MULTI-JET PRODUCTION IN LEPTON-PROTON SCATTERING WITH NEXT-TO-LEADING ORDER ACCURACY

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I summarize the theoretical and experimental status of multijet production in DIS. I present the state-of-the-art theoretical predictions and compare those to the corresponding experimental results obtained by analysing the data collected by the H1 and ZEUS collaborations at HERA. I also show new predictions for three-jet event-shape distributions at the NLO accuracy.

1. Introduction

Deep inelastic lepton-hadron scattering (DIS) has played a decisive role in our understanding of the deep structure of matter. The latest version of the experiments performed with colliding electrons or positrons and protons at HERA yields increasingly precise data so that not only fully inclusive measurements can be used to study the physics of hadronic final states. In fact, the study of jet-rates and event shapes has become an important project at HERA which yields results with continuously increasing accuracy [1]. Thus HERA is considered a machine for performing precision measurements for understanding Quantum Chromodynamics (QCD), the theory of strong interactions.

In order to perform precision measurements one needs precision tools for analysing the data. In the case of studying hadronic final states in high-energy particle collisions such tools have been developed in the framework of perturbative QCD. In order to make precision quantitative predictions in perturbative QCD, it is essential to perform the computations (at least) at the next-to-leading order (NLO) accuracy. Such computations, however, yield reliable predictions only in a limited part of the phase space, where the statistics of the data are relatively small. In order to increase the

predictive power of the theory, the fixed-order predictions must be improved by matching those to predictions obtained by resumming large logarithmic contributions to all orders.

In the case of DIS, fixed-order and resummation computations have so far been completed for one-jet inclusive, 2- or 3(+1 beam[a])-jet cross sections and means or distributions of event shapes. Since the main goal of the experimental analyses is to compare data to precision theoretical predictions (beyond the LO accuracy), this implies that the experimental analysis of multi-jet events is generally constrained to considering three-jet events. Therefore, in this talk "multi" will mean three. This is in contrast to hadron collider studies, where the multi-jet events are backgrounds to various new-particle signatures, therefore, even the predictions at LO are considered valuable information and the construction of parton-level event generators is an important research topic [2]. One of the main lines of this research is the construction of public computer programs that could be used for the automated production of multi-parton events and thus for computing multi-jet cross sections. These computer programs could also be used to study high-multiplicity final states in DIS. Note, however, that the studies that can be made at HERA are not directly applicable at the LHC because the events at HERA have jets of typical energy in the order of 10 GeV while jets at the LHC will be triggered at the order of 100 GeV.

The automation of computing cross sections at the NLO accuracy has also been considered, but has not yet yielded mature results. Thus one has to recourse to programs for specific processes. The state of the art in the fixed-order computations of cross sections in DIS is represented by the NLOJET++ program that can be used for computing two- and three-jet observables [3,4]. Other related programs for leptoproduction of two jets are DISENT, DISASTER++ and JETVIP [5,6,7]. The predictions obtained by the NLOJET++ and DISASTER++ codes for two-jet cross sections agree within statistical accuracy of the numerical integrations.[b] The DISENT code is known to have a small bug [8] leading to slightly different predictions (the cross sections agree within 1–2 % [6]). At the time of the comparison of DISENT and JETVIP the latter code was not able to produce reliable predictions over the whole phase space [9], which was due to a bug in the binning routine that has been corrected since [10].

The state of the art in the resummed predictions is represented by the

[a] In the following, I omit the reference to the beam-jet, i.e., I do not count the beam-jet.
[b] The essential difference is that NLOJET++ is significantly faster.

recent analytic computations of the distribution of the multijet event shape $K_{\rm out}$ [11], the di-jet rates with symmetric $E_{\rm t}$ cuts [12], as well as by the CAESAR program that can be used for computing cross sections of two- and three-jet event shapes in a semi-automatic way [13]. In my talk I shall present NLO predictions of three-jet event-shape distributions for which resummed predictions already exist, but the fixed-order radiative corrections have not been computed before.

2. Fixed-order predictions

There are several process-independent ways to compute QCD radiative corrections. In computing the NLO corrections to multijet cross sections, the dipole subtraction scheme of Catani and Seymour [14] is a convenient formalism. It is used both in the DISENT and the NLOJET++ programs. The comparison of the two-jet predictions of the three programs has been performed in Ref. [3] and complete agreement was found apart from the slight difference in the DISENT predictions mentioned above. These programs use matrix elements that take into account only virtual photon exchange. Neglecting the exchange of the Z^0 boson means that the predictions are not reliable for large Q^2 values around $(90\,\mathrm{GeV})^2$ and above.

The subtraction scheme applied in the NLOJET++ program is modified slightly as compared to the original one in [14] in order to have a better control on the numerical computation. The main idea is to cut the phase space of the dipole subtraction terms as introduced in Ref. [15]. The details of the computations are given in Ref. [16].

Once the phase space integrations are carried out, one can write the NLO jet cross section in the following form:

$$\sigma^{(J)}(p,q) = \sum_a \int_0^1 \mathrm{d}\eta\, f_{a/P}(\eta,\mu_F^2)\, \sigma^{(J)}_{a,\mathrm{NLO}}\left(\eta p, q, \alpha_{\rm s}(\mu_R^2), \mu_R^2/Q_{HS}^2, \mu_F^2/Q_{HS}^2\right) , \tag{1}$$

where p^μ and q^μ are the four-momenta of the incoming proton and the exchanged virtual photon, respectively. The function $f_{a/P}(\eta,\mu_F^2)$ is the density of the parton of type a in the incoming proton at momentum fraction η and factorization scale μ_F. The partonic cross section $\sigma^{(J)}_{a,\mathrm{NLO}}$ represents the sum of the LO and NLO contributions, given explicitly in Ref. [16], with jet function J. In addition to the parton momenta and possible parameters of the jet function, it also depends explicitly on the renormalized strong coupling $\alpha_{\rm s}(\mu_R^2)$, the renormalization and factorization scales $\mu_R = x_R Q_{\rm H.S.}$

and $\mu_F = x_F Q_{\rm H.S.}$, where $Q_{\rm H.S.}$ is the hard scale that characterizes the parton scattering, set event by event. Furthermore, the cross section also depends on the electromagnetic coupling, for which the NLOJET++ code uses $\overline{\rm MS}$ running $\alpha_{\rm EM}(Q^2)$ at the scale of the virtual photon momentum squared $Q^2 = -q^2$.

The publicly available version of the NLOJET++ program [4] is based on the tree-level and one-loop matrix elements given in Refs. [15,17], crossed into the photon-parton channel. It uses a C/C++ implementation of the LHAPDF library [18] with CTEQ6M [19] parton distribution functions and with the corresponding $\alpha_{\rm s}$ expression for the renormalized coupling which is included in this library. The CTEQ6M set was fitted using the two-loop running coupling with $\alpha_{\rm s}(M_{Z^0}) = 0.118$.

3. Comparison of fixed-order predictions to data

During the last few years, the experimental groups at HERA have performed extensive studies of multijet cross sections and compared their results to NLO predictions. The H1 collaboration already presented their results at this workshop four years ago [20]. The analysis was carried out parallel to our theoretical work with Z. Nagy that led to the NLOJET++ code, but without knowing about each other. When we finished testing our program and started to think of what to compute, we learnt about the H1 analysis accidentally. At that time preliminary H1 results showed rather large differences between data and LO predictions as seen in Fig. 1, even in the shapes of distributions, not only the absolute normalization.[c] We decided to make predictions of the same distributions at NLO accuracy.

H1 defined the jets using the inclusive $k_\perp$ algorithm implemented in the Breit frame (the precise definition can be found in Ref. [22]), selected three-jet events and plotted differential distributions of the DIS kinematical variables Q^2, $x_{\rm B}$ and the invariant three-jet mass $M_{3\rm jet}$. We used the same jet algorithm. Furthermore, in our computations we chose the same kinematical region as H1 did [23], namely, for the basic DIS kinematic variables Q^2, $x_{\rm B}$ and $y = Q^2/(s\,x_{\rm B})$ we required $5\,{\rm GeV}^2 < Q^2 < 5000\,{\rm GeV}^2$, $0 < x_{\rm Bj} < 1$, $0.2 < y < 0.6$. Following the H1 analysis, we also restricted the (pseudo)rapidity-range in the laboratory frame and the minimum transverse energy of the jets in the Breit frame as $-1 < \eta^{\rm jet}_{\rm lab} < 2.5$, $E^{\rm jet}_{\rm T,B} > 5\,{\rm GeV}$. For the hard

[c] For obtaining the predictions at LO accuracy we used the CTEQ5L parton distribution functions [21] and the running coupling at one-loop with $\alpha_{\rm s}(M_Z) = 0.127$.

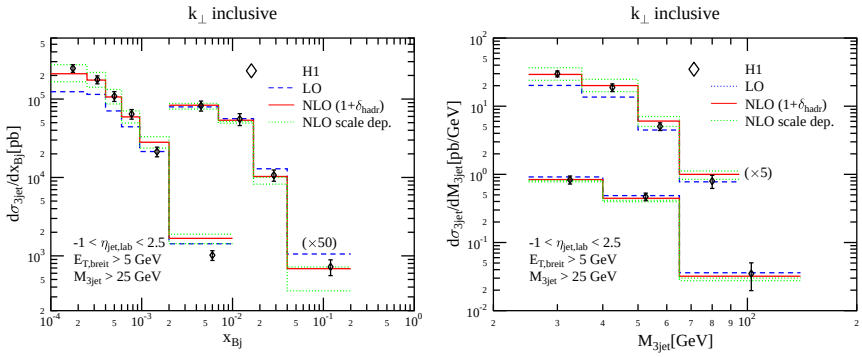

Figure 1. The differential distributions of the Bjorken variable $x_{\rm B}$ and the three-jet invariant mass $M_{3\rm jet}$. The histograms with larger values of the cross section correspond to the range $5\,\mathrm{GeV}^2 < Q^2 < 100\,\mathrm{GeV}^2$ and those with lower values to $150\,\mathrm{GeV}^2 < Q^2 < 5000\,\mathrm{GeV}^2$

scattering scale we chose the average transverse momentum of the jets,

$$Q_{\rm H.S.} = \frac{1}{3}\sum_j E_{\rm T,B}^{{\rm jet},\,j}\ . \tag{2}$$

We also studied the other usual choice, when $Q^2_{\rm H.S.} = Q^2$, but have not found significant differences. Finally, in order to compare our parton-level prediction to the hadron level data, we asked for the bin-wise correction factors of hadronization as estimated by H1 (the correction factors were between 1.2–1.3). With the inclusion of the NLO corrections the improvement in the theoretical description was spectacular, see Fig. 1.

Recently, the ZEUS collaboration has also performed an analysis of the three-jet events [24]. They published measurements of the inclusive three-jet cross section as a function of Q^2, the jet transverse energy in the Breit frame, $E_{\rm T,B}^{\rm jet}$ and the jet pseudorapidity in the laboratory frame $\eta_{\rm lab}^{\rm jet}$ compared to the NLO predictions obtained with the NLOJET++ code, corrected for hadronization (the correction factors $C_{\rm had}$ were in the range of 1.15–1.35.) The NLO QCD predictions were found to describe both the shapes of the predictions as well as the absolute normalization of the measured cross sections. The two competing most significant sources of uncertainty are the energy scale uncertainty from the experimental side and renormalization scale uncertainty from the theoretical side.

Such an agreement between data and theory promised a precise measurement of the strong coupling and its running by fitting the cross section ratio $R_{3/2}$ of the three-jet cross section to the two-jet one as a function

of Q^2. The correlated systematic and renormalization-scale uncertainties mostly cancel in the ratio. According to the studies made by ZEUS [24], the total experimental and theoretical uncertainties are about 5 % and 7 %, respectively. The reduction in the errors is very large. For instance, at low Q^2 (below $100\,\mathrm{GeV}^2$), the theoretical uncertainties in the ratio are a fourth of those in the three-jet distributions. The cited value of $\alpha_\mathrm{s}(M_Z)$ as determined from the measurements of $R_{3/2}$ is

$$\alpha_\mathrm{s}(M_Z) = 0.1179 \pm 0.0013\,(\mathrm{stat.})\,^{+0.0028}_{-0.0046}\,(\mathrm{exp.})\,^{+0.0064}_{-0.0046}\,(\mathrm{theo.})\,.$$

The dominant source of uncertainty is still the theoretical one which calls for further efforts in improving the predictions by computing even higher order corrections.

4. Recent developments: predictions for multi-jet event shapes

There are two directions in computing higher-order corrections. One is the exact fixed-order computations that I discussed previously by considering the NLO corrections. Going beyond the NLO accuracy is very difficult and so far has only been achieved for totally inclusive quantities such as structure functions. For jet cross sections the first step in order to make advances in this direction is the computation of inclusive jet and dijet cross sections at the next-to-next-to-leading order (NNLO) accuracy. The recent advances in computing the NNLO corrections in the crossed channel of electron-positron annihilation into three jets [25] raise hopes that for jet cross sections in DIS the NNLO prediction will also be available in the not too distant future.[d] In order to compute NNLO corrections to the multi-jet cross sections, a major bottleneck is the computation of the necessary virtual corrections, and I do not expect quick progress in this direction.

The other possibility to improve the predictions is to resum the most important logarithmic corrections, due to collinear and soft radiation, to all orders, which leads to predictions at the next-to-leading logarithmic (NLL) accuracy. Such computations are not available for jet rates. However, much progress has been achieved recently in resumming the LL and NLL contributions to multi-jet event-shape distributions [27]. These works lead to much deeper insight about the structure of QCD cross sections. In

[d] Note that the naive iterative extension of the dipole subtraction scheme to NNLO is not possible [26].

particular, prior to these studies it was believed that distributions of DIS observables, measured in the current hemisphere in the Breit frame, were trivially related to their well studied counterparts in electron-positron annihilation, where the resummed logarithms are due to soft radiation over the whole phase space (hence they are called global observables). However, it was found that there were also important single-logarithmic non-global effects due to radiation into one hemisphere [28]. In this talk I want only to collect the currently available theoretical information on multi-jet event shapes without going into the details of the theoretical studies.

There are two multi-jet event shapes computed to NLL accuracy so far. One quantifies the out-of-plane QCD radiation (the sum of the momentum components perpendicular to the event plane), called $K_{\rm out}$, defined precisely in Ref. [11]. The other is the y_3 observable that is defined to be the largest value of the jet resolution variable $y_{\rm cut}$ such that the event is clustered into three jets with $k_\perp$-clustering [29]. The $K_{\rm out}$ distribution has been studied experimentally in Ref. [30], however, with different definition of the observable as done in the resummation computation, therefore, conclusions cannot be drawn from the results.

One may ask why the computation of the NLO corrections is necessary if resummed predictions are known. The reason is that the NLL and NLO predictions are valid in rather distinct parts of the phase space, which can be clearly seen on the left panel of Fig. 2, where the differential distributions in $K_{\rm out}/Q$ at fixed values of $Q^2 = (35{\rm GeV})^2$ and $x_{\rm B} = 0.02$, normalized to the Born cross section, are presented. The dotted line is the LO prediction, the dashed is the NLL one. Expanding the NLL prediction in $\alpha_{\rm s}$ and changing the leading term to the exact LO one, we obtain the matched prediction shown with the dash-dotted line. We see that in the $K_{\rm out}$-region where the best precision experimental data can be collected, neither the fixed-order nor the resummed values are reliable, but one should use the matched prediction. On the right panel, I show the effect of including the power corrections both to the NLL and the matched predictions. The importance of matching is clear also in this case.

In analysing the multihadron data collected in electron-positron annihilation, a very accurate theoretical description of event-shape distributions was found with matched resummed and fixed-order predictions improved with hadronisation corrections (see e.g. [31]). I expect it will also be interesting to compare the HERA results for multi-jet event-shape distributions to predictions of the same level. Conclusions of such studies could also be important for analyses at the LHC, where the presence of the incoming

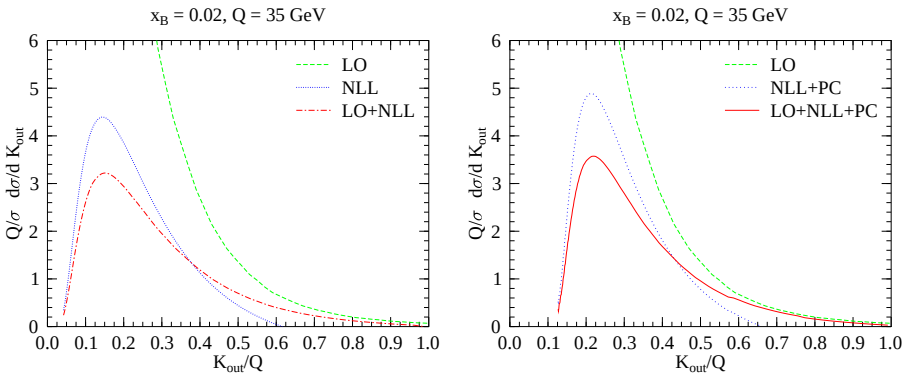

Figure 2. The differential distribution of $K_{\rm out}$. Left panel: perturbative computations. Right panel: perturbative predictions with power corrections.

two hard partons in the event means that even the dijet event shapes need at least four hard partons which is a multi-jet event shape configuration in DIS. Thus dijet event shapes at hadron colliders represent kinematical situations where NLL resummations and power corrections are as yet untested.

The semi-automatic computation of NLL predictions with the program CAESAR is currently being interfaced to the output of the NLOJET++ code. With this interface matched fixed-order and resummed predictions improved with power corrections will be obtained in a semi-automatic way soon [32]. Here I would like to present NLO predictions for the distribution of the event-shape observable $K_{\rm out}$, computed recently [16]. I used the same definitions of the observables and performed the computations at fixed values of the DIS kinematic variables $Q^2 = (35{\rm GeV})^2$, $x_{\rm B} = 0.02$ as in the resummed computations [33]. Figure 3 shows the LO and NLO predictions. The shaded bands in the left panel correspond to the range of scales $1/2 \leq x_R = x_F \leq 2$. We find that the radiative corrections are in general large, thus the scale dependence reduces only relatively to the cross sections. They also increase with increasing value of $K_{\rm out}$ because the phase space for events with large out-of-plane radiation with three partons in the final state (at LO) is much smaller than that with four partons in the final state (real corrections). The boundary of the phase space in $K_{\rm out}$ is about 20 % larger for the NLO computation than at LO. The cross sections decrease rapidly with increasing $K_{\rm out}$. The small cross section for medium or large values of $K_{\rm out}$ leaves the small $K_{\rm out}$-region for experimental analysis.

In the small $K_{\rm out}$-region, the logarithmic contributions of the type

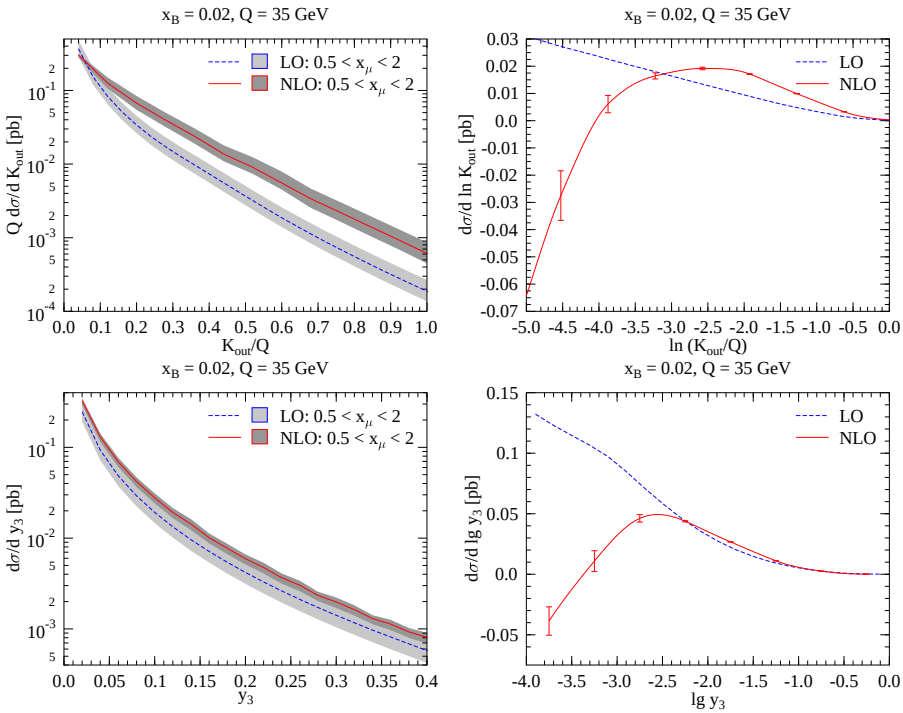

Figure 3. The differential distributions of the $K_{\rm out}$ and y_3 observables at fixed value of $x_{\rm B}$ and Q^2. The left panel shows the distributions as a function of $K_{\rm out}/Q$ and y_3, the right panel shows the distributions as a function of $\ln K_{\rm out}/Q$ and $\lg y_3$ in order to exhibit the logarithmic behaviour for small values of the observable. Both the LO predictions (dashed line) and the NLO predictions (solid line) were obtained with the CTEQ6M parton distribution functions. The error bars indicate the uncertainty of the numerical integration that is negligible for the computation at LO accuracy.

$\ln K_{\rm out}/Q$ are dominant as can be seen on the plot in the right panel. At LO, the logarithmic dominance starts at about $\ln K_{\rm out}/Q = -2$, at NLO, it starts at about $\ln K_{\rm out}/Q = -4$. Below these values the cross section is a linear function of $\ln K_{\rm out}/Q$ and the fixed-order predictions diverge with $K_{\rm out} \to 0$ with alternating signs, which makes the resummation of these large logarithmic contributions mandatory. Reliable theoretical predictions can be obtained by matching the cross sections valid at the NLO and NLL accuracy. This matching is obtained by expanding the NLL prediction in $\alpha_{\rm s}$ and changing the first two terms in that expansion with the exact values of the NLO computation. Qualitatively similar conclusions can be drawn from the distributions for the observable y_3 [16], but the corrections are smaller.

5. Conclusions and Outlook

In this talk I discussed the present status of predicting distributions of multi-jet cross sections in lepton-proton scattering. The only existing program for computing the three-jet observables at NLO accuracy in DIS is the NLOJET++ code. This program is well-tested, but has the slight disadvantage that the Z-boson exchange diagrams are not included. The predictions for three-jet rates agree well with the data collected at HERA, although the main source of uncertainty remains the theoretical one, which calls for taking into account the higher order corrections.

The other option of taking into account higher orders is the matching with resummed predictions valid at the NLL accuracy. Recent years yielded a lot of progress in this area of research. The CAESAR program can be used for computing NLL predictions to multi-jet distributions in a semi-automatic way. I showed the importance of matching the NLO and NLL predictions. Both are available for certain three-jet event-shape observables, like the out-of-plane momentum $K_{\rm out}$and the y_3 variable. The matching of the NLO and NLL predictions is expected to be available soon [32].

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