Screening Effects on F_2 at Low x and Q^2

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Abstract. In this talk we discuss how deeply the region of high parton densities has been studied experimentally at HERA. We show that the measurements of deep inelastic structure functions at HERA confirm our theoretical expectation that at HERA we face a challenging problem of understanding a new system of partons: quarks and gluons at short distances with so large densities that we cannot treat this system perturbatively. We collect all experimental indications and manifestations of specific properties of high parton density QCD.

1 What Are Shadowing Corrections?

In the region of low x and low Q^2 we face two challenging problems which have to be resolved in QCD:

1. The matching of "hard" processes, that can be successfully described in perturbative QCD (pQCD), and "soft" processes, that should be described in non-perturbative QCD (npQCD), but actually, we have only a phenomenological approach for them;
2. The theoretical approach for the high parton density QCD (hdQCD) which we reach in the deep inelastic scattering at low x but at sufficiently high Q^2. In this kinematic region we expect that the typical distances will be small but the parton density will be so large that a new non perturbative approach shall be developed for understanding this system.

We are going to advocate the idea that these two problems are correlated and the system of partons always passes the stage of hdQCD before (at shorter distances) it goes to the black box, which we call non-perturbative QCD and which, practically, we describe in old fashion Reggeon phenomenology. In spite of the fact that there are many reasons to believe that such a phenomenology could be even correct, the distance between the Reggeon approach and QCD is so large we are loosing any taste of theory doing this phenomenology. In hdQCD we still have a small parameter (running QCD coupling α_S) and we can start to approach this problem using the developed methods of pQCD [1]. However, we should realize that the kernel of the hd QCD problems is a non-perturbative one,

and therefore, approaching hdQCD theoretically we are preparing new training grounds for searching methods for npQCD.

First, let me recall that DIS experiment is nothing more than a microscope and we have two variables to describe its work. The first one is the resolution of the microscope, namely, $\Delta x \approx 1/Q$ where Q^2 is the virtuality of the photon. It means that our microscope can see all constituents inside a target with the size larger that Δx. The second variable is time of observation. It sounds strange that we have this new variable, which we do not use, working with a usual light microscope. However, we are dealing here with the relativistic system which can produce hadrons (partons). So, for everyday analogy, we should consider rather a box with flies which multiply and their number is, certainly, different in different moments of time. To estimate this time we can use the uncertainty principle $\Delta t \propto 1/\Delta E$ where ΔE is the change of energy, namely, $\Delta E = E_{initial} - E_{final}$, and for a system of quark and antiquark $\Delta E = q_0 - q_1 - q_2 = q_0 - q = \frac{(q_0-q)(q_0+q)m}{2q_0 m} = \frac{Q^2 m}{W^2} = mx$, where m is the mass of the target and q_0 and q is the energy and momentum of the virtual photon. Finally, $t = 1/mx$ with $x = \frac{Q^2}{W^2}$ where W is the energy of the photon - target interaction.

Therefore, the question, that we are asking in DIS at low x, is what happens with constituents of rather small size after a long time. It is clear that the number of these constituents should increase since in QCD each parton can decay in two partons with the probability $P_i = \frac{N_c \alpha_S}{\pi} \frac{dE_i}{E_i} \frac{d^2 k_{i,t}}{k^2_{i,t}}$ where E_i and $\boldsymbol{k}_i$ are energy and momentum of an emitted parton i.

This growth we can describe by introducing the so called structure function ($xG(x,Q^2)$) or the number of partons that can be resolved with the microscope with definite Q^2 and x. Indeed,

$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln(1/x)\, \partial \ln Q^2} \;=\; \frac{N_c \alpha_S}{\pi} xG(x,Q^2)\;. \tag{1}$$

This equation is the DGLAP [2] evolution equation in the region of low x. It has an obvious solution $xG(x,Q^2) \;\propto\; exp\left(2\sqrt{\frac{N_c \alpha_S}{\pi}\,\ln(1/x)\,\ln(Q^2/Q_0^2)}\right)$. Therefore, we expect the increase of the parton densities at $x \to 0$.

In Fig. 1 we picture the parton distributions in the transverse plane. At $x \approx 1$ there are several partons of a small size. The distance between partons is much larger than their size and we can neglect interactions between them.

However, at $x \to 0$ the number of partons becomes so large that they are populated densely in the area of a target. In this case, you cannot neglect the interactions between them which was omitted in the evolution equations (see Eq. (1)). Therefore, at low x we have a more complex problem of taking into account both emission and rescatterings of partons. Since the most important in QCD is the three parton interaction, the processes of rescattering is actually a process of annihilation in which one parton is created out of two partons (gluons).

Therefore, at low x we have two processes

1. The emission induced by the QCD vertex $G + G \to G$ with the probability which is proportional to $\alpha_S\,\rho$ where ρ is the parton density in the transverse plane , namely

$$\rho = \frac{xG(x,Q^2)}{\pi R^2}\ , \tag{2}$$

where πR^2 is the target area;

2. The annihilation induced by the same vertex $G+G \to G$ with the probability which is proportional to $\alpha_S \sigma_0\,\rho^2$, where α_S is probability of the processes $G + G \to G$, σ_0 is the cross section of two parton interaction and $\sigma_0 \propto \frac{\alpha_S}{Q^2}$. $\sigma_0\,\rho$ gives the probability for two partons to meet and to interact, while $\alpha_S \sigma_0\,\rho^2$ gives the probability of the annihilation process.

Finally, the change of parton density is equal to [1] [3]

$$\frac{\partial^2 \rho(x,Q^2)}{\partial \ln(1/x)\,\partial \ln Q^2} \;=\; \frac{N_c \alpha_S}{\pi}\rho(x,Q^2) \;-\; \frac{\alpha_S^2 \gamma}{Q^2}\,\rho^2(x,Q^2) \tag{3}$$

or in terms of the gluon structure function

$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln(1/x)\,\partial \ln Q^2} \;=\; \frac{N_c \alpha_S}{\pi} xG(x,Q^2) \;-\; \frac{\alpha_S^2 \gamma}{\pi R^2 Q^2}\,\left(xG(x,Q^2)\right)^2\ , \tag{4}$$

where γ has been calculated in pQCD [3].

Therefore, Eq.(4) is a natural generalization of the DGLAP evolution equations. The question arises, why we call such a natural equation for a balance

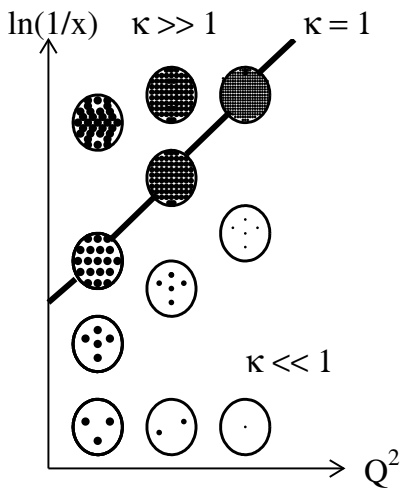

Fig. 1. Parton distribution in the transverse plane

of partons due to two competing processes shadowing and/or screening corrections (SC). To understand this let us consider the interaction of the fast hadron with the virtual photon at rest (Bjorken frame). In the parton model, only the slowest ("wee") partons interact with the photon. If the number of the "wee" partons N is not large, the cross section is equal to $\sigma_0 N$. However, if we have two "wee" partons with the same energies and momenta, we overestimate the value of the total cross section using the above formula. Indeed, the total cross section counts only the number of interactions and, therefore, in the situation when one parton is situated just behind another we do not need to count the interaction of the second parton if we have taken into account the interaction of the first one. It means that the cross section is equal to

$$\sigma_{tot} \;=\; \sigma_0 N \{\; 1 \;-\; \frac{\sigma_0}{\pi R^2} \;\} \;, \tag{5}$$

where R is the hadron radius. One can see that we reproduce Eq.(4) by taking into account that there is a probability for a parton not to interact being in a shadow of the second parton.

2 What Have We Learned about Sreening Corrections?

During the past two decades high parton density QCD has been under the close investigation of many theorists [1] [3][4][5] and we summarize here the result of their activity.

- The parameter which controls the strength of SC has been found and it is equal to

$$\kappa \;=\; \frac{3\,\pi^2\alpha_s}{2Q^2} \;\times\; \frac{xG(x,Q^2)}{\pi\,R^2} \;=\; \sigma_0 \;\times\; \rho(x,Q^2) \;. \tag{6}$$

  The meaning of this parameter is very simple. It gives the probability of interaction for two partons in the parton cascade or, better to say, a packing factor for partons in the parton cascade.

- We know the correct degrees of freedom at high energies: colour dipoles [6]. By definition, the correct degrees of freedom is a set of quantum numbers which mark the wave function that is diagonal with respect to the interaction matrix. Therefore, we know that the size and the energy of the colour dipole are not changed by the high energy QCD interaction.

- A new scale $Q_0^2(x)$ for hdQCD has been traced in the pQCD approach which is a solution to the equation

$$\kappa \;=\; \frac{3\,\pi^2\alpha_s}{2Q_0^2(x)} \;\times\; \frac{xG(x,Q_0^2(x))}{\pi\,R^2} \;=\; 1 \;. \tag{7}$$

  This new scale leads to the effective Lagrangian approach which gives us a general non-perturbative method to study hdCD.

- We know that the GLR equation (see Eqs.(3) and (4)) describes the evolution of the dipole density in the full kinematic region [7]. We understand that the Mueller-Glauber approach for colour dipole rescattering gives the initial condition to the GLR equation.
- The new, non-perturbative approach, based on the effective Lagrangian [5], have been developed for hdQCD which gives rise to the hope that hdQCD can be treated theoretically from first principles.
- We are very close to understanding of the parton density saturation [1].

In general, we think that the theoretical approach to hdQCD in a good shape now.

3 The HERA Puzzle: Where Are the Sreening Corrections?

The wide spread opinion is that HERA experimental data for $Q^2 \geq 1 GeV^2$ can be described quite well using only the DGLAP evolution equations, without any other ingredients such as shadowing corrections, higher twist contributions and so on (see, for example, the reviews [8]). On the other hand, the most important HERA discovery is the fact that the density of gluons (gluon structure function) becomes large in the HERA kinematic region [8][9]. The gluon densities extracted from HERA data are so large that the parameter κ (see Eq.(6)) exceeds unity in a substantial part of the HERA kinematic region (see Fig.2a). Another way to see this is to plot the solution to Eq.(7) (see Fig.2b). It means that in a large kinematic region $\kappa \geq 1$ (to the left from the line $\kappa = 1$ in Fig.2b), we expect that the SC should be large and important for a description of the experimental data. At first sight such expectations are in clear contradiction with the experimental data. Certainly, this fact gave rise to the suspicion or even mistrust that our theoretical approach to SC is not consistent. However, the revision and re-analysis of the SC , as has been discussed in the previous section, have been completed with the result, that κ is responsible for the value of SC.

Therefore, we face a puzzling question: *where are the SC?*. Actually, this question includes, at least, two questions: (i) why SC are not needed to describe the HERA data on F_2, and (ii) where are the experimental manifestation of strong SC. The answers for these two questions you will find in the next three sections but, in short, they are: SC give a weak change for F_2 in the HERA kinematic region, but they are strong for the gluon structure function . We hope to convince you that there are at least two indications in the HERA data supporting a large value of SC to the gluon density:

1. x_P - behaviour of the cross section for diffractive dissociation (σ^{DD}) in DIS;
2. Q^2 - behaviour of F_2 -slope ($\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}$).

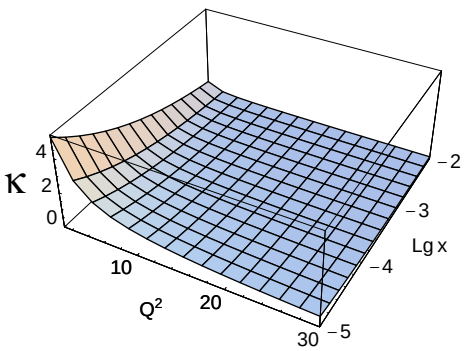

Fig.2a

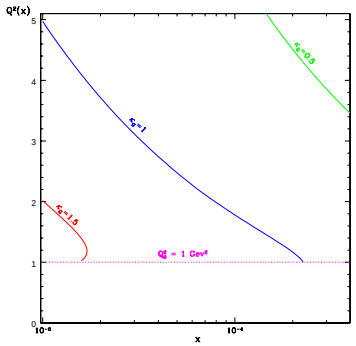

Fig.2b

Fig. 2. Parameter κ and new scale $Q_0^2(x)$.

4 SC for F_2

It is well known, that the γ^* - hadron interaction goes in two stages: (i) the transition from virtual photon to colour dipole and (ii) the interaction of the colour dipole with the target. To illustrate how SC work, we consider the Glauber - Mueller formula which describes the rescatterings of the colour dipole with the target[10]:

$$F_2(x_B,Q^2) \;=\; \frac{N_c}{6\pi^3}\sum_1^{N_f} Z_f^2 \int_{Q_0^2}^{Q^2} dQ'^2 \int db_t^2 \{\; 1 \;-\; e^{-\frac{1}{2}\frac{4}{9}\kappa(x_B,Q'^2)\,S(b_t)}\;\} \;+\; F_2(x_B,Q_0^2)\,, \tag{8}$$

where $S(b_t) = e^{-\frac{b_t^2}{R^2}}$ is the target profile function in the impact parameter representation and $\frac{4}{9}\,\kappa(x_B,Q'^2) = \sigma(x_B, r_\perp^2 = 4/Q^2)$ is the cross section of the dipole scattering in pQCD.

One can see that Eq.(8) leads to

$$F_2(x_B,Q^2) \;\longrightarrow\; \frac{N_c}{6\pi^3}\sum_1^{N_f} Z_f^2\; Q^2R^2\;. \tag{9}$$

However, we are sure that the kinematic region of HERA is far away from the asymptotic one. The practical calculations depend on three ingredients: the value of R^2, the value of the initial virtuality Q_0^2 and the initial F_2 at Q_0^2. We fix them as follows: $R^2 = 10\,GeV^{-2}$ which corresponds to "soft" high energy phenomenology [11], $Q_0^2 = 1\,GeV^2$ and $F_2^{input}(x_B,Q_0^2) = F_2^{GRV'94}(x_B,Q_0^2 = 1\,GeV^2)$. Therefore, the result of the calculation should be read as "SC for colour dipoles with the size smaller than $r_\perp^2 \leq 4/GeV^2$ are equal to ..."

From Fig.3 one can see that the SC are rather small for F_2 but they are strong and essential for the gluon structure function. It means that we have to

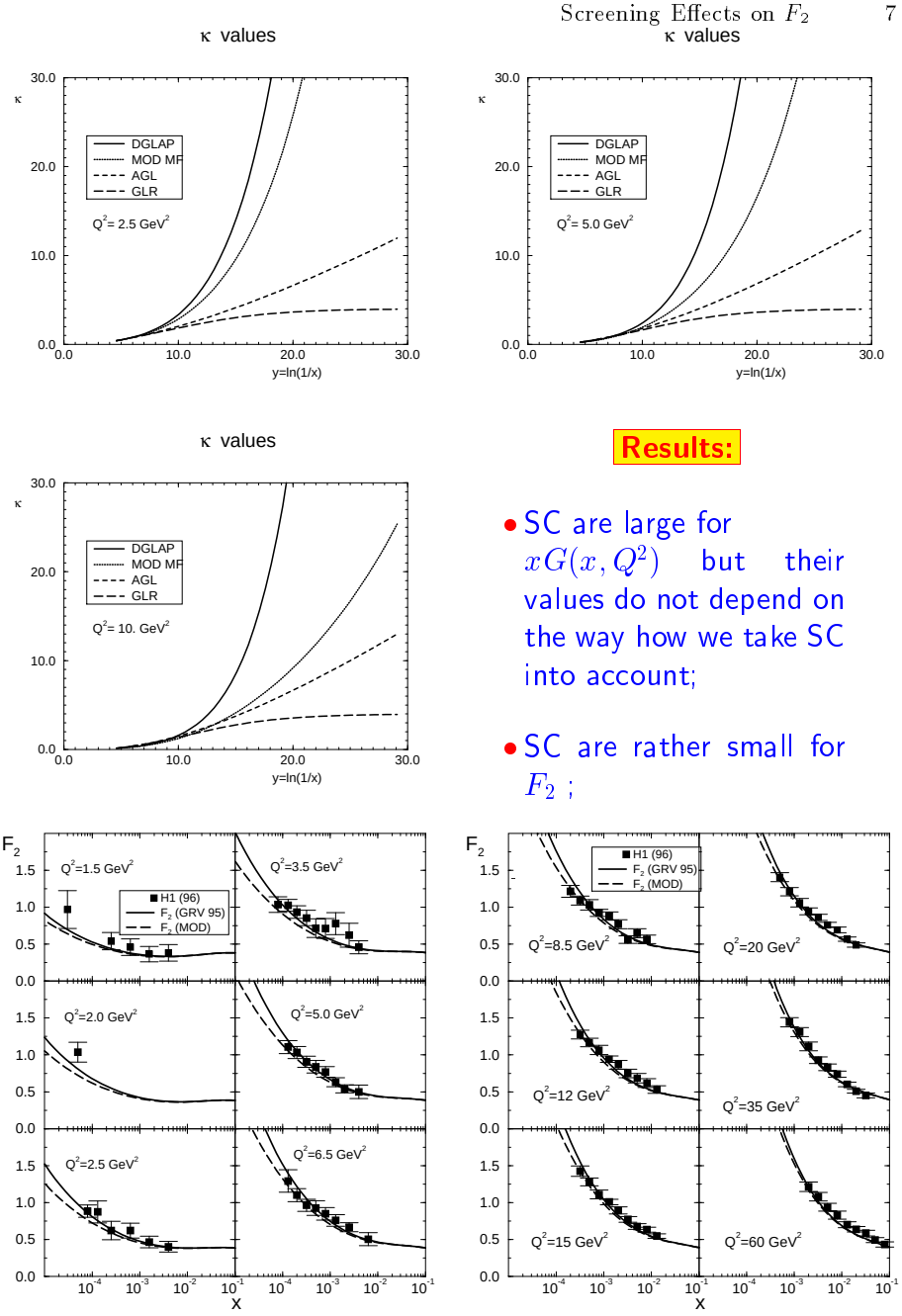

Fig. 3. SC for $xG(x,Q^2)$ and F_2 in the HERA kinematic region .

look for physical observables which will be more sensitive to the value of the gluon structure function than F_2.

5 x_P- Dependence of σ^{DD}

One of such observables is the cross section of the diffractive dissociation and, especially, the energy dependence of this cross section.

Data: Both H1 and ZEUS collaborations [8] found that

$$\sigma^{DD} \;\propto\; \frac{1}{x^{2\Delta_P}}\;, \tag{10}$$

where $\Delta_P = \alpha_P(0) - 1$ and the values of $\alpha_P(0)$ are:

- **H1 [12] :** $\alpha_P(0) = 1.2003 \pm 0.020(\mathrm{stat.}) \pm 0.013(\mathrm{sys})$;
- **ZEUS [13]:** $\alpha_P(0) = 1.1270 \pm 0.009(\mathrm{stat.}) \pm 0.012(\mathrm{sys})$.

It is clear that the Pomeron intercept ($\alpha_P(0)$) for diffractive processes in DIS is higher than the intercept of the "soft" Pomeron [11].

Why is it surprising and interesting? To answer this question we have to recall that the cross sections for diffractive production of a quark-antiquark pair have the following form in pQCD [14] [15]:

$$x_P\,\frac{d\sigma^T_{DD}(\gamma^*\,\to\,q+\bar{q})}{dx_P dt}\;\propto\;\int_{Q_0^2}^{\frac{M^2}{4}}\frac{dk_\perp^2}{k_\perp^2}\;\times\;\frac{\Big(\alpha_S\;x_P\,G(x_P,\frac{k_\perp^2}{1-\beta})\Big)^2}{k_\perp^2}\;; \tag{11}$$

$$x_P\,\frac{d\sigma^L_{DD}(\gamma^*\,\to\,q+\bar{q})}{dx_P dt}\;\propto\;\int_{Q_0^2}^{\frac{M^2}{4}}\frac{dk_\perp^2}{Q^2}\;\times\;\frac{\Big(\alpha_S\;x_P\,G(x_P,\frac{k_\perp^2}{1-\beta})\Big)^2}{k_\perp^2}\;. \tag{12}$$

From Eqs.(11) and (12) you can see that the $k_\perp$ integration looks quite differently for a transverse or a longitudinal polarized photon: the latter one has a typical log integral over $k_\perp$ while the former has the integral which normally converges at small values of $k_\perp$. We have the same property for the production of a more complex system than $q\bar{q}$, for example $q\bar{q}G$ [15]. Therefore, we expect that the diffractive production should come from long distances where the "soft" Pomeron contributes. However, the experiment says a different thing, namely, that this production has a considerable contamination from short distances. How is it possible? As far as we know, there is the only one explanation: SC are so strong that $xG(x,k_\perp^2)\;\propto\;k_\perp^2 R^2$ (see Eq.(9)) Substituting this asymptotic limit in Eq.(11) one can see that the integral becomes convergent and it sits at the upper limit of integration which is equal to $k_\perp^2 = Q_0^2(x)$.

Finally, we have

$$x_P\,\frac{d\sigma_{DD}}{dx_P dt}\;\;\longrightarrow\;\;\Big(x_P\,G(x_P,Q_0^2(x_P))\Big)^2\;\times\;\frac{1}{Q_0^2(x_P)} \tag{13}$$

The calculation for $\frac{\partial xG(x,Q^2)}{\partial \ln(1/x)}$ is given in Fig.4 for the HERA kinematic region using the Glauber-Mueller formula [10] for SC. Taking into account that $Q_0^2(x)$ in Fig.2b can be fitted as $Q_0^2(x) \;=\; 1\; GeV^2 \left(\frac{x}{x_0}\right)^{-\lambda}$ with $\lambda = 0.54$ and $x_0 = 10^{-2}$ we see from Eq.(13) and Fig.4 that we are able to reproduce the experimental value of $\alpha_P(0)$ and conclude that the typical $k_\perp^2$ which are dominant in the integral is not small ($k_\perp^2 \;\approx\; 1 - 2\,GeV^2$ [15]). For the Golec-Biernat Wüsthoff approach, which we will discuss later, $\lambda = 0.288$ and the value of typical $k_\perp^2$ turns out to be higher.

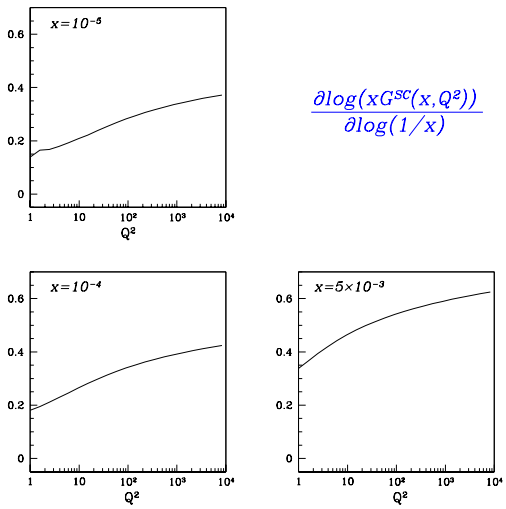

Fig. 4. Effective Pomeron intercept for the gluon structure function calculated using the Glauber-Mueller formula for SC

6 The Q^2- Dependence of the F_2 - Slope.

Data: The experimental data [13] for the F_2 - slope $dF_2(x,Q^2)/d\ln Q^2$ are shown in Fig.5a (Caldwell plot). These data give rise to a hope that the matching between "hard" (short distance) and "soft" (long distance) processes occurs at sufficiently large Q^2 since the F_2-slope starts to deviate from the DGLAP predictions around $Q^2 \approx 5 - 8\,GeV^2$.

F_2-slope and SC: Our principle idea, as we have mentioned in the beginning of the talk, is that matching between "hard" and "soft" processes is provided by the hdQCD phase in the parton cascade or, in other words, due to strong SC. The asymptotic behaviour of $F_2 \propto Q^2R^2$ for $Q^2 \leq Q_0^2(x)$ leads to

$dF_2(x,Q^2)/d\ln Q^2 \propto Q^2R^2$ at $Q^2 \leq Q_0^2(x)$ (see Eq.(9)) and this behaviour supports our point of view [16][17].

However, we have two problems to solve before making any conclusion: (i) the experimental data are taken at different points (x,Q^2) and therefore could be interpreted as the change of x-behaviour rather than Q^2 one; and (ii) the value of the F_2-slope is quite different from the value of F_2 while for the asymptotic solution it should be the same. Therefore, we have to calculate the F_2 - slope to understand them. The result of the calculation using the Glauber-Mueller formula [16] is presented in Fig.5b. One can see that (i) the experimental data show rather the Q^2 - behaviour than the x-dependence, which is not qualitatively influenced by SC; and (ii) SC are able to describe both the value and the Q^2-behaviour of the experimental data. Fig.5b shows also that the ALLM'97 parameterization [18], which can be viewed as the phenomenological description of the experimental data, has the same features as our calculation confirming the fact that the data show the Q^2 - dependence but not the x-behaviour of the F_2-slope.

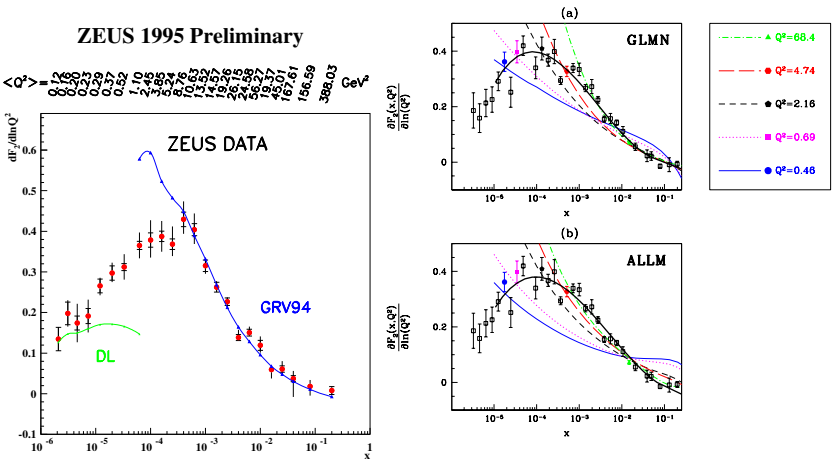

Fig.5a Fig.5b

Fig. 5. F_2-slope: experimental data (Caldwell plot) (Fig.5a) and calculations using the Glauber-Mueller formula (Fig.5b) .

7 Golec-Biernat Wüsthoff Approach

Golec-Biernat and Wüsthoff [19] suggested a phenomenological approach which takes into account the key idea of hdQCD, namely, the new scale of hardness in

the parton cascade. They use for $\gamma^* p$ cross section the following formula [19]

$$\sigma_{tot}(\gamma^* p) \;=\; \int d^2 r_\perp \int_0^1 dz\, |\Psi(Q^2; r_\perp, z)|^2 \;\sigma_{tot}(r_\perp^2, x) \;; \tag{14}$$

$$\sigma(x, r_\perp) \;=\; \sigma_0 \;\{\; 1 \;-\; e^{-\frac{r_\perp^2}{R^2(x)}} \;\}\;; \tag{15}$$

$$R^2(x) \;=\; 1/Q_0^2(x) \;\; with \;\; Q_0^2(x) \;=\; Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda} . \tag{16}$$

Extracting the parameters of their model from fitting of the experimental data, namely, σ_0=23.03 mb, $\lambda = 0.288$, $Q_0^2 = 1GeV^2$ and $x_0 = 3.04\,10^{-4}$, they described quite well all the data on total and diffractive cross sections in DIS (see Fig.6).

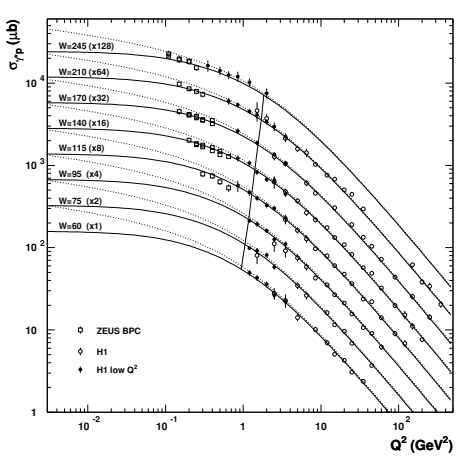

Fig.6a

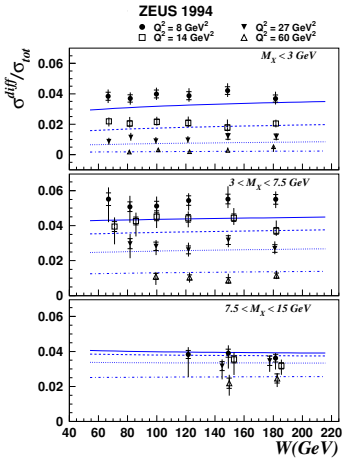

Fig.6b

Fig. 6. $\sigma_{tot}(\gamma^* p)$ and the ratio σ^{DD}/σ_{tot} for DIS in the Golec-Biernat Wüsthoff model. Vertical line in Fig.6a is the $Q_0^2(x)$ given by Eq.(16).

8 Why Have We Only Indications?

The answer is: because we have or can have an alternative explanation of each separate fact. For example, we can describe the F_2 -slope behaviour changing the initial x-distribution for the DGLAP evolution equations [20]. Our difficulties in an interpretation of the experimental data is seen in Fig.6a where the new scale $Q_0^2(x)$ is plotted. One can see that $Q_0^2(x)$ is almost constant in the HERA kinematic region. It means that we can put the initial condition for the evolution equation at $Q_0^2 =< Q_0^2(x) >$ where $< Q_0^2(x) >$ is the average new

scale in the HERA region. Therefore, SC can be absorbed to a large extent in the initial condition and the question, that can and should be asked, is how well motivated these conditions are. For example, I do not think that the initial gluon distribution in the MRST parameterization [20], needed to describe the F_2 - slope data, can be considered as a natural one.

9 Summary

We hope we convinced you that (i) hdQCD is in a good theoretical shape; (ii) the hdQCD region has been reached at HERA; (iii) HERA data do not contradict the strong SC effects; (iv) there are at least two indications on SC effects in the HERA data: the Q^2 behaviour of the F_2 slope and the x_P behaviour of diffractive cross section in DIS; and (v) the HERA data and the hdQCD theory gave an impetus for a very successful phenomenology for matching "hard" and "soft" physics.

We would like to finish this talk with a rather long citation: "Small x Physics is still in its infancy. Its relation to heavy ion physics, mathematical physics and soft hadron physics along with a rich variety of possible signatures makes it central for QCD studies over the next decade" (A.H. Mueller, B. Müller, G. Rebbi and W.H. Smith "Report of the DPF Long Range Planning WG on QCD"). Hopefully, we will learn more on low x physics at the next Ringberg Workshop.

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