Jet Production in DIS

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Abstract. The status of next-to-leading-order corrections to jet cross sections in deeply inelastic scattering is reviewed. I give a brief introduction into the techniques which are used to calculate jet cross sections, and make a few remarks on a recent program comparison. A third part gives a few results of recent measurements of the strong coupling constant $\alpha_s(\mu^2)$ and the gluon density $f_{g/P}(\xi,\mu_f^2)$.

1 Introduction

The increased integrated luminosity at HERA permits the study of hadronic final states of deeply-inelastic processes in great detail [1,2]. Earlier analyses of this kind of process suffered mainly from the poorly understood emission of particles in the forward region. The situation is improved now, because a larger event sample allows for harder cuts on the photon virtuality Q and the transverse momentum E_T of the jets, the latter cut excluding jets in the forward region.

Besides testing QCD, the measurement of (2+1)-jet final states has two main applications: the measurement of the strong coupling constant $\alpha_s(\mu^2)$ and the determination of the proton's gluon density $f_{g/P}(\xi,\mu_f^2)$. Figure 1 shows the basic two leading-order processes which contribute to (2+1)-jet final states. Diagram (a) is the so-called QCD-Compton process, and (b) is called boson–gluon fusion[1]. The corresponding cross sections are of $\mathcal{O}(\alpha_s)$, and thus the strong coupling constant can be measured, preferably via the (2+1)-jet rate $R_{2+1}=\sigma_{2+1}/\sigma_{\rm tot}$, if the parton densities are assumed to be known. Conversely, the cross section of process (b) is a convolution of a hard scattering cross section with the gluon density $f_{g/P}(\xi,\mu_f^2)$. By subtraction of the well-known quark-initiated contribution and a suitable unfolding procedure, the gluon density can be extracted.

Because of the scale dependence of the strong coupling constant α_s and the parton densities $f_{i/P}$, perturbative calculations have to be done beyond the leading order to be physically meaningful. Virtual corrections suffer from two problems. Large loop momenta introduce ultraviolet singularities, which are removed by renomalisation. Small loop momenta lead to infrared singularities. These cancel, in principle, against divergences from the real corrections, but they lead to certain technical problems in the calculations.

The outline of this paper is as follows: in Section 2 I describe the cancellation mechanism of infrared divergences in jet cross sections, and make a few remarks

[1] There are, of course, two additional diagrams: the gluon in (a) attached to the outgoing quark, and the diagram with quark and antiquark exchanged in (b).

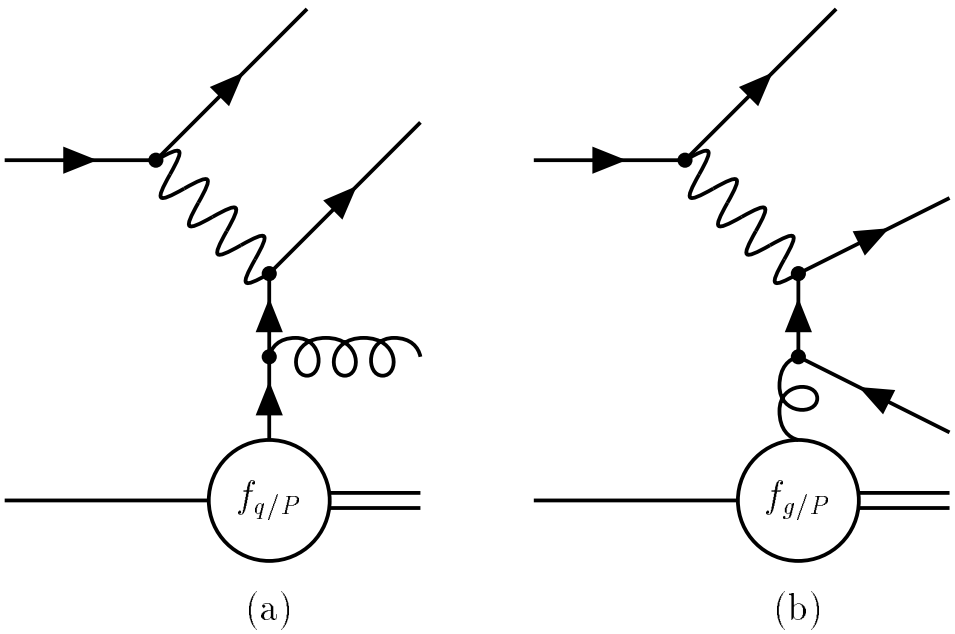

Fig. 1. The leading-order processes for (2+1)-jet production: (a) the QCD-Compton diagram, (b) the boson–gluon fusion diagram.

concerning the techniques used to achieve this. In Section 3, the available calculation for jet cross sections in DIS are discussed and compared. Recent results of data analyses of jets at HERA and a general discussion of scale choices are given in Section 4.

2 QCD at Work

QCD is the microscopic theory of the strong interactions. Perturbative QCD, the evaluation of the theory at large scales for small values of the strong coupling constant, is very successful in describing high-energy scattering processes. Except for very inclusive observables in e^+e^- annihilation, however, perturbation theory is not sufficient to arrive at phenomenological predictions, because non-perturbative input is required. Factorization theorems of QCD [3] guarantee that the non-perturbative part of the description is process-independent. In DIS, for example, parton densities $f_{i/P}(\xi, \mu_f^2)$ are required in order to describe the distribution of partons in the proton.

The standard description of processes in QCD is illustrated in Fig. 2. A hard scale, in DIS usually the photon virtuality $Q^2 \gg \Lambda^2_{\rm QCD}$, allows for a perturbative calculation in fixed order of perturbation theory (I). In the case

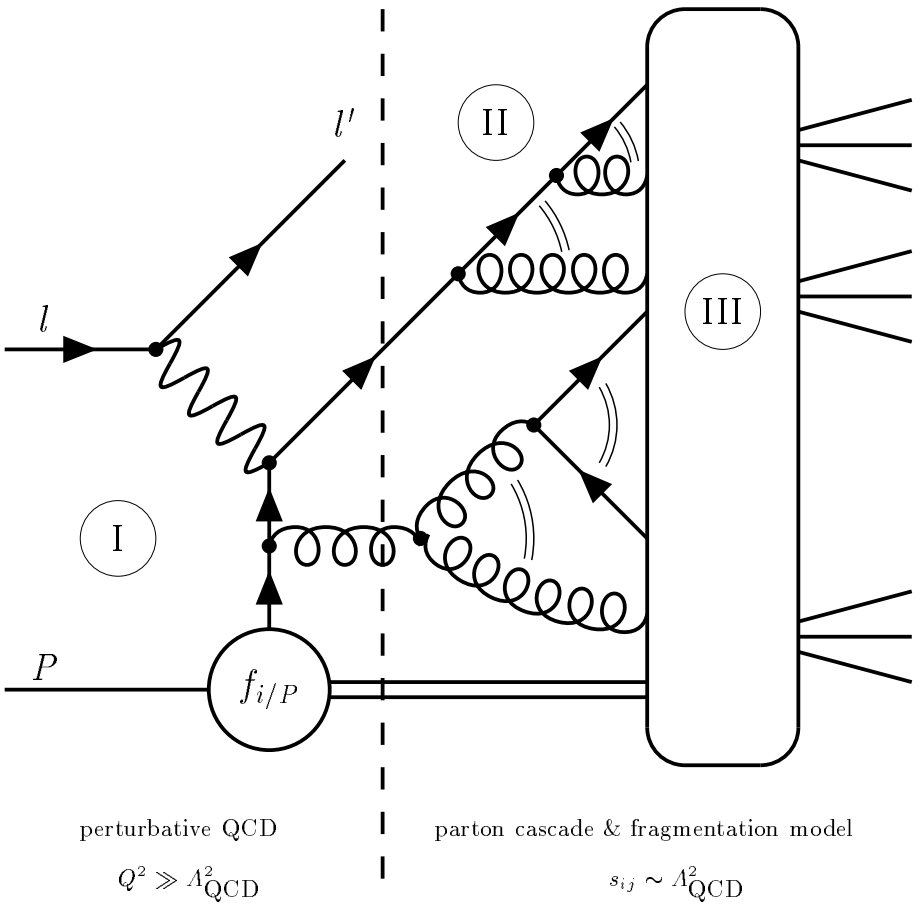

Fig. 2. The standard description of processes in QCD phenomenology consists of three stages. Processes are modelled by a hard scattering process (I), followed by a parton cascade (QCD in the collinear / leading logarithmic approximation, II) and a phenomenological fragmentation model (III). The quantity s_{ij} is the invariant mass of a typical pair of partons after the splitting process.

of generators, a parton cascade (II) is appended, which simulates QCD in the collinear limit of parton branching processes. Finally, in order to obtain final-state hadrons instead of partons, a phenomenological fragmentation model (III), such as the Lund string model [4], is used. Steps (II) and (III) involve certain choices of parameters, for example cut-off parameters for the parton cascade. In contrast, for step (I) there is, except for the parton density parametrization $f_{i/P}$, essentially just one parameter determining the process: the strong coupling

constant. In QCD generators, the matrix elements (I) are usually calculated in leading order only — it is not known how to append a parton cascade in full generality and consistently to a calculation beyond leading order[2]. The virtue of a generator is that it produces a final state as it is seen in an experiment, and that hadronization effects are included. The main drawback is that generator results usually depend on a large number of tuned parameters.

A conceptually cleaner approach is to rely on a fixed-order QCD calculation with a suitable observable that can be evaluated both for parton and hadron final states. This is presently the standard procedure for data analysis of DIS hadronic final states. Generators are used to determine correction factors from the hadron to the parton level. The spread from various fragmentation models gives a contribution to the systematic error of a measurement. Observables are chosen according to the criterion of small hadronization corrections. Another important property for suitable observables is *infrared-safety*. Together with the factorization theorems of QCD, this property makes sure that expectation values of observables calculated in perturbative QCD are finite. Infrared-safe observables behave well in soft and collinear limits: collinear parton splitting and radiation of soft partons do not change the value of an observable. Incidentally, this is also a property that good experimental observables are supposed to have. Calorimetric measurements are insensitive to modifications of the final state of this kind. For more details, I wish to refer the reader to Refs. [6,7].

Cancellation of infrared singularities occurs between real and virtual corrections, a mechanism originally discovered in QED. The main problem in next-to-leading-order calculations is to perform the calculation of the real corrections without knowing the specific form of the observable to be calculated. There are two basic methods to do this: the phase-space-slicing method and the subtraction method. Here I do not have enough space to describe these methods in detail, descriptions can be found in Refs. [8,6] and the references cited therein. I just mention that for phase-space slicing a small technical cut-off parameter $y_{\min}$ has to be introduced. Partons are considered to be technically unresolvable if a quantity such as their scaled invariant mass is smaller than this cut-off. Infrared singularities from unresolved configurations of this kind cancel explicitly. The method is observable-independent, because infrared-safe observables are insensitive to unresolved partons. Contrary to this, the cancellation of IR singularities in the subtraction method is done locally in phase space. No technical cut-off is required, but the construction of the required subtraction terms is cumbersome.

In QCD, there are two distinct types of IR singularities:

- Two massless partons may be collinear (Fig. 3). The real cross section σ_{real} in the collinear limit may be approximated by the expression

$$\int \frac{\mathrm{d}\varphi}{2\pi}\sigma_{\text{real}} \sim \frac{\alpha_s}{2\pi}\,\frac{1}{2p_jp_k}\,\hat{P}_{kj\leftarrow i}(u)\,\sigma_{\text{Born}}. \tag{1}$$

  It is proportional to the Born cross section σ_{Born} and an unsubtracted splitting function $\hat{P}_{kj\leftarrow i}(u)$.

[2] There is a recent proposal for a matching procedure, see Ref. [5].

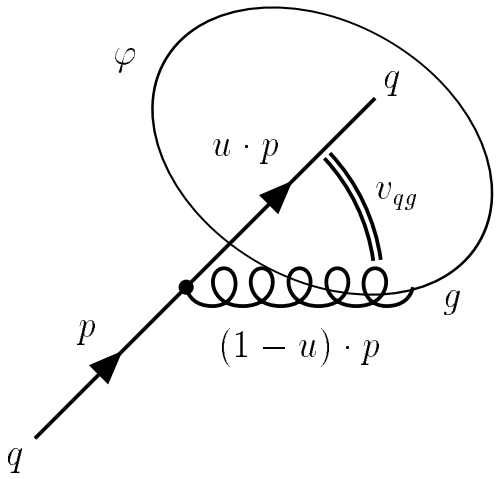

Fig. 3. Collinear limit: splitting of a quark into a quark and a gluon. The incident quark's momentum fraction carried by the outgoing quark in the collinear limit is u, and the azimuthal angle of the gluon with respect to a fixed external axis is φ.

- A soft gluon is radiated (Fig. 4). An eikonal approximation yields an expression of the form

$$\sigma_{\text{real}} \sim \frac{\alpha_s}{2\pi} \sum_{i,j\neq k} C_{ijk} \frac{p_i p_j}{(p_i p_k)(p_j p_k)} \sigma_{\text{Born}} \tag{2}$$

  with fixed coefficients C_{ijk}.

After a separation of overlapping singularities, these expressions are used to generate an approximation term for the subtraction method. For collinear singularities in the case of the splitting $g \to gg$, the subtraction term is actually dependent on the angle φ.

3 Calculations of Jet Cross Sections in DIS

By now there are several calculations for (2+1)-jet processes available with corresponding weighted Monte-Carlo programs. The two early programs `PROJET` [10–12] and `DISJET` [13,15,14] were restricted to the modifed JADE clustering scheme. Subsequent calculations were universal in the sense that in principle arbitrary infrared-safe observables could be calculated:

- `MEPJET` [16]: This is a program for the calculation of arbitrary observables which uses the phase-space-slicing method. The corresponding calculation [17] uses the Giele–Glover formalism [18] for the analytical calculation of

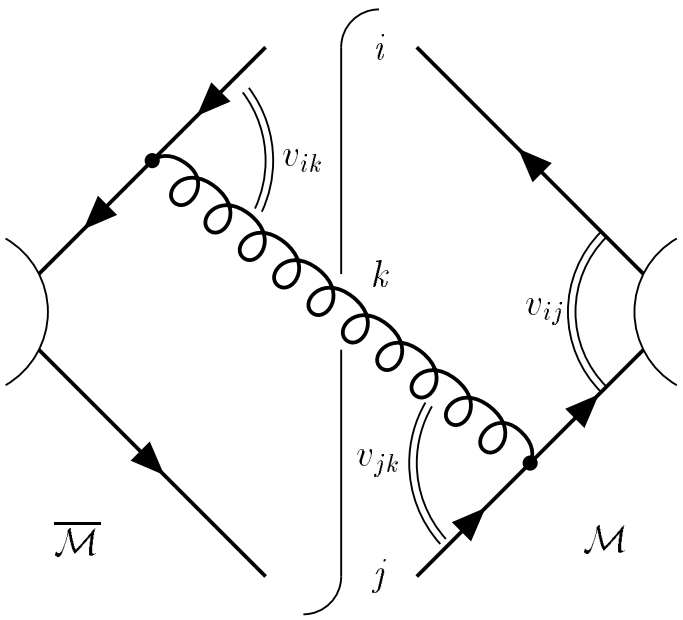

Fig. 4. Soft limit: the gluon attaches to two different quark lines i and j from the matrix element $\mathcal{M}$ and its complex conjugate $\overline{\mathcal{M}}$.

the IR-singular integrals of the real corrections, and the crossing-function technique [19] to handle initial-state singularities. The latter requires the calculation of "crossing functions" for each set of parton densities.

- `DISENT` [20]: This program is based on the subtraction method. The subtraction term is defined by means of the dipole formalism[3] [21,22].
- `DISASTER++` [23]: This is a `C++` class library[4] . The subtraction method is employed, and the construction of the subtraction term resembles the method of Ref. [8], i.e. it is obtained by the evaluation of the residues of the cross section in the soft and collinear limits. Double counting of soft and collinear singularities is avoided by means of a general partial fractions method.
- `JetViP` [24]: This program implements the calculation of [25], which extends the previous calculations into the photoproduction limit $Q^2 \to 0$. The cal-

[3] The subtraction term is written as a sum over dipoles (an "emitter" formed from two of the original partons and a "spectator" parton). Besides the factorization theorems of perturbative QCD, the main ingredient is an exact factorization formula for the three-particle phase space, which allows for a smooth mapping of an arbitrary 3-parton configuration onto the various singular contributions.

[4] The acronym stands for "Deeply Inelastic Scattering: All Subtractions Through Evaluated Residues". The program is written in `C++`. A `FORTRAN` interface is available; thus there is no problem to interface the class library to existing `FORTRAN` code.

culation has been done by means of the phase space slicing method. Up to now, the polarization of the virtual photon is restricted to be longitudinal or transverse.

These four programs have been compared in detail during the recent workshop on Monte Carlo generators for HERA physics [26–28], both for jet cross sections and event shape variables. It turns out that `DISENT` and `DISASTER++` agree well for jet cross sections [27]. The results for event shapes are in agreement as well, except for the current jet broadening variable B [28]. It is not yet known where this residual discrepancy comes from; jet broadening is an infrared-safe shape variable, but it regularizes the (1+1)-jet singularity poorly. It may well be that the different results come from a numerical instability in one of the programs. The `MEPJET` program shows systematic differences with respect to `DISASTER++/DISENT` for jet cross sections (no comparison of shape variables has been done) even for physical setups of integration parameters. The results are stable against variations of the phase-space slicing parameter. The `JetViP` results show deviations from the `DISASTER++/DISENT` results as well, but the differences are smaller than for `MEPJET`. The `JetViP` results depend on the technical phase-space slicing cut-off parameter [27], which is a sign that the final result is not completely stable.

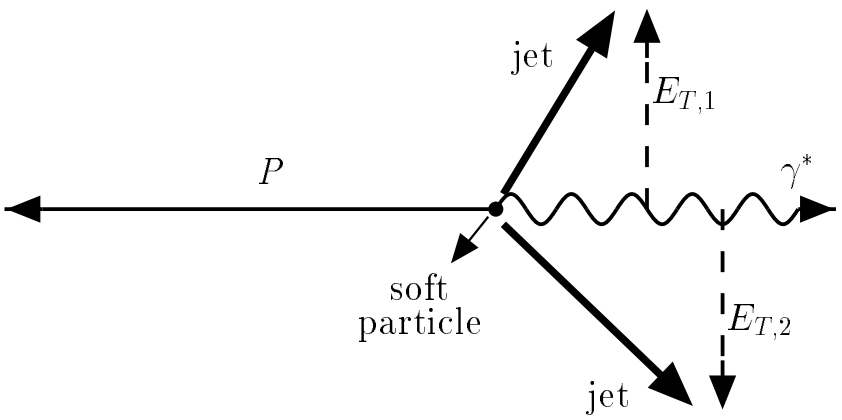

Fig. 5. A (2+1)-jet event in the Breit frame. The transverse momentum is balanced. An additional soft parton of an event close to the transverse momentum cut-off drives the transverse momentum of one of the jets below threshold.

A problem with transverse momentum cuts which has recently been observed is illustrated in Fig. 5. A cut of $E_{T,\mathrm{min}}$ on the transverse momenta of both jets leads to a quantity which is sensitive to the emission of soft particles. If the event is close to the threshold, the emission of a particle with a very small energy removes it from the event sample. This in itself is not critical; after all, this

is what cuts are supposed to do. However, what is affected is the cancellation of infrared singularities. In a next-to-leading-order calculation, a process with two high-E_T partons and a soft gluon contributes to the cancellation of IR singularities from the virtual corrections. The singularities themselves will of course cancel, but the cross section will become very sensitive to other cuts, which is, of course, undesirable. A possible cure for this is to impose asymmetric cuts on the transverse momenta, i.e. conditions such as $E_{T,1} > E_{T,\mathrm{min}}$, $E_{T,2} > E_{T,\mathrm{min}} + \Delta$ for $E_{T,1} > E_{T,2}$, and $\Delta = \mathcal{O}(\text{few GeV})$.

4 Comparison with Data: $\alpha_s(\mu^2)$ and $f_{g/P}(\xi, \mu_f^2)$

Both H1 and ZEUS have published measurements of the strong coupling constant $\alpha_s(\mu^2)$ from jet rates in DIS [1,2]. The H1 results are shown in Fig. 6. Experimental jet cross sections for the inclusive $k_\perp$ algorithm vs. NLO theory (`DISENT`) for `CTEQ5M` parton densities and $\alpha_s(m_Z^2) = 0.118$ are displayed in Fig. 6(a). This impressive agreement suggests that the strong coupling constant may be extracted from hadronic final states in DIS. This is illustrated in Fig. 6(b). The chosen renormalization scale in this case is the transverse momentum E_T. The final result of a combined fit of all data points is $\alpha_s(m_Z^2) = 0.1181 \pm 0.0030(\text{exp.})^{+0.0039}_{-0.0046}(\text{th.})$, which is consistent with the world average value. Moreover, the plots show the running of the coupling constant. A measurement using Q as the renormalization scale (not shown here) leads to the result $\alpha_s(m_Z^2) = 0.1221 \pm 0.0034^{+0.0054}_{-0.0059}$, which is compatible with the result for $\mu = E_T$. According to Ref. [1], the larger theoretical error comes from the larger renormalization scale dependence of the choice $\mu = Q$.

The direct extraction of the gluon density by H1 is shown in Fig. 7 [29][5]. The procedure was to do a fit of quark and gluon densities to a combined data set of the total DIS cross section and the inclusive DIS jet cross section for a given value of α_s; the former constraining the quark densities, and the latter constraining the gluon density. The result for $f_{g/P}(\xi)$ is compatible with the results from global fits.

There is, of course, the question of the right scale choice. Scale dependence comes in because the perturbative series is truncated. Physical results to all orders would be scale-independent. The choice of a scale essentially corresponds to the choice of a renormalization prescription. To a certain degree, perturbation theory takes care of itself: scale variations of a quantity calculated to order α_s^n in perturbation theory have a residual scale dependence of order α_s^{n+1}. This residual scale dependence may be large, which means that the coefficient of this term is large. In practice, there are two approaches for the choice of a scale. The first approach, a procedure employed frequently in experimental analyses, is to choose some central value μ equal to one of the energy scales of the process, and to take the spread of the results in the interval $\mu/2$ to 2μ to be the uncertainty from the theoretical prediction. This is justified by the reasoning that the scale

[5] See also Ref. [30].

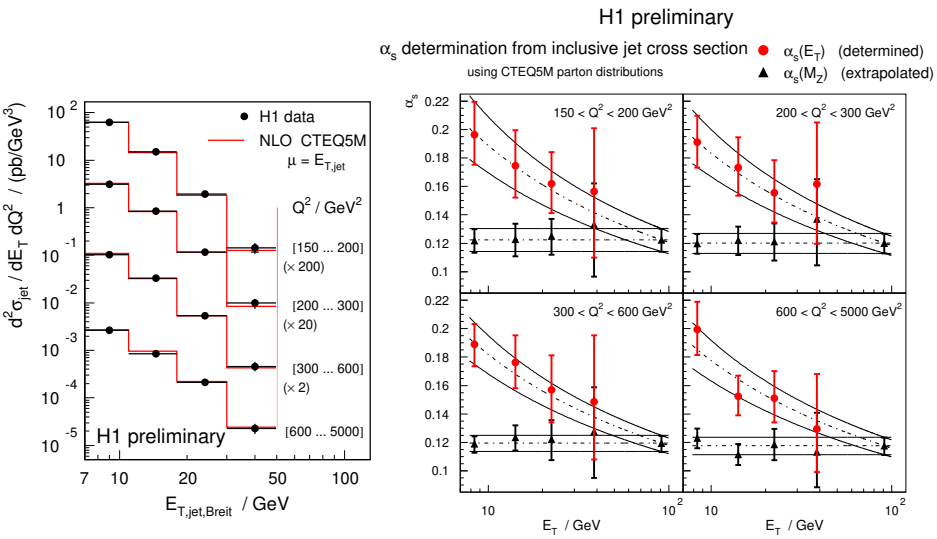

Fig. 6. Preliminary H1 results [1] for jet cross sections and the strong coupling constant: (a) Comparison of data and NLO results (DISENT). (b) Extracted value of $\alpha_s(E_T^2)$ in various bins of Q^2.

dependence of the finite-order result is exactly identical to the scale dependence of the uncalculated higher-order terms. Thus, the scale dependence reflects the size of these terms, and consequently is an estimate of the error made. There are two criticisms: there are scale-independent contributions from higher orders, which are not captured by this procedure, and, in any case, the variation does not determine the absolute size. The second approach invokes some higher principle (such as BLM [31], PMS [32] and FAC [33]), justified in turn either by a physical principle (as for BLM), by an assumption on the behaviour of the perturbative series (for PMS and FAC), or even by a fitted parameter ρ, with, for example, $\mu = \rho Q$. Not all of these scales can be "correct", because they have different values, and what comes out is (necessarily) a typical scale of the process multiplied by a numerical factor. Admittedly, this numerical factor can be different from one. In a certain sense, what is done in the first approach is to estimate the spread of the scale dependence from various *a priori* scenarios.

Typically, there are several physical scales which are relevant in a process, in the case at hand the photon virtuality Q and the transverse momentum E_T of the jets in the Breit frame. Fixed-order perturbation theory breaks down if these scales are very different, because large logarithms of ratios of these scales lead to large coefficients, which in turn spoil the perturbative expansion. The proper way to handle this problem would be resummation. So far, no resummed

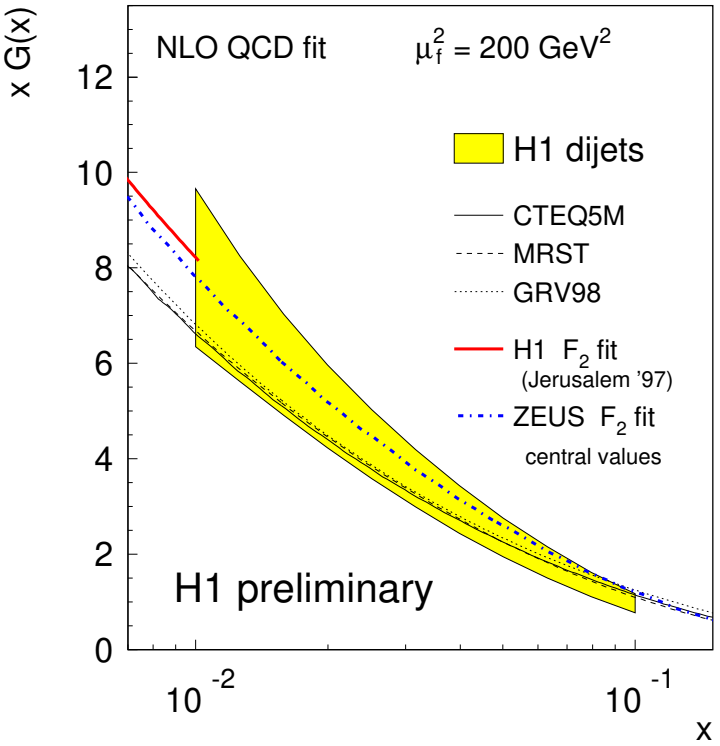

Fig. 7. Preliminary H1 result for the direct extraction of the gluon density [29].

calculation for jet cross sections in DIS is available. Since none of the available scales is to be preferred, the safest procedure at present is to employ all of them in turn, and quote the resulting differences as a theoretical error.

5 Summary

I have given a brief review of the techniques employed in next-to-leading-order QCD calculations. In particular, I have discussed the singularity structure of the real corrections. From a principal point of view, fixed-order perturbation theory is the cleanest approach to theoretical predictions: it depends on the smallest number of input parameters (essentially the strong coupling constant and parton densities), and it is not fragmentation-model dependent.

The situation concerning next-to-leading-order calculations for deeply inelastic processes has improved. Several independent calculations for (2+1)-jet-like quantities are available, with different methods used to extract infrared singular-

ities. The programs have been compared in some detail in the framework of the recent workshop on Monte Carlo generators for HERA. Two of them, `DISENT` and `DISASTER++`, agree well, with the exception of the jet broadening shape variable.

Measurements of α_s and $f_{g/P}$ are by now well established. Compared to earlier analyses, the present measurements are more reliable, mainly due to larger event samples and as a consequence the possibility of stricter cuts on transverse momenta of jets, which reduces the problems with jets in the forward direction.

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