Gauge-gravity duality and high energy collisions

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We propose an overview on the Gauge/Gravity correspondence applied to the dynamics of high-energy collisions, such as the properties and formation of the quark-gluon plasma in heavy-ion collisions. Some recent applications to other high-energy scattering processes are also discussed.

1. Gauge/Gravity correspondence and AdS/CFT

The history of the relations between high-energy collisions and string theory is quite remarkable. It starts with 1968 and the Veneziano formula for 2-particle amplitudes, which theoretical foundation, after some years of intense work, led to the discovery of quantum string theory. The original motivation for the Veneziano amplitude was indeed to describe the strong interaction amplitudes at small transverse momentum, where a perturbative expansion in the coupling seemed to be hopeless. However, in 1974, almost at the same time when this theoretical task was achieved, Quantum Chromodynamics and asymptotic freedom were discovered and found to prevail over string theory as the correct theoretical basis for strong interactions at small coupling. At the same time, string theory shifted from the proton mass scale to the Planck mass, *i.e.* from strong interactions to an unified description of gravity and gauge interactions. Indeed, the consistency for a (super)string theory to be viable required an embedding of the string in a 10-dimensional space, the existence of zero-mass states including the graviton and very probably a supersymmetric framework with all the associated fermions. All those requirements happen to be far from the reality of strong interactions and QCD.

The situation changed quite dramatically around 1998 when a nontrivial correspondence between gauge and gravity interactions was discovered [1]. In this scheme, a gauge interaction in the "physical" world, corresponding to the 3+1-dimensional Minkowski space on a stack of D_3 branes is related to a 9+1-dimensional bulk space of a consistent string theory, hence implying gravity in this higher-dimensional space. This is in fact a "weak-strong" *duality* relation, since a gauge theory at strong coupling can in practice be put in correspondence with a weakly coupled and semi-classical gravitational interaction. A precise calculational framework has been found, and till now studied and extended in numerous works, the paradigmatic case being the AdS/CFT correspondence, where a $\mathcal{N}=4$ supersymmetric gauge theory is related to the type II-B string theory on a particular 10-dimensional background space, namely $AdS_5 \otimes S_5$, that is the tensor product of a 5-dimensional Anti-de Sitter space with the 5-sphere. We will give some more details on this construction in the following.

Gauge/Gravity correspondence for high-energy collisions gives hope to describe the high-energy amplitudes at strong coupling which are beyond reach of the perturbative expansion of QCD, relating them to the semi-classical gravitational regime of the bulk interactions. The goal of our contribution is to explain how high-energy interactions may be formulated and some results obtained thanks to the Gauge/Gravity correspondence and in particular using the AdS/CFT framework. We will see what can be obtained and

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where are the reach and present.

2. AdS/CFT: a brief introduction

The AdS/CFT correspondence allows for a precise computational scheme. It can be considered as a kind of idealized laboratory where some strong coupling investigations on gauge field theories can be fully and rigorously led. It can give some more realistic estimates when its supersymmetric and conformal (no asymptotic freedom) features can be considered less stringent, as seems to be the case for heavy-ion reactions, as we shall see later on. There is however a general argument [2] which gives hope that the duality properties are much more general. Consider Fig. 1, the exchange of a closed string between two branes. It can be equivalently considered as

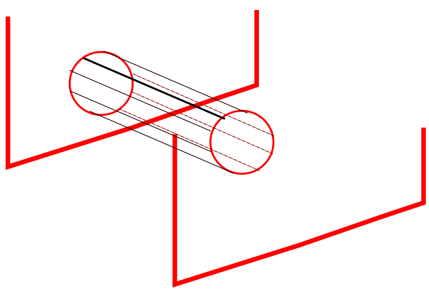

Figure 1. Open/Closed string duality.

a one-loop interaction due to open strings. At long distance, the closed string corresponds to a weakly-coupled, classical, gravitational interaction, while the open-string set-up is expected to be a strongly-coupled, quantum gauge interaction. String theory would then identify the two descriptions, being of identical topology.

The specific $AdS_5 \otimes S_5$ background occurs [1] from the following (here schematic) derivation. One starts from the (super)gravity classical solution of a system of N D_3-branes in a $10-d$ space of the (type IIB) superstrings. The metrics solution of the (super)Einstein equations read

$$ds^2 = f^{-1/2}(-dt^2+\sum_{1-3} dx_i^2)+f^{1/2}(dr^2+r^2 d\Omega_5)\ ,(1)$$

where the first four coordinates are on the brane and r corresponds to the coordinate along the normal to the branes. In formula (1), one defines

$$f = 1 + \frac{R^4}{r^4}\ ; \qquad R = 4\pi g_{YM}^2 \alpha'^2 N\ , \tag{2}$$

where $g_{YM}^2 N$ is the so-called 't Hooft-Yang-Mills coupling equal to the string coupling g_s and α' the string tension.

One considers the limiting behaviour

$$R\ fixed\ ; \frac{\alpha'(\to 0)}{r(\to 0)} \to z\ fixed\ . \tag{3}$$

where one zooms on the neighbourhood of the branes. This, from the second equation of (2) obviously implies

$$\alpha' \to 0\ ,\ g_{YM}^2 N \sim \frac{1}{\alpha'^2} \to \infty\ , \tag{4}$$

i.e. a weak coupling limit for the string theory and a strong coupling limit for the dual gauge field theory. By reorganizing the two parts of the metrics one obtains

$$ds^2 = \frac{1}{z^2}(-dt^2 + \sum_1^3 dx_i^2 + dz^2) + R^2 d\Omega_5\ , \tag{5}$$

which corresponds to the AdS$_5 \otimes S_5$ background structure.

In the case of confining backgrounds, an intrinsic scale breaks conformal invariance and is brought in the dual theory through *e.g.* a geometrical constraint. For instance [1] a confining gauge theory is dual to string theory in an AdS_{BH} black hole (BH) background

$$ds_{BH}^2 = \frac{16}{9}\frac{1}{f(z)}\frac{dz^2}{z^2} + \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{z^2} + \dots \tag{6}$$

where $f(z) = z^{2/3}[1-(z/R_0)^4]$ and R_0 is the position of the horizon.

One fascinating aspect of the Gauge/Gravity duality is the property of *holography*. Qualitatively, it states that the amount of information contained in the boundary gauge theory (on the brane) is the same as the one contained in the bulk string theory. In fact, it acquires the status of a quantitative tool that we shall use in practice. For heavy-ion collisions, for instance, where the energy-momentum tensor of the quark-gluon plasma appears to be an important characteristic of the collision process, a remarkable and useful example is provided by the so-called "holographic renormalization" [3]. Using the Fefferman-Graham coordinate system for the generalized AdS_5 metric in the presence of the plasma (here, in $\mathcal{N}=4$ YM theory),

$$ds^2 = \frac{g_{\mu\nu}(z)\ dx^\mu dx^\nu + dz^2}{z^2} \tag{7}$$

one can write

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(=\eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(=0) + z^4\ \langle T_{\mu\nu}\rangle + \tag{8}$$

where $g_{\mu\nu}$ is the bulk metric in 5 dimensions, $\eta_{\mu\nu}$ is the boundary metric in physical (3+1) Minkowski space and $\langle T_{\mu\nu}\rangle$ is the v.e.v. of the physical energy-momentum tensor. The higher coefficients of the expansion over the fifth dimension z can be obtained by the Einstein equations in the bulk provided the boundary energy-momentum tensor fulfills the zero-trace and continuity equations.

3. Gravity dual of an expanding medium

Among the problems arising from high-energy collisions in the strong coupling regime, the hydrodynamic behaviour of the expanding quark-gluon plasma experienced at RHIC is most striking (see Fig. 2). Numerical simulations show a very small (if not the smallest in the physical world) viscosity over entropy ratio η/s. Quite surprisingly, the Gauge/Gravity correspondence (using again the AdS/CFT laboratory) approach leads to associate the geometry of a 5-dimensional Black Hole (BH) to this physical process. Let us examine in more detail this intriguing correspondence.

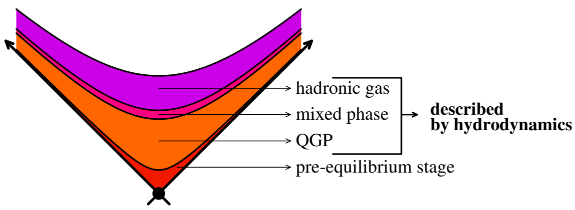

Figure 2. Sketch of a heavy-ion collision in (1+1) configuration space (by courtesy of F.Gélis).

Let us first consider the *static* configuration of perfect fluid with a stress-energy tensor equipped with diagonal elements

$$\langle T_{\mu\nu}\rangle \propto g^{(4)}_{\mu\nu} =$$
$$\mathrm{Diag}\{3/z_0^4 = \epsilon; 1/z_0^4 = p_i, [i=1,2,3]\}\ ,$$

where ϵ is the energy density and $p_1 = p_2 = p_3 = p = \epsilon/3$ is the pressure density. One can resum the whole holographic expansion (8) and, after a change of variable $z \to \tilde{z}$ gives

$$ds^2 = -\frac{1-\tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2}dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1-\tilde{z}^4/\tilde{z}_0^4}\frac{d\tilde{z}^2}{\tilde{z}^2}\ , \tag{9}$$

where one recognizes the BH (in fact an extended black brane) with a static horizon at $\tilde{z}_0$ in the 5th dimension.

In fact there exists a one-to-one correspondence between the thermodynamic properties of the BH and those of the perfect fluid , namely its temperature ($T_{BH} = \epsilon^{\frac{1}{4}} = T_{PF}$) and entropy ($S_{BH} \sim Area = \epsilon^{\frac{3}{4}} = S_{PF}$). It is in this context of a static Black hole configuration that one can go further than the perfect fluid approximation and derive the viscosity [4], using the Kubo formula.

The previous results were obtained for static configurations, *i.e.* for a thermalized QGP at rest. In order to take into account the actual kinematics of a heavy-ion collision, it is required to introduce the proper-time expansion of the plasma. On the gravity side, it calls for studying non-equilibrium geometries of 5d Black Hole configurations, which represent in itself a nontrivial and interesting issue. Let us now sketch how to build [5] the dual geometries of the Bjorken flow, that is the description of a boost-invariant expansion of the QGP, which is expected to correspond to the physical situation in the central rapidity region of the collision. In this context the questions why the QGP fluid appears to be nearly perfect (small viscosity) and why its thermalization time is short can be addressed.

Let us consider the equations obeyed by a physical energy-momentum tensor expressed in the $\{\tau, y, x=x_1=x_2\}$ coordinate system, where $\tau^2 = x_+x_-$ is the proper-time and $2y = \log(x_+/x_-)$ is the space-time rapidity:

$$\begin{aligned} T^\mu_\mu &\equiv -T_{\tau\tau} + \frac{1}{\tau^2}T_{\eta\eta} + 2T_{xx} = 0 \qquad (10) \\ \mathcal{D}_\nu T^{\mu\nu} &\equiv \tau\frac{d}{d\tau}T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2}T_{\eta\eta} = 0 \end{aligned}$$

In a boost-invariant framework, one may consider a general family of solutions of proper-time dependent, boundary energy-momentum tensors

$$\begin{aligned} \langle T_{\mu\nu}\rangle &\equiv \mathrm{Diag}\{f(\tau); -\tau^3\frac{d}{d\tau}f(\tau) - \tau^2 f(\tau)\ ; \\ &\quad \frac{1}{2}\tau\frac{d}{d\tau}f(\tau); \frac{1}{2}\tau\frac{d}{d\tau}f(\tau)\} \qquad (11) \end{aligned}$$

where the function $f(\tau) \propto \tau^{-s}$, satisfying the positivity condition $T_{\mu\nu}t^\mu t^\nu \geq 0 \Rightarrow 0 \leq s \leq 4$ corresponds to an interpolation between different relevant regimes, namely

$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect fluid $\epsilon = p_1 = p_2 = p_3$
$f(\tau) \propto \tau^{-1}$: Free streaming $\epsilon = p_{2,3};\ p_1 = 0$
$f(\tau) \propto \tau^{-0}$: "Full anisotropy" $\epsilon = p_{2,3} = -p_1$

Using the holographic renormalization to compute the coefficients of the corresponding metrics in the expansion on the fifth dimension and after resummation, it was possible to solve the dual geometry for given s at asymptotic proper-time τ. It reveals the existence of a scaling property of the solutions in terms of the proper-time dependent variable

$$v = \frac{z}{\tau^{1/3}}\ .$$

Analyzing the family of solutions as a function of s, it appears that the only nonsingular solution for invariant scalar quantities (here the square of the Ricci tensor $R^2 = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$,) see Fig. 3, is obtained for $s = 4/3$. Indeed, we find asymptotically in τ:

$$ds^2 \sim -\frac{1-\tilde{v}^4/\tilde{v}_0^4}{\tilde{v}^2}dt^2 + \frac{dx^2}{\tilde{v}^2} + \frac{1}{1-\tilde{v}^4/\tilde{v}_0^4}\frac{d\tilde{v}^2}{\tilde{v}^2} \qquad (12)$$

by a suitable change of variable $v \to \tilde{v}$. This metric is similar to the metrics of the static BH (9), but substituting the fixed horizon at z_0 by a *moving* one $z_0 \to z^4/\tau^{1/3}$. This BH solution is unique (*cf.* only for $s = 4/3$) and corresponds to a Black Brane moving away in the fifth dimension when the corresponding plasma cools down. Hence the perfect-fluid case is singled out and the moving BH in the bulk corresponds through duality to the expansion of the QGP taking place in the boundary.

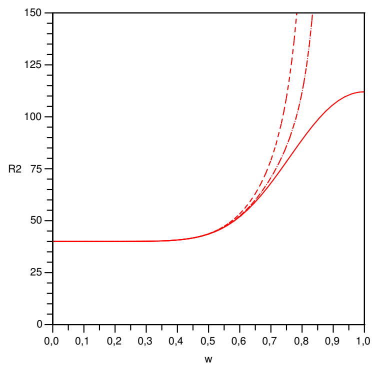

Figure 3. The curvature scaler R^2. The curves correspond to the cases $s = 4/3\pm$, and $s = 4/3$, the only finite case.

Consequently, the BH horizon moves as $z_h(\tau) \propto \tau^{\frac{1}{3}}$, the temperature as $T(\tau) \sim 1/z_h \sim \tau^{-\frac{1}{3}}$, and the entropy stays constant since $S(\tau) \sim Area \sim \tau \cdot 1/z_h^3 \sim const$. Note that again the physical thermodynamical variables of the QGP are the same as those one may attribute to the BH in the bulk (with the reservation that thermodynamics of a moving BH may raise nontrivial interpretation problems).

4. Some recent results

4.1. Beyond the perfect fluid

The geometry (12) is only a solution of Einstein equations in the scaling limit. However with some effort, one can get also the first subleading corrections to the metric. One expects an energy density described by a *viscous* Bjorken expansion, namely

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}}\left(1 - \frac{2\eta_0}{\tau^{\frac{2}{3}}} + \dots\right) \qquad (13)$$

where η_0 is related to the shear viscosity through $\eta = \eta_0/\tau$ (which follows from the scaling $\eta \propto T^3$). Let us show how this arises using the AdS/CFT methods. Using the holographic renormalization, one finds [6] that the curvature scalar has the form

$$\begin{aligned} R^2 &= R_0(v) + \frac{1}{\tau^r} R_r(v) + \frac{1}{\tau^{2r}} R_{2r}(v) \\ &\quad + \frac{1}{\tau^{\frac{4}{3}}} R_2(v) + \ldots \end{aligned} \tag{14}$$

with $R_0(v)$ and $R_r(v)$ being nonsingular, while *a priori*, *both* $R_{2r}(v)$ and $R_2(v)$ turn out to have 4^{th} order pole singularities. In order for them to have a chance to cancel we find the conditions

$$r = \frac{2}{3};\ \eta/s = 1/4\pi \tag{15}$$

which is exactly the scaling of a viscosity correction to Bjorken flow and the value found previously in the static configuration [4]. In a similar manner one can go one-order higher and determines the transport coefficients of second order (nonlinear) hydrodynamics. At that order, it turns out that there remains a leftover logarithmic singularity but it has been proven that it is merely due to a pathology of the Fefferman-Graham expansion and can be avoided when one uses a different metric [7].

4.2. Beyond boost-invariance

The calculations presented in the previous sections were performed for systems with boost invariance symmetry and full translational and rotational symmetry in the transverse plane. This allowed one to perform explicit computations as the symmetry assumptions effectively reduced the calculation to systems of ordinary differential equations. In this manner one obtains directly the solution for gauge theory energy density $\varepsilon(\tau)$. Then, in order to find the link with hydrodynamics, one finds that this solution is a solution of hydrodynamic equations with specific values for the transport coefficients. However, the boost-invariance assumption is crucial to get the results.

This approach has both an advantage and a drawback. The advantage is that one does not presuppose any kind of initial or early-time dynamics and one may try to apply it in contexts very far from equilibrium, where hydrodynamic description does not apply. The drawback is that the appearance of hydrodynamic equations is not transparent and it is difficult to relax the symmetry assumptions.

Recently the latter drawback was addressed and it was shown in general how the equations of hydrodynamics arise from the gravity side [8]. Here we will briefly review this approach. The idea of ref.[8] is to allow u^μ, the local 4-velocity of the fluid and T, the temperature, to be (slowly-varying) functions of the space-time coordinates. Once this is done the geometries (9) or (12) cease to be an exact solution of Einstein equation because of non-vanishing gradients of the parameters u^μ and T. This suggests to perform an expansion of the solution in terms of gradients which has been carried out in [8] up to second order in derivatives. The integration constants arising at each order are again fixed by requiring regularity of the metric at the horizon. The resulting metric is expressed in terms of 4-velocities and temperatures and their derivatives, so when one extracts the energy-momentum tensor it will be given directly in terms of those quantities.

Up to first order the expression is

$$\begin{aligned} T^{\mu\nu} &= \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{\textit{perfect fluid}} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{\textit{viscosity}} + \\ &\underbrace{(\pi T)^2\left(\log 2T_{2a}^{\mu\nu} + 2T_{2b}^{\mu\nu} + (2-\log 2)\left(\frac{1}{3}T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu}\right)\right)}_{\textit{second order hydrodynamics}} \end{aligned}$$

The first term is just the perfect fluid energy momentum tensor, the second term involves the shear viscosity and the third, the other transport coefficients. This result essentially demonstrates how general hydrodynamic equations arise from gravity in AdS/CFT. Once the general form of the gauge theory energy-momentum tensor is fixed, then conservation of energy momentum $\partial_\mu T^{\mu\nu} = 0$ is equivalent to conformal relativistic Navier-Stokes equations. As a byproduct, the above construction also gives a map from solutions of (viscous) hydrodynamics to gravity solutions. However this setup requires that the starting point is not far off from equilibrium. For processes which do not admit a hydrodynamic description (like the early stage of a heavy-ion collision) one has to resort to different methods.

4.3. Beyond hydrodynamics

Far from equilibrium behavior of gauge theories is a fascinating and pretty much open problem of experimental importance, like the early universe or initial stages of heavy ion collisions. The AdS/CFT correspondence is surely capable to shed new light on these problems, or even be understood as a formulation of far from equilibrium gauge theory.

In the context of heavy-ion collisions the most important and probably the most difficult questions concern the issues of early-time dynamics and the transition to an isotropic and thermalized medium, see for instance [9,10]. One of the puzzles here is the short time in which nuclear matter approach local equilibrium. Perhaps some of these questions might be answered by studying collisions of shock-waves in AdS. The geometry corresponding to a projectile in 3+1 dimensions was constructed in [5] using holographic renormalization. The solution is

$$\mathrm{d}s^2 = \frac{-2\mathrm{d}x^+\mathrm{d}x^- + f\left(x^-\right) z^4 \left(\mathrm{d}x^-\right)^2 + \mathrm{d}x_\perp^2 + \mathrm{d}z^2}{z^2} \tag{16}$$

with an arbitrary function $f\left(x^-\right)$.

Choosing $f\left(x^-\right) \propto \delta\left(x^-\right)$ leads to a shock-wave – infinitely thin plane of matter moving at the speed of light, which is a toy-model for a highly boosted nucleus. The idea is to collide two such projectiles and single out the physical behavior of the plasma from the regularity of the dual geometry. This is a difficult problem, but some interesting attempts have been made and are still in progress (see, for instance, [11–13].

5. Gravity dual of scattering amplitudes

5.1. *Holography, Wilson lines and minimal surfaces*

In gauge field theories, scattering amplitudes can be appropriately formulated in terms of expectation values of Wilson loops, which is useful for our purpose. Indeed, the Gauge/Gravity "dictionary" for Wilson loops has been proved to be well suitable for duality properties. Let us thus introduce this dictionary.

Within the general *holographic* framework, Wilson loops in the "boundary" gauge field theory are in correspondence with minimal surfaces in the "bulk".

In order to illustrate the way one may formulate in practice the AdS/CFT correspondence in a context similar to QCD, let us consider first the example of the vacuum expectation value (*vev*) of Wilson loops in a configuration parallel to the time direction of the branes. we consider the large time limit and thus the loops close "near" infinity in the time direction (see Fig. 4). This configuration allows for a determination

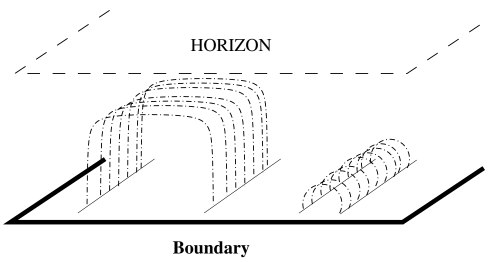

Figure 4. Sketch of minimal surfaces with Wilson lines boundaries. Left: Minimal surface in the presence of a confining background; Right: Minimal surface for an AdS_5-like background.

of the potential between colour charges [14]. The rôle of colour charges is played by open string states elongated between a stack of N_c D_3 branes on one side and one D_3 brane near the boundary of AdS space.

The correspondence can be formulated[2] as follows

$$\langle\, e^{iP\int_C \vec{A}\cdot\vec{dl}}\, \rangle \;=\; \int_\Sigma e^{-\frac{Area(\Sigma)}{\alpha'}} \;, \tag{17}$$

where C is the Wilson loop contour near the D_3 branes and Σ any surface in AdS-space with C as the boundary, see Fig. 4.

In the semi-classical approximation for the right-hand (gravity) side of the relation where the Gauge/Gravity correspondence would give the strong coupling value of the left-hand (gauge) side, the integration over surfaces Σ which boils down to

$$\langle\, e^{iP\int_C \vec{A}\cdot\vec{dl}}\, \rangle \approx e^{-\frac{Area_{min}}{\alpha'}} \times Fluct. \;, \tag{18}$$

where $Area_{min}$ is the minimal surface whose boundary is the gauge-theory Wilson loop. The factor denoted *Fluct.* refers to the fluctuation determinant

[2] For simplicity, an extra singlet term in the left-hand exponent is here neglected.

around the minimal surface, corresponding to the first quantum correction beyond the classical approximation. It gives an interesting calculable semi-classical correction, as we shall see on the example of amplitudes.

In Fig. 4, we have sketched the form of minimal surface solutions for the "confining" AdS_{BH} case, (see above (6)). For large separation of Wilson lines, the minimal surface "feels" the horizon and is consequently curved. At smaller separation, the solution becomes again similar to the conformal case, since the horizon cut-off does not play a rôle.

In gauge theory, the quark-quark potential is known to be obtained from a suitable time-like infinite limit of the quadrangular Wilson loop *vev.* One has

$$V(L) = \lim_{T\to\infty} \frac{1}{T} \times \log\langle\ Wilson\ Loop\ \rangle \tag{19}$$

Thanks to the Gauge/Gravity correspondence (17) and the classical approximation (18), one is able to get the strong coupling limit of the interquark potential from the large time limit of the Wilson loop v.e.v.:

$$\begin{aligned} AdS_5 : \langle Wilson\ Loop\rangle &= e^{TV(L)} \sim e^{\#_1 T/L} \\ AdS_{BH} : \langle Wilson\ Loop\rangle &= e^{TV(L)} \sim e^{\#_2 TL/R_0^2}\ , \end{aligned}$$

where, the potential behaviour obeys the nonconfining Coulomb law $V(L) \propto 1/L$ for the AdS case and the confining law $V(L) \propto L$ for the AdS_{BH} case. An interesting nontrivial square-root dependent coupling appears (here denoted only by $\#_{1,2}$). Note again that, even in the case of a confining geometry with a horizon at R_0, Wilson lines separated by a distance $L << R_0$ do not give rise to minimal surfaces sensitive to the horizon (see Fig. 4), and thus leads to classical solutions similar to the non-confining case, which can give interesting indications for small spatial separation.

The important rôle of fluctuation corrections for scattering amplitudes and the way of computing it in some non-trivial cases is discussed in the following subsection.

5.2. *Dual models for scattering amplitudes*

There are different approaches to scattering amplitudes at strong coupling using gravity duals. We will mention one of them, namely the holographic evaluation of Wilson loop correlators corresponding to dipole-dipole amplitudes [15]. Even if not recent, it starts from a general approach based on Wilson loops which has found recently remarkable applications, namely the formulation of gluon amplitudes at strong coupling in the $SU(N)$ gauge theory with $\mathcal{N}=4$ supersymmetries [16] and the first steps towards a description of deep-inelastic scattering at strong coupling [17] (see also [18] for a different method).

The basic principle of the method is as follows. Since we are interested in the present lecture in the approach to hadronic scattering amplitudes, one is led to search for both a field-theoretical formulation based on QCD and the determination of the gravity duals of the corresponding amplitudes. Concerning the nature of the dual theory, the gravity dual theory of QCD has not yet been identified. More generally, the problem of deriving a correspondence for a confining theory with asymptotic freedom is not yet achieved. In the following we shall use an approach where only generic features of confining backgrounds allow to determine some properties of the amplitudes. The price we pay is that we will only be able to discuss the high-energy behaviour of the amplitudes. Other properties of the amplitudes will not be discussed, and probably are more difficult to derive in the absence of a better determined dual background to QCD. Using the AdS/CFT correspondence, we will find that two-body high energy amplitudes in gauge field theories can be related to specific configurations of minimal surfaces.

Using the Wilson loop properties, it is now possible to formulate the Gauge/Gravity correspondence for the elastic and inelastic scattering amplitude of massive $q\bar{q}$ states forming colourless QCD dipoles. In Fig. 5, one considers the elastic and inelastic ampli-

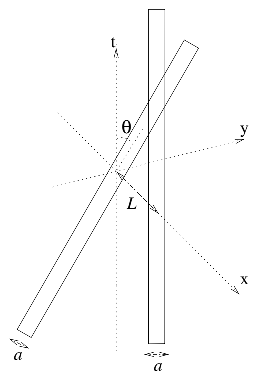

Figure 5. Wilson loops for Dipole-Dipole scattering. The figure is drawn in the physical configuration space (t, x, y, z).

tudes of two dipoles in configuration space, denoted respectively to $A_{\mathcal{P}}^{d-d}(s,q^2)$, and $A_{\mathcal{R}}^{d-d}(s,q^2)$. We will here consider the amplitudes at high energy, i.e. the problem of "Reggeization". Indeed, at high energy, fast moving colour sources propagate along linear trajectories in coordinate space thanks to the eikonal approximation. This important property of high energy propagation of colour sources will be helpful for the evaluation of the amplitudes through Gauge/Gravity duality.

Let us first consider the elastic dipole amplitude $A_{\mathcal{P}}^{d-d}(s,q^2)$, *i.e.* the diagram of Fig. 5. In the gauge field theory, one may write it in terms of a correlator between two Wilson lines in configuration space, namely

$$A_{\mathcal{P}}^{d\text{-}d}(s,q^2) = -2is\int d^2x_\perp e^{iqx_\perp}\left\langle \frac{W_1W_2}{\langle W_1\rangle\langle W_2\rangle}-1\right\rangle \quad (20)$$

where the Wilson loops W_1, W_2 correspond to the two colliding dipoles which follow classical straight lines for quark(antiquark) trajectories and close at infinite time, as for the potential. The normalization $\langle W_1\rangle\langle W_2\rangle$ of the correlator ensures that the amplitude vanishes when the Wilson loops get decorrelated at large distances.

One useful technique is to formulate the duality property in Euclidean $\mathcal{R}^4$ space where it takes the form of a well-defined geometrical interpretation in terms of a minimal surface problem and then the analytic continuation from Euclidean to Minkowski space allows one to find the physical solution.

The Wilson line *vev* can be expressed as a minimal surface problem with (approximately) two copies (for dipole size $a\sim 0$) of a minimal surface whose boundaries are straight lines in a 3-dimensional coordinate space, placed at an impact parameter distance L and rotated one with respect to the other by an angle θ. see Fig. 5. Then the amplitude will be obtained through the analytic continuation

$$\theta \leftrightarrow -i\chi \quad ; \quad t_{Eucl} \leftrightarrow it_{Mink} \ , \quad (21)$$

where $\chi = \log s/m^2$ is the total rapidity interval.

In flat space, with the same boundary conditions, the minimal surface is the *helicoid*. One thus realizes that the problem can be formulated as a minimal surface problem whose mathematically well-defined solution is a *generalized helicoidal* manifold embedded in curved background spaces, such as Euclidean AdS Spaces. Unfortunately, this problem is rather difficult to solve analytically, even in flat space. It is known in mathematics as the Plateau problem, namely the determination of minimal surfaces for given boundary conditions[3].

In fact, the definition of the minimal surface geometry in the conditions of a confined AdS_{BH} metrics (6) appears to be simpler, at least for the leading contribution. Indeed, in the configuration of Wilson lines of a confining theory, the AdS_{BH} metrics is characterized by a singularity at $z=0$ which implies a rapid growth in the z direction towards the D_3 branes, then stopped near the horizon at z_0. Thus, to a good approximation, and for large enough impact parameter (compared to the horizon distance), the main contribution to the minimal area is from the metrics in the bulk near z_0 which is nearly flat. Hence, near z_0, the relevant minimal area can be drawn on a *classical helicoid*, whose analytic expression is known.

After analytic continuation, one obtains

$$\begin{aligned} A_{\mathcal{P}}(s,q^2) &= 2is\int d\vec{l}\; e^{i\vec{q}\cdot\vec{l}-\left\{\frac{\sqrt{2Ng^2_{YM}}}{2R_0^2\chi}\right\}\vec{l}^2} \propto \\ &\propto\ s^{1-q^2\frac{R_0^2}{\sqrt{8Ng^2_{YM}}}} \end{aligned} \quad (22)$$

which represents a Reggeized elastic amplitude, with a linear Regge trajectory

$$\alpha_{\mathcal{P}}(q^2) = \alpha_{\mathcal{P}}(0) - q^2\alpha'_{\mathcal{P}} \equiv 1 - q^2\frac{R_0^2}{\sqrt{8Ng^2_{YM}}} \quad (23)$$

characterized by a "Pomeron" intercept $\alpha_{\mathcal{P}}(0)=1$ and a Regge slope, defined in terms of the gravity dual parameters by $\alpha'_{\mathcal{P}} = \frac{R_0^2}{\sqrt{8Ng^2_{YM}}}$, where $g^2_{YM}N \equiv g_s$ is the string or 't Hooft coupling.

Let us now consider the dipole-dipole inelastic amplitude. The helicoidal geometry remains valid due to the eikonal approximation for the "spectator quarks" while the "exchanged quarks" define a trajectory drawn on the helicoid. This trajectory plays the rôle of a dynamical time-like cut-off which takes part in the minimization procedure. The resulting amplitude reads:

$$\begin{aligned} A_{\mathcal{R}}(s,q^2) &= 2i\int d\vec{l}\; e^{i\vec{q}\cdot\vec{l}-\left\{\frac{\sqrt{8Ng^2_{YM}R_0^2}}{\chi}\right\}\vec{l}^2} \propto \\ &\propto\ s^{-q^2\frac{2R_0^2}{\sqrt{2Ng^2_{YM}}}}\ , \end{aligned} \quad (24)$$

[3]It is interesting to remark that the derivation of the remarkable results of [16] are just related to the nontrivial determination of a minimal surface in AdS_5.

corresponding to a linear Regge trajectory

$$\alpha_{\mathcal{R}}(q^2) = \alpha_{\mathcal{R}}(0) - q^2\alpha'_{\mathcal{R}} \equiv -q^2 \frac{2R_0^2}{\sqrt{2Ng_{YM}^2}} \tag{25}$$

characterized by a "Reggeon" intercept $\alpha_{\mathcal{R}}(0) = 0$ and a Regge slope $\alpha'_{\mathcal{R}} = \frac{2R_0^2}{\sqrt{2Ng_{YM}^2}}$. Note that the slope $\alpha'_{\mathcal{R}}$ is related to the quark potential calculated within the same AdS/CFT framework and, quite interestingly we find $\alpha'_{\mathcal{R}} = 4\alpha'_{\mathcal{P}}$. It is interesting to note the difference between the relation obtained at strong coupling with the expectation for weakly coupled strings, namely $\alpha'_{\mathcal{R}} = 2\alpha'_{\mathcal{P}}$.

Up to now, we restricted ourselves to a classical approximation based on the evaluation of minimal surfaces solutions for the various Wilson loops involved in the preceding calculations. It is interesting to note [19] that a further step can be done by evaluating the contribution of quadratic fluctuations of the string worldsheet around the minimal surfaces in the case where these surfaces are embedded in helicoids, as discussed for the confining backgrounds. The semi classical correction comes from the fluctuations near the minimal surface (see Fig. 6). The main outcome is that this semi classical correction can be computed and is intimately related to the well-known "universal" Lüscher term contribution to the interquark potential .

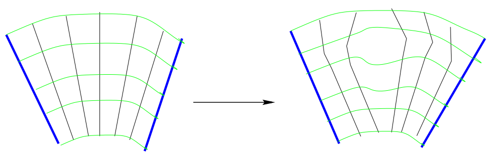

Figure 6. Fluctuations around the minimal helicoid.

After some nontrivial steps, the formulae (23,25) get corrected as follows

$$\begin{aligned} A_{\mathcal{P}}(s,q^2) &\propto s^{\alpha_{\mathcal{P}}(-q^2)} = s^{1+\frac{n_\perp}{96}-q^2\frac{\alpha'_{\mathcal{R}}}{4}} \\ A_{\mathcal{R}}(s,q^2) &\propto s^{\alpha_{\mathcal{R}}(-q^2)} = s^{\frac{n_\perp}{24}-q^2\alpha'_{\mathcal{R}}} \end{aligned} \tag{26}$$

where $n_\perp$ is the number of zero modes of the gravity dual theory in the transverse-to-the-branes directions. The result is just equivalent to the Lüscher term in the potential except that the number of zero modes $n_\perp = D-2$ can be larger than the usual value 2 corresponding to flat $4D$ space.

It is interesting to note that this theoretical feature is in qualitative agreement with the phenomenology of soft scattering. Indeed once we fix the α' from the phenomenological value of the static $q\bar{q}$ potential ($\alpha' \sim 0.9\,GeV^{-2}$) we get for the slopes $\alpha'_R = \alpha' \sim 0.9\,GeV^{-2}$ and $\alpha'_P = \alpha'/4 \sim 0.23\,GeV^{-2}$ in good agreement with the phenomenological slopes.

A second feature is the relation between the Pomeron and Reggeon intercepts. At the classical level of our approach these are respectively 1 and 0. Note that this classical piece is in agreement with what is obtained from simple exchanges of two gluons and quark-antiquark pair, respectively, in the $t \equiv -q^2$ channel. The fluctuation (quantum) contributions to the Reggeon and Pomeron are also related by the factor four.

Adding both classical and fluctuation contributions gives an estimate which is in qualitative agreement with the observed intercepts. Indeed, when calculating the fluctuations around a minimal surface near the horizon in the BH backgrounds there could be $n_\perp = 7, 8$ massless bosonic modes . For $n_\perp = 7, 8$ one gets $1.073 - 1.083$ for the Pomeron and $0.3 - 0.33$ for the Reggeon. This result is in agreement with the observed intercept for the "Pomeron" and somewhat below the intercepts of around 0.5 observed for the dominant Reggeon trajectories. The interesting output of the application of AdS/CFT correspondence to high energy amplitudes at strong coupling is to emphasize the relation between Reggeization and confinement, using the description of two-body scattering amplitudes in the dual string theory. Lattice calculations, which is the only presently known way to evaluate directly QCD observables at strong coupling, are not able to compute high-energy amplitudes.

6. Conclusion

From the present rapid (and partial) survey of some of the results obtained in the AdS/CFT approach, it appears that the Gauge/Gravity correspondence is a promising way to explore some features of QCD at strong coupling. Indeed some general features of this correspondence, relating at long distances the closed and open string geometries are expected to be valid in principle for various dual schemes and thus, hopefully, QCD. However the "Gravity dual of QCD" is not yet understood, in particular the asymptotic freedom property which represents the renormalization flow connecting the strong and weak coupling regimes. However, there exist situations, described in this lecture, where the correspondence may give physical lessons, either because the system is far from its confined phase, or if some general results can be obtained in a confining geometry irrespective of a particular realization.

For the quark-gluon plasma problems, quantitative dual schemes have been more precisely elaborated for the specific AdS/CFT case, *i.e.* the gauge theory with $\mathcal{N} = 4$ supersymmetries. Among the results, it gives a calculable link between the hydrodynamic quasi-perfect fluid behaviour on the gauge theory side with a BH geometry in the higher dimensional gravity side in and AdS background. This relation can be extended from the static case to a dynamical regime reflecting (within the AdS/CFT framework) the relativistic expansion of the corresponding quark-gluon plasma. This, and many other applications, some of them using more complex geometries, less supersymmetric backgrounds and examining other observables, gives hope for the fruitful possibilities of the Gauge/Gravity approach to the QGP formation.

For the scattering amplitude problems, the formalism using Wilson loops and their corresponding semi-classical evaluation through minimal surface problems (+ fluctuations) in the gravity dual seem also to lead to interesting results either connected with Reggeization (cf.[15,19]) or to the strong coupling limit of deep inelastic scattering (cf.[17]) or even, quite remarkably to gluon amplitudes (cf. [16]).

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