Forward jet production as a probe of low x dynamics

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Abstract. The HERA facility at Hamburg has unveiled a wealth of physics spanning a broad kinematic domain. The question of how best to describe data obtained at low x remains open, the main problem being the difficulty of cleanly disentangling perturbative and non-perturbative effects. A recent set of measurements, looking at final states with high transverse momentum produced close to the proton remnant offers fresh insight into low x dynamics. The reasons why these configurations are especially interesting are motivated, and complementary approaches to describing the physics involved are outlined. We illustrate this by comparing predictions based on small x (QCD) dynamics with recent data for deep inelastic events containing forward jets or forward π^0 mesons. We quantify the effect of imposing the (higher order) consistency condition on the BFKL equation and study uncertainties inherent in the QCD predictions.

1 Introduction.

The ability of perturbative QCD to describe a wide variety of HERA phenomena over a broad span of (x, Q^2) kinematic space has been well demonstrated. There is general consensus that the mechanisms which underpin the description of physics within the large Q^2 region, are based on the DGLAP evolution equations which effectively resum the $\alpha_s \log Q^2$ contributions. The small x limit of this approach resums the $\alpha_s \log Q^2 \log(1/x)$ contributions and is known as the Double Leading Log Approximation (DLLA).

In recent years, the focus of experimental and theoretical efforts have shifted to the region of low x and with medium Q^2, which goes beyond DLLA in that we must resum $\alpha_s \log(1/x)$ terms for all Q^2, and not just take the $\log Q^2$ limit. As HERA began to probe at ever decreasing values of Bjorken x ($1/x \sim s$), it became apparent that at high energies, a perturbative expansion containing terms of the form $\alpha_s \log(1/x)$ at *all* Q^2, and not simply the $\log Q^2$ limit, is necessary. That is, for very low x, the small size of the strong coupling α_s is not sufficient to tame large $\log(1/x)$ terms (unaccompanied by $\log Q^2$). These would need to be resummed to all orders. The failure of the standard DGLAP approach to DIS phenomena, wherein large logarithms of $(\alpha_s \log Q^2)^2$ and also $\alpha_s(\alpha_s \log Q^2)^2$ are systematically resummed to all orders, would be a signal for the onset of a distinct low x dynamics. Is there experimental evidence for such effects? This question is not so easy to answer as it first appears.

An early idea, first mooted by Mueller [1], in which one scatters inelastically from a parton, and selects kinematic configurations that can isolate clearly the

low x effects, was enthusiastically taken up by the experimental community at HERA [2,3]. Data of sufficient quality have now been obtained and it is now an interesting phenomenological challenge to see what the measurements imply about the underlying small x dynamics. This is the subject of the current review.

Resumming the large $\log(1/x)$ terms to leading order is performed within the Balitskij, Fadin, Kuraev, Lipatov (BFKL) formalism, first outlined in the late 1970's [4,5]. Since then, leading log BFKL based phenomenology was shown to be insufficient, since the next-to-leading corrections were calculated [6–8] and found to be large. A phenomenological modification to the leading BFKL formalism [9,10] has provided an economical, physically based, description of a variety of phenomena within the low x QCD framework. An alternative formalism [11,12], based on the concept of the parton content of the resolved photon, has shown similar success, over a similarly broad range of phenomena.

Unequivocably describing the physics of low x region, within the bounds of perturbative QCD, would certainly be a worthwhile achievement.

2 BFKL $\log(1/x)$ resummation.

The BFKL prescription for the resummation of small x contributions has been performed for both leading log (LL) [4,5], terms of the form $\alpha_s^n \log^n(1/x)$, and next-to-leading log (NLL) [6–8], terms of the form $\alpha_s^{n+1} \log^n(1/x)$.

All $\log(1/x)$ contributions have to be considered, and so we must retain the full transverse momentum dependence of the gluon distribution, and not simply the $\log Q^2$ dependence arising from the strong ordering in k_T. The ordering in the transverse momentum is relaxed, and an unintegrated gluon distribution, ϕ_i, is introduced, related to the usual gluon distribution, g_i, of a photon of virtuality Q^2 and polarisation $i = T, L$, probed at a scale $\bar{Q}^2$ via:

$$x g_i(x, Q^2, \bar{Q}^2) = \int^{\bar{Q}^2} \frac{dk_T^2}{k_T^2} \phi_i(x, k_T^2, Q^2)$$

x is the longitudinal momentum fraction of the virtual photon carried by the gluon, and k_T denotes its transverse momentum.

We begin with a brief overview of the BFKL technique. It is assumed that the dominant process in the high energy limit is gluon exchange in the t-channel. Higher order QCD corrections are collected together to form an 'effective' diagram. This comprises an effective gluon vertex, which subsumes real gluon emissions, whilst the t-channel gluon reggeizes with the virtual corrections. Then, using this 'rung', one can recursively build up a ladder diagram with reggeized t-channel gluonic propagators emitting regular gluons from effective vertices. The BFKL equation represents the sum of all such ladder diagrams.

$$\phi_i(x,k_T^2,Q^2) = \phi_i^{(0)}(x,k_T^2,Q^2) +$$

$$k_T^2 \int_x^1 \frac{dz}{z} \int_{k_0^2}^{\infty} \frac{dk_T'^2}{k_T'^2} \bar{\alpha_s} \left(\frac{\phi_i(x/z,k_T'^2,Q^2) - \phi_i(x/z,k_T^2,Q^2)}{\mid k_T^2 - k_T'^2 \mid} + \frac{\phi_i(x/z,k_T^2,Q^2)}{(4k_T'^4 + k_T^4)^{\frac{1}{2}}} \right) \tag{1}$$

where $i = T$ or L, the $\phi_i^{(0)}$ represent the driving terms of the equation, $\bar{\alpha}_s = 3\alpha_s/\pi$ and x is the fraction of the virtual photon's longitudinal momentum carried by the gluon. The term proportional to $\phi_i(x/z,k_T'^2,Q^2)$ corresponds to the real gluon emissions and the terms proportional to $\phi_i(x/z,k_T^2,Q^2)$ describe the virtual corrections generating reggeization of the gluons. We see that real and virtual contributions to the integrand cancel in the potentially singular limit of $k_T'^2 \longrightarrow k_T^2$. Notice the equation is invariant under a scaling of the transverse momentum, which allows an analytic solution in the case of fixed α_s to be calculated using Mellin transforms.

Taking α_s fixed, it is possible to calculate the characteristic exponent of the singular power behaviour in x of the unintegrated gluon distribution:

$$\phi \sim x^{-\lambda}$$

with exponents at leading and next-to-leading log, in the $\overline{\rm MS}$ scheme, given by

$$\lambda_{LL} = \bar{\alpha}_s 4 \log 2$$

$$\lambda_{NLL} = \lambda_{LL} \left(1 - 3.4\bar{\alpha}_s - 0.15 \frac{N_f \alpha_s}{\pi} \right) \tag{2}$$

[A brief aside: The BFKL singular behaviour of g_i as $x \longrightarrow 0$ cannot be the complete picture of low x physics. On general conceptual grounds, the Froissart bound forbids the growth of the cross-section faster than $\log^2(s)$. We can motivate this physically by recognising that as we go to higher energies, the density of gluons within the proton will increase to the point that they can no longer be treated as free partons. The gluons will recombine, and the cross-section will be suppressed. At current HERA energies there seems to be no evidence yet for the presence of these *'shadowing'* or *'unitarity'* corrections [13].]

It is clear that the NLL corrections are substantial and thus that LL based phenomenology is strictly unreliable. It is possible to impose higher order, physically motivated restrictions in the governing evolution equation *to all orders*. One important example is that the LL BFKL equation does not respect energy-momentum conservation, the violation being produced by subleading terms. In fact Orr and Stirling [14] are developing a BFKL Monte Carlo which will allow the effect of energy-momentum conservation to be quantified.

There exists a further physical constraint that is more stringent than energy-momentum conservation, and implemented in the BFKL equation rather simply. The *consistency constraint* imposes the physical condition that the virtuality of exchanged gluons along a BFKL chain should arise predominantly from the

transverse component of momentum [9].

$$k_T'^2 < \frac{k_T^2}{z}.$$

This ensures the validity of the small x approximation, by restricting the maximum transverse momentum of an emitted gluon. The BFKL equation was derived assuming this constraint, and in the region of phase space beyond that delimited by the constraint the BFKL equation is inappropriate. Note that, in particular, the gluon propagators which appear in the BFKL equation contain only the transverse momenta squared. It has been shown [9] that including this condition implicitly subsumes the neglected energy-momentum conservation over the greater part of the small x kinematic space.

It can be neatly encapsulated, and incorporated into the LL BFKL equation, through the inclusion of a Θ-function on the kernel component governing real gluon emission:

$$\Theta(k_T^2/k_T'^2 - z)$$

It transpires that limiting the LL BFKL formalism in this way induces an effect at all orders which if truncated to NLL gives the dominant part of the NLL expression of (2). One might conjecture that these represent the dominant higher order effects, and that the modified BFKL evolution resums these. Is it possible to obtain a good description of the HERA data by implementing this type of formalism?

3 Experimental signatures of BFKL-type dynamics.

A definitive experimental identification of the low x physics 'smoking gun' has proven somewhat elusive. There exist several variations on the original 'gold plated' measurements proposed by Mueller[1], that in principle might exhibit the characteristic signs of a rapid rise in gluon distribution at low x, but uniquely separating the $\log(1/x)$ from non-perturbative effects is certainly non-trivial.

3.1 Inclusive quantities.

The first dramatic evidence for the effects of small x resummations came from HERA measurements of the proton structure function $F_2(x, Q^2)$, which exhibit a strong rise with decreasing x. In [15] a quantitative analysis of the F_2 data within a united modified BFKL and DGLAP framework was presented.

The steep increase was initially lauded as a clear demonstration of the BFKL small x resummation effects, but it soon became apparent that it was not possible to ascribe this feature unambiguously to BFKL-type physics. It proved possible to adequately describe the unexpected features in the data as a consequence of DLLA resummations, in which terms of the form $\alpha_s^n \log^n(1/x) \log^n(Q^2)$ are resummed to all orders. There exist sufficient provisos in the theoretical descriptions of F_2 to compromise its discriminatory powers:

- DGLAP evolution, with a sufficiently 'tuned' non-perturbative input distribution can mimic the inverse power behaviour predicted by the BFKL approach.
- The disorder in k_T corresponds to a random walk in transverse momentum along the gluon chain. The kinematic bounds imposed by the F_2 measurement allow for this transverse momentum to significantly diffuse into the infra-red region, blurring clear interpretation.
- The parton distribution functions necessary to an analysis are not sufficiently well determined at all the required kinematical configurations for a quantitative analysis.

A more stringent process is required in order to draw firm conclusions.

3.2 Exclusive quantities.

It is clear that inclusive measurements (F_2 etc...) are unable to disentangle cleanly low x parton dynamics from non-perturbative effects. In attempting to do so, model dependences are necessarily introduced. One should try and utilise more exclusive measurements. More specifically, final states restricted in kinematic (x, Q^2) space by the requirement that they are produced close to the proton remnant, prove to be of particular interest.

Mueller proposed that DIS events containing an identified forward jet would prove an excellent discriminator for low x dynamics. The detailed formalism was given in [16]. These particular measurements were made by the H1 and ZEUS collaborations at HERA [2,3]. Recent analyses can be found in [16,17]. Subsequently, it was realised that this process could be extended, and in some sense refined, by studying energetic forward π^0's produced close to the proton direction [17], and also that the forward dijet rate [18] would yield another possible window on the low x physics.

These processes share the same basic properties that make them a useful probe of low x dynamics:

- Experimentally, one requires that $Q^2 \sim k_T^2$. This ensures that DGLAP type evolution is suppressed, and that features in the data arise solely from small x dynamics.
- By restricting the study to configurations where the final state is produced close to the beam pipe, one insists that the jet carries a large fraction of the longitudinal momentum of the parent proton. This ensures the evolution parameter x/x_j is small, necessary for the validity of the BFKL approach, and also that x_j is sufficiently large for the parton distribution functions to be determined at regions in the phase space where they are well determined from global analyses.
- In principle, the large k_T^2 of the process suppresses the diffusion into the infra-red region due to the random walk in transverse momentum along the gluon chain.

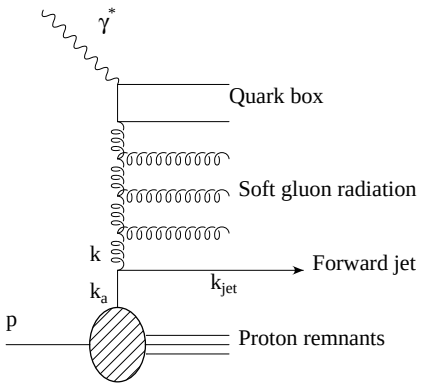

Fig. 1. Generic BFKL formalism for the Mueller process. Virtual photon-gluon fusion realised through the quark box, and soft gluon radiation between that and the struck parton produced close to the proton remnants.

In summary, Mueller's proposal was to perform deep inelastic scattering on parton constituent of the proton, and so to neatly sidestep non-perturbative hadronization problems.

In addition one can examine the subset of events that produce π^0's in the forward direction [2,19]. These measurements possess additional qualities that complement the basic Mueller process:

- The π^0 measurement samples more energetic forward jets than could be measured explicitly in jets. By allowing the struck parton fragment collinearly into a pion with longitudinal momentum fraction $x_\pi < x_j$ and transverse momentum $p_{\pi T} < k_{jT}$ we sample a greater amount of the relevant phase space.
- Experiments take measurements at the *hadron* level, theoreticians make calculations at the *parton* level, so there is a certain amount of ambiguity involved in comparing the predictions with the data... In studying final state particles, hadronisation uncertainties are swept into parameterisations of fragmentation functions, and we can be certain of comparing like quantities.
- Experimentally, it is easier to unambiguously identify a π^0 rather than a jet. The weak model dependence from the algorithms used to specify exactly what constitutes a 'jet' is also eliminated by looking at a specific final state.
- The conceptual 'gains' outlined above must be tempered with a practical 'loss'. By requiring a single energetic fragment of the jet, we reduce the cross-section and so the observed event rate.

Finally, we can obtain an additional measure of the low x region by looking at the subset of forward jet events that contain *two* jets in the forward region.

4 The BFKL approach to forward jet production.

Unlike the previous analyses [16,17], ref. [10] uses the modified BFKL equation to evolve the gluon distribution from the photon end down to the struck parton. The virtual photon is connected to the gluon chain through a quark box, which is calculable in perturbative QCD. This comprises the driving term for the unintegrated gluon distribution, $\phi^{(0)}$ in the BFKL equation. The modified BFKL equation is solved numerically for the unintegrated gluon distribution of the virtual photon, with the quark box as the inhomogeneous driving terms, and allowing α_s to run. Longitudinal momentum fractions are related at the jet vertex through the *strong ordering* condition

$$k_{aT}^2 \ll k_{jT}^2,\ x_j^P \gg x$$

and utilising the jet on-shell condition $k_j^2 = 0$ The singular behaviour of the unintegrated gluon distribution in the BFKL approach then feeds through into observables via k_T factorisation

$$\frac{\partial^2 F_i}{\partial x_j^p \partial k_{jT}^2} = \bar{\alpha}_s k_{jT}^4 \left(\sum_a f_a(x_j^p, k_{jT}^2) \right) \Phi_i \left(\frac{x}{x_j^p}, k_{jT}^2, Q^2 \right) \tag{3}$$

where $i = 1, 2$ and $\sum_a f_a$ is the sum over parton distribution functions in the effective structure function approximation. A recent leading order paramaterisation of the parton distributions is used [20].

These differential structure functions are related to differential cross-sections in the usual way

$$\frac{\partial \sigma_j}{\partial x \partial Q^2 \partial x_j^p \partial k_{jT}^2} = \frac{4\pi\alpha^2}{xQ^4} \left((1-y) \frac{\partial^2 F_2}{\partial x_j^p \partial k_{jT}^2} + \frac{1}{2} y^2 \frac{\partial^2 F_T}{\partial x_j^p \partial k_{jT}^2} \right)$$

The experimental cuts applied by the H1 [2] and ZEUS [3] collaborations in order to restrict the sampling to purely forward jet events are imposed on the numerical evaluation of the cross-sections.

The analgous cross-sections for π^0 production are constructed by convoluting the DIS + forward jet differential cross-sections with the π^0 fragmentation function parameterisations. These are parameterisations of the non-perturbative hadronization of the parton into a final state π^0. We use the LO $\pi^{\pm}$ fragmentaion functions of Binnewies et al. [21], and the π^0 functions are then taken to be $\frac{1}{2}(\pi^+ + \pi^-)$ distributions. The π^0 is assumed to carry a fraction $z = x_\pi / x_j^P$ of the jet's longitudinal momentum in a direction collinear to the parent parton.

A sample of the results are shown in Figs. 2 and 3. Plots are shown for different choices of the scales (in the driving term $\phi^{(0)}$ of (1), and in α_s of (3)) and of the infrared cut-off k_0^2 in (1), taken respectively to be:

$$\begin{array}{lllll} \text{(i)} & (k_T^2 + m_q^2)/4, & k_T^2/4, & k_0^2 = 0.5\text{GeV}^2 & \text{(upper dashed)} \\ \text{(ii)} & (k_T^2 + m_q^2)/4, & k_T^2/4\,, & k_0^2 = 1\text{GeV}^2 & \text{(continuous)} \\ \text{(iii)} & (k_T^2 + m_q^2), & k_T^2, & k_0^2 = 0.5\text{GeV}^2 & \text{(lower dashed).} \end{array} \tag{4}$$

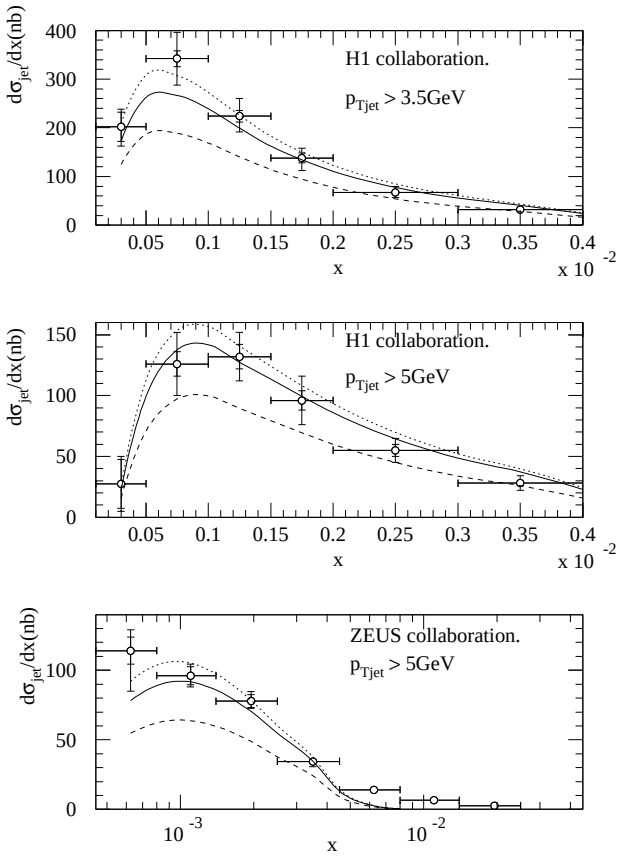

Fig. 2. The DIS + forward jet differential cross-section versus Bjorken-x as measured at the hadron level by the H1 [2] and ZEUS [3] collaborations. The curves are predictions, at the parton level, based on the BFKL formalism including sub-leading corrections, corresponding to the three choices of the scales and infrared cut-off given in (4).

We note that the calculations describe the forward jet, forward π^0 and forward dijet measurements well, for physically reasonably scales and cut-off. We see that they exhibit a weak residual dependence on a transverse momentum cut-off parameter and a stronger dependence on changes in the choice of scale.

In Fig. 4 we contrast the predictions of the modified BFKL formalism with that of the leading-log$(1/x)$ approximation. The data prefer the inclusion of the resummation to all orders of subleading terms through the consistency constraint, especially since converting the data down to the parton level is expected to lead to a decrease in cross-section of some 15-20%.

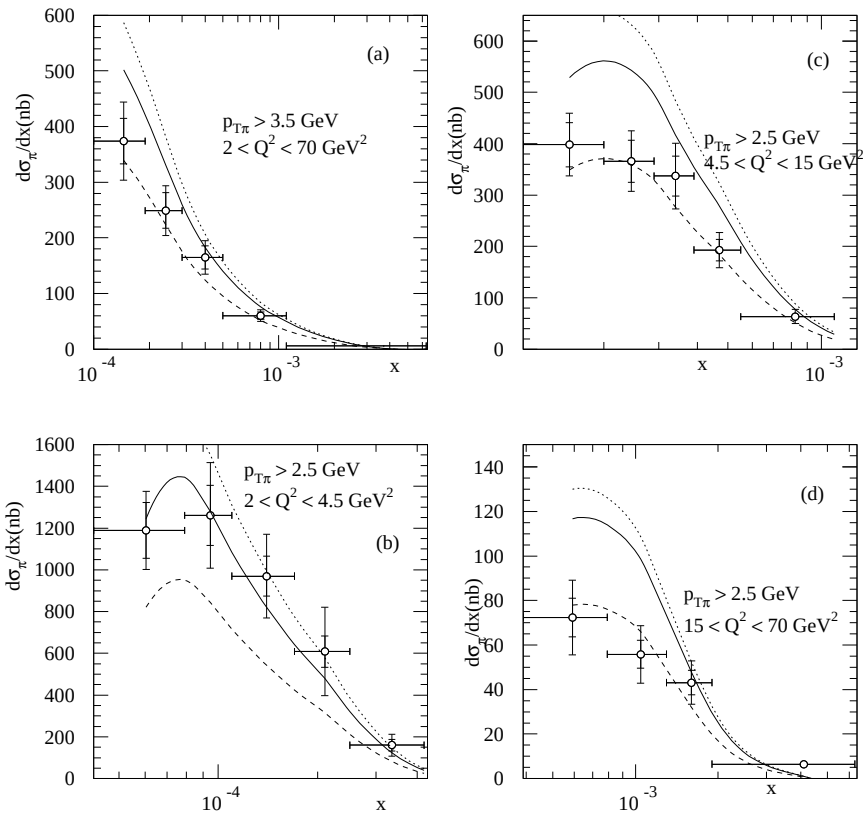

Fig. 3. The π^0 differential cross-section versus Bjorken-x obtained from H1 1996 data [19]. The curves are predictions, based on the BFKL formalism including sub-leading corrections, corresponding to the three choices of the scales and infrared cut-off given in (4).

5 The resolved and direct photon description of forward jet production.

The resolved and direct photon approach splits the process into two distinct mechanisms according to the size of the characteristic scale μ^2 of the subprocess involved. When $\mu^2 < Q^2$, the inverse size of the photon, a standard perturbative approach, with point-like photon interactions is taken. However, when $\mu^2 > Q^2$, the partonic structure of the virtual photon is said to have been resolved. This is then parameterised by a set of parton density functions analgous to the familiar proton parton density functions.

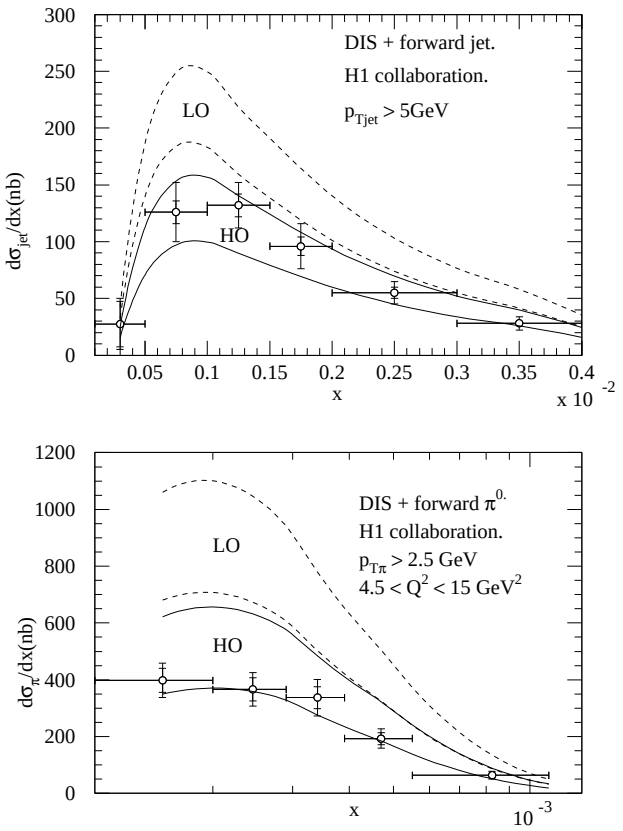

Fig. 4. The differential cross-section for (a) DIS + forward jet, (b) DIS + forward π^0, events. The continuous and dashed curves correspond to the inclusion and omission of the consistency constraint. The lower set of curves corresponds to the scales and infrared cut-off of (i) in (4), and the upper set of curves to (iii). The jet data are at hadron level; converting to the parton level is expected to lower the jet data points by 15-20%.

Two groups [11,12] have implemented the concept of a resolved photon contribution to describe forward jets in DIS. Both have produced Monte Carlo event generators (RAPGAP and JetViP) that describe the surplus of events over the naive (direct, point-like photon) DIS expectation.

RAPGAP [11] uses purely leading order matrix elements, and allows parton showering between both the hard subprocess and the resolved photon, and the subprocess and proton. It is thought that these simulate higher order processes, as the leading order matrix elements alone prove insufficient to describe the data.

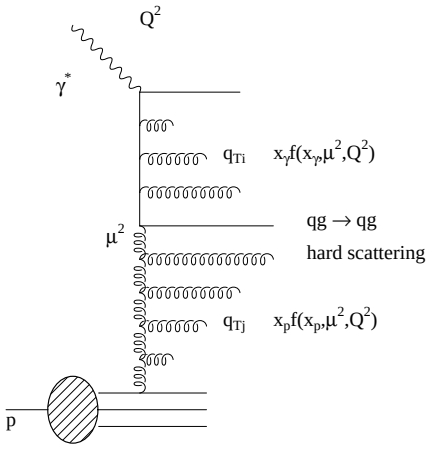

Fig. 5. Deep inelastic scattering and a forward jet within the LO resolved + direct photon formalism of [11], with DGLAP evolution from both the proton and virtual photon ends towards a $qg \longrightarrow qg$ subprocess.

The total cross-section is then calculated as the sum of resolved and direct photon contributions. Whilst we have transverse momentum ordering within each of the DGLAP chains, this formalism allows for an overall disordering in k_T^2 between the virtual photon and proton. This k_T^2 disorder appears to be a 'necessary' condition in producing the rise in the cross-sections at low x. It features in many of the models that best describe the DIS + forward jet data (c.f. BFKL phenomenology, this approach and the color dipole model (CDM) implemented in ARIADNE)

JetViP [12] treats the resolved and direct contributions consistently using NLO matrix elements, taking care to count only once those parts from the NLO direct contribution which are implicitly included from the resolved photon. It was found that the significant part of the NLO corrections arose from the NLO contribution from the resolved photon. This piece corresponds to an extra rung (and so includes one 'disorder') on the gluon ladder from the photon remnant and is essentially calculable within the framework of perturbative QCD.

Both formalisms including a resolved photon contribution into the DIS + forward jet process find good agreement between HERA forward jet data and calculation.

6 Conclusions

One cannot obtain a sufficient increase with decreasing x of the forward jet cross-section from fixed order QCD calculations. NLO predictions [22,23] show a large deficit compared with the data. It is clear that the modified BFKL, and the resolved photon (both NLO and LO + parton showers.) theoretical approaches to processes designed to clarify low x physics provide a good description of HERA

measurements. Resumming subleading effects to *all* orders in BFKL is clearly necessary given the large NLL corrections. Unfortunately, it is not known how to perform such a resummation exactly, indeed it is not clear if such a process is a feasible goal.

The consistency constraint provides a route by which we can circumvent the need for a full resummation, and instead implement the rather simpler all-orders resummation of the *dominant* subleading corrections. It is then possible to make phenomenological predictions for forward jets and π^0's.

Alternatives to this approach exist, based on supplementing the 'traditional' direct photon contribution with a situation where the photon itself exhibits partonic structure and is said to be resolved. The different formalisms, although at first glance appear quite distinct, share some of their conceptual ideas, and it is an informative exercise to contrast them. One might think of the modified BFKL approach, with gluon evolution extending *downwards* from the virtual photon-gluon fusion, as representing in some sense the unintegrated gluon content of the virtual photon.

For the resolved photon approach, the parton showering included at LO level is thought to approximate the explicit higher order effects in the full NLO calculation. In both cases significant enhancement of the cross-section arises from additional gluon emission from virtual photon corrections.

The LO resolved photon formalism can be thought of as mimicking, in a limited way, the complete k_T disordering of the BFKL approach, to the extent that the two approaches are indistinguishable at HERA.

Finally it is worth emphasising here the chief conceptual distinctions. The resolved photon analysis unilaterally divides the underlying mechanism according to the relative sizes of the scales (Q^2 and k_T^2) involved, whereas the modified BFKL approach treats in a unified way all of the possible kinematical configurations. In the BFKL approach, the Q^2 dependence of the parton 'content' of the virtual photon is dynamically generated through a perturbative calculation of the quark box, rather than paramaterised through the virtual photon parton distribution functions.

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