Theory of diffractive structure functions

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Abstract. We discuss the connection between the dipole approach and the DIS structure functions. Particular emphasis is given to DIS diffraction. We report on current work on the NLO photon impact factor. As an application of the dipole approach in DIS diffraction, we discuss the diffractive production of jets and of charm.

1. Introduction

In recent years, the discussion of small-x and low-Q^2 HERA data has used two slightly different languages: the conventual approach based upon the parton picture of deep inelastic scattering [1], in which the virtual photon picks a parton inside the fast moving proton, and the dipole [2, 3] picture in which the fast incoming photon splits into a quark-antiquark pair and interacts with the proton at rest. Both pictures are formulated in different reference frames (in the former one the proton is fast, in the latter the proton is at rest), and they emphasize different features of the same scattering process. In this talk, I want to discuss a few aspects of both pictures; particular emphasis is given to their theoretical basis in QCD.

Let me begin with a brief summary. There is little doubt that within QCD deep inelastic scattering rests on a firm theoretical basis; there exist systematic expansions in powers of $1/Q^2$ (twist expansion) and in powers of α_s, and the linear DGLAP evolution equations provide powerful tools for the definition of parton densities. Because of their universality, the parton densities measured at HERA can be transported to and applied in hadron collisions at the TEVATRON and at the LHC. Nevertheless, for HERA data open questions remain: down to which values in x and Q^2 can we use the DGLAP equations, how to give a physical interpretation to the initial conditions of the evolution equations? And a special question raised by HERA data: how to interpret the diffractive final states as part of the total DIS cross section?

Within the last few years, we have obtained evidence that the use of the dipole picture provides complementary information and thus may help to find answers to these questions. The strongest piece of evidence, maybe, is the phenomenological success of

the simple saturation model [4] which has been formulated within this approach. So far this model has provided the most successful (and economic) description of HERA data at low x and Q^2, in particular for the transition from the parton picture at high Q^2 to the Regge behavior at low Q^2. It has also been successfully applied to diffractive data [5], and it offers a physical picture in which both the total cross section and the diffractive cross section can easily be visualized. At large Q^2 this model is supposed to coincide with the QCD parton picture: this naturally raises the question of the general theoretical status of the dipole approach which provides the framework for this model. How are the formulae used in the dipole approach derived in QCD, what is their relation to the QCD parton densities? Which sort of calculations have to be done in order to clarify these questions?

2. The total cross section: structure functions vs. dipole picture

In this first section let us consider the total cross section and compare the two different formulations. Starting from the usual parton picture formulae we write

$$\begin{aligned} \sigma^{tot}_{\gamma^* p} &= \frac{4\pi^2\alpha}{Q^2} F_2(x,Q^2) \\ F_2(x,Q^2) &= \sum \left(C_i \otimes pdf_i\right)(x,Q^2) \end{aligned} \tag{1}$$

where C_i, pdf_i denote the coefficient function and parton density of parton i, resp., and the convolution denotes the integral over the momentum fraction of the struck parton. This formula is most easily interpreted in a reference frame where the proton is fast, e.g. the Breit frame or the Bjorken frame. In such a frame (Fig.1) the photon with virtuality $q^2 = -Q^2$ selects its parton inside the proton which carries the fraction x of the proton momentum. In the small-x region it is possible to derive a space-time description of this process: starting from the ladder-type Feynman diagrams which in the DIS limit give the leading contributions, and going via Fourier transform from momentum space to configuration space and choosing, for example, as a reference frame the Bjorken frame, one obtains the time sequence illustrated in Fig.1: the breakup of the proton starts very much before the interaction with the photon, and the parton which finally scatters with the photon appears at the end of a sequential decay process.

In the color dipole formulation, on the other hand, one starts from the expression

$$\sigma_{\gamma^* p} = \int dz \int d^2\rho \; \psi(Q,\rho,z)^* \sigma_{q\bar{q}}(\rho,z,x) \psi(Q,\rho,z). \tag{2}$$

Here ψ denotes the photon wave function (where we suppress subscripts for the photon polarization), and $\sigma_{q\bar{q}}$ is the interaction cross section of the quark-antiquark pair (dipole cross section). This formula has its natural interpretation in the rest frame of the proton (Fig.2): long before the incoming photon reaches the proton, it creates a quark-antiquark pair which starts to radiate a gluonic cascade. After several steps of radiation the gluons are losing more and more of the longitudinal momentum fraction

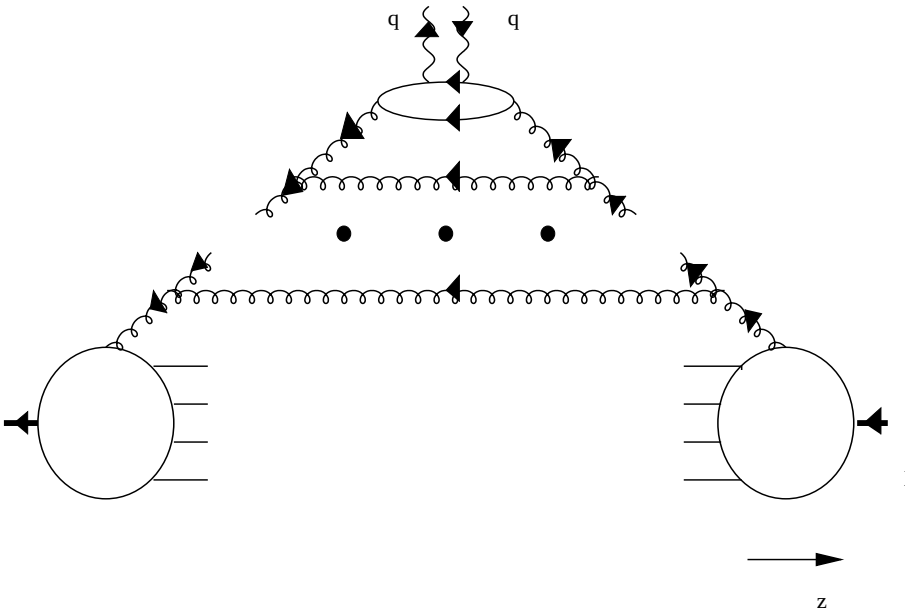

Figure 1. *Space-time diagram of the elastic $\gamma^* p$ scattering process in the deep inelastic (Bjorken) frame.*

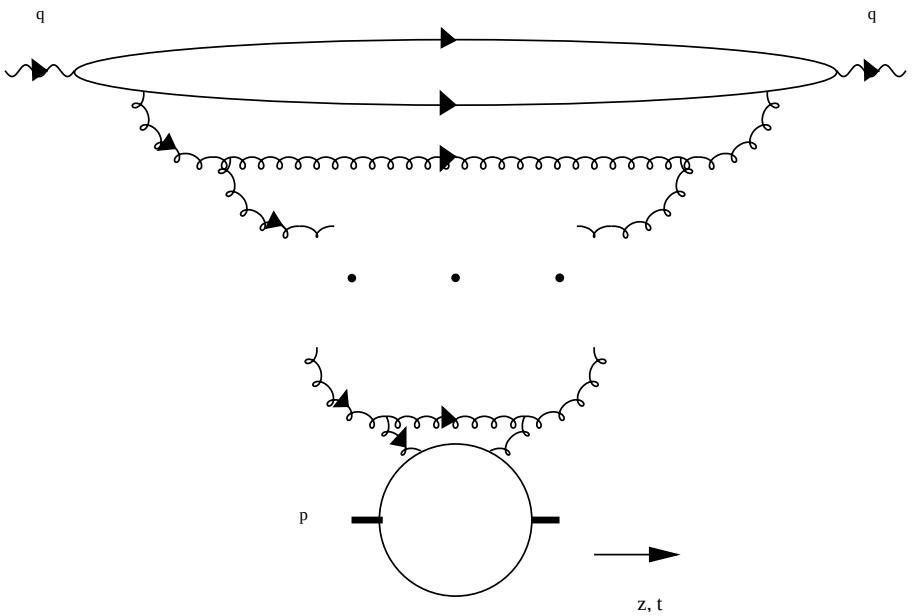

Figure 2. *Space-time diagram of the elastic $\gamma^* p$ scattering process in the proton rest frame.*

of the incoming photon, and at the end of such a cascade the last gluon interacts with the proton at rest. After this interaction the gluon recollects the previously emitted gluons and eventually is reabsorbed by the quark-antiquark pair. It is a remarkable feature of this formula that during this long time the extension of the quark antiquark pair transverse to the direction of flight remains 'frozen', i.e. the ρ coordinate of the outgoing photon wave function ψ^* is the same as for the incoming one. Again, one can derive this space-time picture from the Feynman diagrams in momentum space via Fourier transform by choosing the proton rest frame. Since time is not invariant under Lorentz boosts, one is not surprised to see that, in different Lorentz frames, time plays a different role.

What is the connection between the two formulae (1) and (2)? They both result from QCD, but slightly different kinematic limits have to be taken. This is most easily illustrated if we consider the elastic scattering of a virtual photon on a quark: $\gamma^* q \to \gamma^* q$ to lowest order in α_s. In the high energy limit ($s = (q+p)^2 = Q^2/x \gg Q^2$, $t = -q^2$) one finds for the scattering amplitude:

$$T(s,t) \sim is \ln 1/x \int \frac{d^2k}{(2\pi)^3} \Phi_{\gamma^*}(Q,k,q-k) \frac{1}{k^2(q-k)^2} \Phi_q(k,q-k), \qquad (3)$$

where $\Phi_{\gamma *}$ and Φ_q denote the impact factors of the photon and the quark, resp. The photon impact factor results from the quark loop at the upper end of Fig.2. A closer inspection [2, 3] shows that it factorizes into a product of wave functions for incoming and outgoing photons: in momentum space the quark loop can be written as $\sim \int dz \int d^2l \psi(Q,z,l,k)^* \psi(Q,z,l,k)$, and after Fourier transform with respect to the quark loop momentum l one finds the form (2). Moving on to higher orders in α_s and searching for leading powers in $\ln 1/x$, we find the gluon ladder diagrams with the well known BFKL result

$$\sigma_{q\bar{q}} = \frac{2\pi}{3} \int \frac{d^2k}{(k^2)^2} (1 - e^{ik\rho})(1 - e^{-ik\rho}) \alpha_s f(k,x) \qquad (4)$$

where the BFKL amplitude $f(k,x)$ includes the lower coupling to the quark line.

This brief summary should remind us that the dipole formula emerges from QCD in the leading $\ln 1/x$ approximation. To connect with the DIS limit we have to consider the limit of large Q^2 (leading $\ln Q^2$ approximation): in this limit the ρ integration in (2) will be dominated by small distances. In the double logarithmic approximation (DLLA) formula (2) can easily be shown to reproduce the leading $\ln 1/x$ approximation of the DGLAP formulae. In particular,

$$\sigma_{q\bar{q}}(\rho,z,x) \approx \frac{\pi^2}{3} \alpha_s(\bar{Q}^2) \rho^2 x g(x,\bar{Q}^2) \qquad (5)$$

where g denotes the gluon distribution in the LLA at the scale $\bar{Q}^2 \approx 1/\rho^2$. As a result, the dipole formula (2) and the DGLAP formula (1) agree in the DLLA limit. Beyond this region of overlap, the region of validity of (1) extends into the region of finite x,

whereas (2) is valid also beyond the leading power (or logarithm) of Q^2. It should, however, be stressed that (5), so far, has been shown to be valid only to leading $\ln Q^2$ accuracy.

Having outlined the connection between the two different formulations, we turn to the question whether the dipole formula is valid beyond the leading $\ln 1/x$ approximation and whether the region of common validity (also, the validity of (5)) can be extended beyond this DLLA aproximation. The natural way to answer this question is a study of the process in Fig.3 in next-to-leading order (NLO): consider, again, the limit of high energies and compute the contributions to the total cross section of order α_s^3, proportional to order $(\ln 1/x)^0$ (the contribution of the order $\ln 1/x$ belongs to LLA). The diagrams to be studied follow from those of Fig.2 by adding an extra gluon in all possible ways. Following the high energy factorization ansatz we expect the amplitude to consist of three terms (Fig.3): in the first contribution the extra gluon belongs to the impact factor of the photon (this term has not yet been computed), in the second one the gluon belongs to the quark impact factor at the lower vertex (this contribution has been calculated before [6]), and in the third contribution the gluon is produced in the central region (this belongs to the LLA calculation and has to be removed carefully). The main task, therefore, is the verification, at NLO, of this factorization ansatz and the calculation of the NLO corrections to the photon impact factor. This is presently being pursued by two groups [7, 8, 9], and I want briefly to summarize the status of these investigations.

The NLO photon impact factor can be written as a sum of two pieces: the quark-antiquark intermediate state and the quark-antiquark-gluon state. In the former case, we have a virtual NLO contribution to the $\gamma^* g \to q\bar{q}$ vertex either on the lhs or on the rhs of the energy discontinuity line. Naturally, the calculation of the NLO photon impact factor can be organized in three steps: the computation of the virtual one-loop corrections to the $\gamma^* g \to q\bar{q}$ vertex, the real corrections (which require the tree approximation to the process $\gamma^* g \to q\bar{q}g$), and, in the final step, the integration over the intermediate states. The calculation of the virtual corrections has been completed [7]; the real corrections of the longitudinal photon have been published recently [8], and first steps in dealing with the phase space integrals of the virtual corrections are contained in [9]. Furthermore, in [11] it has been shown that in the sum of real and virtual corrections the infrared singularities cancel. Returning to our question of interest, namely the dipole picture (or, more precisely, the correct generalization of it), we expect to find the following structure of the photon impact factor:

$$\begin{aligned}\sigma_{\gamma^* q} = & \int dz \int d^2\rho \psi^{(0)}(Q,\rho,z)^* \sigma^{(0)}_{q\bar{q}}(\rho,z,x)\psi^{(0)}(Q,\rho,z) \\ & + \int dz \int d^2\rho \psi^{(1)}(Q,\rho,z)^* \sigma^{(0)}_{q\bar{q}}(\rho,z,x)\psi^{(0)}(Q,\rho,z) + c.c. \\ & + \int dz dz' \int d^2\rho d^2\rho' \psi^{(0)}(Q,\rho,z)^* \sigma^{(1)}_{q\bar{q}}(\rho,z,\rho',z',x)\psi^{(0)}(Q,\rho',z')\end{aligned} \tag{6}$$

$$+\int dz_1 dz_2 \int d^2\rho_1 d^2\rho_2 \psi^{(0)}_{q\bar{q}g}(Q,\rho_1,\rho_2,z_1,z_2)^* \sigma^{(0)}_{q\bar{q}g}(\rho_1,\rho_2,z_1,z_2,x)\psi^{(0)}_{q\bar{q}g}(Q,\rho_1,\rho_2,z_1,z_2)$$

In the first line we have included, for completeness, the usual LO term. The next line contains NLO corrections to the $q\bar{q}$ component of the photon wave function; the third line represents corrections to the dipole cross section which may or may not involve a decoupling of the transverse sizes of the $q\bar{q}$ pair in the initial and final state. Finally, the fourth line denotes a new component of the photon wave function: the $q\bar{q}g$ final state. The second and third line result from virtal corrections, the fourth line from real corrections. If this decomposition turns out to be correct, we have to generalize the dipole picture: there will be higher order color multipole components of the photon. From the results that have been obtained so far, we can deduce that the structure contained in the fourth line is correct. In [8] it is shown (for the longitudinal photon) that the real corrections factorize, in the expected manner, into wave functions for incoming and outgoing photons and a cross section for the interaction of the quark-antiquark-gluon system with the quark target.

Let us assume, for the moment, that the structure oulined in (6) will be verified. Then we know that the two approaches - the parton approach with the QCD evolution equations which is visualized most easily in a frame with a fast proton, and the color dipole approach which is visualized most easily in the proton rest system - have a common region of validity. In leading order (LO) the former one sums terms of the order

$$\sum \alpha_s c_n \left(\alpha_s \ln Q^2 (\ln 1/x + ...)\right)^n \tag{7}$$

where the dots indicate corrections in the x dependence beyond the DLLA. The latter approach, on the other hand, sums contributions of the form

$$\sum \alpha_s^2 b_n(Q^2)(\alpha_s \ln 1/x)^n \tag{8}$$

where the coefficients $b_n(Q^2)$ an be expanded in powers of $1/Q^2$ and α_s:

$$b_n(Q^2) = \beta_n (\ln Q^2)^n + ... \tag{9}$$

Here, again, the dots indiate terms beyond the DLLA approximation. The DLLA defines the region where both approaches give the same result: both $\ln Q^2$ and $\ln 1/x$ have to be large. The DGLAP evolution equations are valid also for not so small x, whereas the validity of the dipole approach extends into the region of smaller Q^2. In NLO of the DGLAP approach, eq.(7) receives corrections without the $\ln Q^2$ (i.e. looses one power of $\ln Q^2$). The NLO corrections of the photon impact factor provide corrections in (8) which are down by one power in $\ln 1/x$. As an important task in the calculation of the NLO photon impact factor it has to be verified that, when following the expansion (8), it coincides with the corresponding pieces in the DGLAP evolution kernels and coefficient functions.

After this short discussion of the regions of validity of the two approaches it should become more transparent in what sense they are complementary. In DGLAP one describes F_2 also outside the small-x region, but the formalism cannot be used to go to

very low Q^2 values. The dipole approach is valid for the small-x limit (high energy limit), and it is not restricted to the large Q^2 region. Nevertheless, in the large Q^2 limit where the ρ integral is dominated by small distances we have, because of (5), the firmest basis in pQCD. The continuation into the region of finite and even small values of Q^2, requires modelling of the dipole cross sections $\sigma_{q\bar{q}}$ and $\sigma_{q\bar{q}g}$. The saturation model [4] is just a particular choice of $\sigma_{q\bar{q}}$ in the whole ρ region. Its success suggests that, in the small x and low Q^2 region of HERA, the dipole approach is well suited for finding an interpolation between the large and the low Q^2 regions.

3. The diffractive cross section: structure function vs. dipole picture

Turning now to DIS diffraction I basically repeat the previous discussion of confronting the two different approaches. The concept of diffractive parton densities for the inclusive diffractive cross section which will be discussed in more detail in J.Collins' talk finds its natural interpretation in a frame where the proton is fast: the photon selects a parton with momentum fraction x inside the proton, and the state of the proton is further constrained by the requirement that it contains a rapidity gap of a given size. For the diffractive cross we have a formula similar to (1); for the parton densities 'pdf' one usually makes a special ansatz which models the formation of the rapidity gap. As usual, the densities satisfy the DGLAP evolution equations, and all the physics below the initial evolution scale Q_0^2 is contained in the initial conditions. QCD factorization theorems [12] make no statements on the dynamics which generates the rapidity gap. HERA measurements have shown that the rapidity gap in the final state, when described by the exchange of the Pomeron, leads to a value of the Pomeron intercept $\alpha_P(0)$ which is larger than in hadron-hadron scattering, thus slightly modifying the original Ingelman-Schlein idea.

It is difficult to unify, in this approach, the description of the total cross section $\sigma_{\gamma^* p}$ with that of the diffractive final states. In particular, it is not straightforward to establish the connection between diffractive parton densities and the conventional parton densities entering F_2: a certain fraction of the diffractive cross section can be viewed as arising from the QCD evolution, for which the initial distributions are part of the F_2 initial conditions. But there are also parts of the diffractive cross section which are not taken into account in F_2, and they simply have to be added to the usual DGLAP description of F_2.

Let me list some of the cross section formulae used in DIS diffraction which are based upon the dipole approach. It is instructive to return to the elastic scattering of a virtual photon:

$$T(s,0) = is \int dz \int d^2\rho \;\; \psi(Q,\rho,z)^* \sigma_{q\bar{q}}(\rho,z,x) \psi(Q,\rho,z) \tag{10}$$

which, as we have seen before, is valid for small x. Here we take $t = 0$ and replace the final state photon wave function ψ^* by the wave function of a vector meson (e.g. a ρ-meson): $\psi(Q, \rho, z) \to \psi_V$. At the same time, the dipole cross section has to be modified: although the square of the four-momentum transfer, t, may still remain zero, nonzero longitudinal momentum has to be transported across the rapidity gap (in Fig.3, the two t-channel gluons no longer carry the same longitudinal momentum fractions). In DLA this difference is not important, but beyond this approximation (and in practical applications) it cannot be ignored.

$$\frac{d\sigma_{\gamma^* \to Vp}}{dt}|_{t=0} = \frac{1}{16\pi}| \int dz \int d^2\rho \psi_V(\rho, z)^* \sigma_{q\bar{q}}(\rho, z, x) \psi(Q, \rho, z)|^2 \tag{11}$$

Such a formula for diffractive meson production, which (with some refinements) has successfully been used to describe HERA data, has its natural interpretation in the proton rest frame: at a time $\Delta\tau \sim 1/m_p x$ before the interaction with the proton the photon breaks up into a quark-antiquark pair which, during the interaction with the proton, remains frozen at transverse distance ρ and finally annihilates into the vector particle.

If in (10) we simply remove the wavefunction of the outgoing photon and transform from the transverse distance ρ to transverse momenta k_t of the outgoing quarks we obtain the cross section for the diffractive production of two jets. In DLLA the formula has the form [2, 13]:

$$\frac{d\sigma}{d_t^k dt}|_{t=0} \sim \left(\frac{1}{k_t^2} x_P g(x_P, \frac{k_t^2}{1-\beta} \right)^2 \tag{12}$$

with $\beta = \frac{Q^2}{M^2+Q^2}$. In practice, however, this formula alone is only of limited use. Because of the kinematical relation $k_t^2 = z(1-z)M^2$, two hard jets require a rather large diffractive mass M^2. But for large M^2 the diffractive production of one extra gluon cannot be neglected, i.e. one has to consider in combination the production of two or three jets.

In the next step we take the square of the amplitude of open $q\bar{q}$ production: we integrate over the transverse momenta k_t (or, equivalently, the transverse distances ρ), and we obtain for the $q\bar{q}$-part of the inclusive diffractive cross section the formula

$$\frac{d\sigma}{dt}|_{t=0} = \frac{1}{16\pi} \int dz \int d^2\rho \psi(Q, \rho, z)^* \sigma_{q\bar{q}}(\rho, z, x)^2 \psi(Q, \rho, z) \tag{13}$$

Generalizations to the diffractive production of $q\bar{q}g$ states have also been proposed [14, 15]: they differ in the kinematical limits for which their derivations are valid. For example, in the limit of large diffractive masses the following formula for the integrated $q\bar{q}g$ cross section has been proposed:

$$\frac{d\sigma}{dt}|_{t=0} = \frac{1}{16\pi} \int dz \int d^2\rho_1 d^2\rho_2 \psi_{q\bar{q}g}(Q, \rho_1, \rho_2, z)^* \sigma_{q\bar{q}g}(\rho_1, \rho_2, z, x) \psi_{q\bar{q}g}(Q, \rho_1, \rho_2, z) \tag{14}$$

with

$$\psi_{q\bar{q}g}(Q, \rho_1, \rho_2, z) = \psi_{q\bar{q}}(Q, \rho_1, z) \left(\varphi(\rho_2) - \varphi(\rho_2 - \rho_1)\right)$$

and

$$\sigma_{q\bar{q}g} = \sigma_{q\bar{q}}(\rho_2, z, x) + \sigma_{q\bar{q}}(\rho_2 - \rho_1, z, x) - \sigma_{q\bar{q}}(\rho_2, z, x).$$

This formula can be used only for large diffractive masses. In this region the gluon has a large rapidity separation with respect to the $q\bar{q}$ pair. For a more general formula a complete NLO calculation of diffractive $q\bar{q}$ and $q\bar{q}g$ production is needed: such an investigation requires efforts similar to those for the NLO impact factor.

All these formulae, so far, have been verified only in DLLA. They contain the same dipole cross section as discussed in the previous section. The natural interpretation is done in the proton rest frame: at a time $\tau \sim 1/m_p x$ before the interaction with the proton the photon breaks up into a quark-antiquark pair which, during the interaction with the proton, remains frozen at transverse distance ρ and finally annihilates into the final state photon, vector particle, or quark-antiquark dijet state. The validity of these formulae follows from the previous discussion: for the starting formula (10) we have seen that it is obtained in the high energy limit (large $\ln 1/x$). If, in addition, we also take the limit of large Q^2 (DLLA), the integral over ρ will be dominated by small distances and we have agreement with DGLAP results. For larger ρ-values we have to find some extrapolation of the dipole cross section, i.e. the results for the scattering cross section become model dependent. Moving on to diffractive vector production (11), we have to distinguish between short distance (small ρ) and large distance (large ρ) dominance. The former case belongs to large Q^2 values and longitudinal polarization; also in the case of charmed quarks we are dominated by small ρ. For other cases larger ρ-values play a significant role and we have to rely on our choice of extrapolating the dipole cross section. In the case of diffractive dijet production (12) it is the value of the quark transverse momentum which determines the hardness of the process. For sufficiently large k_t the use of the gluon density in the dipole cross section is well justified. The extrapolation down to smaller k_t values introduces model dependence. In the integrated $q\bar{q}$ cross section (13) the ρ-integral receives large contributions from the region of large ρ (aligned jet configurations): the prediction for the integrated diffractive cross section, therefore, is very much model dependent. Finally, the $q\bar{q}g$ case (14) is analogous to the $q\bar{q}$ configuration: for jets with large transverse momenta all transverse distances are small, and we can rely on the small distance behaviour of the dipole cross section. When integrating over the final state momenta, the gluon prefers to be soft: long transverse distances become important, and we need a model for the long distance extrapolation.

This discussion of the dipole formalism in DIS diffraction shows that, in contrast to the diffractive parton densities which apply to the inclusive diffractive cross section, in this approach it is more natural to consider exclusive final state configurations. As discussed in connection with the total cross section, the dipole formalism makes it attractive to extend its formulae also into the region of larger transverse distances, where the dipole cross sections are no longer given by the gluon density and have to be extrapolated in some model-dependent fashion. The saturation model has been applied to

diffraction [5], and with the same ansatz which has been used for F_2, diffractive HERA data have successfully been explained.

4. Two applications in diffraction

I conclude by mentioning two applications of the dipole approach, diffractive jet production [15, 16] and charm production [18, 19, 17]. Both measurements belong to the class of short-distance dominated processes, and comparisons with data have been made.

For the diffractive jet production we have used the formulae for $q\bar{q}$ and for $q\bar{q}g$ production. The latter one is expected to be important in the region of large diffractive masses. As we have stressed before, the applicability of pQCD and the use of the gluon structure function require all two or three jets to be hard, i.e. strictly speaking we have to exclude any soft Pomeron remnant. With this constraint on the final state kinematics, the dipole cross section can be safely expressed in terms of the gluon structure function ('two gluon model'). The experimental observation that the Pomeron in DIS diffraction is somewhat harder that in soft hadron interaction indicates that also the Pomeron remnant might be somewhat harder. Therefore, in a first approximation, we extrapolate in our $q\bar{q}g$ jet cross section formula the gluon also into the soft region of about $k_t^2 > k_{t\,cut}^2 \approx 1.5$ GeV^2 and use this jet for describing a Pomeron remnant. This model has been compared with H1 data [16] on diffractive dijets (where 'diffractive dijets' may or may not include a Pomeron remnant). A change in the $k_{t\,cut}^2$ value amounts to a change in the overall normalization. Results are shown in Fig.11 of [16]. Both the shapes and the normalization are satisfactory. It should be stressed that, in order to describe H1 data on '3 diffractive jets' (which includes also three diffractive jets plus Pomeron remnant), we need a cross section formula for diffractive $q\bar{q}gg$ production. It is therefore not surprising that the sum of $q\bar{q}$ and $q\bar{q}g$ lies below the data.

Diffractive charm production (diffractive D^* production) still presents some puzzles. In particular, the resolved Pomeron approach overshoots the H1 data by a factor of $2-3$. This makes it particularly interesting to compare the predictions of the dipole approach, as summarized in the previous section, with the H1 data. In a first attempt, this was done for $c\bar{c}$ production: the theoretical prediction was clearly below the data, suggesting that $c\bar{c}g$ production cannot be neglected. In [18] a calculation of $c\bar{c}g$ with massive charm quarks was presented, and the sum of $c\bar{c}$ and $c\bar{g}$ production has been used for comparison with the H1 data. The dipole cross section was treated in the same way as in the case of jet production, with the same cut on the transverse momentum of the gluon. A comparison with the H1 data from 1995-97 is shown Fig.4 [19]. A comparison with the published H1 analysis can be found in Fig.3 of [17]. There is no doubt that the $c\bar{c}g$ component is present in the data, the agreement with data is encouraging but not perfect. In the region of large diffractive mass the theoretical curve is still below the data: maybe the production of more than one extra gluon has to be added (a $c\bar{c}gg$ com-

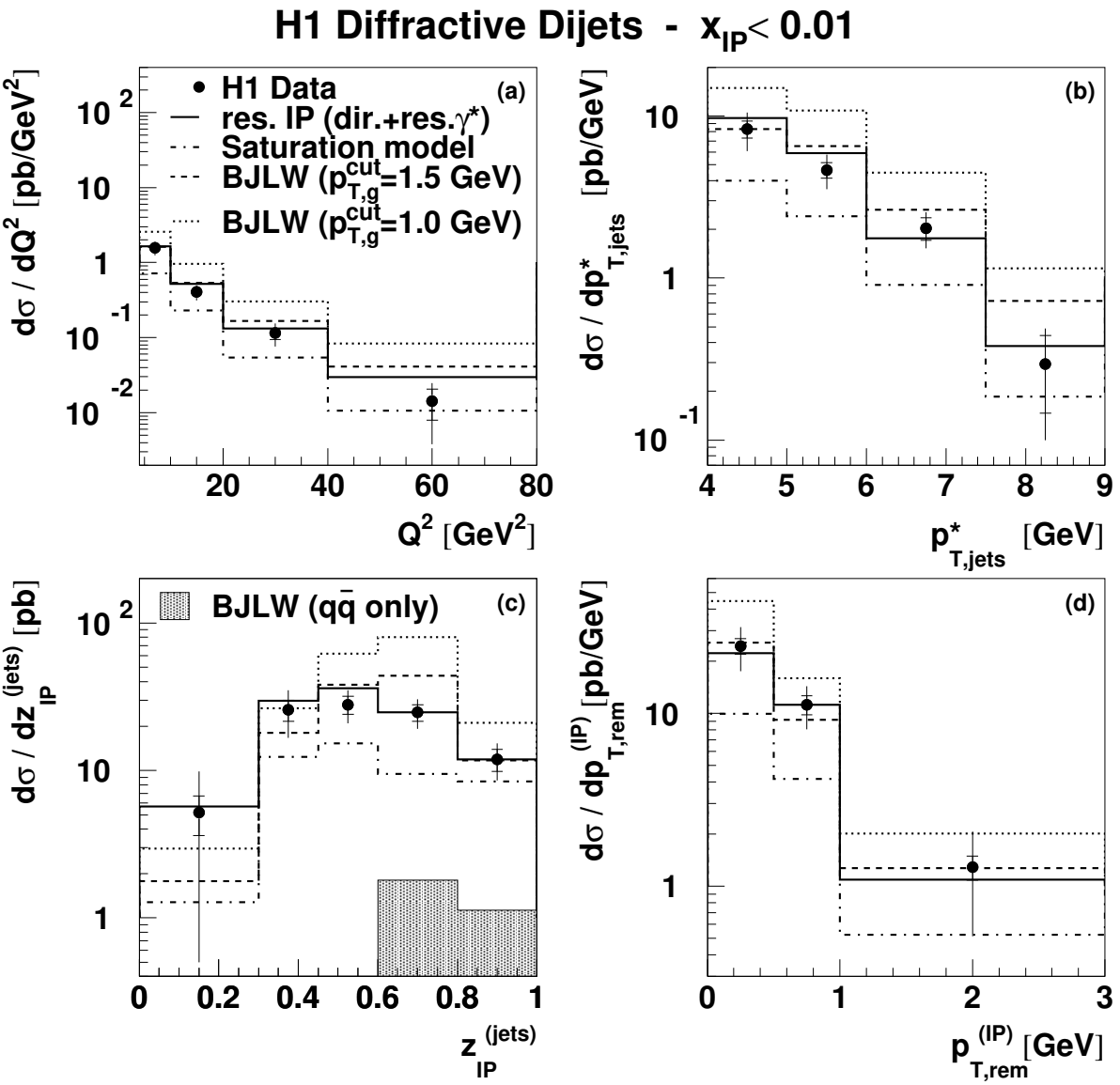

Figure 3. *Figure taken from [16]: Dijet cross sections as function of (a) Q^2, (b) $p^*_{T,jets}$, (c) z_P, (d) $p_{T,rem}$. Our model is denoted by BJLW.*

ponent). Another possible reason for the disagreement is the exchange of secondaries: for not so large rapidity gaps, one needs, in addition to the Pomeron exchange (in our case modelled by the gluon stucture function), the exchange of nonleading secondaries (in pQCD, they could be modelled by the exchange of $q\bar{q}$ states).

In summary, these two examples together with previous analyses of diffractive vector production support the dipole-based approach of analysing HERA data in the small x, low Q^2 region, starting with configurations which are dominated by short transverse distances, and thus provide a justification for the use of perturbative QCD. It seems

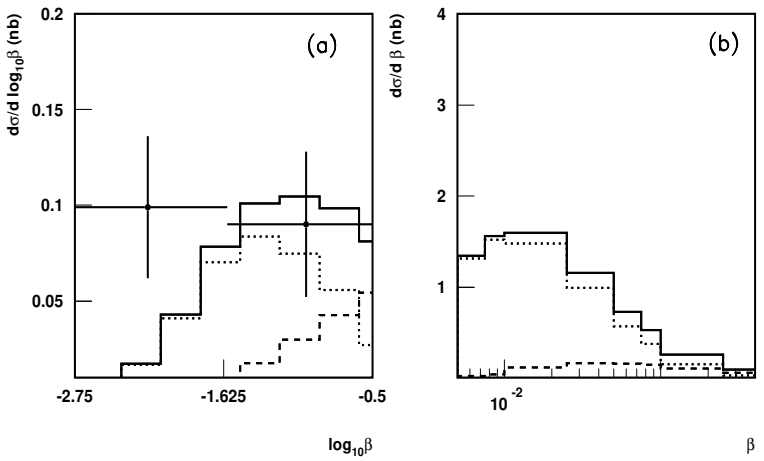

Figure 4. *β distribution of D^* production (dashed: $c\bar{c}$, dotted: $c\bar{c}g$, solid: $c\bar{c}$ + $c\bar{c}g$). The GRV gluon density is used to define the unintegrated gluon structure function.*

possible to extrapolate the perturbative expressions also into the nonperturbative region and to provide adequate descriptions of the data.

5. Summary

For the analysis of HERA data at small x and low Q^2 two complementary approaches have been used, and this talk has attempted to clarify the relation between them. The formulae used in the two approaches are different; they are derived from QCD, using different high energy approximations. There exists a common region of validity, where both $\ln 1/x$ and $\ln Q^2$ are large (DLLA); the latter condition means small transverse distances of the quark-antiquark pair. In the dipole approach, the formula extends also into the region of large transverse distances, but in this region the dipole cross section cannot be derived from perturbative QCD and has to be modelled. A recent model for this dipole cross section has been very successful, and it provides a unified description for both the total cross section and diffractive final states. This suggests that the dipole picture might be a promising starting point for analysing within QCD the transition from perturbative to nonperturbative QCD.

It has also been discussed that the theoretical basis, so far, rests on the leading $\ln 1/x$ approximation. Studies which calculate NLO corrections are under way, and the status of these calculations has been summarized. First results indicate that there exist a systematic expansion: in the leading approximation (LO), the photon wave function has only one component, the $q\bar{q}$ state. In the following step (NLO), a new Fock compo-

nent, a $q\bar{q}g$ component, appears (as well as corrections to the elements of the previous step). This suggests the existence of a 'color multipole expansion' of the photon which describes the spatial distribution of color charge inside the photon.

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