DIJET RATES WITH SYMMETRIC E_T CUTS

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We discuss the physics underlying an all-order resummation of logarithmic enhanced contributions to dijet cross sections, and present preliminary results for the distribution in the dijet transverse energy difference in DIS.

1. Introduction

Clustering hadrons into jets is a very useful tool to make QCD predictions. Measuring jet cross section removes the theoretical uncertainty due to fragmentation functions, and makes it possible to directly compare data with perturbative (PT) QCD predictions, whose degrees of freedom are partons and not hadrons. This comparison works extremely well when considering high transverse energy (E_T) jets. In this kinematical situation one is expected to probe quarks and gluons at small distances. This joint theoretical and experimental effort has led to the measurement of the QCD coupling α_s from jet inclusive E_T spectra and to better constrain the gluon density in the proton[1].

There are actual limitations in the use of perturbation theory. First of all any perturbative series diverges. This is related to the fact that the observed degrees of freedom are hadrons and not partons, and results in an ambiguity in PT predictions that fortunately is suppressed by inverse powers of the hard scale of the process. The second limitation has to do with the fact that the coefficients in the PT expansion can be logarithmically

enhanced due to incomplete real-virtual cancellations in the infrared (IR). This has the consequence that in particular phase space regions one observes a breakdown of the PT expansion, which can be cured only by performing an all-order resummation of such logarithms.

It is therefore mandatory, in order to have predictions valid in the whole of the phase space, to complement any fixed order calculation with the all-order resummation of logarithmic enhanced contributions, and, if possible, to remove the ambiguity in the PT expansion with a power-suppressed non-perturbative (NP) correction, which, unless it can be computed from the lattice, has to be taken as a phenomenological input[2].

2. The observable

The observable we consider is the dijet rate, i.e. the fraction of events with at least two jets. To select hard jets we put a cut on the transverse energy of the two highest E_T jets, requiring $E_{T1} > E_{T2} > E_m$. This particular choice is referred to as "symmetric E_T cuts". It was noted by Klasen and Kramer[3] that symmetric E_T cuts produce IR instabilities in next-to-leading order (NLO) QCD predictions. It was later Frixione and Ridolfi[4] who proposed to perform an asymmetry study by considering

$$\sigma(\Delta) \equiv \sigma(E_{t1} > E_m + \Delta; E_{t2} > E_m)\,. \tag{1}$$

They computed $\sigma(\Delta)$ in photoproduction at NLO, and obtained that while $\sigma(0)$, the total dijet rate with symmetric E_T cuts, is finite, the slope of the curve $\sigma'(\Delta) \equiv d\sigma/d\Delta$ diverges for $\Delta = 0$. This behaviour is not present at all in the data, as one can see from the asymmetry study performed by ZEUS in DIS[5], and reported in fig. 1. There the data is plotted against NLO QCD predictions obtained from the numerical program DISENT[6]. Again, the slope of the NLO curve diverges for $\Delta = 0$, while the data decreases smoothly with increasing Δ. This can be easily understood by noting that $\sigma'(\Delta)$ is just (minus) the differential distribution of the highest E_T jet:

$$\sigma'(\Delta) = -\left.\frac{d\sigma}{dE_{T1}}\right|_{E_{T1}=E_m+\Delta}\,. \tag{2}$$

Since the latter quantity is a physical cross section, the slope $\sigma'(\Delta)$ has to be negative. This corresponds to what is seen in the data, and also implies that any turnover in the NLO curves is unphysical.

To better understand the origin of the divergence in $\sigma'(\Delta)$, we consider the value of the jet transverse energy difference $E_{T1} - E_{T2}$. In the specific case of dijet production in DIS, with an incoming quark p, at the Born level,

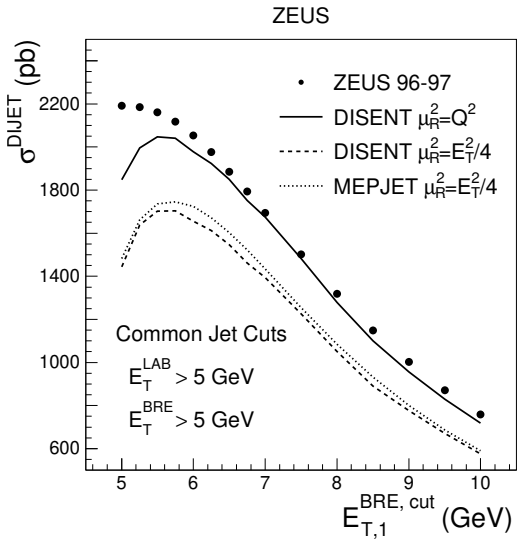

Figure 1. The dijet rate $\sigma(\Delta)$ as a function of E_{T1} in DIS. The dots are the data, while the lines are NLO predictions.

a dijet event consists only of two outgoing hard partons p_1 and p_2, in this case a quark and a gluon. Their transverse momenta $\vec{p}_{t1}$ and $\vec{p}_{t2}$ are back-to-back, so that, defining $E_{Ti} = |\vec{p}_{ti}|$, we have $E_{T1} = E_{T2}$. After emission of a soft gluon k not clustered with any of the two outgoing partons, we obtain from transverse momentum conservation $E_{T1} - E_{T2} \simeq |k_x|$, where k_x is the component of the gluon $\vec{k}_t$ parallel to $\vec{p}_{t1}$. When considering $\sigma'(\Delta)$, E_{T1} is forced to lie on the line $E_{T1} = E_m + \Delta$, but from kinematics $E_{T1} = E_{t2} + |k_x|$. These two lines should then intersect somewhere in the allowed phase space $E_{T1} > E_{T2} > E_m$, and this is possible only for $|k_x| < \Delta$, as can be seen from fig. 2. Squeezing soft radiation is known to give rise to large logarithms in fixed order calculations. Indeed, if we consider the emission of a soft gluon collinear to the incoming quark together with the corresponding virtual correction, we obtain

$$\sigma'(\Delta) = \sigma'_0(\Delta) \times \left(1 - \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{\Delta}\right) , \tag{3}$$

so that the slope $\sigma'(\Delta)$ diverges for $\Delta \to 0$. A quick solution of the problem can be achieved by choosing kinematical cuts such that the Born slope $\sigma'_0(\Delta) \to 0$ for $\Delta \to 0$. This can be achieved for instance by imposing asymmetric E_T cuts[7], or choosing a suitable range for the dijet invariant

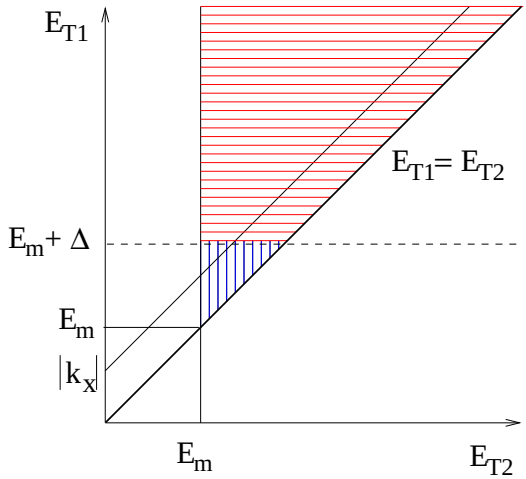

Figure 2. The phase space for the dijet rate in the $E_{T1} - E_{T2}$ plane.

mass and rapidities[1]. However, the actual solution relies on the all-order resummation of soft gluon effects, as will be explained in the next section.

3. Resummation

Considering an arbitrary number of partons one finds $E_{T1} - E_{T2} \simeq |\sum_i' k_{ix}|$, where the primed sum runs on all partons not clustered with the jets. Introducing $S(\vec{k}_t)$, the probability that $\sum_i' \vec{k}_{ti} = \vec{k}_t$, we have $\sigma'(\Delta) = \sigma_0'(\Delta) W(\Delta)$, where the K-factor $W(\Delta)$ is in general given by

$$W(\Delta) = \int d^2\vec{k}_t \, S(\vec{k}_t) \, \Theta(\Delta - |k_x|) = \frac{2}{\pi} \int \frac{db}{b} \sin(b\Delta) \Sigma(b) \, . \qquad (4)$$

For emissions soft and collinear to the incoming quark, no secondary partons are clustered with p_1 or p_2, so that an all-order resummation of such contributions yields $\Sigma(b) = \exp[-C_F \frac{\alpha_s}{\pi} \ln^2 b]$. From eq. (4) we have then $W(\Delta) \sim \Delta$ for small Δ, which roughly corresponds to the behaviour seen in the data.

In general, $\Sigma(b) = \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) \ldots]$, with $L = \ln b$, and we aim at next-to-leading logarithmic (NLL) accuracy, i.e. the knowledge of g_1 and g_2. In order to perform an all-order resummation, one needs to reorganise the PT series and collect all contributions to $\Sigma(b)$ up to a given accuracy. First of all one can see that at NLL accuracy, $\Sigma(b)$ can be interpreted as the probability that $|\sum_i' k_{ix}| < e^{-\gamma_E}/b$. Furthermore, given

a variable V, a generic function of all final state momenta, multi-parton matrix elements and phase space constraints can be approximated by using the following general properties:

(1) all leading logarithms (LL) originate from configurations of soft and collinear gluons for which the hardest emission k_1 dominates, that is $V(k_1, \ldots, k_n) \simeq V(k_1)$;
(2) hard-collinear and soft-large-angle emissions give a NLL contribution, and again one can approximate $V(k_1, \ldots, k_n) \simeq V(k_1)$;
(3) the fact that $V(k_1, \ldots, k_n) \neq V(k_1)$ needs only to be taken into account for soft and collinear emissions; these "multiple emission effects" are NLL, and can be treated either analytically with a suitable integral transform, or numerically with Monte Carlo (MC) techniques;
(4) secondary splittings are accounted for by taking the QCD coupling in the physical CMW scheme, at a scale of the order of the transverse momentum of the parent parton, i.e. $\alpha_s \to \alpha_s(k_t)$.

The strongest implication of these four statements is that at NLL accuracy soft and/or collinear (SC) partons can be considered as emitted independently from the hard parton antenna, in spite of the fact that QCD is a non-abelian gauge theory. This crucial simplification is known as the "independent emission" approximation, and leads to exponentiation of leading logarithms, and factorisation of NL logarithms. If we assume the validity of independent emission, we obtain

$$\Sigma(b) = \frac{f(1/b)}{f(Q)} \otimes e^{-R(b)} , \qquad R(b) = \int [dk]\, w(k)\, \Theta\left(|k_x| - e^{-\gamma_E}/b\right) , \quad (5)$$

where the 'radiator' $R(b)$ is (minus) the contribution to $\Sigma(b)$ from a single SC gluon k emitted with probability $w(k)$. The ratio of parton densities $f(1/b)/f(Q)$ is a general feature of resummations involving incoming partons, and is due to the fact that the scale of the incoming parton density is set by the upper limit of observed parton transverse momenta, which in this case is of order $1/b$.

Unfortunately, independent emission approximation does not hold for all variables, but only for those who are recursively infrared and collinear (rIRC) safe and (continuously) global[8]. The lack of either of these two properties causes one (or more) of the above statements not to be true.

For instance, for the three-jet rate in the JADE algorithm, which is global but not rIRC safe, statement 1 is false, since there are LL contribu-

tions originated by multiple emissions, with the consequence that leading logarithms do not exponentiate[9].

Globalness means simply that V is sensitive to emissions in the whole of the phase space. This is not the case for $E_{T1} - E_{T2}$, since only partons that are not clustered with the two hard jets contribute to the dijet transverse energy difference. The first consequence of non-globalness is that secondary splittings cannot be accounted for by simply setting the proper scale for α_s, because new NLL contributions, called "non-global logs"[10], arise when a cascade of energy ordered partons outside the measure region emits a softer gluon inside. In this case phase space boundaries have to be taken into account exactly for all emissions, so that one has to rely on MC methods. Moreover, due to the complicated colour structure of multi-gluon matrix elements, non-global logs at the moment can be resummed only in the large N_c limit.

In our case, if particles are clustered into jets with a cone algorithm, after having specified a procedure to deal with overlapping cones, one has that to NLL accuracy a jet consists of all particles that flow into a cone around p_1 or p_2, and equation (5) gets modified as follows[11]:

$$\Sigma(b) = \frac{f(1/b)}{f(Q)} \otimes e^{-R(b)} \times S[t(b)] \,, \qquad t(b) = \frac{1}{2\pi} \int_{1/b}^{Q} \frac{dk_t}{k_t} \alpha_s(k_t) \,, \qquad (6)$$

where $S(t)$ represents the contribution of non-global logs, and can be computed with a MC procedure as a function of the evolution variable t.

If jets are clustered with the k_t algorithm[12], further complications arise. In this case the fact that a particle belongs to a jet depends on all other emitted particles. This has interesting implications for non-global logs. Dasgupta and Salam[13] have shown that the dominant contribution to non-global logs arises when the observed gluon is close to the boundary of the measure region. Appleby and Seymour[14] noted then that the k_t algorithm requirement forces such gluons to be clustered with gluons outside the measure region, thus reducing the magnitude of non-global logs.

One can also ask whether for the independent emission contribution to $\Sigma(b)$, the exponential form of equation (5) is still valid. This is true for cones (the "unclustered" case) since the phase space constraint is factorised, but should be checked for the k_t algorithm (the "clustered" case). In order to answer this question we considered a much simpler variable, the transverse energy flow away from the jets. Given a pair of hard jets (taken back-to-back for simplicity), one defines a region Ω away from the jet axis (which in e^+e^- annihilation roughly coincides with the thrust axis), and the away-

from-jet E_T flow

$$E_{T,\Omega} = \sum_{i \in \Omega} k_{ti} \,, \tag{7}$$

with k_{ti} the transverse momentum of the i-th *jet* with respect to the jet axis[13,14]. The quantity $\Sigma(Q, Q_\Omega)$, the probability that $E_{T,\Omega} < Q_\Omega$, in the region $Q_\Omega \ll Q$ is sensitive only to soft gluons at large angles. One then wishes to resum LL contributions to $\Sigma(Q, Q_\Omega)$, whose order is $\alpha_s^n \ln^n(Q/Q_\Omega)$.

At LL accuracy $\Sigma(Q, Q_\Omega)$ is given by

$$\Sigma(Q, Q_\Omega) = \Sigma_{\Omega,P}(t) \cdot S(t) \,, \tag{8}$$

where $t \equiv t(Q_\Omega)$ is the evolution variable defined in equation (6). Gluons emitted directly from the two hard partons give rise to $\Sigma_{\Omega,P}(t)$, while non-global logs are embodied in $S(t)$. Appleby and Seymour[14] assumed that also for the clustered case $\Sigma_{\Omega,P}(t) = e^{-R(t)}$, that is the single gluon contribution to $\Sigma(Q, Q_\Omega)$ exponentiates. This naive expectation can be motivated by the fact that if emissions are assumed to be independent, multiple emission effects are usually relevant only for soft and collinear gluons (statement 3). However, since $E_{T,\Omega}$ is manifestly non-global, one cannot exclude multiple emission effects coming from soft gluons at large angles. Consider for instance two gluons k_1 and k_2, with $\omega_1 \gg \omega_2$, k_2 inside Ω and k_1 outside. It is possible that the jet algorithm clusters k_2 with k_1, thus spoiling the exponentiation of the single gluon result[15]. This is indeed seen when comparing the resummed expression for the differential distribution $\sigma^{-1} d\sigma/dL$, with $L = \ln(Q_\Omega/Q)$, with the NLO program EVENT2[6]. If LL are correct, the difference of the two distributions should go to a constant for large (negative) L. This happens for the coefficient of $C_F C_A \alpha_s^2$, indicating that both in the unclustered and the clustered case non-global logarithms are correctly taken into account. If one however uses for the clustered case the same expression for $\Sigma_{\Omega,P}$ as for the unclustered case, one finds a discrepancy in the coefficient of $C_F^2 \alpha_s^2$, the plot of fig. 3. In the clustered case $\Sigma_{\Omega,P}$ must be corrected by taking into account the fact that the softest gluon, in spite of the fact that it is emitted in Ω, can be nevertheless clustered with the hard jets, and therefore does not contribute to $\Sigma_{\Omega,P}$. Even for primary emissions this constitutes a LL contribution. The resummation of these multiple emission effects can be performed with the same MC used for non-global logs, and is the last bit that is needed to resum the dijet rate with the k_t algorithm at NLL accuracy.

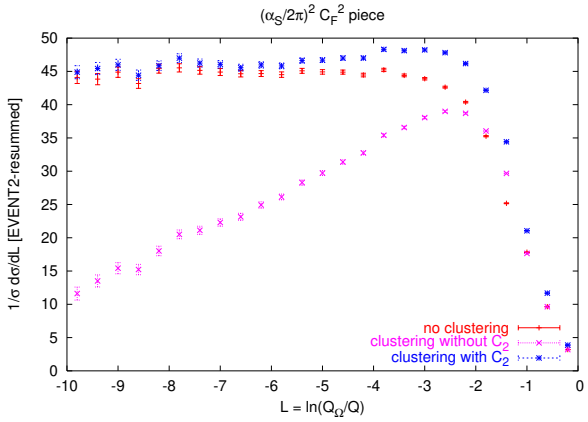

Figure 3. The coefficient of $(\alpha_s/2\pi)^2 C_F^2$ for the difference between resummed and NLO predictions for $\sigma^{-1} d\sigma/dL$, as explained in the text. Jets are clustered with the k_t algorithm with $R = 1$, and Ω is a rapidity slice with $\Delta\eta = 1$. C_2 is the correction that needs to be applied at order α_s^2 to take into account multiple emission effects.

4. Phenomenology of dijet observables

Instead of going through the details of the resummed calculation for the dijet rate in DIS, we discuss what information on QCD dynamics we can gain from allowing symmetric E_T cuts. Since in the region $\Delta \to 0$ we are almost in a three-jet configuration, this measurement is complementary to three-jet event shapes, and allows one to investigate the coherence properties of QCD emission from a three parton antenna. This observable has also the advantage that NP corrections are smaller than in event shapes distributions. This is mainly due to the fact that setting $\Delta \to 0$ does not put a direct veto on emitted parton transverse momenta, but rather on their vector sum. We have then emissions with large transverse momenta contributing also in the extreme $\Delta \to 0$ region. This fact also affects the magnitude of non-global logs, which here[11] give a correction of order 10%, while for event shapes[2] their contribution can be as large as 30%.

One can consider other observables that have the same resummation as the dijet rate and can be more easily handled theoretically or experimentally. Among these are the distributions in the transverse energy difference $\Delta = E_{T1} - E_{T2}$ and in the azimuthal angle $\Delta\phi$ between the jets. For any of these variables V, one studies

$$\frac{1}{\sigma}\frac{d\sigma}{dV} = \frac{d}{dV}\Sigma(V)\,. \tag{9}$$

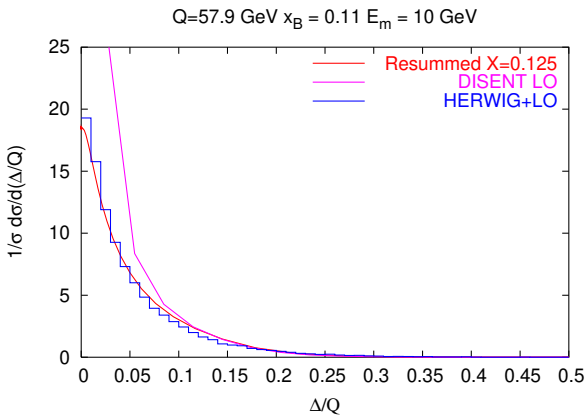

Figure 4. Preliminary plots for the transverse energy differential distribution in DIS. The parameter X is used to estimate NNLL effects.

As already stated, no further effort is needed to resum these distributions, since $\Sigma(V) = \sigma_0(0)W(V)$, where W is the same as in eq. (4). Since from the previous analysis we have seen that $\Sigma(V) \sim V$ for small V, one expects any of these differential distributions to approach a constant for $V \to 0$. This is what is already seen in the data for the $\Delta\phi$ distribution in hadronic dijet production at the Tevatron[16]. This observable is particularly interesting because it gives access to the QCD *fifth form factor*, which is present when there are at least four hard emitting partons[17].

A plateau for small Δ is also seen in the differential distribution in the dijet transverse energy difference in DIS computed with HERWIG[18]. Fig. 4 shows the comparison between the resummed result and that obtained from HERWIG. In this case jets are identified with the k_t algorithm, and in the resummation non-global logs have been approximated with their value for the cone algorithm, which has been taken as an upper bound. The pretty good agreement between the two predictions, together with the fact that HERWIG does not fully contain the matrix elements giving rise to non-global logs, suggests that the observable is dominated by soft and collinear emissions.

Another interesting possibility could be that of making $E_{T1} - E_{T2}$ global by modifying the particle recombination scheme. Actually, in a massless E_0 scheme, one finds that emissions clustered with the jets do contribute to $E_{T1} - E_{T2}$, so that one can use the program CAESAR[8] to obtain automatically the resummed expression for its distribution.

The concluding remark of this overview is that pushing measurements in phase space regions where fixed order predictions are not expected to give an accurate description of the data opens up the possibility to investigate properties of QCD otherwise unaccessible. In particular, cross sections for jets with almost equal E_T's represent a yet poorly explored field, which could be complementary to the traditional event-shape measurements.

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