimated $g_{\rm G}^2 \propto T^3$ for the t-J model,[7] which implies the result (3.25). By repeating the same argument as Ref.[7], we obtain the same result $g_{\rm G}^2 \propto T^3$ for the present gauge model, which is consistent with the condition (3.22).

E. Numerical results

Let us estimate $T_{\rm PFS}(\nu)$ numerically by using (3.24) with $V_0(T)$ determined by (3.2). We consider the following choice of the Coulomb coupling;

$$g = (0.1 \sim 1) \times \frac{e^2}{\epsilon \ell}, \tag{3.26}$$

and the parameters,

$$\begin{aligned} &a = \ell, \quad B^{\rm ex} = 10[{\rm T}], \\ &m = 0.067 \times m_{\rm electron}, \quad \epsilon = 13. \end{aligned} \tag{3.27}$$

The parameter γ is ν-independent and calculated by (2.37) as

$$\gamma = 0.96 \sim 0.69, \tag{3.28}$$

for (3.26).

We plot $T_{\rm PFS}(\nu)$ at various fillings $\nu = p/(2pq \pm 1)$ for $g = 0.1e^2/(\epsilon\ell)$ in Fig.2 and for $g = e^2/(\epsilon\ell)$ in Fig.3. At $\nu = 1/2$,

$$T_{\rm PFS}(1/2) = 5.7 \sim 6.7{\rm K}. \tag{3.29}$$

$T_{\rm PFS}(\nu)$ seems consistent with the experiments.[22] We note that the highest temperature at which FQHE is observed is $T_{\rm BC}(< T_{\rm PFS})$ since Bose condensation of fluxons is necessary for FQHE (See Sec.IV C for details).

Let us estimate the effect of phase fluctuations of U_{xj} upon PFS. For this purpose, we compare $T_{\rm PFS}$ with its "mean-field" value T_{V_0}. T_{V_0} is defined by the temperature at which V_0 starts to develop and is calculated by setting the coefficient of the V_0^2 term in $A_{\rm eff}$ with $|U_{xj,0}| = 1$ to vanish, i.e., $-\beta/(2ma^2) + \beta^2 D_2 = 0$ as

$$T_{V_0} = \frac{1}{2ma^2 k_{\rm B}} \Big(\frac{n(1-n)}{\gamma^2} + \gamma^2 n(1+n) \Big). \tag{3.30}$$

This $T_{V_0}(\nu)$ is also plotted in Fig.2,3. At $\nu = 1/2$,

$$T_{V_0}(1/2) = 16 \sim 19{\rm K}, \tag{3.31}$$

which is about 3 times larger than $T_{\rm PFS}(1/2)$ of (3.29). This demonstrates the importance of fluctuations of the gauge-field A_{xj} in PFS, which reduce the critical temperature of PFS significantly from T_{V_0} down to $T_{\rm PFS}$. The similar large effect of gauge field was found also in CSS.[8]

Let us next consider the masses of chargeon and fluxon, $m_\eta(\nu)$ and $m_\phi(\nu)$. To calculate m_η and m_ϕ, one may set $V_{xj} \to V_0$ in the $J_{xj}^\dagger V_{xj}$ term in Lagrangian (2.24) at low T. Then this term gives rise to the hopping terms of η_x and ϕ_x, from which one obtains

$$m_\eta = \frac{\gamma}{V_0} m, \ \ m_\phi = \frac{1}{\gamma V_0} m. \tag{3.32}$$

Their values for (3.26) and (3.27) are plotted in Fig.4,5, which show $m_\eta, m_\phi > m$. At $\nu = 1/2$,

$$m_\eta(1/2) = (6.6 \sim 4.5)m, \quad m_\phi(1/2) = (7.2 \sim 9.5)m. \tag{3.33}$$

Experimentally, the mass of CF, $m_{\rm CF}$, is determined by equating the observed activation energy $E_{\rm ac}$ in the resistivity $\rho_{xx} \propto \exp(-E_{\rm ac}/2k_{\rm B}T)$ to be an energy gap $\omega_{\Delta B} \equiv e\Delta B/m_{\rm CF}$ between the lowest and the next LL formed by the residual magnetic field ΔB felt by CFs, i.e., $m_{\rm CF} = e\Delta B/E_{\rm ac}$. In our theory, chargeons just describe Jain's CFs, so one has $m_\eta = m_{\rm CF}$. The estimate (3.33) of m_η is consistent with the experiments.[22]

One may argue that the scale of the CF mass, m_η, should be set by the Coulomb energy, and not by the electron band mass m. For example, even for the limit $m \to 0$ at which the energy gap between neighboring LL's of electrons diverges, the CF should have a finite mass. The expressions of $m_{\eta,\phi}$ in (3.33) are free from this criticism. They hold just for the physical value of the electron band mass m, and do not imply $m_{\eta,\phi}$ are proportional to m. (The ratios $m_{\eta,\phi}/m$ vary as m varies.) In our approach, we argued in Sec.II E that the residual interactions H_4 of η_x and ϕ_x, which involves m, and the short-range Coulomb interaction $H_{\rm int}$ should relate as (2.36). Thus the formula above may be interpreted as $m_{\eta,\phi} \sim e^2/(\epsilon\ell)$.

In this section, we studied the phase structure of the effective gauge theory, which is defined on a lattice. One may wonder whether the PFS and the CDPT studied on the lattice model survive in the "continuum limit". The CDPT at finite T was first discovered by Polyakov and Susskind[19] in lattice gauge theory. After that, more detailed investigations, including numerical studies and renormalization-group (RG) analyses, confirm the existence of this CDPT in the continuum. The lattice models are regarded there as the effective models of RG, and the transition tepmerature is a RG-invariant quantity. Moreover, recent studies on gauge theories in the continuum spacetime, which is closely related to the present gauge system, support the existence of CDPT at finite T. For example, in Ref.[23], the $U(1)$ gauge theory in $(2+1)$ dimensions coupled with multi-fermions is studied by the

RG method. There a nontrivial infrared fixed point is found. This result indicates that a deconfinement phase is realized at low T. In Ref.[24], finite$-T$ properties of the compact $U(1)$ gauge theory is studied by using the Georgi-Glashow model in $(2+1)$ dimensions. From the instanton calculation, it is shown that a CDPT occurs at finite T as predicted by the lattice gauge theory.[19] These facts support that our results of the PFS and CDPT obtained in this section remain valid even in the "continuum limit".

IV. GROUND STATE, LOW-ENERGY EXCITAITONS, EM TRANSPORT

A. Ground state

In the previous section, we showed that the PFS takes place at low $T < T_{\rm PFS}$. In the PFS states, the chargeons and fluxons are interacting only weakly through the gauge field A_{xj}. In this perturbative phase of gauge dynamics, one can return to the continuum notation. (Recall that the lattice regularization was essential for the strong-coupling phase of gauge dynamics.) Let us study the ground state by putting $V_{xj} = V_0$. Due to the separation pheomenon PFS, the Hamiltonian may be separated effectively into the chargeon part H_η and the fluxon part H_ϕ,

$$H_{\rm eff} = H_\eta + H_\phi, \tag{4.1}$$

where we have safely neglected perturbative mixing effects mediated by A_{xj}. Then the ground state of electrons $|G\rangle_C$ is given by a direct product of the ground state of H_η, $|G\rangle_\eta$, and the ground state of H_ϕ, $|G\rangle_\phi$, i.e.,

$$|G\rangle_C = |G\rangle_\eta |G\rangle_\phi. \tag{4.2}$$

Thus the electron wave function is expressed in the continuum notation as

$$\begin{aligned}
\Psi_e(x_1,\cdots,x_N) &\equiv {}_C\langle 0|C(x_1)\cdots C(x_N)|G\rangle_C \\
&= \Psi_\eta(x_1,\cdots,x_N)\cdot\tilde\Psi_\phi(x_1,\cdots,x_N), \\
\Psi_\eta(x_1,\cdots,x_N) &\equiv {}_\eta\langle 0|\eta(x_1)\cdots\eta(x_N)|G\rangle_\eta, \\
\tilde\Psi_\phi(x_1,\cdots,x_N) &\equiv {}_\phi\langle 0|\phi(x_1)\cdots\phi(x_N)\times \\
&\quad \prod_k \exp[2iq\int d^2y\theta(x_k-y)\phi^\dagger(y)\phi(y)]|G\rangle_\phi,
\end{aligned} \tag{4.3}$$

where $\theta(x)$ is the azimuthal angle of x.

The fluxon part of Lagrangian density is given for $a_{xj}=0$ by

$$\begin{aligned}
\mathcal{L}_\phi &= -\phi^\dagger(\partial_\tau - \mu_\phi)\phi \\
&\quad -\frac{V_0\gamma}{2m}|(\partial_j + iA_j^\phi)\phi|^2 - H_\phi^{\rm LR}(\phi^\dagger\phi),
\end{aligned} \tag{4.4}$$

with

$$A_j^\phi(x) \equiv 2q\int d^2y \frac{\partial}{\partial x_j}\theta(x-y)(\phi^\dagger\phi(y)-\rho). \tag{4.5}$$

Here we have included the term $H_\phi^{\rm LR}$ to $\mathcal{L}_\phi$, which expresses a long-range part of the Coulomb repulsion between fluxons. We partition the full long-range Coulomb repulsion as follows;

$$\begin{aligned}
H^{\rm LR} &= \frac{g}{2}{\int}' d^2x d^2y (C^\dagger C(x)-\rho)\frac{\ell}{|x-y|}(C^\dagger C(y)-\rho) \\
&\quad \Rightarrow H_\eta^{\rm LR} + H_\phi^{\rm LR}, \\
H_\eta^{\rm LR} &= \frac{g_\eta}{2}{\int}' d^2x d^2y (\eta^\dagger\eta(x)-\rho)\frac{\ell}{|x-y|}(\eta^\dagger\eta(y)-\rho), \\
H_\phi^{\rm LR} &= \frac{g_\phi}{2}{\int}' d^2x d^2y (\phi^\dagger\phi(x)-\rho)\frac{\ell}{|x-y|}(\phi^\dagger\phi(y)-\rho), \\
g_\eta + g_\phi &= g \sim \frac{e^2}{\epsilon\ell},
\end{aligned} \tag{4.6}$$

where the prime indicates that the integration range is $|x-y|>a$. We note that the recent experiment[25] suggests that the CFs (i.e., chargeons η_x) have residual long-range Coulomb repulsion, i.e., $g_\eta \neq 0$.

The above fluxon Lagrangian $\mathcal{L}_\phi$ has just the same form as the Lagrangian of the composite bosons (CB) in the CB theory of FQHE[18], except that each fluxon here carries $2q$ CS flux quanta, while each CS boson carries $2q+1$ flux quanta. (We shall discuss the related chargeon-fluxon formalism of CB systems in Sec.V B, where $\phi(x)$ carries $2q+1$ flux quanta.) In Ref.[18], the ground state is given by the Bose condensation of CBs and the wave function is calculated explicitly. Then one may consider $|G\rangle_\phi$ also as the Bose-condensed state of $\phi(x)$ as in the CB theory. By following the derivation in Ref.[18], we obtain

$$\begin{aligned}
{}_\phi\langle 0|\phi(x_1)\cdots\phi(x_N)|G\rangle_\phi &= \prod_{i<j}|z_i-z_j|^{2q}\ e^{-\frac{1}{4\ell_\phi^2}\sum_{j=1}^N|z_j|^2}, \\
\ell_\phi \equiv \frac{1}{\sqrt{eB_\phi}},\ B_\phi &= \frac{4\pi q\rho}{e},
\end{aligned} \tag{4.7}$$

where z_j's are the complex coordinates of N fluxons, $z_j \equiv x_{j1}+ix_{j2}$. The CS factor $\exp[2iq\int d^2y\theta(x-y)\phi_y^\dagger\phi_y]$ in $\tilde\Psi_\phi$ of (4.3) produces a phase factor of $|z_i-z_j|^{2q}$, changing $|z_i-z_j|^{2q}$ in (4.7) to $(z_i-z_j)^{2q}$ in $\tilde\Psi_\phi$ of (4.3). Thus the fluxon part $\tilde\Psi_\phi$ in (4.3) becomes

$$\tilde\Psi_\phi(x_1,\cdots,x_N) = \prod_{i<j}(z_i-z_j)^{2q}\ e^{-\frac{1}{4\ell_\phi^2}\sum|z_j|^2}. \tag{4.8}$$

From (4.8), one sees that fluxons give rise to the factor $\prod_{i<j}(z_i-z_j)^{2q}$ that describes *correlation holes* as it is expected.

Let us turn to the chargeon part $\Psi_\eta(x_1,\cdots,x_N)$. As explained in Sec.II D, at $\nu = p/(2pq\pm1)$, the chargeons η_x feel the residual field

$$\Delta B = B^{\rm ex} - B_\phi = \pm \frac{2\pi\rho}{ep}, \tag{4.9}$$

and they fill up the p LL's formed by ΔB, giving rise to IQHE. This observation clearly implies that the chargeons are nothing but Jain's CFs.[1] The wave function Ψ_η in (4.3) is known for $p = 1$ as the Slater determinant,

$$\Psi_\eta(x_1, \cdots, x_N) = \prod_{i<j} (z_i - z_j)\ e^{-\sum |z_j|^2/(4\ell_\eta^2)},$$
$$\ell_\eta \equiv \frac{1}{\sqrt{e\Delta B}}. \tag{4.10}$$

Thus (4.8) becomes just the Laughlin's wave function for $\nu = 1/(2q+1)$,

$$\Psi_e(x_1, \cdots, x_N) = \prod_{i<j} (z_i - z_j)^{2q+1}\ e^{-\sum |z_j|^2/(4\ell^2)},$$
$$\frac{1}{\ell_\phi^2} + \frac{1}{\ell_\eta^2} = \frac{1}{\ell^2} \equiv eB^{\rm ex}. \tag{4.11}$$

For $p \neq 1$, one needs the wave function Ψ_η of IQHE with the filling factor $\nu_\eta = p = 2, 3, \cdots$ to obtain the full wave function Ψ_e.

At $\nu = 1/(2q)$ $(p = \infty)$, $\Delta B = 0$, so chargeons behave as quasi-free fermions in vanishing magnetic field. Beyond MFT, fluctuations of A_{xj} meditate interactions among chargeons (and fluxons), and may generate non-Fermi-liquid-like behaviors.[23]

B. Low-energy excitations

Let us study the low-energy excitations in the PFS state with $\nu = p/(2pq \pm 1)$ at which FQHE is observed experimentally.

We first consider the excitations in the chargeon sector. In the leading approximation for PFS in which the interactions by the gauge field are ignored, the chargeons move in a reduced magnetic field ΔB of (2.31) as explained and fill up the first p LL's formed by ΔB. So the possible excitations are the inter-LL excitations from the p-th LL to the $p+1$-th LL, hence their energy gap is $\omega_\eta \equiv e\Delta B/m_\eta$. It is estimated as

$$\omega_\eta = \frac{e\Delta B}{m_\eta} = \frac{\Delta B}{B^{\rm ex}} \frac{m}{m_\eta} \omega_B \simeq \frac{\nu}{p} \cdot \frac{\nu}{3.2} \cdot 200{\rm K}$$
$$< T_{\rm PFS} \sim 10\nu \text{ K for } \frac{\nu}{p} < 0.16, \tag{4.12}$$

where we used the rough estimation, $m_\eta/m \sim 3.2/\nu$ (Fig.4), $T_{\rm PFS} \sim 10\nu$ (Fig.2) for $g = 0.1e^2/(\epsilon\ell)$, and $\omega_B \simeq 200$K. Since ω_η can be smaller than $T_{\rm PFS}$, these chargeon excitations may be genuine excitations connecting PFS states.

In the fluxon sector, there are two types of excitations. The first one is described by small fluctuations φ_x around the fluxon condensate (here we consider $T \simeq 0$) as

$$\phi(x) = \sqrt{\rho} + \varphi(x). \tag{4.13}$$

To calculate their energy gap, we follow the similar calculation in CB theory.[18,26] By substituting (4.13) into the Lagrangian (4.4), expanding it up to $O(\varphi^2)$, and making a Bogoliubov transformation to diagonalize it, one finds that the field $\varphi(x)$ describes excitations with an energy gap $\omega_\phi \equiv eB_\phi/m_\phi$. It is estimated for $g = 0.1e^2/(\epsilon\ell)$ as

$$\omega_\phi = \frac{eB_\phi}{m_\phi} = \frac{B_\phi}{B^{\rm ex}} \frac{m}{m_\phi} \omega_B$$
$$\simeq \frac{2pq}{2pq \pm 1} \cdot \frac{\nu}{3} \cdot 200{\rm K} \simeq 60\nu \text{ K}, \tag{4.14}$$

which is always larger than $T_{\rm PFS} \sim 10\nu$. Therefore these "excitations" are not genuine ones in the PFS states.

The proper fluxon excitations in the PFS states are described by the second type of excitations; i.e., vortices. As in the CB theory of FQHE[18], they are given by the configurations as

$$\phi(x) \simeq \sqrt{\rho}\ e^{iN\theta(x)}, \quad |x| \gg \ell, \tag{4.15}$$

where N is an integer. A single vortex carries the electric charge $ceN/(2q)$. Their kinetic energy vanishes due to cancellation with $A_{xj}^{\rm CS}$ in the covariant derivative, so an energy gap is supplied from the interaction $H_\phi^{\rm LR}$ in (4.6). The vortex energy is to be obtained by substituting the vortex configuration (4.15) into $H_\phi^{\rm LR}$. In the CB theory for the states of FQHE at $\nu = 1/(2q+1)$, a similar estimation of the vortex energy was done[27], giving rise to the single-vortex energy E_V for $N = 1$ as

$$E_V \simeq 0.04 \frac{e^2}{\epsilon\ell} \simeq 6{\rm K}, \tag{4.16}$$

with $e^2/(\epsilon\ell) \simeq 160$K. Because this E_V is of the same order as $T_{\rm PFS}$, they are in charge of genuine excitations.

The estimation (4.16) is in agreement with the numerical calculation by using the Laughlin wave function.[27,28] However, there are still some discrepancy with the experimentally observed activation energies in FQH states, which are smaller than (4.16) by a factor $2 \sim 3$. From this fact, more careful studies are required to calculate the energy gap in the fluxon sector in the present formalism.

Let us note that the above chargeon excitations with the energy gap ω_η and the fluxon vortices with the energy gap E_V are *two independent types* of excitations, both of which are supported in the PFS states. As we shall see in Sec.IV C, chargeons and fluxons contribute to the resistivity ρ_{xx}, ρ_{xy} in a different manner. Therefore, by measuring activation energies in two quantities ρ_{xx} and ρ_{xy}, one may identify two independent excitations. Such excitations should be identified with the above chargeon excitations and fluxon vortices. Further quantitative study of this problem is welcome to show the appropriateness of the present chargeon-fluxon theory of CFs.

Finally, we comment on the case of $\nu = 1/(2q)$. In this case, chargeons move in a vanishing magnetic field $\Delta B = 0$, so they form a Fermi line if the interactions with the gauge field are ignored. Although, the perturbative gauge interactions may give rise to a non-Fermi-liquid-like behavior,[23] the chargeon excitations may remain gapless as in a usual Fermi-liquid theory. Thus the states are no more incompressive and no FQH effect will be observed (See next subsection). In the fluxon sector, the fluxon vortices with the energy (4.16) exist as low-energy excitations with a gap as in the previous case of $\nu = p/(2pq \pm 1)$.

C. EM transport property

Let us consider the EM transport properties of the PFS states. The response functions of electrons is calculated from the EM effective action $A_{\rm EM}[a_{xj}]$ defined by

$$\int [dU] \exp(A_{\rm eff}[a_{xj}, V_0 U_{xj}]) = \exp(A_{\rm EM}[a_{xj}]), \qquad (4.17)$$

where we have set $\theta_x = 0$ as in the previous calculations and also showed the dependence of a_{xj} explicitly. In the PFS states, fluctuations of the dynamical gauge field A_{xj} are small, so $A_{\rm eff}[a_{xj}, V_0 U_{xj}]$ can be expanded in powers of A_{xj} up to $O(A^2)$ as

$$A_{\rm eff}[a_{xj}, V_0 U_{xj}] = -\sum_{x,y,i,j} \Big[(A + cea)_{xi} \Pi^{ij}_{\phi\, xy} (A + cea)_{yj}$$
$$+ (A + (1-c)ea)_{xi} \Pi^{ij}_{\eta\, xy} (A + (1-c)ea)_{yj} \Big], \qquad (4.18)$$

where $\Pi^{ij}_{\phi(\eta)}$ is the polarization tensor of $\phi_x(\eta_x)$. $A_{\rm EM}[a_{xj}]$ is obtained by making Gaussian integration over $A_{xj} (\in \mathbf{R})$ as

$$A_{\rm EM}[a_{xj}] = -e^2 \sum a_{xi} \Pi^{ij}_{xy} a_{yj}\,,$$
$$\Pi = (\Pi_\phi^{-1} + \Pi_\eta^{-1})^{-1}, \qquad (4.19)$$

where Π is nothing but the response function of electrons. Then we obtain the simple formula for the resistivity,

$$\rho = \rho_\eta + \rho_\phi\,,$$
$$\rho \equiv \frac{1}{e^2 \Pi},\ \rho_\eta \equiv \frac{1}{e^2 \Pi_\eta},\ \rho_\phi \equiv \frac{1}{e^2 \Pi_\phi}, \qquad (4.20)$$

where $\rho_{(\eta,\phi)}$ are 2×2 *resistivity tensor* of electrons, chargeons, and fluxons, respectively. Both chargeons and fluxons contribute additively to ρ. This result obviously reflects the composite nature (2.12) even though the PFS takes place.

The parameter c in (2.25) expresses arbitrariness to choose the reference state from which the relative EM charges (2.25) are measured. Then it is natural that the formula (4.20) does not depend on c. In high-$T_{\rm c}$ cuprates, there exists exactly the same arbitrariness in the EM charges of holons and spinons.[9] There, the formula of ρ that corresponds to (4.20) is known as the Ioffe-Larkin formula.[29]

What is the contribution to the electric transports from fluxons? Let us consider the FQH state at $\nu = p/(2pq \pm 1)$ first. In the CB theory of FQHE[18], CBs form a Bose condensate and each CB carries $(2q+1)$-flux quanta and gives rise to $\rho_{xy} = (2q+1)h/e^2$. Because each fluxon in the present formalism carries $2q$ flux quanta and certainly contributes to ρ_{xy} in the same manner as those CBs do, we have $\rho_{\phi\, xy} = 2qh/e^2$. Likewise, $\rho_{\phi\, xx} = 0$ because of the superfluidity of the fluxon condensation. On the other hand, the chargeons fill up the p LL's of ΔB and so contribute with $\rho_{\eta\, xy} = \pm h/(pe^2)$ and $\rho_{\eta\, xx} = 0$ as in the IQHE. Thus, from (4.20), we obtain

$$\rho_{xy} \frac{e^2}{h} = 2q \pm \frac{1}{p} = \frac{1}{\nu}, \quad \rho_{xx} = 0, \qquad (4.21)$$

which are actually observed in the experiments.

We stress that chargeons exhibit IQHE as long as $T < T_{\rm PFS}$ whatever fluxons behave as explained at the end of Sec.II D. However, to exhibit FQHE like the Laughlin state (4.8) and the resistivity (4.21), fluxons must Bose-condense. Thus FQHE takes place not at $T < T_{\rm PFS}$ but $T < T_{\rm BC} (< T_{\rm PFS})$.[30]

How about the resistivity in the compressible states like $\nu = 1/(2q)$? The fluxons contribute to $\rho_{\phi\, xy} = 2qh/e^2$ as before, whereas the chargeons are effectively in the vanishing magnetic field and form a Fermi line as explained before. For such a fermionic system, $\rho_{\eta xx} \neq 0$, and $\rho_{\eta xy} = 0$. This result shall persist although the gauge field may influences the transport properties of the chargeons. Therefore, the formula (4.20) becomes

$$\rho_{xy} \frac{e^2}{h} = 2q = \frac{1}{\nu}, \quad \rho_{xx} \neq 0, \qquad (4.22)$$

as is also observed experimantally.

Finally, let us discuss the behavior of ρ_{xx} in the region $T_{\rm BC} < T < T_{\rm PFS}$. In this region, vortices of fluxons are activated and Bose condensation diappears. Thus fluxons give rise to a nonvanishing contribution to the resistivity, $\rho_{xx} \neq 0$, so the FQH states disappear as we mentioned above. There may be some anomalous behavior of ρ_{xx} as a function of T that starts to appear at $T_{\rm PFS}$, which reflects the change of quasiparticles at $T_{\rm PFS}$. In fact, at $T < T_{\rm PFS}$, ρ_{xx} has contributions from chargeons and (uncondensed) fluxons (and perturbative gauge bosons), whereas at $T > T_{\rm PFS}$, ρ_{xx} has a contribution from electrons, the quasiexcitations there, which may interact with impurities. We recall a similar change in high-$T_{\rm c}$ cuprates, where anomalous behaviors of various physical quantities start to appear at $T = T_{\rm CSS}$, below which the anomalous metallic phase is realized. For example, the dc resistivity exhibits a linear-T behavior below $T_{\rm CSS}$, which is consistent with the perturbative calculations using a system of bosons interacting with a

massless gauge field. In the present FQH states, the chargeons still give rise to $\rho_{\eta\, xx} = 0$ for $T_{\rm BC} < T < T_{\rm PFS}$ due to IQHE, while fluxons interact with a massless perturbative gauge field *and* fluxons themselves. Thus we expect that the linear-T behavior in $\rho_{\phi\, xx}$ caused by the coupling to gauge field shall be modified further by the self-interactions. We conjecture that the conductivity $\sigma_{\eta\, xx}$ acquires a reduction factor, which gives rise to a reduction factor in $\rho_{\eta\, xx}$. In Fig.6 the expected T-dependence of ρ_{xx} is illustrated.

V. DISCUSSION

A. Conclusion

In this paper we studied the low-energy quasiexcitations of the fermionic CS gauge theory for the CF. We show that phenomenon which we call PFS is essential for the CF to appear as a quasiparticle and the PFS takes place at low $T < T_{\rm PFS}$. The PFS can be understood as a deconfinement transition of the dynamical gauge field. As a result of the PFS, an electron or a CS fermion splinters off a chargeon and a fluxon. Below $T_{\rm BC}$, fluxons Bose condense, cancelling the external magnetic field partly and producing correlation holes in the desired form. The chargeons move in a reduced manetic field ΔB to form an incompressible fluid as the Jain's CFs. The system below $T_{\rm BC}$ exhibit FQHE. We estimated the transition temperature $T_{\rm PFS}$ as well as the CF and fluxon masses. From the local gauge invariance with the dynamical gauge field, chargeons and fluxons contribute to the resistivity tensor of electrons additively. This formula describes well the observed results of resistivity. For the important problem in future, we consider the calculation of $T_{\rm BC}$. It is not only important for comparison with experiments but also challenging theoretically. To compare with the experiments, we have calculated $T_{\rm PFS}$, m_η, m_ϕ. Also we conjectured ρ_{xx} should have an extra damping factor below $T_{\rm PFS}$. Finally we stress that PFS is closely related to CSS in high-$T_{\rm c}$ cuprates. Actually, theoretical techniques are common and both phenomena are understood as separations of the degrees of freedom describing electrons. In Fig.7 we illustrate the phase strucutres of the gauge dynamics for the present FQH system and the t-J model of high-$T_{\rm c}$ cupper oxides.[7–9] We note that a deconfinement phase may be further classified into two independent phases; (I) Coulomb phase in which the gauge bosons are massless and (II) Higgs phase in which the gauge bosons acquire a finite mass. The two phase strucures in Fig.7 look almost similar each other, except that the present FQH system has one Higgs phase that is generated by the Bose condensation of fluxons via the well-known Anderson-Higgs mechanism, while the t-J model has two Higgs phases reflecting the two different mechanisms[7–9]; (I) Higgs I induced by the spin-gap order parameter and (II) Higgs II induced by the Bose condensation of holons.

B. PFS in composite bosons

Let us comment on the CB approach to the FQHE in terms of the "chargeon" and "fluxon". First we intorduce the CS boson operator $\psi^{\rm B}_x$ from the electron operator C_x by the following CS transformation;

$$C_x = \exp[(2q+1)i\sum_y \theta_{xy}\psi^{\rm B\,\dagger}_y\psi^{\rm B}_y]\psi^{\rm B}_x. \tag{5.1}$$

Each CS boson is viewed as a composite of an electron and $(2q+1)$ quanta of CS fluxes. Then the "chargeon" operator $\eta^{\rm B}_x$ and the "fluxon" operator $\phi^{\rm B}_x$ are intoroduced as constituents of a CS boson just like the case of CS fermion in (2.12) as follows;

$$\psi^{\rm B}_x = \phi^{\rm B}_x\eta^{\rm B}_x, \tag{5.2}$$

where both $\phi^{\rm B}_x$ and $\eta^{\rm B}_x$ are canonical bosons. Each fluxon $\phi^{\rm B}_x$ carries an *odd* number $(2q+1)$ of flux quanta. One may follow the analyses of Sec.II and Sec.III and obtain a critical temperature $T^{\rm B}_{\rm PFS}$ below which the gauge dynamics is realized in a deconfinement phase and ther PFS takes place.

In the FQH states, both chargeons and fluxons Bose condense and fluxons create the correlation holes. In this sense, the fluxon field corresponds to the longitudinal gauge field in the SM formalism of the CB. However, the PFS is essential for the chargeon and fluxon to behave as weakly interacting quasiexcitations and to satisfy the usual canonical commutation relations.

C. Comment on the theory of Shanker and Murthy

As we mentioned in the introduction, recently there appeared some papers for the low-energy quasiexcitations in the FQH state by using the CS theory.[4] Let us briefly discuss the SM's approach among them, in particular, the treatment of the CS constraint and field operators which they introduced in order to describe quasiparticles.

Lagrangian of the CS theory is given as follows,

$$L = i\bar\psi\partial_0\psi + a_0\Big(\frac{\vec\nabla\times\vec a}{2\pi l} - \bar\psi\psi\Big) - \frac{1}{2m}|(-i\vec\nabla + e\vec A + \vec a)\psi|^2, \tag{5.3}$$

where ψ is the CS particle field (a boson or fermion), $\vec A$ is the external magnetic field and a_0 is the Lagrange multiplier for the CS constraint,

$$\frac{\vec\nabla\times\vec a}{2\pi l} - \bar\psi\psi = 0. \tag{5.4}$$

Then SM introduced *a composite particle operator* ψ_{CP};

$$\psi(x,y,t) = \exp[i\Theta(x,y,t)]\psi_{CP}(x,y,t), \tag{5.5}$$

$$\Theta(x,y,t) = -\int_{-\infty}^{t} a_0(x,y,t')dt'. \tag{5.6}$$

The Lagrangian (5.3) is rewritten in terms of ψ_{CP},

$$L = i\bar{\psi}_{CP}\partial_0\psi_{CP} + a_0\frac{\vec{\nabla}\times\vec{a}}{2\pi l} - \frac{1}{2m}|(-i\vec{\nabla} + e\vec{A} + \vec{a} + 2\pi l\vec{P})\psi_{CP}|^2, \tag{5.7}$$

where

$$\vec{P}(x,y,t) = \frac{1}{2\pi l}\vec{\nabla}\Theta(x,y,t). \tag{5.8}$$

From (5.6) and (5.8),

$$\vec{\nabla}^{-1}\cdot\partial_0\vec{P}(x,y,t) = -\frac{1}{2\pi l}a_0(x,y,t). \tag{5.9}$$

By substituting (5.9) into the second term of (5.7), it is easily seen that the CS gauge field $\vec{a}$ and $\vec{P}$ become canonical conjugate variables with each other. More precisely let us define the following variables after going into the momentum space $\vec{q}$,

$$P = -i\frac{\vec{q}}{q}\cdot\vec{P}, \quad \delta\vec{a} = e\vec{A} + \vec{a}, \quad \delta a = i\frac{\vec{q}}{q}\times\delta\vec{a}. \tag{5.10}$$

Then $[\delta a(\vec{q}_1), P(\vec{q}_2)] = (2\pi)^2\delta^2(\vec{q}_1+\vec{q}_2)$.

The local CS constraint on "physical states" appears from the invariance under time-independent gauge transformations; $\psi_{CP}\to e^{i\Lambda}\psi_{CP}$ and $\vec{P}\to\vec{P}-\frac{1}{2\pi l}\vec{\nabla}\Lambda$,

$$\Big(\frac{\vec{\nabla}\times\delta\vec{a}}{2\pi l} - :\psi_{CP}^\dagger\psi_{CP}:\Big)|\text{phys}\rangle = 0, \tag{5.11}$$

where $:\psi_{CP}^\dagger\psi_{CP}: \equiv \psi_{CP}^\dagger\psi_{CP} - \rho$ and SM used (5.11) for deriving the Laughlin's wave function, etc.[4](see later discussion).

In the "new" Lagrangian (5.7), SM treated ψ_{CP} and $\vec{P}$ as *independent commuting dynamical variables* instead of the "original" ones ψ and a_0. But this treatment is *not* legitimate by the following reason; From (5.5) and the fact that $\vec{P}$ and $\vec{a}$ are conjugate operators, the new variable ψ_{CP} must satisfy *a nontrivial nonlocal commutation relation* with the CS gauge field $\vec{a}$ or δa defined by (5.10). Therefore ψ_{CP} cannot be treated as an independent variable that commute with other operators. In other words, SM have *changed* the system from the original one.

From (5.5), it is obvious that the above local gauge symmetry comes from the invariance of the original field ψ under a simultanoeous phase rotation of ψ_{CP} and $\exp(i\Theta)$. Therefore, this gauge symmetry is close to the one in the chargeon-fluxon approach in this paper. As we discussed in this paper, the most important point in the gauge theory is how the above local gauge symmetry and the constraint (5.11) are realized in the ground state and low-energy excitations.[26] This dynamical problem is essential and must be clarified. For example in QED, the Gauss' law constrant, which is similar to (5.11), is *not* satisfied by the low-energy excitations, i.e., the electron and photon. Similarly, in the fermionic CS gauge theory, the multiplier a_0 often acquires a "mass term" from the radiative corrections of the fermion ψ and the CS constraint becomes irrelevant at low energies.

Let us consider the case of CS bosons and examine the SM's derivation of the Laughlin's wave function for $\nu = 1/3$. By ignoring the interaction terms of the "gauge field" (a,P) and the composite boson ψ_{CP} as SM did, Hamiltonian is given as follows from (5.7),

$$H_0 = \frac{1}{2m}|\vec{\nabla}\psi_{CP}|^2 + \frac{\rho}{2m}(\delta a^2 + (6\pi)^2P^2). \tag{5.12}$$

If we *assume* that ψ_{CP} and $(\delta a, P)$ are independent variables as SM did, wave functional of the lowest-energy state of the Hamiltonian (5.12), $\Psi[a,\psi_{CP}]$, is readily obtained. The boson field ψ_{CP} Bose condenses and the system of (a,P) is just a harmonic oscillator (here we ignore quantum fluctuations of ψ_{CP} as SM did),

$$\Psi[a,\psi_{CP}] = \prod_x \delta(\psi_{CP}(x) - \sqrt{\rho})\cdot\prod_{\vec{q}} e^{-\delta a^2(\vec{q})}. \tag{5.13}$$

However it is obvious that the wave functional (5.13) does *not* satisfy the constraint (5.11). The Bose condensation of ψ_{CP} breaks the gauge invariance. Moreover, the Hamiltonian H_0 in (5.12) itself does *not* respect the local gauge symmetry generated by the operator $\Big(\frac{\vec{\nabla}\times\delta\vec{a}}{2\pi l} - :\psi_{CP}^\dagger\psi_{CP}:\Big)$. SM simply ignored the ψ_{CP} part of the wave functional and simply put $\Psi[a,\psi_{CP}] = 1\cdot\prod_{\vec{q}} e^{-\delta a^2(\vec{q})}$ where 1 stands for "wave functional" of the Bose condenesd state. This manipulation is essential for SM's derivation of the Laughlin's wave function. Obviously they confused the field operator in the second quantization and the wave function.[31] From the above discussion, it is obvious that SM's treatment of the CS constraint is *not* satisfactory at all.

Contrary to the SM approach, new dynamical degrees of freedom for the CS fluxes is introduced in the present formalism, i.e., the fluxon field. The fluxon field can be quanatized as canonical bosons and it commutes with other fields like the chargeon and the gauge field. As the phase factor $\exp(i\Theta)$ in the SM approach, the fluxon produces the correlation holes and contains the reminiscence of the inter-LL excitations.

As we stressed in the present paper, PFS is essential to assure the validity of Jain's CF theory (such as the stability of CFs), and to identify the conditions with which PFS is possible is a purely dynamical problem. In this context, we recall there is a similar dynamical problem of gauge theory that is studied as "hidden gauge

symmetry"[32] in a context of composite partcle theory in high-energy particle physics.

Acknowledgement

We dedicate this paper to the memory of Professor Bunji Sakita. He has influenced us through his attitude to research and his character.

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[16] The fluxons can be consistently quantized as hard-core bosons instead of ordinary bosons.[5,6] In this case, the condition (2.37) is changed to $(2ma^2)^{-1}(\gamma^2+\gamma^{-2}) \simeq e^2/(\epsilon\ell)$.
[17] Here we once again emphasize that ambiguity of the parameters γ, g_1, g_2 corresponds to the freedom of in definitions of chargeons and fluxons. We have determined these parameters by the condition that the quasiparticles must be free as much as possible.
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[20] Strictly speaking, to obtain the full Ginzburg-Landau theory of U_{xj}, one needs to calculate the full propagators of η_x and ϕ_x that include the effects of all the possible hoppings with arbitrary numbers of steps. The present hopping expansion keeps just the leading terms with the shortest pathes. The effect of long contours may renormalize the quantitative results such as $T_{\rm PFS}$ obtained in the hopping expansion, although the qualitative results are unchanged.
[21] We note that, in the case that the fluxons are hard-core bosons[6] instead of canonical bosons, D_2 in A_2 of (3.19) is to be replaced by $D_2' \equiv n(1-n)(\gamma^2+\gamma^{-2})/(4m^2a^4)$. In Ref.[10], the expression of $S_2(\equiv -A_2)$ for the canonical fluxons, Eq.(13), contained D_2' incorrectly, which should read D_2. Therefore, the numerical results of $T_{\rm PFS}, T_{V_0}, m_\eta, m_\phi$ for $\nu = 1/2$ given in Ref.[10] should be replaced by the correct values of (3.29,3.31,3.33) calculated in Sect.III E. However, the differences are almost negligible.
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[30] In this paper, we have assumed $T_{\rm BC} < T_{\rm PFS}$, which is logically necessary for the PFS theory. One may think that the BC occurs only at $T=0$ in the pure $2D$ systems. However if the boson couples with a CS gauge field and there is a replusion between bosons as in the present fluxon system, all excitations have energy gaps. Then it is possible that $T_{\rm BC} \neq 0$. Actually, as far as we know, there is no reliable estimation of $T_{\rm BC}$ for such systems. Both $T_{\rm PFS}$ and $T_{\rm BC}$ are controlled by the repulsive interactions between electrons, and we need a convincing calculation of $T_{\rm BC}$.
[31] In the first-quantization representation, one-body wave functions of the Hamiltonian $m^{-1}|\vec{\nabla}\psi_{CP}|^2$ are obtained as $\psi_{CP} = e^{i\vec{q}\cdot\vec{x}}$. Then the wave function of the Bose-condensed state in many-body system, $\Phi(\vec{x}_i)$, is simply given as $\Phi(\vec{x}_i) = 1$. However in this representation, the most important aspect of the Bose condensation, i.e., the spontaneous breaking of the gauge symmetry, cannot be clearly seen.
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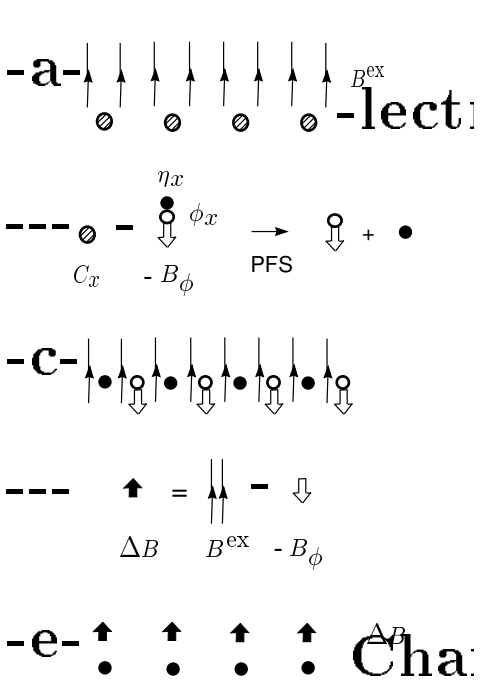

FIG. 1. Illustration of chargeon-fluxon theory of CF and PFS. (a) Electrons in a magnetic field $B^{\rm ex}$. Thin arrows are $B^{\rm ex}$ and hatched circles are electrons C_x. (b) As described in (2.6), each electron is regarded as a composite of CS fluxes $-B_\phi$ (a thick arrow) expressed by the phase factor in (2.6) and a CS fermion ψ_x. Then, as shown in (2.12), ψ_x is viewed as a composite of a fluxon ϕ_x (an open circle) and a chargeon η_x (a filled circle). Each fluxon is accompanied with CS fluxes. When the PFS takes place at $T < T_{\rm PFS}$, chargeons and fluxons with CS fluxes dissociate and behave independently. (c) In PFS states, chrgeons feel a reduced magnetic field (d) ΔB (a thick black arrow) given by (2.20) and (2.31). (e) At $T < T_{\rm BC} (< T_{\rm PFS})$ fluxons Bose-condense and the system reduces to fermionic chargeons in a constant magnetic field ΔB. At $T < T_{\rm BC}$, FQHE takes place at $\nu = p/(2pq \pm 1)$ as a joint effect of (I) IQFE of chargeons in ΔB and (II) Bose condensation of fluxons.

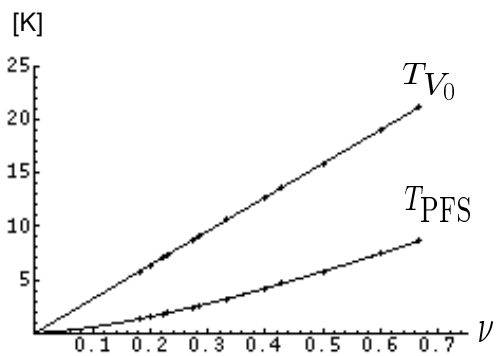

FIG. 2. $T_{\rm PFS}$ and T_{V_0} versus ν for $g = 0.1e^2/(\epsilon\ell)$ calculated by using (3.24) and (3.30). The dots represent the cases of $\nu = 2/3$, $3/5$, $1/2$, $3/7$, $2/5$, $1/3$, $2/7$, $3/11$, $3/13$, $2/9$, $1/5$, $2/11$ (Same in Fig.3, 4, 5).

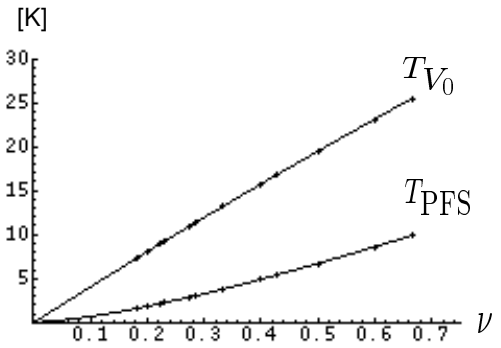

FIG. 3. $T_{\rm PFS}$ and T_{V_0} versus ν for $g = e^2/(\epsilon\ell)$.

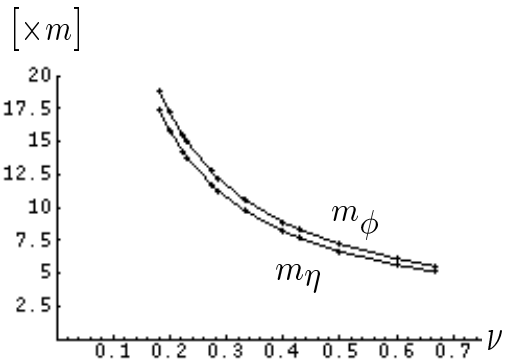

FIG. 4. m_η, m_ϕ for $g = 0.1e^2/(\epsilon\ell)$ calculated by using (3.32). The curves are roughly fitted by $m_\eta \sim 3.2\nu^{-1}$, $m_\phi \sim 3.5\nu^{-1}$.

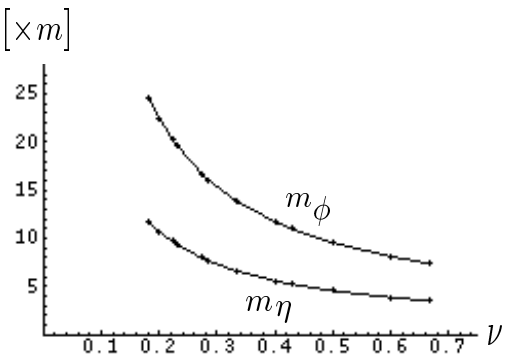

FIG. 5. m_η, m_ϕ for $g = e^2/(\epsilon\ell)$.

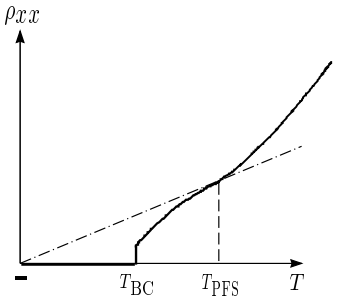

FIG. 6. Expected T-dependence of ρ_{xx}.
For $T < T_{\rm PFS}$, the chargeon contribution $\rho_{\eta\,xx}$ vanishes due to IQHE of chargeons in ΔB, while the fluxon contribution $\rho_{\phi\,xx} \neq 0$. For $T_{\rm BC} < T < T_{\rm PFS}$ interactions between fluxons and gauge bosons may give rise to the linear-T behavior (dashed line) of $\rho_{\phi\,xx}$ and the self-interactions of fluxons may reduce $\rho_{\phi\,xx}$ further (real line). Below $T_{\rm BC}$, fluxons Bose condense and $\rho_{\phi\,xx}$ vanishes, so $\rho_{xx} = 0$.

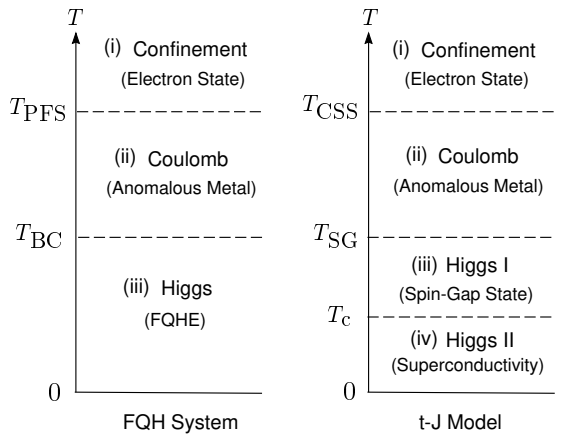

FIG. 7. Phase structure of the gauge dynamics along the temperature axis T for the FQH system studied in the present paper (left) and the t-J model of high-$T_{\rm c}$ cuprates[7] (right). In the t-J model, (i) holons and spinons are confined in electrons above $T_{\rm CSS}$; (ii) CSS takes place below $T_{\rm CSS}$, where holons and spinons are deconfined, exhibitting anomalous-metallic behaviors;. (iii) the spin-gap develops below $T_{\rm SG}$; (iv) holons Bose-condense below $T_{\rm c}$ and induce the superconductivity (for some values of holon density). In FQH system, (i) chargeons and fluxons are confined in electrons above $T_{\rm PFS}$; (ii) chargeons and holons are deconfined below $T_{\rm PFS}$ and some anomalous behaviors are expected; (iii) fluxons Bose condense and FQHE takes place. There is an almost complete correspondence between the gauge dynamics of these two models.