Power Corrections to the Parton Model in DIS

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Abstract. We review the techniques to evaluate the $1/Q^2$ corrections to the Deep Inelastic structure functions. The importance of the power corrections is that they indicate the kinematical region beyond which the leading-twist calculations are no longer valid.

1 Introduction

This Workshop is to discuss the HERA physics, which is primarily the physics of the Deep Inelastic scattering at small x. We approach the DIS at small x from the pure perturbative side and represent a generic structure function F_i as

$$F_i(x,Q^2) \;=\; F_i^{p.th}(x,Q^2) + \frac{\Lambda^2_{QCD}}{Q^2} h_i(x) \tag{1}$$

where $F_i^{p.th}$ is the leading twist contribution. Note that the $1/Q^2$ term is considered to be a correction to the leading twist. However, if the power correction reaches, say, 10% at some "moderate" Q^2, it blows up fast at smaller Q^2. Note also that we do not reserve for a possible $\ln Q^2$ dependence of the power correction but this is just for simplicity.

One of the basic questions addressed by the theory of the DIS at small x is when one is to stop using the leading order perturbative calculation which might extend much further than one would expect naively (for a recent review and further references see [1]). There do exist estimates of the higher twist effects which show that the $1/Q^2$ corrections are pretty large at small x [3,4]:

$$\frac{\delta F_2(x\sim 10^{-3},Q^2)}{F_2(x\sim 10^{-3},Q^2)} \;\approx\; -\frac{1\;GeV^2}{Q^2} \tag{2}$$

where we picked up $x\sim 10^{-3}$ for the sake of definiteness. The result has been found for any x [3,4].

If true, the estimate (2) is an important piece of information. There has been not much discussion yet of this estimate in the literature. Thus, we are going to provide an overview of the assumptions made to derive (2). Responding to the request of the organizers, we will try to make the presentation understandable to a broader audience.

2 General Remarks on the Power Corrections

We consider QCD and processes determined by physics at short distances. Which means that there is a generic large mass scale, $Q \gg \Lambda_{QCD}$ where Λ_{QCD} is the position of the Landau pole in the coupling:

$$\alpha_s(Q^2) \approx \frac{1}{b_0 ln Q^2/\Lambda^2_{QCD}}. \tag{3}$$

In case of DIS, Q is the 4-momentum of the virtual photon.

Moreover, we consider power corrections $(\Lambda^2_{QCD}/Q^2)^k$ to the parton model. Note first of all that the power corrections appear to be a pure non-perturbative effect. Indeed, on one hand we have

$$\left(\frac{\Lambda_{QCD}}{Q}\right)^k = exp(-k/2b_0\alpha_s(Q^2)). \tag{4}$$

On the other hand, the function $exp(-const/\alpha)$ with a positive $const$ is a classical example from the math courses of a function which has a trivial Taylor expansion at $\alpha = 0$:

$$exp(-const/\alpha)|_{\alpha=0} = 0 + 0 \cdot \alpha + 0 \cdot \alpha^2 + ... \tag{5}$$

since the function itself and all its derivatives vanish at $\alpha = 0$. Thus, this function, being a non-zero, vanishes identically as a perturbative expansion, which is the expansion at $\alpha = 0$.

It could be quite well that the power corrections, unlike pure perturbative contributions, are sensitive to the mechanism of confinement (for a recent review and further references see, e.g., [5,6]). However, here we would not go into this issue since the interest in the estimates like (2) is rather pragmatic than theoretical.

The only point we would like to mention is that we are assuming the validity of the standard picture for the non-perturbative effects. While calculating the perturbative expansions is a well defined procedure in QCD, at least as a matter of principle, the definition of the non-perturbative terms, at first sight is close to saying that these are unknown terms, the rest of the amplitudes upon subtraction of the perturbative contributions. In other words, working with the power corrections relies to a great extent on intuition and heuristic models. In particular, we would not like to use any concrete form of the non-perturbative fluctuations since very little is known about this. Nevertheless, there is one basic assumption underlying all further analysis. Namely, we will assume here that the non-perturbative fields are soft. In other words, the typical size of the non-perturbative fluctuations is of order Λ^{-1}_{QCD}. One could challenge this assumption as well [5] but so far there were no alternative models for the power corrections in DIS developed.

Finally, let us mention that actually working with an infinite perturbative expansion would be awfully difficult in practice. Thus, in reality one is always

relying on a kind of a *truncated* series, keeping, say, only the leading perturbative plus power corrections. The assumption behind this truncation is that the power corrections are somehow *enhanced* numerically. There is no proof of this enhancement but it is an indispensable ingredient of any phenomenology based on the power corrections. Historically, this assumption was made first in case of the so called QCD sum rules and worked well. It might fail, however, in other cases.

If there exist high-precision data one may try to check this assumptions varying the number of terms in the perturbative expansion kept explicit and determining the corresponding variation in the fitted values of the power corrections. So far, this procedure was implemented in the most careful way in case of the DIS at large x [7]. The analysis certainly indicates that the fitted value of h_i diminishes quite dramatically with inclusion of higher terms in the perturbative expansion. A similar conclusion is reached in a very detailed model study of power corrections to the thrust [8]. Although such conclusions might seem quite disturbing, we would like to treat them again rather pragmatically. Namely, we would understand the estimate (2) as a correction to the leading log approximation and leave open the question whether it would be absorbed into higher order perturbative corrections.

3 Operator Product Expansion

The basic theoretical tool which allows to exploit the idea that the non-perturbative fields are soft is the Operator Product expansion (OPE). In particular, it allows for a systematic treatment of the power corrections in DIS [9]. The most important outcome of the analysis is that the power corrections to the structure functions start with Q^{-2} terms and we will outline briefly the derivation.

The OPE corresponds to a particular way of preparing ordinary Feynman graphs [10]. The basic ingredient is to separate hard and soft lines. By hard lines we understand the lines which carry virtual momentum of order Q while soft lines carry momentum of order Λ_{QCD}. Then, the hard lines are treated according to the standard Feynman rules. Moreover, in a space-time picture of the interaction all the points along these lines are close to each other, $\Delta x \sim 1/Q$ and, in the first approximation can be reduced to a single point. Note that it is true only in the Euclidean space, while in the Minkowskian space there could be large cancellations between space- and time- components of the space-time interval. The relation of the results obtained within the OPE to the physical processes (measurable in the Minkowski space) is provided by dispersion relations.

On the other hand, soft lines are not integrated out at all and their effect is absorbed into matrix elements of the corresponding operators. The logic behind this step is that the soft lines with $p \sim \Lambda_{QCD}$ are modified drastically by confinement effects. Note that in the approximation that all the hard lines are reduced to a single point the soft lines originate and end up at the same point. Thus, instead of propagators of soft particles which are unknown functions of soft mo-

menta we have now only matrix elements of local operators, that is numbers, not functions.

In this way, the route of the flow of the large momentum Q becomes crucial. For a given order of perturbation theory all the possibilities for this flow should be tried. Examples of the OPE approach to some Feynman graphs in the case of DIS is given in Fig. 1.

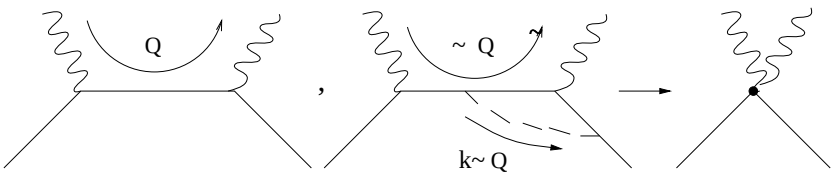

Fig. 1. Examples of applying the OPE to Feynman graphs. One follows the route of the large momentum Q flow. All the points along these, hard lines are reduced to a single point indicates by a black blob. The large momentum Q is brought in and taken away by the virtual photons, denoted by wavy lines. The dashed line denotes gluon and solid lines are quarks.

Applying furthermore the standard dispersion relations for the forward scattering of the virtual photon off the nucleon one arrives to the well-known predictions for the moments:

$$M_n(Q^2) \equiv \int F_i(x,Q^2)x^N dx = M_n(Q_0^2)\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_N} \tag{6}$$

where γ_N are calculable and reflect the effect of the hard lines while $M_n(Q_0^2)$ are to be borrowed once from experiment for some Q_0^2, as a reflection of our inability to evaluate the effect of the soft quark lines. Moreover, for each moment we have a new matrix operator and the corresponding matrix element.

Note that radiative corrections are dominated by virtual momenta $k \sim Q$, as indicated in Fig. 1. However, there is also a possibility that the gluon momentum k is small, $k^2 \ll Q^2$. Then the effect of the gluon exchange is to be included into new matrix elements, as indicated in Fig. 2

Restricting momenta of virtual gluons to the infrared region costs suppression of the corresponding contributions by powers of Q^{-2}, due to phase space (and gauge invariance). Applying the OPE allows to immediately find out this power. Indeed, the leading twist (see Fig. 1) is related to the matrix elements of operators bilinear in the quark fields, which can generically be represented as $\bar{q}\Gamma q$ where Γ is some matrix in the spinor and color spaces. If a gluon line is added, see Fig. 2, then the corresponding operator becomes $\bar{q}F\Gamma q$ where F is the gluon field strength tensor (and we suppress all Lorentz and color indices). The dimension of this extra factor, F, is $d = 2$ and it is to be compensated by

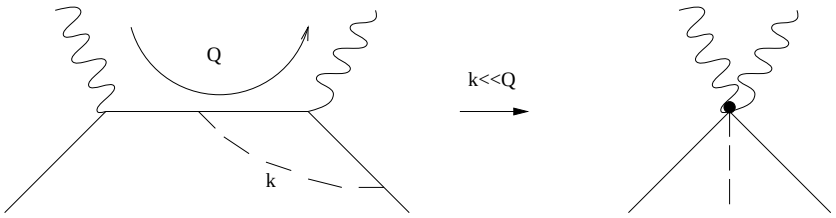

Fig. 2. Emergence of higher twists. If the virtual gluon is soft, $k \sim \Lambda_{QCD}$ its effect cannot be evaluated but is rather to be included into matrix elements of operators containing an extra power of the gluon field strength tensor.

the corresponding factor Q^{-2}:

$$\delta M_n \;\sim\; M_n^{(0)} \frac{\beta_n \Lambda_{QCD}^2}{Q^2}. \tag{7}$$

To summarize, the OPE allows to parameterize in a systematic way the infrared sensitive contributions to the ordinary Feynman graphs. In particular, in case of the DIS one readily concludes that the leading power corrections are of order Q^{-2}. These corrections are associated with matrix elements of operators bilinear in quark fields and linear in the gluon field. Borrowing the matrix elements from experiment amounts to introducing new structure functions. Without this input, the use of the OPE does not allow for any concrete numerical predictions for the power corrections.

4 Renormalons or Infrared Sensitive Gluon Mass

Renormalons (for review see, e.g., [11]) represent a particular model for the power corrections which avoids the introduction of a new structure function. Qualitatively, the basic idea can be explained as follows. The general formulation of the OPE outlined above relates the Q^{-2} correction to an admixture of gluons in the nucleon wave function. One could argue, however, that the nucleon wave function is build on the valence quarks alone and all the gluon effects can be understood in terms of the gluon bremsstrahlung from the valence quarks.

In fact this idea, to reduce the gluon structure functions to the quark structure functions has been tried first in the leading twist case [12]. Namely, one assumes that at some $Q_0^2 \sim \Lambda_{QCD}^2$ there are only quark distributions, with no gluons. Gluons evolve at larger Q^2, as a result of bremsstrahlung. The model is not without a success although one cannot go too far because of the matching point at low Q_0^2.

Now, we need technical means to realize the idea that the soft gluons responsible for the power-like corrections (see Fig. 2 and discussion of it) are produced exclusively through bremsstrahlung. Introduce a non-vanishing gluon mass squared λ^2 and try first to expand in λ^2, see Fig. 3. The expansion amounts

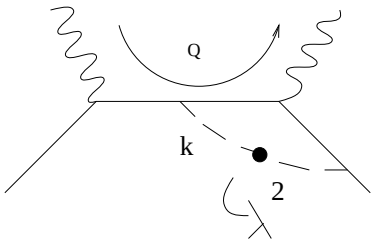

Fig. 3. Introduction of a (fictitious) gluon mass $\lambda^2 \to 0$ into the radiative corrections to the parton model. One evaluates contribution proportional to $\lambda^2 \ln \lambda^2$.

to replacing the gluon propagator by $(k^2+\lambda^2)^{-1} \to k^{-2}\lambda^2 k^{-2}$. Upon this replacement the integral over the momentum of the gluon k becomes logarithmically divergent in the infrared and as a result we come to terms of order $\lambda^2 \ln \lambda^2$. Indeed,

$$\lambda^2 \int_\lambda \frac{d^4k}{k^4} \ \sim \ \lambda^2 \ln \lambda^2, \tag{8}$$

where the lower bound of the integration $k \sim \lambda$ is introduced because at such k the expansion in λ^2 is no longer valid.

This estimate illustrates a general point that terms non-analytical in λ^2, like $\lambda^2 \ln \lambda^2$ can develop only through contributions of soft gluons with momenta $k \sim \lambda$. Thus, non-analyticity in λ^2 can be used to mark the infrared sensitive part of the Feynman integrals [13]. To make connection with QCD one replaces

$$\lambda^2 \ln \lambda^2 \ \to \ \Lambda^2_{QCD}, \tag{9}$$

with the overall coefficients in this relation left as a fitting parameter. Indeed, for the sake of parameterization of the contribution of momenta $k \sim \Lambda_{QCD}$ we can simply adjust $\lambda \sim \Lambda_{QCD}$. In the particular case of the DIS, detailed calculations along these lines can be found in [14].

Note that so far we have not mentioned renormalons. Actually, this is another way, consistent with the gauge invariance, to isolate contributions of soft gluons. Namely, one inserts fermionic bubbles into the gluon line, see Fig. 4. It can be readily seen that if the number of fermionic bubbles is large, the virtual momentum of the gluon, k_{eff} dominating the Feynman integrals is small. Indeed, each bubble is proportional to $b_0 \ln(k^2/Q^2)$ where b_0 is the fermionic contribution to the first coefficient in the β-function and the log factor vanishes at the gluon momentum $k = Q$ because of normalizing the running coupling at $k = Q$. Moreover, assume that the number of the fermionic bubbles, n is large, $n \gg 1$. Then the integral over the gluon line takes the form

$$\int \frac{d^4k}{k^2} \left(b_0 \ln(k^2/Q^2)\right)^n \ \sim b_0^n n! \ . \tag{10}$$

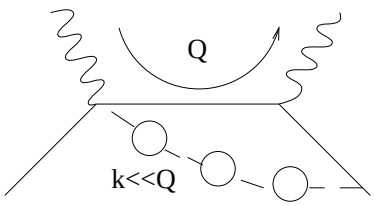

Fig. 4. Renormalon chain inserted into a gluon line. The number of the fermionic bubbles is assumed to be large large, $n \gg 1$.

and is large at large n. What is even more important, it is dominated by low momenta

$$(k^2)_{eff} \sim Q^2 \cdot exp(-n).$$

The type of graphs represented in Fig. 4 is called renormalon chain. The motivation to use the renormalon chain is exactly the same as to isolate the terms of order $\lambda^2 \ln \lambda^2$, see above. Namely, the renormalon chain enhances greatly infrared sensitive contributions to the Feynman graphs and allows for their parameterization. Using the renormalons one comes to the same results for the power corrections as by using the infinitesimal gluon mass, see above.

To summarize, renormalons allow to parameterize contribution of soft gluons to the radiative corrections in DIS. The method is equivalent to introducing an infinitesimal gluon mass λ and following the terms non-analytical in λ^2. The model has much greater predictive power than the general OPE since no independent quark-gluon structure function is introduced (see Fig. 2). Instead, the power correction to a structure function is proportional to the structure function itself (see Figs. 3,4).

5 Reservations, Successes, Predictions

The great simplification brought by the renormalon model comes not without a price tag. The basic new assumption is that external quark lines are considered hard with respect to the gluon lines even if both are in fact soft. We called this assumption a valence quark approximation, and it might be valid only for rough estimates.

There are some further caveats as well. In particular, the effect of, say, two soft gluon emission is not suppressed compared to a single soft gluon considered so far. Indeed, the emission of soft gluons is governed by $\alpha_s(A^2)$ which is not small. Moreover, in case of some observables one can prove that the multiple gluon emission is important numerically [15].

Thus, the renormalon approach is of openly heuristic and approximate nature and can be justified only by its success, if any, in the description of the data. On the other hand renormalons do fit nicely the x dependence of the power corrections at large x, see, e.g., [2,16]. This is especially true if one truncates

both the perturbative expansion and expansion in powers of Q^{-2} to their first respective terms.

This agreement of the theory and data allows to fix the unknown overall coefficient in front of the power corrections and extend the calculation to the region of small x as well. Since small x are dominated by the singlet structure functions one has to include the effect of the infinitesimal gluon mass in graphs of the type represented in Fig. 5 [3,4]. The prediction for the x dependence of

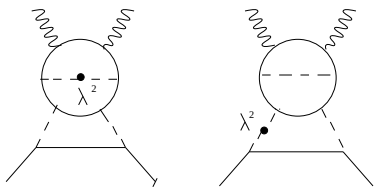

Fig. 5. Power corrections to the singlet structure functions. The black blob indicates insertion of the infinitesimal gluon mass.

the power corrections appears to be highly non-trivial. The correction practically dies away at moderate x but blows up again, with a negative sign at small x [3,4]. I would expect that there should be a simple kinematical reason for such a behaviour but I am not in the position to identify it at the moment.

Thus, we come back to the estimate (2) and may conclude that despite many potential uncertainties the estimate looks quite conservative and might well be realistic.

6 Acknowledgments

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