Open heavy flavor production – conceptual framework and implementation issues

Wu-Ki Tung, Stefan Kretzer, Carl Schmidt

Department of Physics/Astronomy, Michigan State University

E-mail: Tung@pa.msu.edu

Abstract. Heavy flavor production is an important QCD process both in its own right and as a key component of precision global QCD analysis. Apparent disagreements between fixed-flavor scheme calculations of b-production rate with experimental measurements in hadro-, lepto-, and photo-production provide new impetus to a thorough examination of the theory and phenomenology of this process. We review existing methods of calculation and place them in the context of the general PQCD framework of Collins. A distinction is drawn between scheme dependence and implementation issues related to quark mass effects near threshold. We point out a so far overlooked kinematic constraint on the threshold behavior, which greatly simplifies the variable flavor number scheme. It obviates the need for the elaborate existing prescriptions and leads to robust predictions. It can facilitate the study of current issues on heavy flavor production as well as precision global QCD analysis.

1. Introduction

Conventional perturbative Quantum Chromodynamics (PQCD) theory is formulated most simply in terms of zero-mass partons. For processes depending on only one hard scale Q, the well-known factorization theorem provides a straightforward procedure for order-by-order perturbative calculations, as well as an associated intuitive parton picture interpretation of the perturbation series. Heavy quark production presents a challenge in PQCD because the heavy quark mass, m_H ($H = c, b, t$), provides an additional hard scale which complicates the situation. The perturbative series must be organized in different ways depending on the relative magnitudes of m_H and Q.

A reliable formulation of heavy quark production is important for high energy physics because of its intrinsic value as a fundamental process, as well as because of its significant contribution to total inclusive cross-sections at high energies. Recent heightened interest has been directed to this process on both of these accounts. First, evidence is accumulating from several processes in hadro- [1, 2], photo- [3], lepto-, and $\gamma\gamma$-production of bottom [4], that experimental cross-sections are uniformly larger than existing calculations, by roughly a factor of 2. This is in contrast to charm production where no such gross discrepancy is seen. Second, the global QCD analysis of the very accurately measured deep inelastic scattering structure functions now definitely demands

a quantitative treatment of charm production which incorporates heavy quark mass effects in a reliable way.

We first briefly review the various approaches to heavy quark production [5, 6, 7, 8, 9, 10], and describe their synthesis in the general framework of Collins [11]. This provides a context to distinguish between issues relating to the choice of factorization scheme and those relating to implementation prescriptions allowed within a given scheme. The choice of prescriptions, such as that associated with mass threshold behavior, can matter in physical applications. In principle, the possible choices may be equivalent within the PQCD formalism; but in practice, some are natural and robust, while others appear to be more ad hoc and volatile. We examine this practical problem, and point out a so far overlooked kinematic constraint which greatly simplifies the calculation of subprocesses with heavy quark initial states. This leads to a efficient formalism which also yields very robust predictions. It can be used to address the current challenges described in the previous paragraph.

2. Conventional approaches

To see the basic physics ideas, let us focus explicitly on the production of charm ($H = c$) in deep inelastic scattering (DIS). All considerations apply to a generic heavy quark, and to hadro-production processes. The two standard methods for PQCD calculation of heavy quark processes represent two diametrically opposite ways of reducing the two-scale problem to an effective (hence approximate) one-scale problem.

2.1. 4-flavor zero-mass scheme

In the conventional parton-model approach, used in many global QCD analyses of parton distributions (e.g. EHLQ, MRS, CTEQ) and Monte Carlo programs (e.g. ISAJET, PYTHIA, HERWIG), the textbook zero-mass parton approximation is applied to a heavy quark calculation as soon as the typical energy scale of the physical process Q† is above the mass scale m_c. This leaves Q as the only apparent hard scale in the problem. The LO and NLO production mechanisms for charm are given by Fig. 1, where the solid

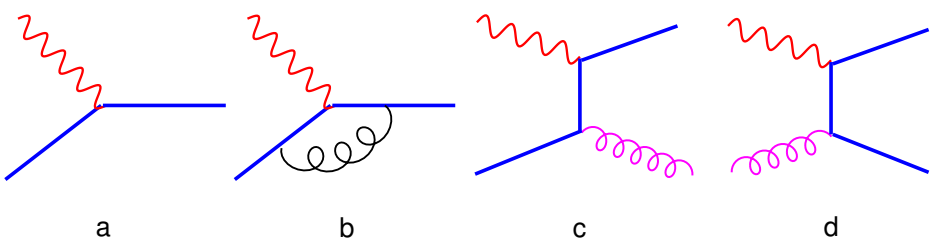


Figure 1. Partonic processes for charm production to NLO in the 4-flavor scheme.

† We use Q as the generic name for a typical kinematic physical scale. It could be Q, W, or p_T, depending on the process.

lines represent the charm quark. Note that the NLO diagrams are of order α_s , just as for the familiar case of total inclusive DIS structure functions. This is the most natural calculational scheme to adopt at high energies when $Q \gg m_c$. However, as we go down the energy scale toward the charm production threshold ($W = 2m_c$) region, it becomes unreliable because the approximation $m_c = 0$ deteriorates as Q becomes of the same order of magnitude as m_c. In this limit, all terms in Fig. 1 become effectively of order α_s (assuming no non-perturbative charm), the formal "NLO" designation loses meaning. This point is illustrated qualitatively in Fig. 2a as an uncertainty band marked by vertical hashes which is narrow at large Q but is expected to widen as Q decreases.

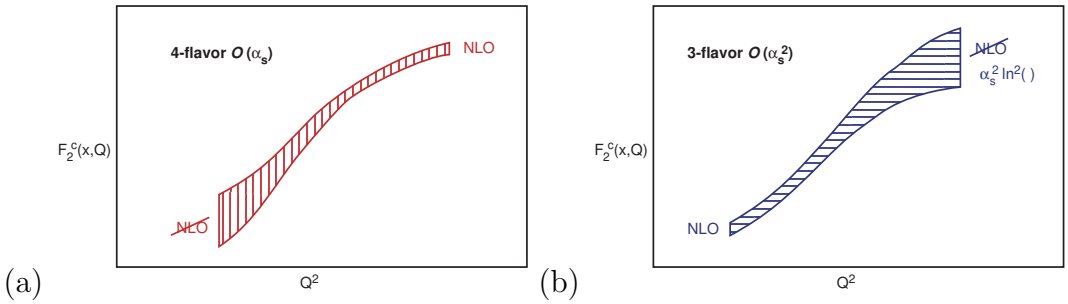


Figure 2. Expected regions of applicability and uncertainty of the 4-flavor (a) and 3-flavor (b) schemes. Note: (i) the power of α_s for "NLO" is different in the two schemes due to the resummation of perturbation series; and (ii) the reliability of the calculation in each scheme depends on the scale Q.

2.2. 3 (Fixed)-flavor scheme

In the heavy quark approach which played a dominant role in "NLO calculations" of the production of heavy quarks [12], the quark is always treated as a "heavy" particle and never as a parton. The mass parameter m_c is explicitly kept along with Q as if they are of the same order, *irrespective of their real relative magnitudes*. This is usually referred to as the fixed flavor-number (FFN) scheme. The LO and NLO partonic processes in this scheme are exemplified by the type of diagrams shown in Fig. 3. In this case, the NLO diagrams are of order α_s^2, which are much more complicated to calculate. Near threshold $W = 2m_c$, it is natural to consider the charm quark as a heavy particle, hence the NLO calculation in this scheme is reliable (unless there is non-perturbative charm). However, as Q becomes large compared to m_c, the FFN approach becomes unreliable since the perturbative expansion contains terms of the form $\alpha_s^n \log^n (m_c^2/Q^2)$ at any order n, which ruin the convergence of the series. These terms are *not infra-red safe* as $m_c \to 0$ or $Q \to \infty$. Furthermore, the calculation is no longer NLO in accuracy, in spite of the hard $\mathcal{O}(\alpha_s^2)$ calculation when $\alpha_s \log(m_c^2/Q^2) \sim 1$. This is illustrated in Fig. 2b as an uncertainty band marked by horizontal hashes which is narrow near threshold but is expected to widen as Q/m_c increases.

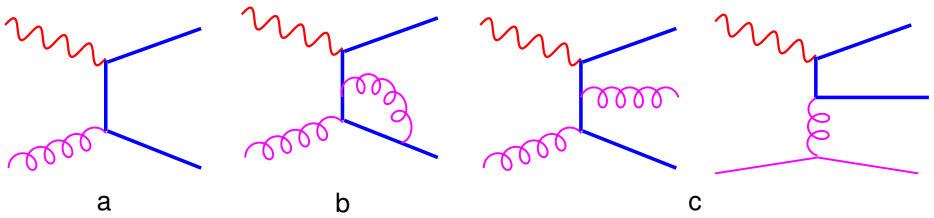


Figure 3. Partonic processes for charm production to NLO in the 3-flavor scheme.

PQCD does not predict at what energy scale Q the large logarithm terms actually become a problem [8, 13]. In practice, the 3-flavor scheme has worked well in charm production phenomenology so far [14]. However, for reasons touched upon in the introduction (and to be discussed in detail in subsequent sections), it is important to examine the general picture.

3. The unified framework of Collins – factorization with massive partons

It is obvious from Figs. 2a,b that the two conventional approaches are individually unsatisfactory over the full energy range, but are mutually complementary. Therefore, the most reliable PQCD prediction for the physical $F_2(x, Q)$ overall can be obtained by combining the two, utilizing the most appropriate scheme at a given energy scale Q. This results in a composite scheme, as represented by the cross-hashed region in Fig. 4, which is simply a composite of the two figures of Fig. 2. The use of a composite scheme consisting of different numbers of flavors in different energy ranges, rather than a fixed number of flavors, is familiar in the conventional zero-mass parton picture. The new formalism espoused in Refs. [5, 6] provides a quantum field theoretical basis [11] for this intuitive picture in the presence of non-zero quark mass. The 4-flavor scheme component of the general formalism *includes the full charm quark mass effects, after the infra-red unsafe part has been resummed*. It represents a substantial improvement over the conventional 4-flavor formalism in the region where $ln(Q^2/m_c^2)$ is not very large, which includes a substantial fraction of the current experimental range. This general approach has now been adopted, in different guises, by most recent papers on heavy quark production in PQCD. [7, 8, 9, 10, 15]

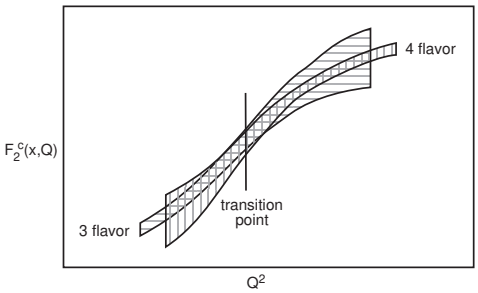


Figure 4. Intuitive picture of the general (composite) scheme.

The intuitively "obvious" general formalism is also technically precise: the order-by-order rules of calculation can be stated succinctly [16]; and the validity of the factorization theorem which underlies it can be established to all orders of perturbation theory [11]. The essential ingredients of this formalism are:

• *3-flavor scheme* at physical scales $Q \sim m_c$ and higher;

- *4-flavor $m_c \neq 0$ scheme* at asymptotic $Q \gg m_c$ and lower;
- a set of *matching conditions* which relate the two schemes at some scale μ_m;
- a suitable *transition* scale μ_t at which one switches from one scheme to the other.

There is considerable inherent flexibility in the choice of μ_m and μ_t, which partially account for the apparent differences in recent papers on this subject. For a detailed discussion see Refs.[16, 11]. We note that this composite scheme is just an extension of the conventional (zero-mass) variable flavor number scheme (VFNS) to include heavy quark mass effects in a rigorous way. We will simply refer to it as the VFNS in the following discussions.

4. Comparison to recent HERA data on inclusive charm production and importance of the non-zero-mass 4-flavor scheme

With the use of the general formalism, the theory of inclusive structure functions, including heavy quark mass effects, is on firm ground. The comparison of the charm component of this, $F_2^c(x,Q)$, to measurement is, however, subject to some theoretical and experimental subtleties [16, 19]. Previous comparison between the NLO 3-flavor calculation (of order α_s^2 in this scheme) [17] with recent HERA data [18, 14] showed good agreement. One can also compare the NLO 4-flavor calculation (of order α_s in this scheme) with the same data [16]. The results of this calculation are presented in Fig. 5, along with previous results. We see that the agreement with data is also excellent. This comparison tells us that, at least within the current experimental kinematic region, both 3-flavor and 4-flavor schemes are robust, in the sense that both can be applied to the full range without explicit evidence of the inadequacies expected from the theoretical considerations described in the previous section. In other words, both work better than expected. This is not guaranteed to hold indefinitely, however, for future expanded kinematic ranges. In fact, as we shall see, calculations show clear discrepancies between the two schemes at moderate x (say between 0.01 and 0.2) and large Q.

It is encouraging that the NLO 4-flavor (order α_s) calculation in the general formalism appears to maintain good accuracy approaching the threshold region from above, since it enjoys the advantage of being much simpler than the 3-flavor NLO (order α_s^2) calculation. This is important for phenomenology. For instance, in Global QCD Analysis of parton distributions, the charm contribution to the total inclusive cross-section is quite significant – up to 25% at small x. Since the total inclusive structure functions are always treated in the variable flavor number scheme (VFNS), the charm component is naturally treated the same way.

Furthermore, the 4-flavor scheme is inherently more versatile than the 3-flavor scheme. It can accommodate, in principle, one more non-perturbative degree of freedom – a charm component inside the nucleon – which is non-existent in the 3-flavor scheme. In view of the dilemma confronting the phenomenology of bottom production described in the introduction, it is important to leave open the possibility of unexpected non-perturbative heavy quark contribution to the structure of the nucleon. Only detailed

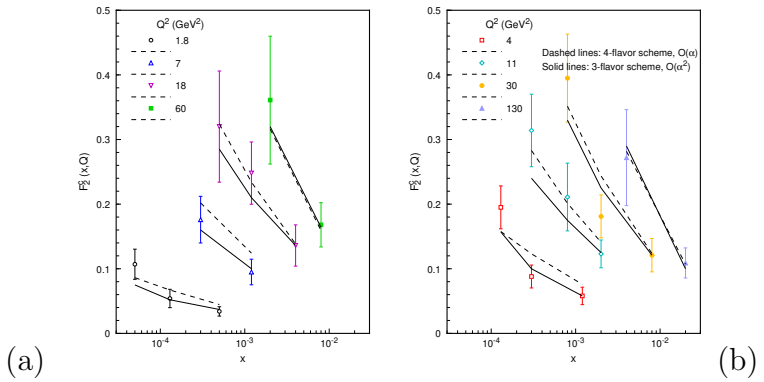


Figure 5. Comparison of the inclusive charm production data of Zeus [18] with: (i) order α_s^2 3-flavor (NLO) calculation (solid lines); and order α_s 4-flavor (also NLO) calculation ($m_c \neq 0$) in the general formalism (dashed lines). The various Q bins are alternately put into two separate plots to avoid overlapping points and curves. Both schemes appear to be robust within the experimental kinematic range.

phenomenological study done in the VFNS will be able to tell whether such components actually exist in nature.

These considerations suggest that it is important to examine in more detail implementation issues of the VFNS schemes (in which heavy quarks participate as partons) – issues associated with the choice of prescriptions near the threshold region which affect the predictive power of the calculation.

5. Implementation issues associated with variable flavor schemes

In this section, we will quickly summarize the general features of the various existing calculations of open heavy flavor production in the VFNS, then focus on the practical issues related to implementation choices. All the approaches should agree at high energies, within the accuracy of the perturbative approach. The differences are mainly in the threshold region, where heavy quark partons are less well-defined (as discussed in Sec. 2), hence the perturbation theory contains more prescription dependence. We will show, however, that other relevant physical considerations, especially kinematic constraints of the overall heavy flavor production process, can provide valuable input which considerably improves the predictive power of the calculation.

The fixed-flavor-number scheme is the scheme of choice in the threshold region, provided there is no non-perturbative heavy flavor component of the nucleon. It provides more definitive predictions in this region, and it respects the heavy quark production kinematics. Hence its results can be used for comparing and calibrating the different prescriptions for performing variable flavor calculations. This will guide our analysis which follows.

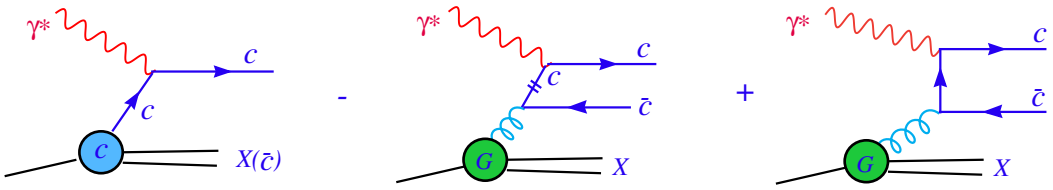
For simplicity, we continue to restrict ourselves to the case of 4-flavor scheme calculation of charm production in DIS to order α_s. All ideas apply to higher orders, and to bottom production.

5.1. General features of the 4-flavor calculation

Since the original formulation of (non-zero mass) 4-flavor scheme [5, 6], a variety of variable flavor number scheme (VFNS) calculations have been applied to photo- [15], lepto- [7, 8, 9, 19], and hadro-production [10] of charm. In spite of apparent differences in formulation and implementation, the underlying ideas described in Sec. 2 are adopted by all these calculations. The general formula written down in [6] which embodies these ideas has the structure,

$$[\text{LO 4-flv scheme term}] \text{ - } [\text{asymptotic/subtract term}] + [\text{LO 3-flv scheme term}] \quad (1)$$

Pictorially, these terms can be represented, for DIS charm production at the lowest non-trivial order as,



This basic structure appears in all the VFNS calculations in different guises. The first (LO 4-flv scheme) term is variously called the flavor-excitation, or quark-scattering, or resummed term – as the 4-flv scheme resums the large logarithms associated with the mass of the charm quark. The third (LO 3-flv scheme) term is variously referred to as the flavor-creation, or gluon-fusion, or fixed-flavor-number term – since the charm quark never becomes an active parton flavor. The middle (asymptotic/subtract) term represents the overlap between the LO 3-flv scheme and LO 4-flv scheme terms; hence it needs to be subtracted in order to avoid double-counting.

At high energies, the asymptotic/subtract term contains the logarithmic mass singularities of the 3-flv scheme calculation. It constitutes the subtraction which is needed to make the latter infra-red safe. Together, they form part of the next-order correction to the dominant resummed LO 4-flv scheme term, which gives the most accurate physical picture at high energies (cf. Sec. 1). In contrast, near the threshold region, the same subtraction term overlaps strongly with the first (flavor excitation) term, since in this region the charm parton arises primarily from a single gluon splitting, as seen in the pictorial illustration. In this region, the LO 3-flv scheme (third) term gives the best representation of the correct physics, as discussed in Sec. 1. In a consistent application of the PQCD formalism, the subtraction term is automatically generated by the renormalization and factorization procedure, as shown in [5, 6]. However, this procedure does not dictate every detail of the implementation. It allows some degree of prescription-dependence, which is in addition to the already well-known scheme dependence of massless PQCD.

The prescription dependence associated with non-zero charm quark mass is most noticeable at a low energy scale, not far above threshold. In principle, this dependence can be minimized by choosing the transition scale μ_t from the 3-flv scheme to the 4-flv scheme (cf. Sec. 3) to be relatively high. However, in practice, it is desirable to use the

4-flv scheme even at lower energy scales for reasons discussed in the introduction and at the end of Sec. 4. Accordingly, we will examine in some detail the implementation issues of the 4-flv scheme, using PQCD as well as other applicable physical constraints, and see to what extent its range of predictions can be narrowed.

5.2. Constraints on the Implementation of the 4-flavor scheme

Consider one of the inclusive structure functions in DIS. The factorization theorem, including quark mass effects [11], takes the form

$$F(x,Q) = \int \frac{dz}{z}\, f_a(z,\mu)\, \hat{\omega}^a(\frac{x}{z}, \frac{Q}{\mu}, \frac{m_H}{\mu}, \alpha_s(\mu)) \;+\; O(\alpha_s^{n+1}, \frac{\Lambda^2}{Q^2}, \frac{\Lambda^2}{m_H^2}) \quad (2)$$

where f_a is the parton distribution function, $\hat{\omega}^a$ is the hard scattering amplitude calculated in PQCD to some power of α_s, (say n), and m_H is the heavy quark mass. The prescription-dependence allowed by the PQCD formalism is associated with possible implementations of the first term of Eq. 2 within the accuracy specified by the order of magnitude of the remaining term.

For the case we use as an illustration, the factorization formula consists of the three terms given in Eq. 1. The explicit expressions are:

$$\begin{aligned} c(\zeta,\mu)\,\omega^0 \;-\; \alpha_s(\mu)\ln(\frac{\mu}{m_H}) \int_\zeta^1 \frac{dz}{z} g(z,\mu) P_{g\to c}(\frac{\zeta}{z})\,\omega^0 \\ + \alpha_s(\mu) \int_\chi^1 \frac{dz}{z} g(z,\mu) \omega^1(\frac{\chi}{z}, \frac{m_H}{Q}) \end{aligned} \quad (3)$$

The third, LO 3-flv scheme (gluon fusion), term is well-defined in the threshold region. The kinematic variable $\chi = x(1{+}4m_H^2/Q^2)$ can be interpreted as the "rescaling variable" for creating a pair of heavy quarks from massless partons (cf. the graph associated with this term in Eq. 1), and the hard cross-section $\omega^1(\frac{\chi}{z}, \frac{m_H}{Q})$ contains the full heavy quark mass dependence. Note that, at high Q values ($m_H^2/Q^2 \ll 1$), the rescaling variable reduces to x, $\chi \to x$.

The first two terms of Eq. 3 (the LO resummed and the subtraction terms) contain the main prescription-dependent mass effects.‡ Different versions of the VFNS scheme in the literature mostly amount to different prescriptions to implement heavy quark mass effects of these terms or their equivalents (cf. Sec. 6). It is therefore important to examine the origin of the prescription dependence, and to identify implementations which are physically reasonable and which lead to numerically stable results.

Prescription dependence related to quark mass effect enter Eq. 3 in two ways: the so-far unspecified scaling variable ζ, and the mass-dependence of the LO 4-flv scheme hard cross-section ω^0. From the PQCD factorization viewpoint, the requirements are that $\zeta \to x$ and $\omega^0(m_H^2/Q^2) \to \omega^0(0)$ when $Q^2 \gg m_H^2$. Thus, at high energies, the charm quark behaves just like a light quark. At low energies, the exact treatment of ζ and ω^0 is not prescribed by factorization; however, it makes sense to use the same ζ and

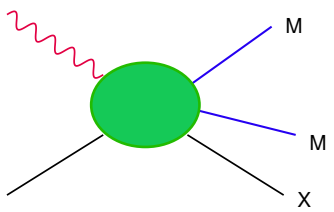
‡ These are in addition to the scheme- and scale-dependence known in the massless theory.

ω^0 in the first two terms of Eq. 3. This ensures the desired cancellation between these terms (in the absence of non-perturbative charm), leaving the 3-flavor (gluon fusion) term as the dominant contribution near the threshold region, as expected from physical considerations, cf. Sec. 2.

It has been known since the original ACOT paper [6] that the naive choice of kinematic variable ζ based on the LO 4-flv scheme $2 \to 1$ process leads to unnatural results (because kinematical effects due to the unobserved heavy quark in the target fragment are ignored). The various proposals to remedy this problem in the literature either exploit PQCD features (such as ad hoc choices of the scale variable μ [6, 8, 19]) or rely on ad hoc threshold regulating factors [15, 10] or matching conditions [9].§

Since the main arbitrariness of the first two terms in Eq. 3 is of kinematic origin, its resolution is most naturally obtained by a kinematic treatment (so far unexplored). In the next subsection, we show how this can be easily accomplished. Then, in Sec. 6, we will compare the results from this treatment with those from the more elaborate ones in the existing literature.

5.3. Kinematic solution to the threshold problem



It is well known that, in a generic process, producing a $H\bar{H}$ pair, the final-state phase space close to threshold is proportional to $\Delta = 1 - 4m_H^2/W^2$ where W is the CM energy ($W^2 = Q^2(x^{-1} - 1)$ in DIS). This factor arises algebraically from the phase space calculation in either of two ways: (i) in a final transverse momentum integral, it arises from $\int_0^{p_{CM}^2} \frac{dk_t^2}{k_t^2+m_H^2}|\mathcal{M}|^2$, where p_{CM} is the CM 3-momentum and $p_{CM}^2 \propto \Delta$; or in a final longitudinal momentum fraction integral, it arises from $\int_\chi^1 \frac{d\xi}{\xi}|\mathcal{M}|^2$ and the integration range $1 - \chi \propto \Delta$. This generic phase-space factor is a necessary consequence of the fact that the heavy quark is produced in pairs.

In DIS, $\chi = x(1 + 4m_H^2/Q^2) = 1 - \Delta(1 - x)$; it is the same as the χ variable appearing in the gluon fusion term in Eq. 3. We see that $\chi \to 1$ as $W \to 2m_H$ (the threshold) from above. This fact guarantees that the cross-section for the gluon fusion term vanishes at threshold, as it should. On kinematical grounds, the same condition should also be satisfied by the first two terms of Eq. 3 – the final state does consist of two heavy quarks, even though only one explicitly appears in the hard scattering part. We can implement the correct kinematics for the resummed and the subtraction terms by choosing the variable $\zeta = \chi$. This choice is also logical from the PQCD point of view: since the subtraction term represents the part of the gluon fusion term which is singular at high energies, the integration range is naturally chosen to be the same to ensure smooth matching.

§ The sensitivity to the choice of mass dependence of $\omega^0(m_H^2/Q^2)$ is less significant than that to the choice of ζ. The easiest choice is to let $\omega^0 = \omega^0(m_H = 0)$ [20]. We shall not discuss this issue in this talk.

This simple prescription leads to the following expression for the charm contribution to the inclusive DIS structure function, from Eq. 3:

$$[c(\chi,\mu) \; - \; \alpha_s(\mu)\ln(\frac{\mu}{m_H})\int_\chi^1 \frac{dz}{z} g(z,\mu) P_{g\to c}(\frac{\chi}{z})]\;\omega^0 + \alpha_s(\mu)\int_\chi^1 \frac{dz}{z}\; g(z,\mu)\;\omega^1(\frac{\chi}{z},\frac{m_H}{Q}) \quad (4)$$

where $\chi = x(1+4m_H^2/Q^2)$. We shall call this the ACOT(χ) prescription for implementing the VFNS. In this implementation, the structure function satisfies the kinematic requirements of charm pair production even in the presence of non-perturbative charm, provided $c(\chi,\mu) \to 0$ as $\chi \to 1$. If there is no non-perturbative charm, the first two terms will nearly cancel in the threshold region, leaving the gluon fusion term as the dominant contribution, as expected from 3-flavor calculation.

The use of the rescaling variable $\xi = x(1+m_H^2/Q^2)$ for the LO ($s \to c$) mechanism in neutrino DIS charm production has been known for a long time. The use of the χ rescaling variable for neutral current c-pair creation discussed here is similar in principle. However, in the neutrino scattering case, the initial state partons are all light; the kinematics is obvious. For neutral current scattering, because of the unobserved heavy quark in the target fragment, the need for a similar treatment of the kinematics of the ($c \to c$) term has apparently been overlooked until now.

Our proposed prescription is in fact applicable beyond the LO flavor excitation ($c \to c$) term. We have emphasized the generality of the kinematic argument, the importance of using the same choice also in the subtraction term in the NLO implementation, and the relation of this natural choice to the matching gluon fusion term which contains the correct kinematics. In fact, although we motivated the ACOT(χ) prescription by the specific example of NLO 4-flv scheme DIS charm production,‖ it can be applied to (N_f+1) - scheme calculation of heavy quark production in general. (N_f is the number of light quark flavors.) Roughly speaking, the rule is: for gluon-initiated subprocesses, use the full m_H kinematics and matrix elements; for heavy-quark initiated subprocesses, use the rescaling variable appropriate for heavy quark pair production to restore the correct kinematics.¶

6. Results and Comparisons

We now show some typical results on the charm contribution to the total inclusive structure function $F_2(x,Q)$ in the HERA kinematic range. Fig. 6a,b compares results obtained using two different choices of the scale μ. The naive LO 4-flavor result ($\propto c(x,\mu)$, with Bjorken x) is extremely sensitive to the choice of μ. By itself, it is clearly unphysical in the threshold region. However, once we adopt the rescaling variable χ in place of x, even this LO 4-flavor term (labelled LO 4-flv ACOT(χ)) behaves sensibly

‖ The full NLO 4-flv scheme calculation includes, in addition to the terms shown in Eq. 1, an order α_s quark-initiated term with the associated subtraction term. For simplicity, they have been left out of the qualitative discussions. They are numerically small in the current kinematic region. Nonetheless, they have been included in our calculations shown below.

¶ It is permissible to set $m_H = 0$ in the hard matrix element, cf. footnote on page 9.

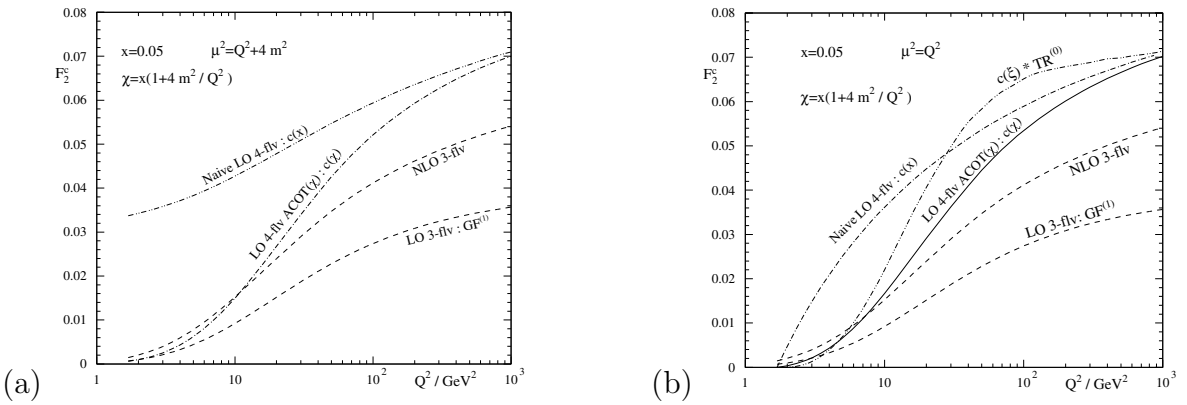


Figure 6. Comparison of F_2^c calculations vs. Q^2 with two different choices of μ. The curve labelled "$c(\xi)$) TR" in (b), is the LO result using the prescription of Thorne-Roberts [9].

compared to the LO 3-flavor and NLO 3-flavor results (which represent the "right physics" in the threshold region, assuming no non-perturbative charm). In addition, the behavior is much less sensitive to the choice of scale μ, as seen by comparing the corresponding curves in the two figures.

The original ACOT paper [6] attempted to regulate the threshold behavior by exploiting the freedom to choose the renormalization and factorization scale μ. By hindsight, that procedure as well as others which rely on ad hoc threshold regularizing schemes [10, 9, 19] seem contrived since they do not address the underlying kinematic constraint in a direct way. Consequently, all lead to results which are sensitive to the specifics of the prescription. As an example, in Fig. 6b, we compare our results with the LO 4-flavor calculation of Thorne-Roberts [9] (the curve labelled $c(\xi)$*TR) which was designed to remedy the threshold behavior of earlier implementations. This is a rather elaborate scheme, involving lengthy integral-differential formulas even for the LO term. In the figure, we see that the threshold behavior of our simple LO term, given by $c(\chi)$, and that of TR are indistinguishable. Above threshold, the ACOT(χ) curve naturally interpolates between the correct 3-flavor results at threshold and the zero-mass 4-flavor ($c(x)$) results at large values of Q. The TR curve, on the other hand, rises steeply above the threshold region, overtaking the zero-mass 4-flavor result, then approaches the same limit from above at high energies. The sharp rise in the intermediate region appears to be artificial; and, given the size of the effect, it will have phenomenological consequences.

Fig. 7 gives an overview of the predictions of the LO and NLO ACOT(χ) calculation compared to the LO and NLO 3-flavor calculation from $x = 10^{-3}$ to $x = 0.1$, in the Q/W range of the HERA measurements. As before, using this prescription, we observe the correct physical threshold behavior both at LO and at NLO. In addition, the fractional change due to the NLO correction is relatively small over the full range of W/Q; i.e. the perturbative series is "radiatively stable". The results are also stable with respect to the choice of scale μ. More comprehensive results on the ACOT(χ) calculation and its

applications to global analysis and heavy flavor production will be presented in [21].

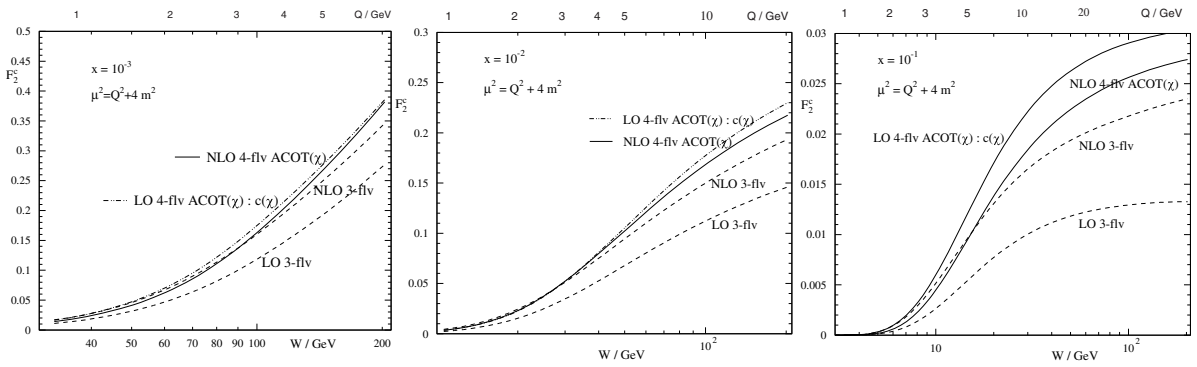


Figure 7. F_2^c vs. W (Q) for three x values, to give an overall view of the behavior of the LO and NLO ACOT(χ) calculations, and their comparison to the LO and NLO results of the 3-flavor scheme. See discusions in the text.

7. Summary

We conclude by summarizing the main points of this talk.

The (fixed) 3-flavor scheme calculation for charm production in DIS has worked well phenomenologically up to now. But, (i) it is inadequate for the more general purpose of global QCD analysis since other hard processes require a more general VFNS; (ii) it will become unreliable at some higher energy scale which cannot be predicted by PQCD; and (iii) it is intrinsically unequipped to include non-perturbative heavy flavor degrees of freedom should these occur in nature.

The (fixed) 4-flavor scheme calculation for bottom production fails in every experimental comparison performed to date. Among other possibilities, this could be the smoking gun for non-perturbative effects beyond fixed order (radiatively-generated) heavy flavor formalisms. This problem has to be studied in detail, including both charm and bottom.

In order to study the underlying physics of heavy flavor production in a systematic way, it is important to have a general PQCD formalism which is applicable over the full range of energy scales. Such a framework ("VFNS" with non-zero quark masses) exists [6], and is on a firm theoretical basis [11]. It is conceptually simple. The universal parton distributions satisfy the usual mass-independent evolution equation. Matching between different flavor number schemes is known to NLO [19].

The non-zero mass VFNS provides the possibility of incorporating additional non-perturbative degrees of freedom associated with heavy quarks. Currently, there exist several different versions of the VFNS of varying degree of complexity and naturalness. All implementations approach the same high energy limit given by the conventional zero-mass PQCD. However, so far, most prescriptions are sensitive in the threshold region

to the ad hoc choices made in their implementation; some exhibit artificial features in this region as a result.

We point out in this talk that the main source of these problems lies in the lack of proper treatment of kinematics in the resummed term(s) of the VFNS, due to the neglect of the missing heavy particle in the target fragment. Once this neglect is remedied (by the use of the natural rescaling variable), a simple and general method to implement the VFNS emerges. This method yields amazingly stable and accurate results for charm production already at order α_s (which is NLO in this scheme) – remaining uncertainties due to scale choice and higher-order corrections are shown to be small. We call this the ACOT(χ) scheme. It can provide a practical tool for the systematical investigation of the non-perturbative heavy flavor degrees of freedom and for global QCD analysis with full treatment of heavy-flavor mass effects.

Details of the material presented here as well as references which cannot be included in these pages can be found in [16, 21] and references cited therein.

Acknowledgments

We would like to thank John Collins and Frederick Olness for many discussions and for fruitful collaborations on many aspects of the subjects discussed in the review part of this talk.

References

[1] F. Abe *et al.* [CDF Collaboration], Phys. Rev. Lett. **75**, 1451 (1995), [hep-ex/9503013].
[2] B. Abbott *et al.* [D0 Collaboration], Phys. Lett. B **487**, 264 (2000), [hep-ex/9905024].
[3] J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. C **18**, 625 (2001) [hep-ex/0011081].
[4] For a brief review, see F. Sefkow, these proceedings, hep-ex/0109038.
[5] J. C. Collins and W. Tung, Nucl. Phys. **B278**, 934 (1986); F. I. Olness and W. Tung, Nucl. Phys. **B308**, 813 (1988). M. A. Aivazis, F. I. Olness and W. Tung, Phys. Rev. Lett. **65**, 2339 (1990).
[6] M. A. Aivazis *et al.*, Phys. Rev. **D50**, 3102 (1994); and refs. cited therein.
[7] B. A. Kniehl, M. Kramer, G. Kramer and M. Spira, Phys. Lett. B **356**, 539 (1995), [hep-ph/9505410].
[8] M. Buza *et al.*, Eur. Phys. J. **C1**, 301 (1998) ; M. Buza *et al.*, Phys. Lett. **B411**, 211 (1997).
[9] R. S. Thorne and R. G. Roberts, Phys. Rev. **D57**, 6871 (1998); Phys. Lett. B **421**, 303 (1998) [hep-ph/9711223].
[10] M. Cacciari *et al.*, JHEP **9805**, 007 (1998).
[11] J. Collins, F. Wilczek, and A. Zee, Phys. Rev. D **18**, 242 (1978).
J. C. Collins, Phys. Rev. **D58**, 094002 (1998).
[12] P. Nason, S. Dawson, R.K. Ellis, *Nucl. Phys.* **B327**, 49 (1989), Erratum: B335, 260 (1990). W. Beenakker *et al.*, Phys. Rev. **D40**, 54 (1989); E. Laenen *et al.*, Nucl. Phys. **B392**, 162 (1993); and Nucl. Phys. **B392**, 229 (1993). For an extensive review, see: S. Frixione *et al.*, in *Heavy Flavors II*, edited by A.J. Buras and M. Lindner, World Scientific, 1998.
[13] M. Gluck, E. Reya and M. Stratmann, Nucl. Phys. B **422**, 37 (1994).
[14] I. Redondo [H1 Collaboration], these proceedings, hep-ex/0109030.
[15] M. Cacciari, S. Frixione and P. Nason, JHEP **0103**, 006 (2001) [hep-ph/0102134].
[16] J. Amundson, C. Schmidt, W. K. Tung and X. Wang, JHEP **0010**, 031 (2000) [hep-ph/0005221].

[17] B. W. Harris and J. Smith, Nucl. Phys. **B452**, 109 (1995); Phys. Rev. **D57**, 2806 (1998); and B. W. Harris, hep-ph/9909310.
[18] J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. **C12**, 35 (2000).
[19] A. Chuvakin *et al.*, Phys. Rev. **D61**, 096004 (2000).
[20] M. Kramer, F. I. Olness and D. E. Soper, Phys. Rev. D **62**, 096007 (2000), [hep-ph/0003035].
[21] Stefan Kretzer, Carl Schmidt and Wu-Ki Tung, under preparation.