Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B Proceedings Supplement 00 (2012) 1–8

Nuclear Physics B Proceedings Supplement

World Summary of α_s (2011)

Siegfried Bethke

Max-Planck-Institute for Physics, Föhringer Ring 6, 80805 Munich, Germany

---

Abstract

The most significant determinations of the strong coupling strength, α_s, are summarised and a new world average value of $\alpha_s(M_Z)$ is determined, using a new method of pre-averaging results within classes of measurements like hadronic τ decays, deep inelastic scattering processes, determinations on the lattice, electro-positron annihilation processes and electro-weak precision fits. The overall result is

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007 ,$$

unchanged from it's corresponding value obtained in 2009.

Keywords:
strong interactions, QCD, strong coupling.

---

1. Introduction

The numerical size of the strong coupling strength, α_s, like other fundamental "constants" of nature, is not given by theory, but must be determined by experiment. Quantum Chromodynamics (QCD) [1], the gauge field theory describing the Strong Interaction between quark and gluons, the fundamental constituents of hadronic matter, allows to predict physical cross sections and many other observables $\mathcal{R}$ of particle reactions involving quarks and gluons, in terms of - basically - one single parameter, α_s. Assuming $\alpha_s < 1$ and applying perturbation theory, predictions are typically given by a power series in α_s, like:

$$\begin{aligned}\mathcal{R} &= P_l \sum_n R_n \alpha_s^n \\ &= P_l \left(R_0 + R_1 \alpha_s + R_2 \alpha_s^2 + ... \right) \qquad (1)\end{aligned}$$

where R_n are the n_{th} order coefficients of the perturbation series and $P_l R_0$ denotes the lowest-order value of $\mathcal{R}$. This allows to measure α_s in a large variety of particle reactions.

QCD also predicts the energy dependence of α_s through the "beta function",

$$Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = \beta\left(\alpha_s(Q^2)\right) , \qquad (2)$$

where Q^2 is the squared energy scale or the momentum transferred of the particle reaction under study, and any energy scale dependence of $\mathcal{R}$ is determined by the energy dependence of α_s[1] Measurements of α_s from different particle reactions and scattering processes, performed at different energy scales Q^2, therefore *test* the global nature of QCD and, in particular, its characteristic prediction of "Asymptotic Freedom", which determines that α_s is small and asymptotically tends to zero at large energy scales (or at small distances), while it is large at small energy scales (large distances), explaining the "confinement" of quarks and gluons inside hadrons.

Experimental determinations of α_s were regularly summarised and reviewed in the past, see e.g. [2, 3, 4].

---

[1] For processes with initial state hadrons, also the coefficients R_n may depend on α_s, through the effects of parton density functions.

These references also contain the definition of basic equations and fomulae which shall not be repeated here. In 2009, the world average value of α_s, expressed at a common energy scale corresponding to the rest mass of the Z boson, determined from a set of most recent determinations from many different processes at a large range of energy scales, converged to

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007 ,$$

where the overall error includes experimental as well as (dominating) systematic and theoretical uncertainties [3]. In the following, this summary is updated with the newest results in this field.

This review is based on the α_s section of the upcoming 2012 Review of Particle Physics (RPP) of the Particle Data Group, and the results and the summary presented below are identical to those reported in the 2011 RPP partial web update [5]. While this corresponds to a slight refinement and continuation of the work actually presented at this workshop, compatibility of results presented here with those in [5] is ensured.

2. Selected results

The set of results used in this review is restricted to those

- which were published in peer-reviewed journals until middle of 2011, and
- which use QCD perturbation theory to at least full next-to-next-to-leading order perturbation[2] .

These requirements exclude e.g. results from jet production in deep inelastic lepton-nucleon scattering (DIS) at HERA and from hadron collisions at the Tevatron, as well as those from heavy quarkonia decays for which calculations are available in NLO only. While being excluded from calculating a world average value of α_s, NLO results will nevertheless be cited in this review as they are important ingredients for demonstrating the experimental evidence of the energy dependence of α_s, i.e. for Asymptotic Freedom, one of the key features of QCD.

Furthermore, here we add an intermediate step of pre-averaging results within well defined sub-fields like e^+e^--annihilation, DIS and hadronic τ-decays. The overall world average will then be calculated from those pre-averages rather than from individual measurements. This is done because in a number of sub-fields, different determinations of the strong coupling from substantially similar datasets lead to values of α_s that are only marginally compatible with each other, or with the final world average value, which may be a reflection of the challenges of evaluating systematic uncertainties. In such cases, a pre-average value is determined, with a symmetric, overall error that encompasses the central values of all individual determinations.

2.1. Hadronic τ decays

Determinations of α_s from hadronic τ lepton decays continues to be one of the most actively studied fields to measure this basic quantity. The small effective energy scale, $Q = M_\tau = 1.78$ GeV, small nonperturbative contributions to an inclusive and well defined experimental observable, and the availability of perturbative predictions which are complete to N^{3}LO determine the importance and large interest in this particular field. Several re-analyses of the hadronic τ decay width [6, 7, 8, 9, 10, 11] were performed, using different approaches of treating perturbative (fixed order or contour improved perturbative expansions) and non-perturbative contributions. The result of [6] includes both, fixed order and contour improved perturbation, while the others adhere to either one or the other of the two. These results are summarised in Fig. 1(a).

There are two more studies of α_s from τ-decays, [12] and [13], which were not yet available as peer-reviewd publications. They are, however, compatible with the overall picture which is summarized in 1(a). Another very recent study [14] argues that an improved treatment of non-perturbative effects results in values of α_s which are systematically lower than those discussed above.

The pre-average result from τ-decays, to be used for calculating the final world average of $\alpha_s(M_Z^2)$, is determined using the simple method mentioned above, i.e. defining one central value with symmetric overall error bars which include the smallest as well as the largest of all results, as $\alpha_s(M_\tau^2) = 0.330 \pm 0.014$. This value of $\alpha_s(M_\tau^2)$ corresponds, when evolved to the scale of the Z-boson, using the QCD 4-loop beta-function plus 3-loop matching at the charm- and the bottom-quark masses (see [3, 2, 4] for relevant equations), to $\alpha_s(M_Z^2) = 0.1197 \pm 0.0016$, unchanged from its value in the 2009 review.

2.2. Lattice QCD

There are several recent results on α_s from lattice QCD, see also Sec. *Lattice QCD* in [5]. The HPQCD collaboration [15] computes short distance quantities

---

[2] NNLO; for observables with QCD contributions starting at leading order, that is $O(\alpha_s^3)$.

like Wilson loops with lattice QCD and analyzes them with NNLO perturbative QCD. This yields a value for α_s. The lattice scale must then be related to a physical energy/momentum scale. This is achieved with the Υ'-Υ mass difference, however, many other quantities could be used as well [16]. HPQCD obtains $\alpha_s(M_Z^2) = 0.1184 \pm 0.0006$, where the uncertainty includes effects from truncating perturbation theory, finite lattice spacing and extrapolation of lattice data. An independent perturbative analysis of a subset of the same lattice-QCD data yields $\alpha_s(M_Z^2) = 0.1192 \pm 0.0011$[17]. Using another, independent methodology, the current-current correlator method, HPQCD obtains $\alpha_s(M_Z^2) = 0.1183 \pm 0.0007$ [15]. A more recent result in [18], which avoids the staggered fermion treatment of [15], finds $\alpha_s(M_Z^2) = 0.1205 \pm 0.0008 \pm 0.0005\,^{+0.0000}_{-0.0017}$[18], where the first uncertainty is statistical and the others are from systematics. Since this approach uses a different discretization of lattice fermions and a different general methodology, it provides an independent cross check of other lattice extractions of α_s. Finally, the JLQCD collaboration - in an analysis of Adler functions - obtains $\alpha_s(M_Z^2) = 0.1181 \pm 0.0003\,^{+0.0014}_{-0.0012}$[19].

These results are summarized in Fig. 1(b). Since they are compatible with and largely independent from each other, a pre-average of lattice results is calculated using the same method as applied to determine the final world average value α_s, i.e. calculate a weighted average and a (correlated) error such that the overall χ^2 equals unity per degree of freedom - rather than using the simple method as applied in the case of τ decays. This gives $\alpha_s(M_Z) = 0.1185 \pm 0.0007$ which is taken as result from the sub-field of lattice determinations.

2.3. Deep inelastic lepton-nucleon scattering (DIS)

Studies of DIS final states have led to a number of precise determinations of *as*:
A combination [20] of precision measurements at HERA, based on NLO fits to inclusive jet cross sections in neutral current DIS at high Q^2, quotes a combined result of $\alpha_s(M_Z) = 0.1198 \pm 0.0032$, which includes a theoretical uncertainty of ±0.0026. A combined analysis of non-singlet structure functions from DIS [21], based on QCD predictions up to N^3LO in some of its parts, gave $\alpha_s(M_Z) = 0.1142 \pm 0.0023$, including a theoretical error of ±0.0008. Further studies of singlet and non-singlet structure functions, based on NNLO predictions, resulted in $\alpha_s(M_Z) = 0.1129 \pm 0.0014$ [22] and in $\alpha_s(M_Z) = 0.1158 \pm 0.0035$ [23]. The MSTW group [24], also including data on jet production at the Tevatron, obtains, in NNLO[3], $\alpha_s(M_Z) = 0.1171 \pm 0.0014$.

Summarizing these results from world data on structure functions, applying the same method as in the case of summarizing results from τ decays, leads to a pre-average value of $\alpha_s(M_Z) = 0.1150 \pm 0.0021$ (see Fig. 1(c)).

Note that criticism has been expressed on some of the above extractions. Among the issues raised, we mention the neglect of singlet contributions at $x \geq 0.3$ in pure non-singlet fits [25], the impact and detailed treatment of particular classes of data in the fits [26][27][25] and possible biases due to insufficiently flexible parametrizations of the PDFs [27]. Most recently, the NNPDF group [28] has presented a result which is more in line with the one from the MSTW group [24].

2.4. Heavy quarkonia decays

The most recent extraction of the strong coupling constant from an analysis of radiative Υ decays [29] resulted in $\alpha_s(M_Z) = 0.119^{+0.006}_{-0.005}$. This determination is based on QCD in NLO only, so it will not be considered for the final extraction of the world average value of α_s; it is, however, an important ingredient for the demonstration of Asymptotic Freedom as given in Fig. 3.

2.5. Hadronic final states of e^+e^- annihilations

Re-analyses of event shapes in e^+e^--annihilation, measured at the Z peak and LEP2 energies up to 209 GeV, using NNLO predictions matched to NLL resummation, resulted in $\alpha_s(M_Z) = 0.1224 \pm 0.0039$ [30], with a dominant theoretical uncertainty of 0.0035, and in $\alpha_s(M_Z) = 0.1189 \pm 0.0043$ [31]. Similarly, an analysis of JADE data [32] at center-of-mass energies between 14 and 46 GeV gives $\alpha_s(M_Z) = 0.1172 \pm 0.0051$, with contributions from hadronization model (perturbative QCD) uncertainties of 0.0035 (0.0030). A precise determination of α_s from 3-jet production alone, in NNLO, resulted in $\alpha_s(M_Z) = 0.1175 \pm 0.0025$ [33]. Computation of the NLO corrections to 5-jet production and comparison to the measured 5-jet rates at LEP [34] gave $\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$. More recently, a study using the world data of Thrust distributions and soft-collinear effective theory, including fixed order NNLO, gave $\alpha_s(M_Z) = 0.1135 \pm 0.0010$ [35].

Note that there is criticism on both classes of α_s extractions just described: those based on corrections

---

[3] Note that for jet production at the hadron collider, only NLO predictions are available, while for the structure functions full NNLO was utilized.

of non-perturbative hadronisation effects using QCD-inspired Monte Carlo generators (since the parton level of a Monte Carlo is not defined in a manner equivalent to that of a fixed-order calculation), as well as the studies based on effective field theory, as their systematics have not yet been verified e.g. by using observables other than Thrust.

A summary of the e^+e^- results based on NNLO predictions is shown in Fig. 1(d). They average, according to the simple procedure defined above, to $\alpha_\mathrm{s}(M_\mathrm{Z}) = 0.1172 \pm 0.0037$.

2.6. *Hadron collider jets*

A determination of α_s from the p_T dependence of the inclusive jet cross section in $p\overline{p}$ collisions at $\sqrt{s} = 1.96$ TeV, in the transverse momentum range of $50 < p_T < 145$ GeV, based on NLO ($O(\alpha_\mathrm{s}^3)$) QCD, led to $\alpha_\mathrm{s}(M_\mathrm{Z}) = 0.1161^{+0.0041}_{-0.0048}$ [36], which is the most precise α_s result obtained at a hadron collider. Experimental uncertainties from the jet energy calibration, the p_T resolution and the integrated luminosity dominate the overall error.

2.7. *Electroweak precision fits*

The N^{3}LO calculation of the hadronic Z decay width was used in a recent revision of the global fit to electroweak precision data [37], resulting in $\alpha_\mathrm{s}(M_\mathrm{Z}) = 0.1193 \pm 0.0028$, claiming a negligible theoretical uncertainty. Note that this result from electroweak precision data, however, strongly depends on the strict validity of Standard Model predictions and the existence of the minimal Higgs mechanism to implement electroweak symmetry breaking. Any - even small - deviation of nature from this model would strongly influence this extraction of α_s.

3. Determination of the world average value of $\alpha_\mathrm{s}(M_\mathrm{Z})$

A non-trivial exercise consists in the evaluation of a world-average value for $\alpha_\mathrm{s}(M_\mathrm{Z})$. A certain arbitrariness and subjective component is inevitable because of the choice of measurements to be included in the average, the treatment of (non-Gaussian) systematic uncertainties of mostly theoretical nature, as well as the treatment of correlations among the various inputs, of theoretical as well as experimental origin. In earlier reviews [3, 4, 2] an attempt was made to take account of such correlations, using methods as proposed, e.g., in [38], and - likewise - to treat cases of apparent incompatibilities or possibly underestimated systematic uncertainties in a meaningful and well defined manner:

The central value is determined as the weighted average of the different input values. An initial error of the central value is determined treating the uncertainties of all individual measurements as being uncorrelated and being of Gaussian nature, and the overall χ^2 to the central value is determined. In case this initial χ^2 is larger than the number of degrees of freedom, i.e. larger than the number of individual inputs minus one, then all individual errors are enlarged by a common factor such that $\chi^2/d.o.f.$ equals unity. If the initial value of χ^2 is smaller than the number of degrees of freedom, an overall, a-priori unknown correlation coefficient is introduced and determined by requiring that the total $\chi^2/d.o.f.$ of the combination equals unity. In both cases, the resulting final overall uncertainty of the central value of α_s is larger than the initial estimate of a Gaussian error.

This procedure is only meaningful if the individual measurements are known not to be correlated to large degrees, i.e. if they are not - for instance - based on the same input data, and if the input values are largely compatible with each other and with the resulting central value, within their assigned uncertainties. The list of selected individual measurements discussed above, however, violates both these requirements: there are several measurements based on (partly or fully) identical data sets, and there are results which apparently do not agree with others and/or with the resulting central value, within their assigned individual uncertainty. Examples for the first case are results from the hadronic width of the τ lepton, from DIS processes and from jets and event shapes in e^+e^- final states. An example of the second case is the apparent disagreement between results from the τ width and those from DIS [21] or from Thrust distributions in e^+e^- annihilation [35].

Due to these obstacles, we have chosen to determine pre-averages for each class of measurements, and then to combine those to the final world average value of $\alpha_\mathrm{s}(M_\mathrm{Z})$, using the methods of error treatment as just described. The five pre-averages are summarized in Fig. 2; we recall that these are exclusively obtained from extractions which are based on (at least) full NNLO QCD predictions, and are published in peer-reviewed journals at the time of completing this review. From these, the new central and world average value of

$$\alpha_\mathrm{s}(M_\mathrm{Z}) = 0.1184 \pm 0.0007\,, \qquad (3)$$

is determined, with an uncertainty of well below 1 %.[4] This world average value is - in spite of several new

---

[4] The weighted average, treating all inputs as uncorrelated measurements with Gaussian errors, results in $\alpha_\mathrm{s}(M_\mathrm{Z}) = 0.11844 \pm$

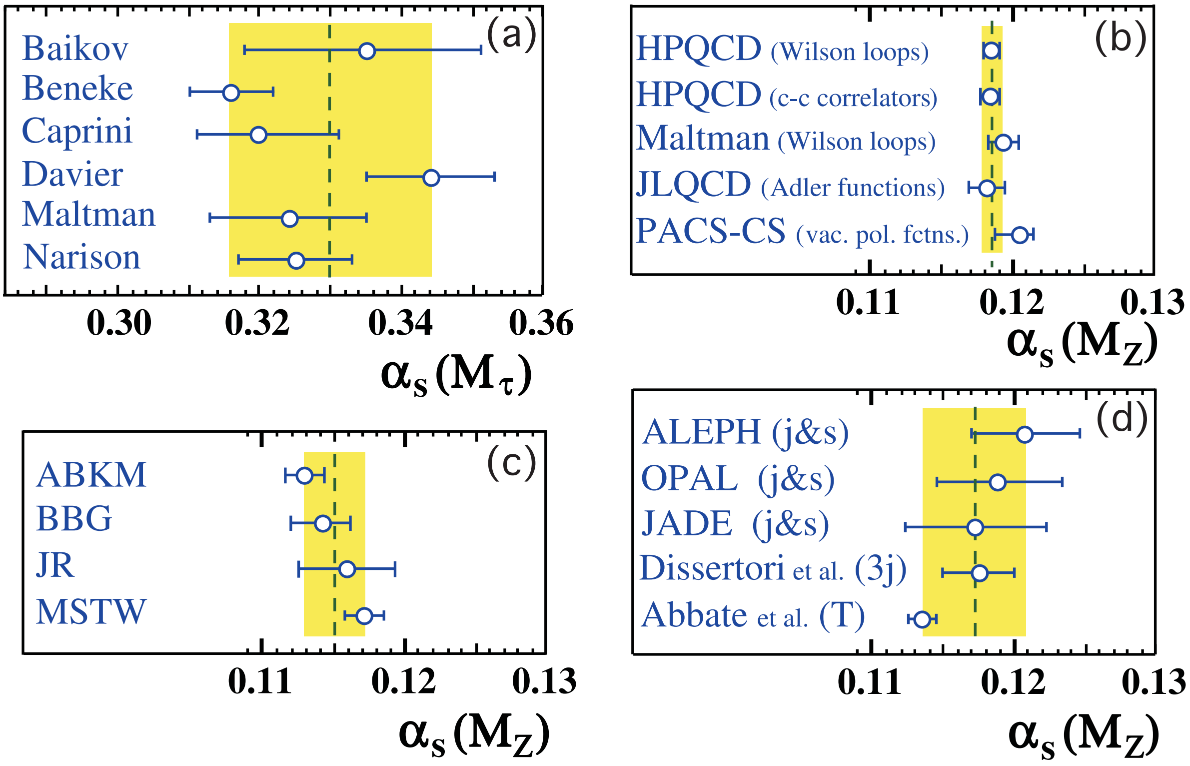

Figure 1: Summary of determinations of α_s from hadronic τ-decays (a), from lattice calculations (b), from DIS structure functions (c) and from event shapes and jet production in e^+e^--annihilation (d). The shaded bands indicate the average values chosen to be included in the determination of the new world average of α_s. Figure taken from [5].

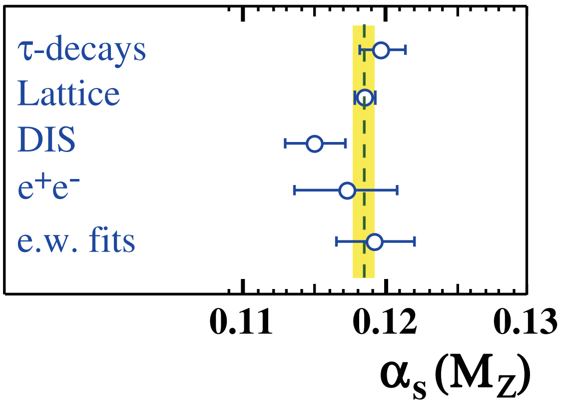

Figure 2: Summary of values of $\alpha_s(M_Z)$) obtained for various sub-classes of measurements (see Fig. 1 (a) to (d)). The new central and world average value of $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ is indicated by the dashed line and the shaded band. Figure taken from [5].

contributions to this determination - identical to and thus, in excellent agreement with the 2009 result [3, 4]. For convenience, we also provide corresponding values for $\Lambda_{\overline{MS}}$ suitable for use with the common Λ-parametrisation of α_s, see e.g. Eq. 6 in [3]:

$$\Lambda^{(5)}_{\overline{MS}} = (213 \pm 8)\ \mathrm{MeV}\ , \quad (4)$$

$$\Lambda^{(4)}_{\overline{MS}} = (296 \pm 10)\ \mathrm{MeV}\ , \quad (5)$$

$$\Lambda^{(3)}_{\overline{MS}} = (339 \pm 10)\ \mathrm{MeV}\ , \quad (6)$$

for $N_f = 5$, 4 and 3 quark flavors, respectively.

In order to further test and verify the sensitivity of the new average value of $\alpha_s(M_Z)$ to the different pre-averages and classes of α_s determinations, we give each of the averages obtained when leaving out one of the five input values:

$$\begin{aligned}
\alpha_s(M_Z) &= 0.1182 \pm 0.0007 \ \ (\text{w/o } \tau \text{ results}),\\
\alpha_s(M_Z) &= 0.1181 \pm 0.0012 \ \ (\text{w/o lattice}),\\
\alpha_s(M_Z) &= 0.1187 \pm 0.0009 \ \ (\text{w/o DIS}),\\
\alpha_s(M_Z) &= 0.1184 \pm 0.0006 \ \ (\text{w/o } e^+e^-), \text{ and}\\
\alpha_s(M_Z) &= 0.1184 \pm 0.0006 \ \ (\text{w/o e.w. prec. fit}).
\end{aligned}$$

They are well within the error of the overall world average quoted above. Most notably, the result from lattice calculations, which has the smallest assigned error, agrees well with the exclusive average of the other results. However, it largely determines the size of the (small) overall uncertainty.

There are apparent systematic differences between the various structure function results, and also between the new result from Thrust in e^+e^- annihilation and the other determinations. Expressing this in terms of a χ^2 between a given measurement and the world average as obtained when *excluding* that particular measurement, the largest values are $\chi^2 = 12.6$ and $\chi^2 = 16.1$, corresponding to 3.5 and 4.0 standard deviations, for the measurements of [22] and [35], respectively. We note that such and other differences between some of the measurements have been extensively discussed at a recent workshop on measurements of α_s, however none of the explanations proposed so far have obtained enough of a consensus to definitely resolve the tensions between different extractions [39].

Notwithstanding these open issues, a rather stable and well defined world average value emerges from the compilation of current determinations of α_s:

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007\ .$$

The results also provide a clear signature and proof of the energy dependence of α_s, in full agreement with the QCD prediction of Asymptotic Freedom. This is demonstrated in Fig. 3, where results of $\alpha_s(Q^2)$ obtained at discrete energy scales Q, now also including those based just on NLO QCD, are summarized and plotted.

Acknowledgments: I wish to thank the organizers of this workshop for their kind invitation to present these results. I am grateful to G. Dissertori and to G. Salam, my co-authors for the QCD section of the upcoming edition of the Review of Particle Physics, for their valuable collaboration in discussing, optimising and streamlining the α_s summary and its presentation. All results and large parts of the text of this contribution were actually adapted from the 2011 web update of RPP [5] which was scrutinized by many other colleagues, for which I also wish to express my gratitude.

References

[1] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B47, (1973) 365;
D.J. Gross, F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; Phys. Rev. D8 (1973) 3633;
H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.
[2] S. Bethke, Prog. Part. Nucl. Phys. 58 (2007) 351.
[3] S. Bethke, Eur. Phys. J. C64 (2009) 689.
[4] K. Nakamura et al. (Particle Data Group), J. Phys. G37, 075021 (2010).
[5] 2011 partial web update and 2012 version of RPP, see http://pdg.lbl.gov/ .

0.00060 with χ^2/d.o.f. = 3.1/4. Requiring χ^2/d.o.f. to reach unity leads to a common correlation factor of 0.21 which increases the overall error to 0.00074.

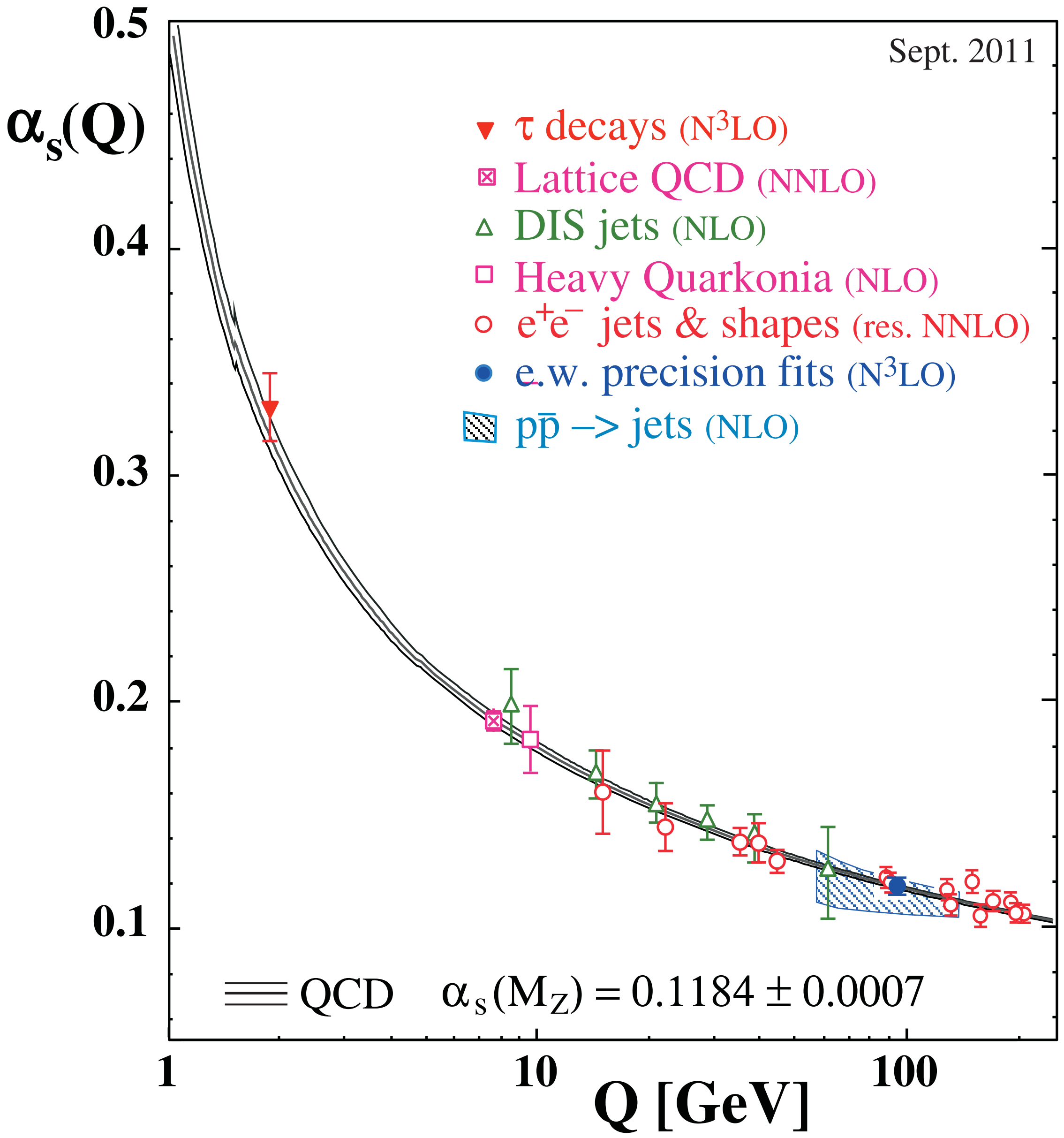

Figure 3: Summary of measurements of α_s as a function of the respective energy scale Q. The respective degree of QCD perturbation theory used in the extraction of α_s is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; N^3LO: next-to-NNLO). Figure taken from [5].

[6] P.A. Baikov, K.G. Chetyrkin, and J.H. Kuhn, Phys. Rev. Lett 101, 012002 (2008), arXiv:0801.1821 [hep-ph].
[7] M. Beneke and M. Jamin, JHEP 0809, 044(2008) arXiv:0806.3156 [hep-ph].
[8] M. Davier et al., Eur. Phys. Jour. C56, 305 (2008), arXiv:0803.0979 [hep-ph].
[9] K. Maltman and T. Yavin, Phys. Rev. D78, 094020 (2008), arXiv:0807.0650 [hep-ph].
[10] S. Narison, Phys. Lett. B673, 30 (2009), arXiv:0901.3823 [hep-ph].
[11] I. Caprini and J. Fischer, Eur. Phys. Jour. C64 (2009) 35, arXiv:0906.5211 [hep-ph].
[12] S. Menke, arXiv:0904.1796 [hep-ph].
[13] A. Pich, arXiv:1107.1123 [hep-ph].
[14] D. Boito et al., arXiv:1110.1127 [hep-ph].
[15] C. McNeile et al., [HPQCD Collaboration], Phys. Rev. D82, 034512 (2010), arXiv:1004.4285 [hep-lat].
[16] C.T.H. Davies et al., [HPQCD Collaboration, UKQCD Collaboration, and MILC Collaboration], Phys. Rev. Lett. 92, 022001 (2004), arXiv:hep-lat/0304004.
[17] K. Maltman, et al., Phys. Rev. D78, 114504 (2008), arXiv:0807.2020 [hep-lat].
[18] S. Aoki et al., [PACS-CS Collaboration], JHEP 0910, 053 (2009), arXiv:0906.3906 [hep-lat].
[19] E. Shintani et al., [JLQCD Collaboration], Phys. Rev. D82, 074505 (2010), arXiv:1002.0371 [hep-lat].
[20] C. Glasman [H1 Collaboration and ZEUS Collaboration], J. Phys. Conf. Ser. **110** 022013 (2008), arXiv:0709.4426 [hep-ex]
[21] J. Blumlein, H. Bottcher, and A. Guffanti, Nucl. Phys. B774, 182 (2007), arXiv:hep-ph/0607200.
[22] S. Alekhin, J. Blumlein, S. Klein and S. Moch, Phys. Rev. D81, 014032 (2010), arXiv:0908.2766 [hep-ph]
[23] P. Jimenez-Delgado and E. Reya, Phys. Rev. D79, 074023 (2009), arXiv:0810.4274 [hep-ph]
[24] A. D. Martin, W. J. Stirling, R. S. Thorne, G. Watt, Eur. Phys. Jour. C64, 653 (2009), arXiv:0905.3531 [hep-ph]
[25] R.S. Thorne, G. Watt, JHEP 1108, 100 (2011) , arXiv:1106.5789 [hep-ph]
[26] S. Alekhin, J. Blumlein, S.Moch, Eur. Phys. Jour. C71, 1723 (2011), arXiv:1101.5261 [hep-ph]
[27] R.D. Ball et al., Phys. Lett. B704, 36 (2011), arXiv:1102.3182 [hep-ph]
[28] R.D. Ball et al., arXiv:1110.2483 [hep-ph]
[29] N. Brambilla et al., Phys. Rev. D75, 074014 (2007), arXiv:hep-ph/0702079
[30] G. Dissertori et al., JHEP 0908, 036 (2009), arXiv:0906.3436 [hep-ph]
[31] G. Abbiendi et al. [OPAL Collab.], Eur. Phys. Jour. C71, 1733 (2011), arXiv:1101.1470 [hep-ex]
[32] S. Bethke et al. [JADE Collaboration], Eur. Phys. Jour. C64, 351 (2009), arXiv:0810.1389 [hep-ex]
[33] G. Dissertori et al., Phys. Rev. Lett. 104, 072002 (2010), arXiv:0910.4283 [hep-ph].
[34] R. Frederix, S. Frixione, K. Melnikov, G. Zanderighi, JHEP 1011, 050 (2010), arXiv:1008.5313 [hep-ph]
[35] R. Abbate et al., Phys. Rev. D83, 074021 (2011), arXiv:1006.3080 [hep-ph].
[36] M. Abazov et al. [D0 Collaboration], Phys. Rev. D80, 111107 (2009), arXiv:0911.2710 [hep-ex].
[37] H. Flacher et al., Eur. Phys. Jour. C60, 543 (2009), arXiv:0811.0009 [hep-ph].
[38] M. Schmelling, Phys. Scripta 51, 676(1995)
[39] S. Bethke et al., Workshop on precision measurements of α_s, Munich, Feb. 9-11, 2011; arXiv:1110.0016 [hep-ph].