Azimuthal asymmetries in semi-inclusive leptoproduction

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Abstract. We point out the role of intrinsic transverse momentum of partons in the study of azimuthal asymmetries in deep-inelastic 1-particle inclusive leptoproduction. Leading asymmetries often appear in combination with spin asymmetries. This leads not only to transverse momentum dependence in the parton distribution functions, but also to functions beyond the ones known from inclusive deep-inelastic scattering (DIS). We use Lorentz invariance and the QCD equations of motion to study the evolution of functions that appear at leading (zeroth) order in a $1/Q$ expansion in azimuthal asymmetries. This includes the evolution equation of the Collins fragmentation function. The moments of these functions are matrix elements of known twist two and twist three operators. We give the evolution in the large N_c limit, restricted to the non-singlet case for the chiral-even functions.

1. Inclusive deep-inelastic scattering

In inclusive deep-inelastic scattering (DIS) the neutral current (photon exchange) cross section can be expressed in terms of two structure functions $F_1(x_{_B}, Q^2)$ and $F_2(x_{_B}, Q^2)$, which if the momentum transfer squared Q^2 is large can be expressed as a weighted sum over quark distributions $q(x)$ for the various flavors, $2\,F_1(x_{_B}, Q^2) = F_2(x_{_B}, Q^2)/x_{_B} = \sum_q e_q^2\, q(x_{_B})$. In polarized deep-inelastic scattering with both lepton and target polarized again two structure functions $g_1(x_{_B}, Q^2)$ and $g_2(x_{_B}, Q^2)$ appear, of which for large Q^2 the first one can be expressed as a weighted sum over polarized quark distributions $\Delta q(x)$ for the various flavors.

In hard processes such as DIS the effects of hadrons can be studied via quark and gluon correlators. In inclusive deep inelastic scattering (DIS), these are lightcone‡

‡ For inclusive leptoproduction the lightlike directions $n_\pm$ and lightcone coordinates $a^\pm = a \cdot n_\mp$ are defined through hadron momentum P and the momentum transfer q,

$$P = \frac{Q}{x_{_B}\sqrt{2}}\, n_+ + \frac{x_{_B} M^2}{Q\sqrt{2}}\, n_-,$$
$$q = -\frac{Q}{x_{_B}\sqrt{2}}\, n_+ + \frac{Q}{\sqrt{2}}\, n_-.$$

correlators depending on $x \equiv p^+/P^+$ of the type

$$\Phi_{ij}(x) \equiv \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi} \langle P,S|\,\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{LC}. \tag{1}$$

where the subscript 'LC' indicates $\xi^+ = \xi_T = 0$ and $\mathcal{U}(0,\xi)$ is a gauge link with the path running along the minus direction. The use of lightlike directions enters naturally in a deep-inelastic process in which the hadrons, compared to the scale Q^2, are massless.

The parametrization for DIS at leading (zeroth) order in a $1/Q$ expansion is

$$\Phi^{\text{twist}-2}(x) = \frac{1}{2}\left\{f_1(x)\,\not{n}_+ + S_L\, g_1(x)\,\gamma_5\not{n}_+ + h_1(x)\,\gamma_5\not{S}_T\not{n}_+\right\}, \tag{2}$$

where longitudinal spin S_L refers to the component along the same lightlike direction as defined by the hadron§. Specifying also the flavor one also encounters the notations $q(x) = f_1^q(x)$, $\Delta q(x) = g_1^q(x)$ and $\delta q(x) = \Delta_T q(x) = h_1^q(x)$. The evolution equations for these functions are known to next-to-leading order and for the singlet f_1 and g_1 there is mixing with the unpolarized and polarized gluon distribution functions $g(x)$ and $\Delta g(x)$, respectively.

Instead of using the compact Dirac notation, it is useful to write explicitly the production matrix $M^{\text{prod}} = (\Phi\gamma^+)^T$ that appears in leading order DIS calculations. In hard processes only two Dirac components are relevant, *one* of them righthanded and *one* lefthanded ($\psi_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi$). The advantage of the matrix representation is that M^{prod} is a matrix of which any diagonal element corresponds to a positive definite momentum distribution. Restricting ourselves to the two relevant (right-/lefthanded) quark states and using instead of the spin vector S explicitly two nucleon spin states, the matrix in the 4×4 quark $\otimes$ nucleon spin space becomes [1]

$$M^{(\text{prod})} = \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix} \begin{matrix} \text{R} \\ \text{R} \\ \text{L} \\ \text{L} \end{matrix} \tag{3}$$

$$\begin{matrix} \text{R} & \text{R} & \text{L} & \text{L} \end{matrix}$$

From the matrix the physical interpretation of f_1 as quark momentum density and the interpretation of g_1 as the difference of chiral densities (often sloppily referred to as helicity distribution) is seen. As can be seen the function $h_1(x)$ involves a matrix elements between left- and right-handed quarks, it is chirally odd [2]. This implies that it is not accessible in inclusive DIS, where the hard scattering part does not change chirality except via (irrelevant) quark mass terms. This third distribution function, however, is needed for the complete characterization of the (collinear) spin state of a proton as probed in hard scattering processes. By choosing a different basis (actually

§ The spin vector is parametrized $S = S_L\,\frac{P^+}{M}n_+ - S_L\,\frac{M}{2P^+}\,n_- + S_T$.

of transversely polarized quarks and transversely polarized nucleons) h_1 appears on the diagonal and its interpretation as tranverse spin polarization in a transversely polarized target (often referred to as transversity) is seen.

From the fact that any forward matrix element of the above matrix represents a density, one derives positivity bounds

$$f_1(x) \geq 0 \tag{4}$$

$$|g_1(x)| \leq f_1(x) \tag{5}$$

$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right) \leq f_1(x). \tag{6}$$

For the last bound [3] one explicitly needs the semi-definiteness of the matrix.

For DIS up to order $1/Q$ one needs also the M/P^+ parts in the parameterization of $\Phi(x)$. Not imposing time-reversal invariance one obtains

$$\begin{aligned}\Phi^{\text{twist}-3}(x) = &\frac{M}{2P^+}\left\{e(x) + g_T(x)\,\gamma_5 \not{S}_T + S_L\, h_L(x)\,\gamma_5 \frac{[\not{n}_+, \not{n}_-]}{2}\right\} \\ &+ \frac{M}{2P^+}\left\{-i\,S_L\, e_L(x)\gamma_5 - f_T(x)\,\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} + i\,h(x)\frac{[\not{n}_+, \not{n}_-]}{2}\right\}\end{aligned} \tag{7}$$

The functions e, g_T and h_L are T-even, the functions e_L, f_T and h are T-odd. They vanish for the matrix element Φ in Eq. 1 , but we keep them because in the analogous situation of fragmentation functions, to be discussed further down, they become very important. The leading order evolution of e, g_T and h_L is known [4] and for the non-singlet case this also provides the evolution of the T-odd functions e_L, f_T and h respectively, for which the operators involved differ only from those of the T-even functions by a γ_5 matrix.

The twist assignment is more evident by connecting these twist-3 functions to matrix elements of the form $\langle P, S|\overline{\psi}_j(0)\,\mathcal{U}(0,\eta)\,iD_T^\alpha(\eta)\,\mathcal{U}(\eta,\xi)\,\psi_i(\xi)|P,S\rangle$ via the QCD equations of motion.

Of the twist-3 functions, e and h_L are chiral-odd, while g_T is chiral-even. Only these latter functions g_T^q, reinstating flavor indices, appears in the polarized DIS cross section in the structure function $g_T(x_B, Q^2) = (g_1 + g_2)(x_B, Q^2) = \frac{1}{2}\sum_q e_q^2\, g_T^q(x_B)$. The calculation actually requires taking into account not only the handbag diagram, shown as the left part in Fig. 1, but also diagrams in which an additional gluon emerges from the soft part. These additional contributions with matrix elements containing both quark and gluon fields can be rewritten into the twist-3 functions by the use of QCD equations of motion.

2. Semi-inclusive leptoproduction

Semiinclusive DIS (SIDIS), in particular one-particle inclusive DIS, can be used for additional flavor identification. Instead of weighing quark flavors with the quark charge squared e_q^2 one obtains a weighting with $e_q^2\, D_1^{q\to h}(z_h)$, where $D_1^{q\to h}$ is the usual unpolarized fragmentation function for a quark of flavor q into hadron h, experimentally

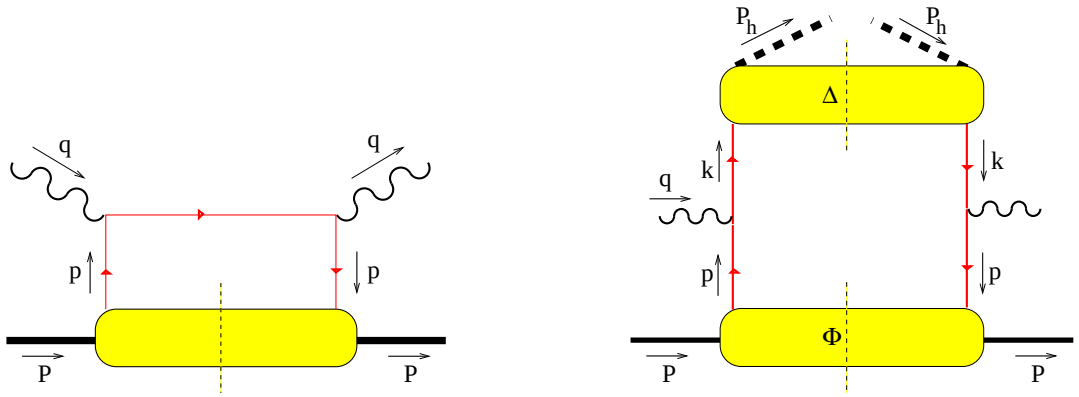

Figure 1. The leading contributions to inclusive (left) and 1-particle inclusive (right) leptoproduction.

accessible at $z_h = P{\cdot}P_h/P{\cdot}q$. In this contribution, we want to focus on the possibilities to study the intrinsic transverse momentum of partons, quarks and gluons. This is possible via azimuthal asymmetries, often appearing in combination with spin asymmetries.

Azimuthal asymmetries appear in hard scattering processes with at least two relevant hadrons. By relevant hadrons we mean hadrons used as target or detected in the final state. Factorization crucially depends on the presence of a large energy scale in the process, such as the space-like momentum transfer squared $q^2 = -Q^2$ in leptoproduction. In this paper we will be concerned with functions that appear in hard processes but which have, apart from such a hard scale, an additional soft momentum scale, related to the transverse momentum of the partons. In one-hadron inclusive leptoproduction such a scale is accessible because one deals with three momenta: the large momentum transfer q, the target momentum P and the momentum of the produced hadron P_h. The noncollinearity at the quark level appears via $q_T = q + x_B\, P - P_h/z_h$, where $x_B = Q^2/2P\cdot q$ and $z_h = P\cdot P_h/P\cdot q$ are the usual semiinclusive scaling variables, at large Q^2 identified with lightcone momentum fractions. The hadron momenta P and P_h define in essence two lightlike directions‖ n_+ and n_-, respectively. The soft scale is $Q_T^2 = -q_T^2$.

While the distribution functions in DIS in leading order could be obtained from the lightcone correlation function in Eq. 1, one encounters in SIDIS two types of lightfront

‖ For SIDIS the lightlike directions $n_\pm$ and lightcone coordinates $a^\pm = a\cdot n_\mp$ are naturally defined through hadron momenta P and P_h, in which case the momentum transfer q requires a transverse component

$$P = \frac{Q}{x_B\sqrt{2}}\, n_+ + \frac{x_B M^2}{Q\sqrt{2}}\, n_-,$$
$$q = -\frac{Q}{\sqrt{2}}\, n_+ + \frac{Q}{\sqrt{2}}\, n_- + q_T,$$
$$P_h = \frac{M_h^2}{Z_h\, Q\sqrt{2}}\, n_+ + \frac{z_h\, Q}{\sqrt{2}}\, n_-.$$

(where only one lightcone component vanishes) correlation functions (see right part of Fig. 1), involving also transverse momenta of partons as first pointed out by Ralston and Soper [5, 6] One part is relevant to treat quarks in a hadron

$$\Phi_{ij}(x, \boldsymbol{p}_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P, S|\overline{\psi}_j(0)\psi_i(\xi)|P, S\rangle\Big|_{\xi^+=0}, \tag{8}$$

depending on $x = p^+/P^+$ and the quark transverse momentum $\boldsymbol{p}_T$ in a target with $P_T = 0$. A second correlation function [7]

$$\Delta_{ij}(z, \boldsymbol{k}_T) = \sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi} \langle 0|\psi_i(\xi)|P_h, X\rangle\langle P_h, X|\overline{\psi}_j(0)|0\rangle\Big|_{\xi^-=0}, \tag{9}$$

describes fragmentation of a quark into a hadron. It depends on $z = P_h^-/k^-$ and the quark transverse momentum k_T when one produces a hadron with $P_{hT} = 0$. A simple boost shows that this is equivalent to a quark producing a hadron with transverse momentum $P_{h\perp} = -z\, k_T$ with respect to the quark.

The parametrization of the x and p_T dependent correlators becomes [5, 8, 9]

$$\begin{aligned}\Phi(x, \boldsymbol{p}_T) = \frac{1}{2}\Bigg\{ & f_1(x, \boldsymbol{p}_T^2)\, \not{n}_+ + f_{1T}^\perp(x, \boldsymbol{p}_T^2)\, \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} \\ & - g_{1s}(x, \boldsymbol{p}_T)\, \not{n}_+\gamma_5 - h_{1T}(x, \boldsymbol{p}_T^2)\, i\sigma_{\mu\nu}\gamma_5 S_T^\mu n_+^\nu \\ & - h_{1s}^\perp(x, \boldsymbol{p}_T)\, \frac{i\sigma_{\mu\nu}\gamma_5 p_T^\mu n_+^\nu}{M} + h_1^\perp(x, \boldsymbol{p}_T^2)\, \frac{\sigma_{\mu\nu} p_T^\mu n_+^\nu}{M}\Bigg\}. \end{aligned} \tag{10}$$

We used the shorthand notation $g_{1s}(x, \boldsymbol{p}_T) \equiv S_L\, g_{1L}(x, \boldsymbol{p}_T^2) + \frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M}\, g_{1T}(x, \boldsymbol{p}_T^2)$, and similarly for $h_{1s}^\perp$. The parameterization contains two T-odd functions, the functions $f_{1T}^\perp$ and $h_1^\perp$. The whole treatment of the fragmentation functions is analogous with dependence on the quark momentum fraction $z = P_h^-/k^-$ and k_T. In the notation for fragmentation functions we employ the replacements $f_{...} \to D_{...}$, $g_{...} \to G_{...}$ and $h_{...} \to H_{...}$. One also encounters a hat notation.

Again we give the matrix representation which is useful for the interpretation of the functions and which can be used to derive positivity constraints [1]. Including transverse momentum dependence one finds the production matrix $M^{(\mathrm{prod})}$ being

$$\begin{pmatrix} f_1 + g_1 & \frac{|p_T|}{M} e^{i\phi}\, g_{1T} & \frac{|p_T|}{M} e^{-i\phi}\, h_{1L}^\perp & 2\, h_1 \\ \frac{|p_T|}{M} e^{i\phi}\, g_{1T}^* & f_1 - g_1 & \frac{|p_T|^2}{M^2} e^{-2i\phi}\, h_{1T}^\perp & -\frac{|p_T|}{M} e^{-i\phi}\, h_{1L}^{\perp *} \\ \frac{|p_T|}{M} e^{i\phi}\, h_{1L}^{\perp *} & \frac{|p_T|^2}{M^2} e^{2i\phi}\, h_{1T}^\perp & f_1 - g_1 & -\frac{|p_T|}{M} e^{i\phi}\, g_{1T}^* \\ 2\, h_1 & -\frac{|p_T|}{M} e^{i\phi}\, h_{1L}^\perp & -\frac{|p_T|}{M} e^{-i\phi}\, g_{1T} & f_1 + g_1 \end{pmatrix}, \tag{11}$$

to be compared with Eq. 3. We have omitted here the T-odd functions $f_{1T}^\perp$ and $h_1^\perp$ appearing as imaginary parts of g_{1T} and $h_{1L}^\perp$, respectively. Using time-reversal invariance all the distribution functions appearing in this matrix are expected to be real, leaving aside mechanisms such as discussed in Refs [10]. For $\Delta(z, k_T)$, a correlator that contains

explicit out-states $|P_h, X\rangle$, T-reversal cannot be used, in contrast to Φ, a correlator that only contains plane-wave hadron states [11, 12, 13]. Thus in that case one finds the two T-odd fragmentation functions [14, 8] appearing as imaginary parts of the complex off-diagonal (p_T-dependent) functions in the matrix representation. The matrix corresponds for $\Delta(z, k_T)$ actually to a semi-definite quark decay matrix.

3. Azimuthal asymmetries

The possibility to access the full (transverse momentum dependent) spin structure of the nucleon is in my opinion one of the most exciting possibilities offered by 1-particle inclusive leptoproduction. It requires measurements of azimuthal asymmetries, which for the leading cross sections often appear only in combination with spin asymmetries. The T-odd fragmentation functions play a special role since they appear in single-spin asymmetries. Recent experimental results have clearly shown the presence of azimuthal asymmetries including single-spin asymmetries [15, 16, 17, 18]

At measured q_T one deals with a convolution of two transverse momentum dependent functions, where the transverse momenta of the partons from different hadrons combine to q_T [5, 8, 19]. A decoupling is achieved by studying cross sections weighted with the momentum q_T^α, leaving only the directional (azimuthal) dependence. For the quark distribution and fragmentation functions this implies a weighting with the transverse momentum, which is also important for proper theoretical handling of the functions [19].

We give a few explicit examples. For that we define weighted cross sections such as

$$\int d^2\boldsymbol{q}_T \, \frac{Q_T^2}{MM_h} \, \sin(2\phi_h^\ell) \, \frac{d\sigma_{OL}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{q}_T} \equiv \left\langle \frac{Q_T^2}{MM_h} \, \sin(2\phi_h^\ell) \right\rangle_{OL}, \tag{12}$$

where the angle ϕ_h^ℓ is the azimuthal angle between the hadron production plane and the lepton scattering plane. The subscripts O, L and T are used to refer to polarization of lepton and target respectively. The subsequently used angle ϕ_S^ℓ is the azimuthal angle between the transverse target spin vector and the lepton scattering plane.

For polarized targets, several azimuthal asymmetries arise already at leading order. For example the following possibilities were investigated in Refs [20, 14, 21, 22].

$$\left\langle \frac{Q_T}{M} \, \cos(\phi_h^\ell - \phi_S^\ell) \right\rangle_{LT} =$$
$$\frac{2\pi\alpha^2 \, s}{Q^4} \, \lambda_e \, |\boldsymbol{S}_T| \, y(2-y) \sum_{a,\bar{a}} e_a^2 \, x_B \, g_{1T}^{(1)a}(x_B) D_1^a(z_h), \tag{13}$$

$$\left\langle \frac{Q_T^2}{MM_h} \, \sin(2\phi_h^\ell) \right\rangle_{OL} =$$
$$-\frac{4\pi\alpha^2 \, s}{Q^4} \, \lambda \, (1-y) \sum_{a,\bar{a}} e_a^2 \, x_B \, h_{1L}^{\perp(1)a}(x_B) H_1^{\perp(1)a}(z_h), \tag{14}$$

$$\left\langle \frac{Q_T}{M_h} \, \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OT} =$$

$$\frac{4\pi\alpha^2\, s}{Q^4}\,|\boldsymbol{S}_T|\,(1-y)\sum_{a,\bar{a}} e_a^2\, x_B\, h_1^a(x_B) H_1^{\perp(1)a}(z_h). \tag{15}$$

The latter two are single spin asymmetries involving the T-odd fragmentation function $H_1^{\perp(1)}$. The last one was the asymmetry proposed by Collins [14] as a way to access the transverse spin distribution function h_1 in pion production. In the weighted cross sections, the socalled transverse moments show up, defined as

$$f^{(n)}(x) = \int d^2p_T \left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x,\boldsymbol{p}_T). \tag{16}$$

These transverse moments only depend on x and are contained in $\Phi_\partial^\alpha(x) \equiv \int d^2p_T\, \frac{p_T^\alpha}{M}\, \Phi(x,\boldsymbol{p}_T)$ which projects out the functions in $\Phi(x,\boldsymbol{p}_T)$ where p_T appears linearly,

$$\Phi_\partial^\alpha(x) = \frac{1}{2}\Big\{-g_{1T}^{(1)}(x)\, S_T^\alpha \not{n}_+\gamma_5 - S_L\, h_{1L}^{\perp(1)}(x)\,\frac{[\gamma^\alpha, \not{n}_+]\gamma_5}{2} - f_{1T}^{\perp(1)}(x)\,\epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho - i\, h_1^{\perp(1)}(x)\,\frac{[\gamma^\alpha, \not{n}_+]}{2}\Big\}. \tag{17}$$

4. The operator structure of transverse moments [23]

To study the scale dependence we use the moments in both p_T and x of the p_T-dependent functions, employ Lorentz invariance and use the QCD equations of motion. The moments in x for leading (collinear) distribution functions (appearing for instance in inclusive leptoproduction) are related to matrix elements of twist two operators. On the other hand, for the transverse moments entering the azimuthal asymmetry expressions of interest, one finds relations to matrix elements of twist two *and* twist three operators, for which the evolution, however, is known. In the large N_c limit this evolution becomes particularly simple.

We first invoke Lorentz invariance to rewrite some functions discussed so far. All functions in $\Phi(x)$ and $\Phi_\partial^\alpha(x)$ involve nonlocal matrix elements of two quark fields. Before constraining the matrix elements to the light-cone or lightfront only a limited number of amplitudes can be written down. This leads to the following Lorentz-invariance relations

$$g_T = g_1 + \frac{d}{dx}\, g_{1T}^{(1)}, \qquad h_L = h_1 - \frac{d}{dx}\, h_{1L}^{\perp(1)}, \tag{18}$$

$$f_T = -\frac{d}{dx}\, f_{1T}^{\perp(1)}, \qquad h = -\frac{d}{dx}\, h_1^{\perp(1)}. \tag{19}$$

From these relations, it is clear that the $\boldsymbol{p}_T^2/2M^2$ moments of the p_T-dependent functions, appearing in $\Phi_\partial^\alpha(x)$, involve both twist-2 and twist-3 operators.

Another useful set of functions is obtained as the difference between the correlator Φ_D, which via equations of motion is connected to $\Phi^{\rm twist-3}$, and the correlator Φ_∂. This difference corresponds in $A^+ = 0$ gauge to a correlator Φ_A, involving $\langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\eta)A_T^\alpha(\eta)\,\mathcal{U}(\eta,\xi)\,\psi_i(\xi)|P,S\rangle$. Via the parametrisation of Φ_A one defines interaction-dependent (tilde) functions,

$$x\, g_T(x) - \frac{m}{M}\, h_1(x) - g_{1T}^{(1)}(x) + i\left[x\, f_T(x) + f_{1T}^{\perp(1)}(x)\right] \equiv x\,\tilde{g}_T(x) + ix\,\tilde{f}_T(x),$$

$$x\,h_L(x) - \frac{m}{M}\,g_1(x) + 2\,h_{1L}^{\perp(1)}(x) - ix\,e_L(x) \equiv x\,\tilde{h}_L(x) - ix\,\tilde{e}_L(x), \tag{20}$$

$$x\,e(x) - \frac{m}{M}\,f_1(x) + i\left[x\,h(x) + 2\,h_1^{\perp(1)}(x)\right] \equiv x\,\tilde{e}(x) + ix\,\tilde{h}(x). \tag{21}$$

$$\tag{22}$$

Using the equations of motion relations in Eqs. (20) - (22) and the relations based on Lorentz invariance in Eqs. (18) - (19), it is straightforward to relate the various twist-3 functions and the $\boldsymbol{p}_T^2/2M^2$ (transverse) moments of p_T-dependent functions. The results for the functions are (omitting quark mass terms)

$$g_T(x) = \int_x^1 dy\ \frac{g_1(y)}{y} + \left[\tilde{g}_T(x) - \int_x^1 dy\ \frac{\tilde{g}_T(y)}{y}\right], \tag{23}$$

$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 dy\ \frac{g_1(y)}{y} - \int_x^1 dy\ \frac{\tilde{g}_T(y)}{y}, \tag{24}$$

$$h_L(x) = 2x\int_x^1 dy\ \frac{h_1(y)}{y^2} + \left[\tilde{h}_L(x) - 2x\int_x^1 dy\ \frac{\tilde{h}_L(y)}{y^2}\right], \tag{25}$$

$$\frac{h_{1L}^{\perp(1)}(x)}{x^2} = -\int_x^1 dy\ \frac{h_1(y)}{y^2} + \int_x^1 dy\ \frac{\tilde{h}_L(y)}{y^2}, \tag{26}$$

$$f_T(x) = \left[\tilde{f}_T(x) - \int_x^1 dy\ \frac{\tilde{f}_T(y)}{y}\right], \tag{27}$$

$$\frac{f_{1T}^{\perp(1)}(x)}{x} = \int_x^1 dy\ \frac{\tilde{f}_T(y)}{y}, \tag{28}$$

$$h(x) = \left[\tilde{h}(x) - 2x\int_x^1 dy\ \frac{\tilde{h}(y)}{y^2}\right], \tag{29}$$

$$\frac{h_1^{\perp(1)}(x)}{x^2} = \int_x^1 dy\ \frac{\tilde{h}(y)}{y^2}. \tag{30}$$

The relation for g_T is just the Wandzura-Wilczek result [24], having a complete analogue in the relation for h_L discussed in Ref. [2]. In slightly different form the result for $g_{1T}^{(1)}$ has been discussed in Ref. [25]. Actually, we need not consider the T-odd functions separately. They can be simply considered as imaginary parts of other functions, when we allow complex functions. In particular one can expand the correlation functions into matrices in Dirac space [1] to show that the relevant combinations are $(g_{1T} - i\,f_{1T}^{\perp})$ which we can treat together as one complex function g_{1T}. Similarly we can use complex functions $(h_{1L}^{\perp} + i\,h_1^{\perp}) \to h_{1L}^{\perp}$, $(g_T + i\,f_T) \to g_T$, $(h_L + i\,h) \to h_L$, $(e + i\,e_L) \to e$. The functions f_1, g_1 and h_1 remain real, they don't have T-odd partners.

5. Evolution of transverse moments [23]

As mentioned the evolution of the twist-2 functions and the tilde functions is known. The twist-2 functions have an autonomous evolution of the form

$$\frac{d}{d\tau} f(x,\tau) = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y}\, P^{[f]}\left(\frac{x}{y}\right) f(y,\tau), \tag{31}$$

where $\tau = \ln Q^2$ and $P^{[f]}$ are the splitting functions. In the large N_c limit, also the tilde functions have an autonomous evolution. Using the known splitting functions and the relations given in the previous section, we then find the evolution of the transverse moments,

$$\frac{d}{d\tau} g_{1T}^{(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi} N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{x^2+xy}{y^2(y-x)_+}\right] g_{1T}^{(1)}(y,\tau) + \frac{x^2}{y^2}\, g_1(y,\tau)\right\}, \tag{32}$$

$$\frac{d}{d\tau} h_{1L}^{\perp(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi} N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{3x^2-xy}{y^2(y-x)_+}\right] h_{1L}^{\perp(1)}(y,\tau) - \frac{x}{y}\, h_1(y,\tau)\right\}. \tag{33}$$

One can also analyse the fragmentation functions or use some specific reciprocity relations [23]. Furthermore, we note that apart from a γ_5 matrix the operator structures of the T-odd functions $f_{1T}^{\perp(1)}$ and $h_1^{\perp(1)}$ are in fact the same as those of $g_{1T}^{(1)}$ and $h_{1L}^{\perp(1)}$ (or as mentioned before, they can be considered as the imaginary part of these functions [1]). With these ingredients one immediately obtains for the non-singlet functions the (autonomous) evolution of the T-odd fragmentation functions. In particular we obtain for the Collins fragmentation function (at large N_c),

$$\frac{d}{d\tau} z H_1^{\perp(1)}(z,\tau) = \frac{\alpha_s}{4\pi}\, N_c \int_z^1 dy \left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+}\right] y H_1^{\perp(1)}(y,\tau), \tag{34}$$

which should prove useful for the comparison of data on Collins function asymmetries from different experiments, performed at different energies.

6. Summary

In this overview we have presented the p_T-dependent quark distribution and fragmentation functions that appear in azimuthal asymmetries in one-particle inclusive leptoproduction. We have shown how appropriately weighted cross sections contain socalled transverse moments, including the T-odd fragmentation functions relevant in single-spin asymmetries. Using Lorentz invariance and the QCD equations of motion, these transverse moments can be related to the twist-2 and twist-3 functions appearing in inclusive leptoproduction. This also enables us to find evolution equations for the p_T-dependent functions that appear in azimuthal asymmetries

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