Soft and Hard Pomerons

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Abstract. Regge theory provides an excellent description of small-x structure-function data from $Q^2 = 0$ up to the highest available values. The large-Q^2 data should also be described by perturbative QCD: the two descriptions must agree in the region where they overlap. However, at present there is a serious lack in our understanding of how to apply perturbative QCD at small x. The usual lowest-order or next-to-lowest order expansion is not valid, at least not until Q^2 becomes much larger than is usually assumed; a resummation is necessary, but as yet we do not know how to do this resummation.

1 Introduction

Perturbative QCD has become a well-established description of hard processes. But it is not complete: it must be supplemented with other descriptions.

Regge theory is an example of another description. It was extensively developed some 40 years ago [1] and is based on our knowledge of analyticity properties of scattering amplitudes. It relates the high-energy behaviour of scattering amplitudes to exchanges of known particles. In order to describe the data, it also introduces extra terms that, at least so far, are not related to exchanges of known particles. These extra terms are called pomeron-exchange terms, after the Russian physicist Isaac Pomeranchuk. The notion of the pomeron is a very old one. We do not know whether it too describes the exchange of particles, but if it does there is general agreement that these are likely to be glueballs.

Regge theory gives an excellent description not only of soft hadronic proceses [1][2][3], but also of the behaviour of the structure function $F_2(x, Q^2)$ at small x, all the way from real-photon-induced events ($Q^2 = 0$) up to very deeply inelastic ones (large Q^2). The structure function describes an example of a semihard process. Another example of a semihard process is the quasi-elastic reaction $\gamma\ p \to J/\psi\ p$. For such semihard processes, the data reveal [4] that there exists a second pomeron, the "hard" pomeron, in addition to the original "soft" pomeron which enters into purely hadronic reactions. The soft pomeron is certainly nonperturbative in origin, though it is possible that the hard pomeron is associated with the perturbative BFKL equation [5]. However, there are many serious problems [6][7] with the BFKL equation, and so this is not sure.

Certainly, one would hope that at small x and large Q^2 Regge theory can be made to agree with perturbative QCD, in particular with DGLAP evolution [8]. However, there are problems with the DGLAP equation too at small x. The equation involves a kernel or splitting function $P(z, \alpha_S(Q^2))$. If one expands

this in powers of α_S, each term is singular at $z = 0$. However [9] it is rather sure, from general considerations of the known analyticity in Q^2 of the structure function, that $P(z, \alpha_S(Q^2))$ is not singular at $z = 0$. The singularities in the terms of the expansion are a signal that the expansion is illegal near $z = 0$, and a resummation is needed to remove them. At present, we do not know exactly how to do this. It is possible that the presence or absence of the singularity is immaterial if Q^2 is large enough [10], but for most applications in the literature [11] it is likely to be a real problem.

2 Regge theory

Through the optical theorem, the structure function $F_1(x, Q^2)$ is the imaginary part of the virtual Compton amplitude $T_1(\nu, t, Q^2)$ evaluated at zero momentum transfer, $t = 0$. Here $2\nu = 2p.q = Q^2/x$. Regge theory begins [1] by considering the crossed-channel process $\gamma^*\gamma^* \to p\bar{p}$, for which $\sqrt{t}$ is the centre-of-mass energy. It first makes a partial-wave series expansion in this channel, in terms of partial-wave amplitudes $a_\ell(t, Q^2)$ and Legendre polynomials $P_\ell(\cos\theta_t)$, where θ_t is the crossed-channel scattering angle. This expansion has a definite, but limited, region of convergence in the space of the three variables $\cos\theta_t, t, Q^2$ or, equivalently, ν, t, Q^2. The partial-wave amplitude $a_\ell(t, Q^2)$ is defined initially for physical values of the angular momentum, $\ell = 0, 1, 2, \ldots$. By a well-defined procedure, its definition is extended to all values of ℓ, both real and complex, and the partial wave series is converted to an integral:

$$T_1(\nu, t, Q^2) = \frac{1}{2i}\int_C d\ell \frac{(2\ell+1)P_\ell(-\cos\theta_t)}{\sin\pi\ell} a(\ell, t, Q^2) \qquad (1)$$

The contour C is initially that of figure 1a, wrapped arround the positive real-ℓ

Fig. 1. Contour C in the complex ℓ plane for the integral (1)

axis on which are located the zeros of the denominator $\sin\pi\ell$. But the properties of $a(\ell, t, Q^2)$ allow one to use Cauchy's theorem to distort it to become parallel to the imaginary-ℓ axis, as in figure 1b. It turns out that this extends the region of convergence beyond that of the original series, and the integral can be continued

analytically to where we need it to calculate the structure function $F_1(x,Q^2)$:

$$t=0 \qquad\qquad 2\nu > Q^2 \geq 0 \tag{2}$$

As we continue analytically into this region, the singularities of $a(\ell,t,Q^2)$ will move around in the complex ℓ-plane, and one of them may try to cross the contour C. We must distort the contour again so as to avoid this happening. Depending on whether the singularity is a branch point or a pole, we then have either figure 2a or 2b.

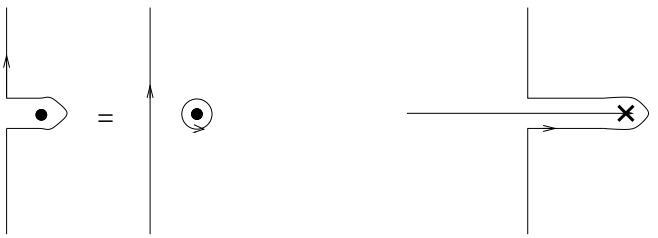

Fig. 2. Distortion of the contour C caused by (a) a pole, or (b) a branch point, of $a(\ell,t,Q^2)$ trying to cross it

From what is known about the analyticity structure, it is rather sure that the positions of the singularities of $a(\ell,t,Q^2)$ in the complex ℓ plane do not depend on Q^2, only on t. In the case of a pole

$$a(\ell,t,Q^2) \sim \frac{\beta(Q^2,t)}{\ell-\alpha(t)} \tag{3}$$

the integration around it yields a contribution

$$\frac{\pi\beta(Q^2,t)\; P_{\alpha(t)}(-\cos\theta_t)}{\sin\pi\alpha(t)} \tag{4}$$

to $T_1(\nu,t,Q^2)$. Now when $t=0$ the analytic continuation of $\cos\theta_t$ is

$$\cos\theta_t = \frac{\nu}{iQm_p} \tag{5}$$

and when this is large the analytic continuation of the Legendre polynomial has the simple behaviour

$$\frac{\pi P_\ell(-\cos\theta_t)}{\sin\pi\ell} \sim -\frac{\Gamma(-\ell)\Gamma(\ell+\frac{1}{2})}{\sqrt{\pi}}(-2\cos\theta_t)^\ell \qquad (\mathrm{Re}\;\ell > \tfrac{1}{2}) \tag{6}$$

So, with (5), we see that the integral (1) becomes just a Mellin transform, and the "Regge trajectory" $\alpha(t)$ contributes at $t=0$

$$b_1(Q^2)\nu^{\alpha(0)} \tag{7}$$

to the large-ν behaviour of $T_1(\nu,0,Q^2)$. Here, $b_1(Q^2)$ is a constant multiple of $\beta(Q^2,0)$. In the case of $T_2(\nu,0,Q^2)$ its definition includes a kinematic factor which reduces the power of ν by one unit, so since $\nu = Q^2/2x$ this gives

$$F_2(x,Q^2) \sim f(Q^2)x^{-\epsilon} \tag{8}$$

with

$$\epsilon = \alpha(0) - 1 \tag{9}$$

Regge theory gives no information about the function $f(Q^2)$, other than that it is an analytic function with singularities whose locations are known [12]. The power $(1-\alpha(0))$ is independent of Q^2.

3 Fit to data

In the case where the singularity that crosses the contour C is a branch point instead of a pole, dragging a branch cut with it as shown in figure 2b, the simple power of x in (8) is replaced with something more complicated. One knows, from unitarity [1], that if there are poles in the complex ℓ plane, there must certainly also be branch points.

On the principle that it is usually the best strategy to try the simplest possible assumption first, Donnachie and I tested the hypothesis that at $t = 0$ the contribution from branch points is much weaker than from poles. Not everybody agrees with this strategy [13], but we applied this several years ago [3] to purely hadronic total cross-sections and found that they are all described well by a sum of just two powers ν^{ϵ_1} and ν^{ϵ_2} The two powers are

$$\begin{array}{ll} \epsilon_1 = 0.08 & (\text{“soft pomeron” exchange}) \\ \epsilon_2 = -0.45 & (\rho,\omega,f,a \text{ exchange}) \end{array} \tag{10}$$

More recently [4], we tested the hypothesis that also the contribution to the structure function F_2 at small x from branch points is much weaker than from poles. We made a fit of the form

$$F_2(x,Q^2) \sim \sum_{i=0}^{2} f_i(Q^2)x^{-\epsilon_i} \tag{11}$$

We fixed the values of the powers ϵ_1 and ϵ_2 to be the same as in (10), and left ϵ_0 as a free parameter to be determined from the small-x structure-function data. Our fitting procedure had three stages. First, we arrived at a provisional value for ϵ_0 by using data only in the region

$$x < 0.07 \qquad\qquad 0 \le Q^2 < 10 \text{ GeV}^2 \tag{12}$$

which gave us

$$\epsilon_0 \approx 0.4 \tag{13}$$

Next, with this value for ϵ_0, we fitted the data to a sum (11) of three powers of x for each values of Q^2 for which there exist data, still restricting x to less than 0.07 . This gave us the plots shown in figure 3 of the hard and soft pomeron coefficient functions $f_0(Q^2)$ and $f_1(Q^2)$ as functions of Q^2. The data do not constrain the (f,a)-exchange coefficient function $f_3(Q^2)$ at all well, so we retained the simple form we chose for it in the first stage of our fitting procedure.

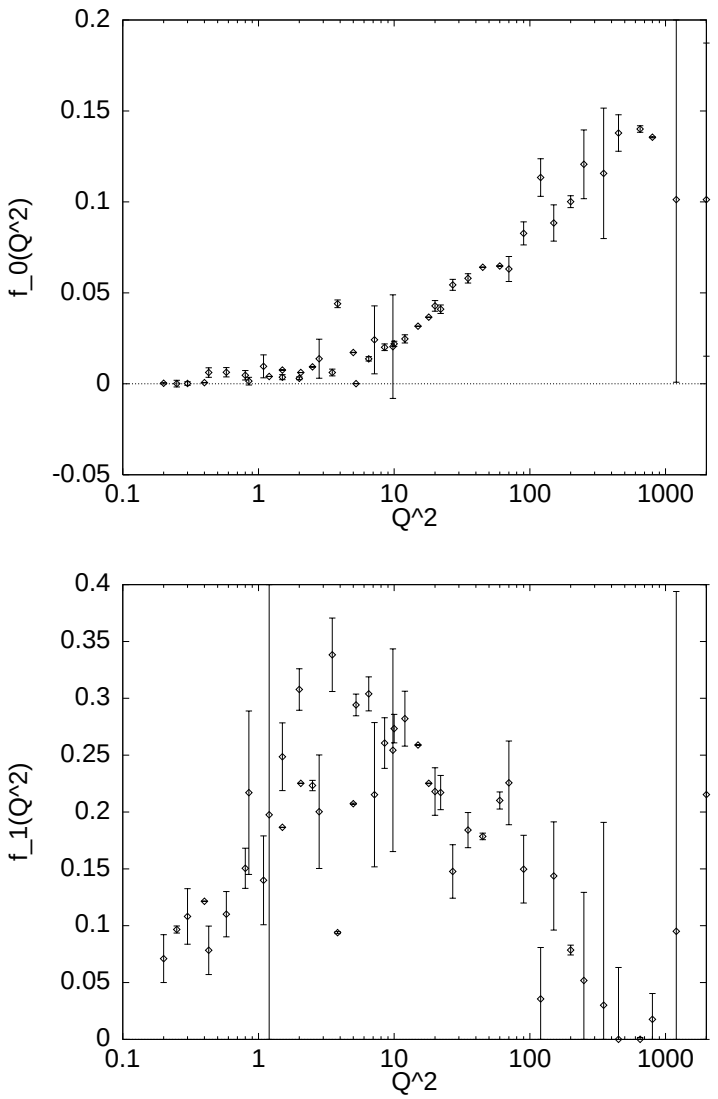

Fig. 3. The hard and soft pomeron coefficient functions $f_0(Q^2)$ and $f_1(Q^2)$ extracted from the data

We know from gauge invariance that F_2 must vanish linearly in Q^2 as $Q^2 \to 0$ at fixed ν. We see that the hard-pomeron coefficient function $f_0(Q^2)$ recovers from this rather slowly as Q^2 increases, until it begins at $Q^2 \approx 10$ GeV^2 to rise quite rapidly. On the other hand, the soft–pomeron coefficient function $f_1(Q^2)$ rises rapidly away from $Q^2 = 0$, until it peaks at between 5 and 10 GeV^2. It then falls again. This was a surprise to us: the soft-pomeron contribution to the small-x structure function apparently is higher twist.

The third part of our fitting procedure chooses functions that have the general shape of the plots in figure 3, with a number of parameters. It still retains the same form for $f_2(Q^2)$, which matters significantly only for the real-photon data at low energy — our fit includes these for $\sqrt{s} > 6$ GeV, and including the f_2 term is essential to fit them. So we use (11) with

$$f_0(Q^2) \;= A_0 \left(\tfrac{Q^2}{Q^2+Q_0^2}\right)^{1+\epsilon_0} \left(1+\tfrac{Q^2}{Q_0^2}\right)^{\frac{1}{2}\epsilon_0} \tag{14}$$

$$f_1(Q^2) = A_1 \left(\tfrac{Q^2}{Q^2+Q_1^2}\right)^{1+\epsilon_1} \left(1+\sqrt{\tfrac{Q^2}{Q_S^2}}\right)^{-1} \tag{15}$$

$$f_2(Q^2) \qquad\quad = A_2 \left(\tfrac{Q^2}{Q^2+Q_2^2}\right)^{1+\epsilon_2} \tag{16}$$

The hard-pomeron power is now again a free parameter, together with the three coefficients A_i, the mass scales Q_i^2, and Q_S^2. With these 8 parameters, we obtain a χ^2 per data point of 1.0 for 595 data points. These data points have $x < 0.07$ and range from $Q^2 = 0$ to 2000 GeV^2. Sample plots are shown in figure 4.

I want to make a number of comments on these fits:

- The choice (16) for the analytic form of the coefficient functions is an economical set that describes the data well, but there are other choices that agree also with the data extracted in figure 3. Using different choices results in different values for the hard-pomeron power ϵ_0, but they are all within 10% of 0.4.
- The choice (16) makes $f_0(Q^2) \sim Q^{\epsilon_0}$ at large Q^2, which corresponds to $\beta(Q^2, 0)$ in (3) becoming constant for large Q^2. There is no general theory that explains this, though it has been predicted [14] from the BFKL equation. Parametrisations of $f_0(Q^2)$ that behave logarithmically at large Q^2 can also fit the data satisfactorily.
- The choice (16) makes $f_1(Q^2) \sim 1/Q$ at large Q^2. Fits that make it instead behave as $1/Q^2$ are also acceptable, though less good. A $1/Q$ behaviour has been predicted [15] from a combination of the BFKL equation and infrared renormalons, though this is controversial [16]. Fits in which $f_1(Q^2)$ does not go to zero at high Q^2 do not work: it really does seem that the soft-pomeron contribution to $F_2(x, Q^2)$ is higher twist. At, say, $Q^2 = 5$ GeV^2 soft-pomeron exchange dominates in $F_2(x, Q^2)$ until x is less than about 10^{-3}. The consequence of this for conventional structure-function fits [11] needs discussion.
- Without the contribution from the hard pomeron, the fit to the real-photon data agrees well [3] with the measured HERA points. However, it is seen

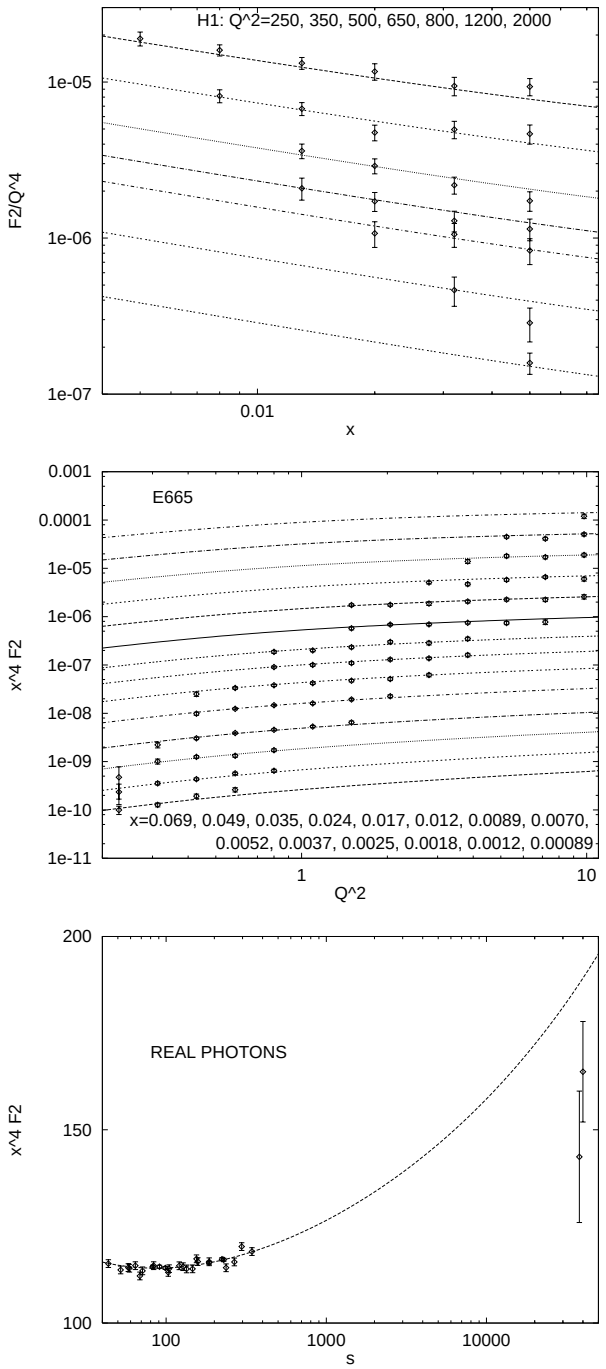

Fig. 4. The Regge fit compared with some of the data, at the largest available Q^2, small Q^2, and $Q^2 = 0$

in figure 4 that including the hard pomeron makes the fit pass significantly above these.

- Figure 5 shows a fit to the preliminary ZEUS data for the charm structure function. It shows that, to a good approximation, these data may be described well by hard-pomeron exchange alone. The fit is actually a single-parameter fit; it includes the constraint that at high Q^2 the hard pomeron is flavour blind.
- Figure 6 shows that the data for the semihard process $\gamma\, p \to J/\psi\, p$ are well described by a mixture of soft and hard pomeron exchange [4]. The figure is for $Q^2 = 0$; the HERA data indicate that, as Q^2 increases, the hard-pomeron component becomes relatively more important. One might guess that the data for $\gamma\, p \to \rho\, p$ may be described similarly, though at $Q^2 = 0$ the soft pomeron dominates.

4 Perturbative evolution

At high Q^2, parton distributions evolve with Q^2 according to a perturbative evolution equation, called the DGLAP equation. In the singlet channel, the equation is written [8] in terms of a 2-component vector $\mathbf{u}$ whose elements are the singlet quark distribution and the gluon distribution:

$$\dot{\mathbf{u}}(x,t) = \int_x^1 \frac{dz}{z}\, \mathbf{P}(z,t)\, \mathbf{u}(x/z,t) \tag{17}$$

On the left-hand side, $\mathbf{u}$ appears differentiated with respect to

$$t = \log(Q^2/\Lambda^2) \tag{18}$$

On the right-hand side $\mathbf{P}$ is the 2×2 splitting matrix.

The splitting matrix $\mathbf{P}$ may be expanded as power series in $\alpha_S(Q^2)$. It was observed long ago [17] that if one uses just the first term in this expansion,

$$\mathbf{P}(z,t) = \frac{\beta_0}{\log(Q^2/\Lambda^2)}\, \mathbf{p}(z) \tag{19}$$

and assumes that each element of $\mathbf{u}(x,t)$ has simple power behaviour in x as in (8), then the DGLAP equation gives a simple differential equation for the coefficient matrix $\mathbf{f}(Q^2)$ that multiplies the power. Its solution is

$$\mathbf{f}(Q^2) = \mathbf{f}(\Lambda^2) \left(\log \frac{Q^2}{\Lambda^2} \right)^{\gamma(\epsilon)} \tag{20}$$

where $\gamma(N)$ is the eigenvalue of $\beta_0 \mathbf{p}(N)$. Here $\mathbf{p}(N)$ is the Mellin transform of $\mathbf{p}(N)$:

$$\mathbf{p}(N) = \int_0^1 dz\, z^N\, \mathbf{p}(z) \tag{21}$$

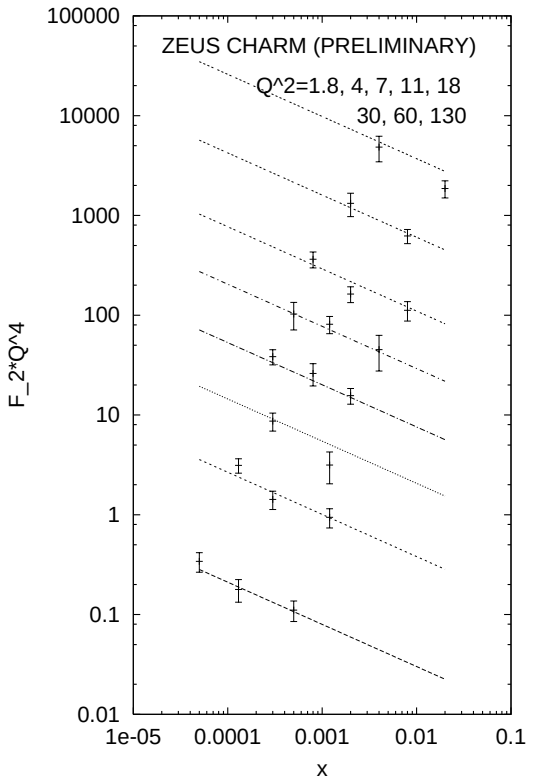

Fig. 5. Hard-pomeron fit to preliminary ZEUS data for F_2^c

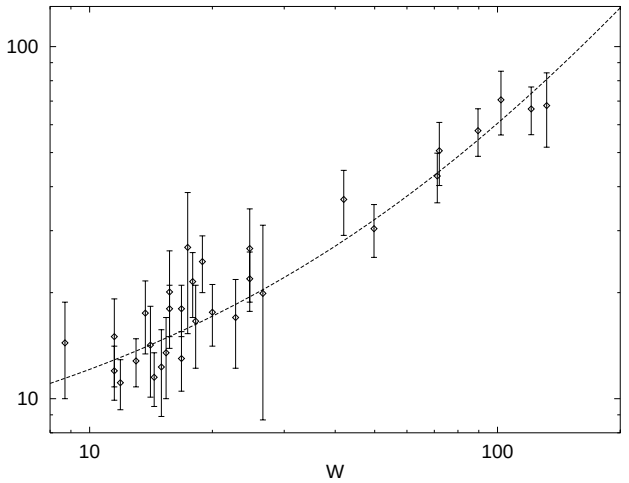

Fig. 6. Two-pomeron fit to data for $\gamma\, p \to J/\psi\, p$

For $\epsilon = 0.4$ the eigenvalue $\gamma(\epsilon)$ is close to 3. One may fit the data satisfactorily by requiring the hard-pomeron coefficient function $f_0(Q^2)$ to have this behaviour at large Q^2, though not as well as with the power behaviour of (16). However, in any case it is not valid to use the approximation (19) at low x, even if it is supplemented by adding in higher-order terms in the perturbative expansion. Such an expansion is not legal at small x, even though it is widely used [11].

The problem is that two of the elements p_{qG} and p_{GG} of the matrix $\mathbf{p}(z)$ have poles [8] at $z = 0$, so that their Mellin transforms have poles at $N = 0$. For example,

$$\pi\, p_{GG}(z) \sim \frac{C_A}{z} \qquad\qquad \pi\, p_{GG}(N) \sim \frac{C_A}{N} \tag{22}$$

If one includes higher-order terms in the perturbative expansion, all the elements of the Mellin transform of the splitting matrix are singular at $N = 0$. The consequence is that, as soon as Q^2 is large enough for the DGLAP equation to be applicable, $Q^2 > Q_0^2$ say, the Mellin transforms of the parton distributions acquire rather nasty singularities at $N = 0$, something like $\exp(1/N)$. This conflicts with what we believe we know about analyticity properties in Q^2. As I explained in section 2, the Regge amplitude at high energy (or small x) is essentially just the Mellin transform: the relationship is

$$\ell = N + 1 \tag{23}$$

So a singularity at $N = 0$ corresponds to one at $\ell = 1$. However, the analyticity properties of the amplitude whose imaginary part is $F_2(x, Q^2)$ were well studied nearly 40 years ago [1] and there was never any suggestion that it has a singularity at $\ell = 1$, let alone a nasty one. If there is no singularity when Q^2 is small, it is not possible that one suddenly appears when we continue analytically in Q^2 to $Q^2 > Q_0^2$. That is, there can be no singularity of the splitting matrix at $N = 0$ — and probably not, indeed, at any other value of N.

The appearance of the singularity at $N = 0$ arises because of the perturbation expansion. Compare, for illustration, the function

$$\phi(N, \alpha_S) = N - \sqrt{N^2 - \alpha_S} \tag{24}$$

Its expansion in powers of α_S is

$$\phi(N, \alpha_S) = \frac{\alpha_S}{2N} + \frac{\alpha_S^2}{8N^3} + \ldots \tag{25}$$

Each term in the expansion is singular at $N = 0$, but manifestly the complete function $\phi(N, \alpha_S)$ is not. The expansion is illegal near $N = 0$.

The timelike splitting matrix is very similar [18] in form to $\phi(N, \alpha_S)$, but in the spacelike region things are a little more complicated. If one combines the DGLAP equation with the BFKL equation, which could well be a valid thing to do when x or N is small, one finds that the element P_{GG} of the spacelike

splitting matrix is given [19] in terms of the Lipatov characteristic function [5] $\chi(\omega, \alpha_S)$:

$$\chi(P_{gg}(N,t), \alpha_S) = N \tag{26}$$

To lowest order in α_S,

$$\chi(\omega, \alpha_S) = \frac{3\alpha_s}{\pi}\left[2\psi(1) - \psi(\omega) - \psi(1-\omega)\right] \tag{27}$$

One may easily verify [9] that, if one inserts (27) into (26), each term in the expansion of $P_{gg}(N,t)$ in powers of α_S is singular at $N = 0$, but the complete function $P_{gg}(N,t)$ is finite there. Some authors [20] therefore advocate that one should use the solution to the full equation (26) and avoid expanding it.

This resummation is obviously sensible, but there are still serious unsolved problems. Firstly, it is highly doubtful that it is valid to use the uncorrected BFKL equation [6]: it assumes that nonperturbative effects cause no complications, and does not properly take account of energy conservation. Secondly, even if one ignores these difficulties, using the lowest-order approximation (27) to $\chi(\omega, \alpha_S)$ is not valid: the next-to-leading-order correction is huge [7]. There have recently been some interesting attempts to solve this problem [21], but more work remains to be done.

So at present, although we know that the elements of the splitting matrix cannot diverge at $N = 0$, we cannot calculate them. Any application of the DGLAP equation in which they are expanded perturbatively, and in which use is made of the fact that the terms in this expansion are large, cannot be trusted.

5 Summary

- When we use the DGLAP equation at small x it is not valid to use an unresummed perturbative expansion of the splitting matrix — at least, not until Q^2 is somewhat larger than is normally assumed. At present, we do not know how to perform the necessary resummation properly.
- Regge theory with two pomerons fits the small-x data for $F_2(x, Q^2)$ extremely well up to the highest available values of Q^2. It strongly suggests that at, say, $Q^2 = 5$ GeV^2, most of $F_2(x, Q^2)$ at small x is higher twist.
- Perturbative QCD and Regge theory are not rival theories. They complement each other and we have to learn how to make them fit together.

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