Unintegrated parton densities applied to heavy quark production in the CCFM approach

H. Jung

Physics Department, Lund University, Box 118, S-221 00 Lund, Sweden

E-mail: Hannes.Jung@desy.de

Abstract. The application of k_t - factorization supplemented with the CCFM small-x evolution equation to heavy quark production is discussed. The $b\bar{b}$ production cross sections at the TEVATRON can be consistently described using the k_t - factorization formalism together with the unintegrated gluon density obtained within the CCFM evolution approach from a fit to HERA F_2 data. Special attention is drawn to the comparison with measured visible cross sections. The visible measured cross sections at HERA are compared to the hadron level Monte Carlo generator CASCADE.

1. Introduction

The calculation of inclusive quantities, like the structure function $F_2(x, Q^2)$ at HERA, performed in NLO QCD is in perfect agreement with the measurements. However, Catani argues that the NLO approach, although phenomenologically successful for $F_2(x, Q^2)$, is not fully satisfactory from a theoretical viewpoint, because *"the truncation of the splitting functions at a fixed perturbative order is equivalent to assuming that the dominant dynamical mechanism leading to scaling violations is the evolution of parton cascades with strongly ordered transverse momenta"* [1]. As soon as exclusive quantities like jet or heavy quark production are investigated, the agreement between NLO coefficient functions convoluted with NLO DGLAP [2–5] parton densities and the data is not at all satisfactory: large so-called K-factors (normalization factors) [6–9] are needed to bring the NLO calculations close to the data ($K \sim 2 - 4$ for bottom production at the TEVATRON), indicating that in the calculations a significant part of the cross section is still missing.

At small x the structure function $F_2(x, Q^2)$ is proportional to the sea quark density, and the sea-quarks are driven via the DGLAP evolution equations by the gluon density. The standard QCD fits determine the parameters of the initial parton distributions at a starting scale Q_0. With help of the DGLAP evolution equations these parton distributions are then evolved to any other scale Q^2, with the splitting functions still

truncated at fixed $O(\alpha_s)$ (LO) or $O(\alpha_s^2)$ (NLO). Any physics process in the fixed order scheme is then calculated via collinear factorization into the coefficient functions $C^a(\frac{x}{z})$ and collinear (independent of k_t) parton density functions: $f_a(z, Q^2)$:

$$\sigma = \sigma_0 \int \frac{dz}{z} C^a(\frac{x}{z}) f_a(z, Q^2) \tag{1}$$

At large energies (small x) the evolution of parton densities proceeds over a large region in rapidity $\Delta y \sim \log(1/x)$ and effects of finite transverse momenta of the partons may become increasingly important. Cross sections can then be k_t - factorized [10] into an off-shell (k_t dependent) partonic cross section $\hat{\sigma}(\frac{x}{z}, k_t)$ and a k_t - unintegrated parton density function $\mathcal{F}(z, k_t)$:

$$\sigma = \int \frac{dz}{z} d^2k_t \hat{\sigma}(\frac{x}{z}, k_t) \mathcal{F}(z, k_t) \tag{2}$$

The unintegrated gluon density $\mathcal{F}(z, k_t)$ is described by the BFKL [11–13] evolution equation in the region of asymptotically large energies (small x). An appropriate description valid for both small and large x is given by the CCFM evolution equation [14–17], resulting in an unintegrated gluon density $\mathcal{A}(x, k_t, \bar{q})$, which is a function also of the additional evolution scale $\bar{q}$ described below.

In [18] Catani argues that by explicitly carrying out the k_t integration in eq.(2) one can obtain a form fully consistent with collinear factorization: the coefficient functions and also the DGLAP splitting functions leading to $f_a(z, Q^2)$ are no longer evaluated in fixed order perturbation theory but supplemented with the all-order resummation of the $\alpha_s \log 1/x$ contribution at small x.

In this paper heavy quark production at the TEVATRON and at HERA is investigated using the k_t - factorization approach. The unintegrated gluon density has been obtained previously in [19] from a CCFM fit to the HERA structure function $F_2(x, Q^2)$. All free parameters are thus fixed and absolute predictions for bottom production can be made. As both TEVATRON and HERA data can be described, this shows for the first time evidence for the universality of the unintegrated CCFM gluon distribution.

First the basic features of the CCFM evolution equation are recalled and the unintegrated gluon density is investigated. Then the calculations for $b\bar{b}$ production at the TEVATRON are presented as well as calculations of the visible cross section for $b\bar{b}$ production at HERA.

2. The CCFM evolution equation

A solution of the CCFM evolution equation, which properly describes the inclusive structure function $F_2(x, Q^2)$ and also typical small x final state processes at HERA

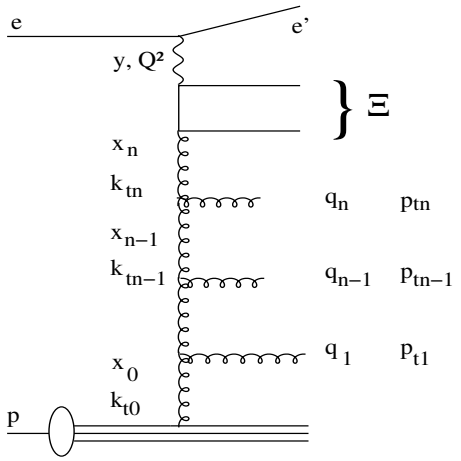

Figure 1. *Kinematic variables for multi-gluon emission. The t-channel gluon four-vectors are given by k_i, and the gluons emitted in the initial state cascade have four-vectors p_i. The upper angle for any emission is obtained from the quark box, as indicated with Ξ.*

has been presented in detail in [19]. Figure 1 shows the pattern of QCD initial-state radiation in a small-x lepto-production process, together with labels for the kinematics. According to the CCFM evolution equation, the emission of partons during the initial cascade is only allowed in an angular-ordered region of phase space. The maximum allowed angle Ξ is defined by the hard scattering quark box, producing the heavy quark pair. In terms of Sudakov variables the quark pair momentum is written as:

$$p_q + p_{\bar{q}} = \Upsilon(p_p + \Xi p_e) + Q_t, \tag{3}$$

where p_e (p_p) are the incoming electron (proton) momenta, respectively, and Q_t is the transverse momentum of the quark pair in the laboratory frame. Similarly, the momenta p_i of the gluons emitted during the initial state cascade are given by (here treated massless):

$$p_i = \upsilon_i(p_p + \xi_i p_e) + p_{ti} \ , \ \ \xi_i = \frac{p_{ti}^2}{s\upsilon_i^2}, \tag{4}$$

with $\upsilon_i = (1 - z_i)x_{i-1}$, $x_i = z_i x_{i-1}$ and $s = (p_e + p_p)^2$ being the squared center of mass energy. The variable ξ_i is connected to the angle of the emitted gluon with respect to the incoming proton and x_i and υ_i are the momentum fractions of the exchanged and emitted gluons, while z_i is the momentum fraction in the branching $(i-1) \rightarrow i$ and p_{ti} is the transverse momentum of the emitted gluon i.

The angular-ordered region is then specified by (Fig. 1):

$$\xi_0 < \xi_1 < \cdots < \xi_n < \Xi \tag{5}$$

which becomes:

$$z_{i-1}q_{i-1} < q_i, \tag{6}$$

where the rescaled transverse momenta q_i of the emitted gluons are defined by:

$$q_i = x_{i-1}\sqrt{s\xi_i} = \frac{p_{ti}}{1-z_i} \tag{7}$$

The CCFM equation for the unintegrated gluon density can be written [17,19–21] as an integral equation:

$$\mathcal{A}(x,k_t,\bar{q}) = \mathcal{A}_0(x,k_t,\bar{q}) + \int \frac{dz}{z}\int \frac{d^2q}{\pi q^2}\Theta(\bar{q}-zq)\Delta_s(\bar{q},zq)\tilde{P}(z,q,k_t)\mathcal{A}\left(\frac{x}{z},k_t',q\right) \tag{8}$$

with $\vec{k}_t' = |\vec{k}_t + (1-z)\vec{q}|$ and $\bar{q}$ being the upper scale for the last angle of the emission: $\bar{q} > z_n q_n$, $q_n > z_{n-1}q_{n-1}$, ..., $q_1 > Q_0$. Here q is used as a shorthand notation for the 2-dimensional vector of the rescaled transverse momentum $\vec{q} \equiv \vec{q}_t = \vec{p}_t/(1-z)$. The splitting function $\tilde{P}(z,q,k_t)$ and the Sudakov form factor $\Delta_s(\bar{q},zq)$ are given explicitly in [19].

2.1. The unintegrated gluon density

In [19] the unintegrated gluon density $x\mathcal{A}(x,k_t^2,\bar{q})$ has been obtained from a fit to the structure function $F_2(x,Q^2)$ ‡.

In Fig. 2 the CCFM unintegrated gluon density distribution as a function of x and k_t^2 is shown and compared to

$$\mathcal{F}(x,k_t^2) \simeq \left.\frac{dxG(x,\mu^2)}{d\mu^2}\right|_{\mu^2=k_t^2} \tag{9}$$

with $xG(x,\mu)$ being the collinear gluon density of GRV 98 [23] in LO and NLO.

The unintegrated gluon density can be related to the integrated one by:

$$xG(x,\mu)|_{\mu=\bar{q}} \simeq \int_0^{\bar{q}^2} dk_t^2 x\mathcal{A}(x,k_t^2,\bar{q}). \tag{10}$$

Here the dependence on the scale of the maximum angle $\bar{q}$ is made explicit: the evolution proceeds up to the maximum angle $\bar{q}$, which plays the role of the evolution scale in the collinear parton densities. This becomes obvious since

$$\bar{q}^2 = x_g^2\Xi s = yx_g s = \hat{s} + Q_t^2. \tag{11}$$

The last expression is derived by using $p_Q + p_{\bar{Q}} \simeq x_g p_p + y p_e + Q_t$, $\Xi \simeq y/x_g$ and $\hat{s} = yx_g s - Q_t^2$. This can be compared to a possible choice of the renormalization and factorization scale μ^2 in the collinear approach with $\mu^2 = Q_t^2 + 4\cdot m_Q^2$ and the similarity between μ and $\bar{q}$ becomes obvious.

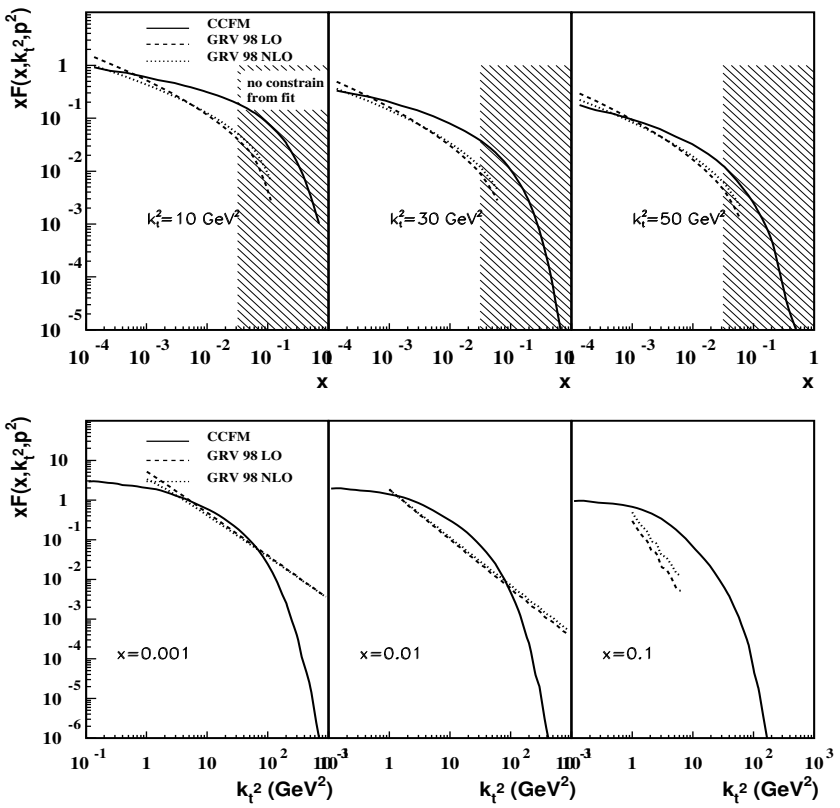

Figure 2. The *CCFM* k_t dependent (unintegrated) gluon density [19,22] at $\bar{q} = 10$ GeV as a function of x for different values of k_t^2 (upper) and as a function of k_t^2 for different values of x (lower) compared to $\frac{dxG(x,\mu^2)}{d\mu^2}$ with $xG(x,\mu)$ being the collinear gluon density of GRV 98 [23] in LO and NLO.

In Fig. 3 the CCFM gluon density integrated over k_t according to eq.(10) is compared to the gluon densities of GRV 98 [23] in LO and NLO. It is interesting to note that the CCFM gluon density is flat for $x \to 0$ at the input scale $Q = 1$ GeV. Even at larger scales the collinear gluon densities rise faster with decreasing x than the CCFM gluon density. However, after evolution and convolution with the off-shell matrix element the scaling violations of $F_2(x, Q^2)$ and the rise of F_2 towards small x is reproduced, as shown in Fig. 4 of [19]. A similar trend is observed in the collinear fixed order calculations, when going from LO to NLO: at NLO the gluon density is less steep at small x, because part of the x dependence is already included in the NLO P_{qg} splitting function, as argued in [1].

From Fig. 3 one can see that the integrated gluon density from CCFM is larger in the medium x range, than the ones from the collinear approach. Due to an additional

‡ A Fortran program for the unintegrated gluon density $x\mathcal{A}(x, k_t^2, \bar{q})$ can be obtained from [22]

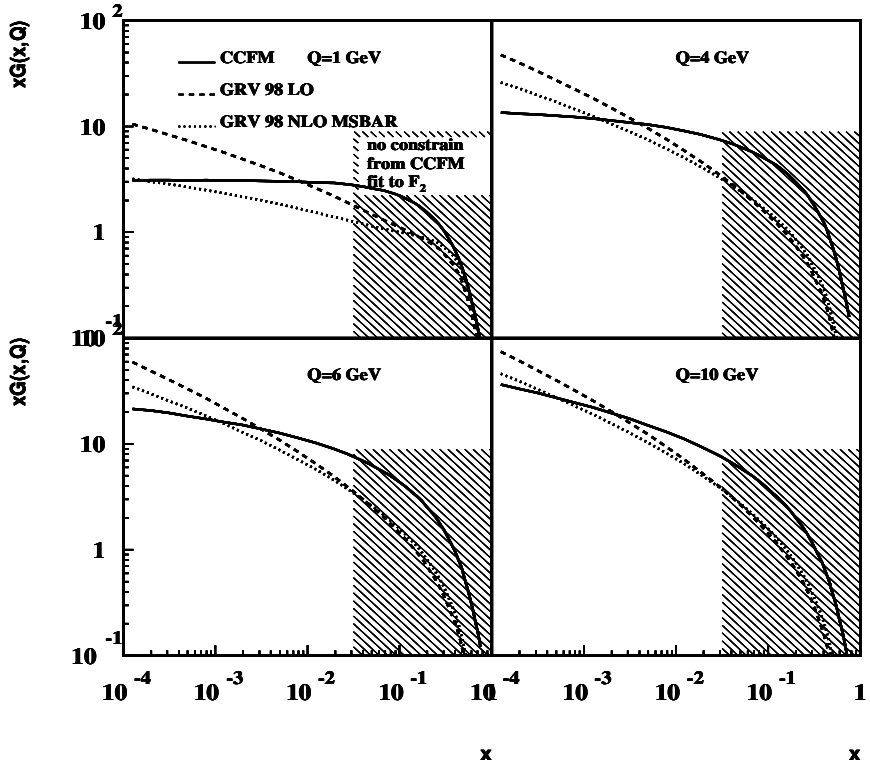

Figure 3. The *CCFM* gluon density (solid line) integrated over k_t as a function of x for different values of Q. For comparison the GRV 98 [23] gluon density in LO (dashed line) and NLO (dotted line) is also shown.

$1/k_t^2$ suppression in the off-shell matrix elements, the gluon density obviously has to be larger to still reproduce the same cross section. In addition only gluon ladders are considered in the k_t - factorization approach used here, which means that the sea quark contribution to the structure function $F_2(x, Q^2)$ comes entirely from boson gluon fusion, without any contribution from the intrinsic quark sea. One also should remember that the relation in eq.(10) is only approximately true, since the gluon density itself is not a physical observable.

3. $b\bar{b}$ production at the TEVATRON

The cross section for $b\bar{b}$ production in $p\bar{p}$ collision at $\sqrt{s} = 1800$ GeV is calculated with CASCADE [19,22], which is a Monte Carlo implementation of the CCFM approach described above. The off-shell matrix element as given in [10] for heavy quarks is used with $m_b = 4.75$ GeV. The scale μ used in $\alpha_s(\mu^2)$ is set to $\mu^2 = m_T^2 = m_b^2 + p_T^2$ (as in [19]), with p_T being the transverse momentum of the heavy quarks in the $p\bar{p}$ center-of-mass

frame. In Fig. 4 the prediction for the cross section for $b\bar{b}$ production with pseudo-rapidity $|y^b| < 1$ is shown as a function of p_T^{min} and compared to the measurement of D0 [7]. Also shown is the NLO prediction from [24], taken from [7].

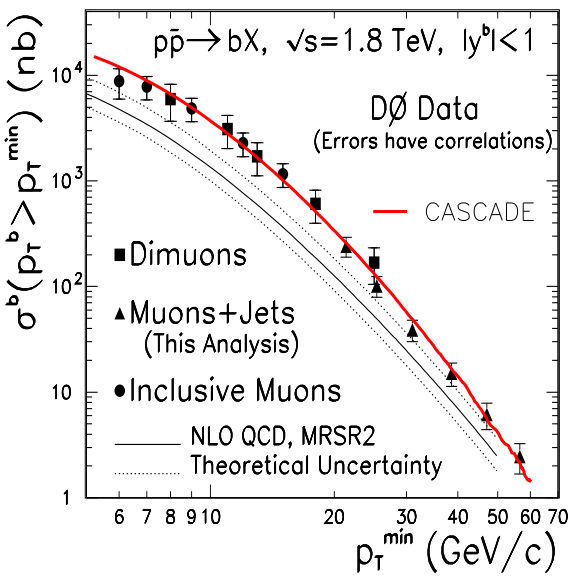

Figure 4. *Cross section for $b\bar{b}$ production with $|y^b| < 1$ as a function of p_T^{min}. Shown are the D0 [7] data points, the fixed order NLO prediction, and the prediction of* Cascade.

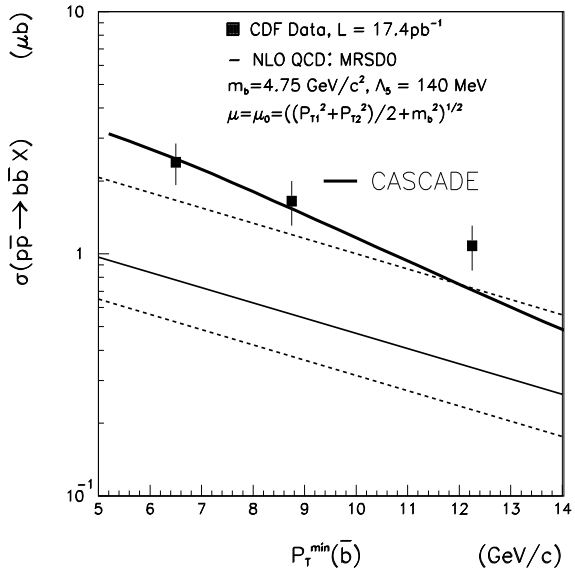

Figure 5. *Cross section for $b\bar{b}$ production with $|y_b| < 1$ and $p_T^{min}(b) > 6.5$ GeV as a function of $p_T^{min}(\bar{b})$. Shown are the CDF [6] data points, the fixed order NLO prediction, and the prediction of* Cascade.

In Fig. 5 the measured cross section of CDF [6] is shown for 3 values of $p_T^{min}(\bar{b})$ with the kinematic constraint of $|y^b|, |y^{\bar{b}}| < 1$ and $p_T^{min}(b) > 6.5$ GeV together with the prediction from Cascade and the NLO calculation from [25], taken from [6]. In all cases the NLO calculation used $m_b = 4.75$ GeV and the factorization and renormalization scales were set to $\mu^2 = m_T^2 = m_b^2 + p_T^2$. Both, D0 and CDF measurements are above the NLO predictions by a factor of ~ 2. The Cascade predictions are in reasonable agreement with the measurements. A similarly good description of the D0 and CDF data has been obtained in [26] using also k_t - factorization but supplemented with a BFKL type unintegrated gluon density. It is interesting to note that the CCFM unintegrated gluon density has been obtained from inclusive $F_2(x, Q^2)$ at HERA. In this sense, the prediction of Cascade is a parameter free prediction of the $b\bar{b}$ cross section in $p\bar{p}$ collisions. This also shows for the first time evidence for the universality of the unintegrated CCFM gluon distribution.

In Fig. 6 the x_n and k_{tn} distributions of the gluons entering the hard scattering process $g+g \to b+\bar{b}$ (see Fig. 1) are shown and the predictions from the k_t - factorization approach (Cascade) are compared to the standard collinear approach (here Pythia

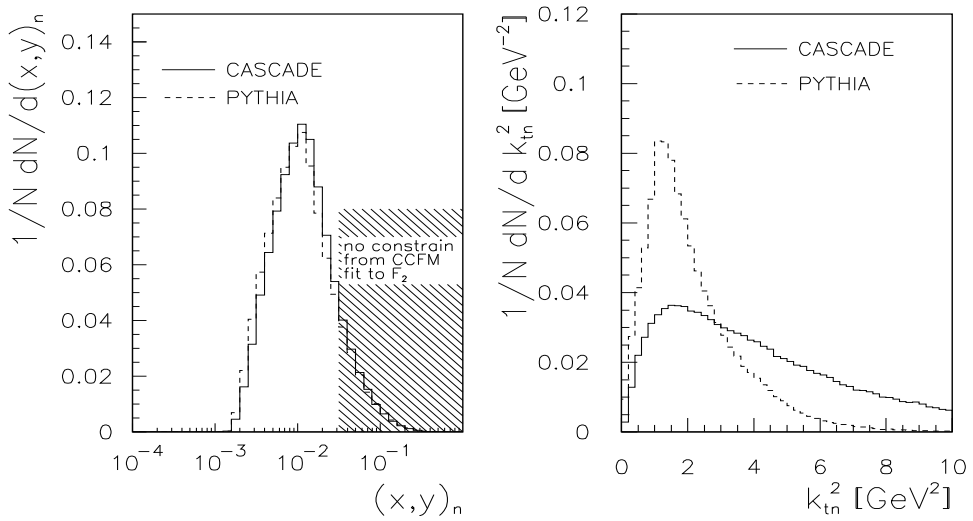

Figure 6. *Comparison of x_n and k_{tn} distributions of the gluons entering the process $g + g \to b + \bar{b}$. Shown are the predictions from* Cascade *representing k_t-factorization with the CCFM unintegrated gluon density and also from* Pythia *representing the collinear approach supplemented with initial and final state DGLAP parton showers to cover the phase space of p_t ordered QCD cascades.*

[27] with LO $gg \to b\bar{b}$ matrix elements supplemented with DGLAP parton showers to simulate higher order effects). Whereas the x_n distributions agree reasonably well, a significant difference is observed in the k_{tn} distribution. However this is not surprising: the k_t factorization approach includes a large part of the fixed order NLO corrections (in collinear factorization). Such corrections are $gg \to Q\bar{Q}g$, where the final state gluon can have any kinematically allowed transverse momentum, which could be regarded as a first step toward a non-p_t ordered QCD cascade.

4. $b\bar{b}$ production at HERA

In [19] the prediction of Cascade for the total $b\bar{b}$ cross section was compared to the extrapolated measurements of the H1 [8] and ZEUS [9] experiments at HERA. Since Cascade generates full hadron level events, a direct comparison with measurements can be done, before extrapolating the measurement over the full phase space to the total $b\bar{b}$ cross section. ZEUS [9] has measured the dijet cross section which can be attributed to bottom production by demanding an electron inside one of the jets. In the kinematic range of $Q^2 < 1$ GeV^2, $0.2 < y < 0.8$, at least two jets with $E_T^{jet1(2)} > 7(6)$ GeV and $|\eta^{jet}| < 2.4$ and a prompt electron with $p_T^{e^-} > 1.6$ GeV and $|\eta^{e^-}| < 1.1$, ZEUS [9] quotes the cross section as:

$$\sigma^{b\to e^-}_{e^+p\to e^+ + dijet + e^- + X} = 24.9 \pm 6.4^{+4.2}_{-7.3} \text{ pb} \qquad \text{(ZEUS [9])} \tag{12}$$

with the statistical (first) and systematic (second) error given. Within the same kinematic region and applying the same jet algorithm CASCADE predicts:

$$\sigma^{b\to e^-}_{e^+p\to e^+ + dijet + e^- + X} = 20.3^{+1.6}_{-1.9} \text{ pb}, \tag{13}$$

where the error reflects the variation of $m_b = 4.75 \mp 0.25$ GeV. This value agrees with the measurement within the statistical error. The $b\bar{b}$ cross section, extrapolated from the measured jet cross section to the region $p_t^b > 5$ GeV and $|\eta^b| < 2$ using CASCADE is given by:

$$\sigma(ep \to e' b\bar{b} X) = 1.07 \pm 0.27\ ^{+0.18}_{-0.3} \text{ nb} \qquad \text{(ZEUS using CASCADE)}, \tag{14}$$

which can be compared with the CASCADE prediction $\sigma(ep \to e' b\bar{b} X) = 0.87 \pm 0.08$ nb using $m_b = 4.75 \mp 0.25$ GeV. The NLO prediction is $\sigma(ep \to e' b\bar{b} X) = 0.64$ nb. When using HERWIG [28] for the extrapolation, ZEUS quotes an extrapolated cross section of [9]:

$$\sigma = 1.6 \pm 0.4(stat.)^{+0.3}_{-0.5}(syst.)^{+0.2}_{-0.4}(ext.) \text{ nb}$$

The quoted extrapolation uncertainty includes the extrapolation with PYTHIA, which is similar to the one obtained with CASCADE. Thus the extrapolated cross section includes large model uncertainties and the comparison of extrapolated cross sections with predictions from NLO calculations are questionable. This also shows the advantage of a full hadron level simulation in form of a Monte Carlo program over a fixed order parton level prediction.

It is interesting to note that the CASCADE results agree well with the PYTHIA results for the dijet plus electron cross section, if heavy quark excitation is included, as well as with the extrapolation factor. As in the case of $b\bar{b}$ production at the TEVATRON higher order QCD effects are important, which are already included in the k_t - factorization approach.

In Fig. 7 within the same kinematic range the differential cross section for heavy quark decays (charm and bottom) as a function of x_γ predicted by CASCADE is compared with the measurement of ZEUS [9]. A similar trend as in the case of charm photoproduction is observed, namely a significant fraction of the cross section with $x_\gamma < 1$. In LO in the collinear factorization approach this is attributed to resolved photon processes. However, in k_t - factorization, the x_γ distribution is explained naturally because gluons in the initial state need not to be radiated in a p_t ordered region and therefore can give rise to a high p_t jet with transverse momentum larger than that of the heavy quarks.

The prediction of CASCADE has also been compared to the measurement of H1 [8] for the electroproduction cross section in $Q^2 < 1$ GeV^2, $0.1 < y < 0.8$, $p_\perp^\mu < 2$ GeV and $35^o < \theta^\mu < 130^o$:

$$\sigma(ep \to e' b\bar{b} X \to \mu X') = 0.176 \pm 0.016(stat.)^{+0.026}_{-0.017}(syst.) \text{ nb}$$

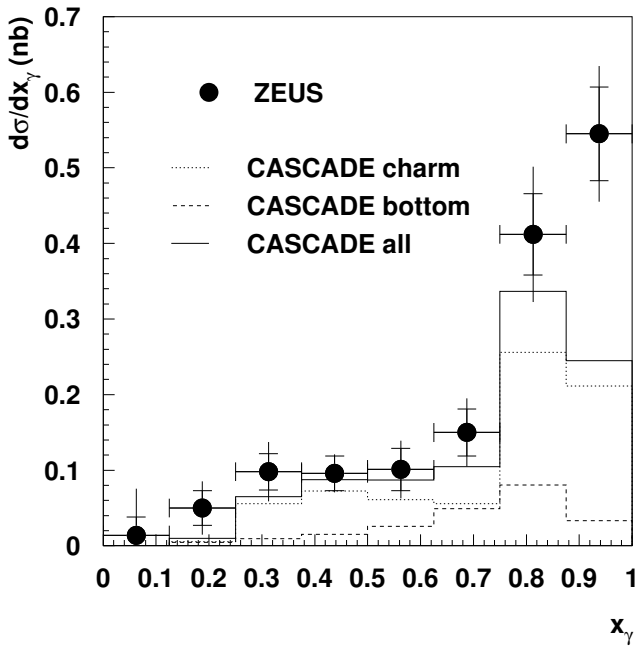

Figure 7. *The differential cross section $d\sigma/dx_\gamma^{obs}$ for heavy quark decays as measured by ZEUS [9] and compared to* CASCADE. *Shown are the charm, bottom and the sum of both contributions to the cross section (Note there is no additional K factor applied).*

This cross section already includes the extrapolation from measured jets to the muon. In the same kinematic range CASCADE predicts:

$$\sigma(ep \to e' b\bar{b}X \to \mu X') = 0.066^{+0.009}_{-0.007} \text{ nb}.$$

which is a factor ~ 2.6 below the measurement. It is interesting to note that the ratio

$$R(\text{H1}) = \frac{\sigma(\text{CASCADE})}{\sigma(\text{NLO})} = \frac{0.066}{0.054} = 1.2$$

is similar to the one obtained for the ZEUS extrapolated measurements: $R(\text{ZEUS}) = 1.4$. The measured cross sections of H1 and ZEUS cannot be compared directly because different kinematic ranges and decay channels were used. To compare both experiments, a ratio R_{MC} is defined:

$$R_{MC} = \frac{\sigma_{measured}}{\sigma_{MC}} \tag{15}$$

Using CASCADE, the ratios are:

$$R_{MC}(\text{H1}) \quad = \frac{\sigma_{measured}}{\sigma_{MC}} = 2.7 \pm 0.25^{+0.4}_{-0.26} \tag{16}$$

$$R_{MC}(\text{ZEUS}) = \frac{\sigma_{measured}}{\sigma_{MC}} = 1.2 \pm 0.32^{+0.21}_{-0.37}, \tag{17}$$

where the error comes only from $\sigma_{measured}$. This exercise shows that both experiments differ by a factor of more than 2 in the published measurements, when compared to CASCADE.

5. Conclusion

Bottom production at the TEVATRON can be reasonably well described using the k_t-factorization approach with off-shell matrix elements for the hard scattering process. One essential ingredient for the satisfactory description is the unintegrated gluon density, which was obtained by a CCFM evolution fitted to structure function data at HERA, showing evidence for the universality of the unintegrated gluon density. The comparison with the data was performed with the CASCADE Monte Carlo event generator, which implements k_t-factorization together with the CCFM unintegrated gluon density.

Measurements of bottom production at HERA are also compared to predictions from CASCADE. The visible dijet plus electron cross section attributed to b-production as measured by ZEUS could be reproduced within the statistical error. It was pointed out that the extrapolation from the measured to the total $b\bar{b}$ cross section contains large model dependencies. If CASCADE or PYTHIA was used for extrapolation, the cross section was found to agree with CASCADE and even with NLO calculations within the combined statistical and systematic error.

However, the situation is different with the H1 measurement: the visible muon cross section is already a factor of 2.7 above the prediction from CASCADE. Comparing both HERA measurements with Monte Carlo predictions of CASCADE, it could be shown that both experiments, H1 and ZEUS, differ in their measurements by a factor of more than two. Further measurements, also differential, are desirable to clarify the situation at HERA.

In general the k_t-factorization approach has now proven to be successful even in a kinematic region, where typical small-x effects are expected to be small. It is the advantage of that approach that important parts of NLO and even NNLO contributions are consistently included due to the off-shell gluons, which enter into the hard scattering process.

6. Acknowledgments

Many thanks go to the organizers of this very nice workshop: G. Grindhammer, B. Kniehl, G. Kramer and W. Ochs. I am grateful to G. Salam for all his patience and his help in all different kinds of discussions concerning CCFM and a backward evolution approach. I am grateful to E. Elsen, L. Gladilin, G. Ingelman, L. Jönsson, P. Schleper for careful reading and comments on the manuscript. No words are adequate for everything I have with Antje. I also want to thank the DESY directorate for hospitality and support.

References

[1] S. Catani, Aspects of QCD, from the Tevatron to LHC, in *Proceedings of the International Workshop* Physics at TeV Colliders (Les Houches, France, 8-18 June,

1999), hep-ph/0005233.

[2] V. Gribov, L. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 438 and 675.

[3] L. Lipatov, *Sov. J. Nucl. Phys.* **20** (1975) 94.

[4] G. Altarelli, G. Parisi, *Nucl. Phys.* **B 126** (1977) 298.

[5] Y. Dokshitser, *Sov. Phys. JETP* **46** (1977) 641.

[6] CDF Collaboration; F. Abe et al., *Phys. Rev.* **D 55** (1997) 2546.

[7] D0 Collaboration; B. Abbott et al., *Phys. Lett.* **B 487** (2000) 264.

[8] H1 Collaboration; C. Adloff et al., *Phys. Lett.* **B 467** (1999) 156, and erratum ibid.

[9] ZEUS Collaboration; J. Breitweg et al., *Eur. Phys. J.* **C 18** (2001) 625.

[10] S. Catani, M. Ciafaloni, F. Hautmann, *Nucl. Phys.* **B 366** (1991) 135.

[11] E. Kuraev, L. Lipatov, V. Fadin, *Sov. Phys. JETP* **44** (1976) 443.

[12] E. Kuraev, L. Lipatov, V. Fadin, *Sov. Phys. JETP* **45** (1977) 199.

[13] Y. Balitskii, L. Lipatov, *Sov. J. Nucl. Phys.* **28** (1978) 822.

[14] M. Ciafaloni, *Nucl. Phys.* **B 296** (1988) 49.

[15] S. Catani, F. Fiorani, G. Marchesini, *Phys. Lett.* **B 234** (1990) 339.

[16] S. Catani, F. Fiorani, G. Marchesini, *Nucl. Phys.* **B 336** (1990) 18.

[17] G. Marchesini, *Nucl. Phys.* **B 445** (1995) 49.

[18] S. Catani, k_t-factorisation and perturbative invariants at small x, in *Proceedings of the International Workshop on Deep Inelastic Scattering,* DIS 96 (Rome, Italy, 15-19 April, 1996), hep-ph/9608310.

[19] H. Jung, G. Salam, *Eur. Phys. J.* **C 19** (2001) 351, hep-ph/0012143.

[20] G. Bottazzi, G. Marchesini, G. Salam, M. Scorletti, *JHEP* **12** (1998) 011, hep-ph/9810546.

[21] J. Kwiecinski, A. Martin, P. Sutton, *Phys. Rev.* **D 52** (1995) 1445.

[22] H. Jung, *The CCFM Monte Carlo generator* CASCADE *for lepton - proton and proton - proton collisions*, Lund University, 2001, hep-ph/0109102, DESY 01-114, `http://www.quark.lu.se/~hannes/cascade/`.

[23] M. Glück, E. Reya, A. Vogt, *Eur. Phys. J.* **C 5** (1998) 461, hep-ph/9806404.

[24] S. Frixione, M. Mangano, *Nucl. Phys.* **B 483** (1997) 321.

[25] M. Mangano, P. Nason, G. Ridolfi, *Nucl. Phys.* **B 373** (1992) 295.

[26] P. Hagler et al., *Phys. Rev.* **D62** (2000) 071502.

[27] T. Sjöstrand, *Comp. Phys. Comm.* **82** (1994) 74.

[28] G. Marchesini et al., *Comp. Phys. Comm.* **76** (1992) 465, hep-ph/9912396.