UNINTEGRATED PARTON DENSITIES AND APPLICATIONS

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Different formalisms for unintegrated parton densities are discussed, and some results and applications are presented.

1. Introduction

Calculations based on *collinear factorization* work very well in many applications, including e.g.:
- DIS at large Q^2.
- High $p_\perp$ jets in $p\bar{p}$ collisions.
- Inclusive observables.

In these cases DGLAP evolution works, and $k_\perp$-ordered chains up to a hard subcollision or a highly virtual photon dominate the parton evolution. The unintegrated parton density, $\mathcal{F}(x, k_\perp^2)$, then satisfies the relations

$$F(x, Q^2) = \int^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \mathcal{F}(x, k_\perp^2); \quad \mathcal{F}(x, k_\perp^2) = \left. \frac{\partial F(x, Q^2)}{\partial \ln Q^2} \right|_{Q^2 = k_\perp^2}. \quad (1)$$

This formalism has, however, problems for observables which are sensitive to the $k_\perp$ of the "active" quark. Some examples are:
- Transverse momentum unbalance in 2-jet events.
- Heavy quark production.
- Forward jets at small x_{Bj}.

In many of these cases calculations, which allow for one extra gluon, e.g. LO pQCD + parton showers or NLO DGLAP calculations, are able to give a good description of the data, but also these calculations have problems for observables which involve a large rapidity separation. In the following I will discuss different formalisms for $k_\perp$-factorization, non-$k_\perp$-ordered evolution and some applications.

2. Non-$k_\perp$-ordered Evolution and $k_\perp$-Factorization

At small x and limited Q^2 non-$k_\perp$-ordered chains give important contributions. In a formalism based on $k_\perp$-factorization the non-integrated pdfs $\mathcal{F}(x, k_\perp^2, Q^2)$ may depend on 2 scales, the $k_\perp$ of the active parton and the virtuality of the photon or the hard scattering. The second scale is then related to a limiting angle for the emissions, as discussed further below. It is also important to remember that the gluon distribution is not an observable, and depends on the theoretical formalism.

2.1. *BFKL*

In the BFKL evolution[1], accurate to leading $\log 1/x$, $\mathcal{F}(x, k_\perp^2)$ depends only on a single scale $k_\perp^2$, and satisfies an integral equation with a kernel $K(\mathbf{k}_\perp, \mathbf{k}'_\perp)$:

$$\mathcal{F}(x, k_\perp^2) = \mathcal{F}_0 + \frac{3\alpha_s}{\pi}\int\frac{dz}{z}\int d^2k'_\perp K(\mathbf{k}_\perp, \mathbf{k}'_\perp)\mathcal{F}(\frac{x}{z}, k_\perp'^2). \tag{2}$$

We note that the dominant leading log behaviour originates from the $1/z$ pole in the splitting function. This leading order result has problems because the NLO corrections are very large. There are e.g. large effects from energy conservation, as demonstrated e.g. in MC studies by J. Andersen *et al.*[2], which imply that analytic calculations often are unreliable.

The kernel K in Eq. (2) describes the emission of a quasi-real gluon with transverse momentum $\mathbf{q}_\perp$ from a virtual link with momentum $\mathbf{k}'_\perp$, which after the emission gets momentum $\mathbf{k}_\perp = \mathbf{k}'_\perp - \mathbf{q}_\perp$. The kernel has the property that small values of $\mathbf{q}_\perp$ are suppressed. Such emissions are compensated by virtual corrections, and in the BFKL formalism this is taken into account by treating the links as Reggeized gluons.

We can compare this situation with e^+e^--annihilation. Here the total cross section is determined by the lowest order diagram (α_s^0). The lowest order contribution to 3-jet events is $\mathcal{O}(\alpha_s)$, and to this order there are negative contributions to σ_{2jet}, such that σ_{tot} is approximately unchanged. In this case the gluon emissions can be treated with Sudakov form factors.

For a link in the BFKL chain the $\mathcal{O}(\alpha_s)$ corrections give a compensation for emissions for which $q_\perp < k_\perp,\ k'_\perp$. These soft emissions give no contribution to the inclusive cross section, i.e. to F_2. They must, however, be added for exclusive final states, with appropriate Sudakov form factors. The net result of this is that downward steps in $k_\perp$ are suppressed by a factor $k_\perp^2/k_\perp'^2$. In the relevant variable $\ln k_\perp^2$, this corresponds to an exponential suppression allowing downward steps within ~ 1 unit in $\ln k_\perp^2$ [3].

2.2. *The CCFM model*

An evolution equation which interpolates between BFKL and DGLAP was formulated by Catani, Ciafaloni, Fiorani, and Marchesini[4]. This formalism, the CCFM model, is based on a different separation between initial and final state radiation (denoted ISR and FSR respectively). Some soft emissions are included in the ISR, but this increase is compensated by "non-eikonal" form factors. The ISR ladder satisfies the following constraint:

In the ISR emissions *the colour order agrees with the order in energy* (or order in lightcone momentum $p_+ = p_0 + p_L$). (Thus all emissions which in colour order are followed by a more energetic one are treated as FSR.) With this constraint colour coherence implies that *this ordering also coincides with the ordering in angle, or rapidity.*

This angular ordering implies that the emission angle for the last emission must be known, when a new step is taken in the evolution. This angle therefore appears as a limiting angle in the non-integrated distribution function used in the evolution. Thus the distribution $\mathcal{F}(x, k_\perp^2, \bar{q})$ depends on *two* scales, $k_\perp^2$ and $\bar{q}$, where the limiting angle is specified by $y_{\text{limit}} = \ln(xM_p/\bar{q})$ (in the proton rest frame).

For exclusive final states, the final state radiation has to be added in appropriate kinematical regions. As the initial state radiation is ordered in p_+ but not in p_-, also the final state radiation is unsymmetric with respect to the initial proton and photon directions.

We can compare with the BFKL and DGLAP formalisms, where the parton distributions depend on a single scale. The initial state radiation is strongly ordered either in q_+ (for BFKL), or in $q_\perp$ (for DGLAP). In both cases this ordering also implies a strong ordering in y. CCFM interpolates between the two regions at the cost of a more complicated formalism.

That more emissions are included in the ISR implies that the average step is shorter. Therefore the angular ordering constraint becomes more important and implies a strong dependence upon the scale $\bar{q}$. This feature also implies that it has not been easy to implement the CCFM model in an event generator, but such a program, CASCADE by Hannes Jung[5], is now available. The CCFM model does not include quark links in the parton chains. The original model, and also the first version of the CASCADE program, referred to as JS, also included only the singular term $\propto 1/z$ in the splitting function. To include the non-singular terms is not straight forward, but one possible solution is implemented in a new fit, called J2003[6].

In this fit set 1 includes only the singular terms, while in set 2 also the non-singular terms in the splitting function are included.

2.3. *The Linked Dipole Chain Model*

The Linked Dipole Chain Model, LDC[7], is a reformulation and generalisation of the CCFM model. It is based on a different separation between initial and final state radiation. The ISR is ordered in both q_+ and q_-, and satisfies the constraint $q_{\perp i} > \min(k_{\perp i}, k_{\perp i-1})$. Softer emissions are treated as final state emissions. In this respect the LDC formalism is more similar to BFKL. We note that the ordering in q_+ and q_- also implies an ordering in angle. An important property is also that the parton chain is fully symmetric with respect to the two ends of the chain.

The fact that fewer emissions are treated as ISR implies that a single chain in LDC corresponds to the collective contributions from several possible chains in CCFM. Now it turns out that summing over all possible emissions in CCFM, the non-eikonal form factors exactly cancel. The result is a simple evolution equation in terms of a *single scale* unintegrated density function $\mathcal{F}(x, k_\perp^2)$. In the MC implementation it is, however, also possible to add an angular cut and thus obtain results for a two-scale distribution.

We note that to leading order the LDC and CCFM formalisms give the same result for the integrated structure functions. The parton chains and the unintegrated distributions differ, however, and only after addition of final state emissions in the different relevant kinematic regions do the two formalisms also give the same result for exclusive final parton states (to leading order).

Thus the LDC formalism results in a much simplified evolution equation. Other merits of the formalism include:

- It contains the same chains as in DGLAP for Q^2 large, which makes it easier to interprete the differences between large and small Q^2.

- There is a natural generalization to include subleading terms, e.g. quark links, non-singular terms in the splitting functions, and a running α_s.

- It is suitable for implementation in a MC, and the event generator LDCMC is produced by Lönnblad and Kharraziha[8].

- The MC gives very good fits to experimental F_2 data, and it also agrees well with MRST and CTEQ results for the *integrated* gluon distribution[9].

2.4. *Other formalisms for unintegrated parton densities*

A different formalism, which also interpolates smoothly between BFKL and DGLAP, was formulated by Kwieciński, Martin, and Staśto (KMS)[10]. This formulation is based on a single scale evolution equation. The contributions from $k_\perp^2 > Q^2$ are neglected, and thus the gluon distribution satisfies Eq. (1). This formalism has been further developed by Kimber, Martin and Ryskin (KMR)[11]. In their approach the single scale KMS evolution is used, but an angular constraint is applied for the last step. The result is therefore a density distribution, which depends on two scales. Since the underlying KMS evolution does not include the soft initial state radiation in the CCFM model, it also takes fewer and larger steps, which implies that the dependence on the limiting angle is significantly smaller than for the CCFM formalism.

3. Comparison between results from different formalisms

We here want to compare the parton densities obtained in different formalisms, and also study the effects of the non-singular terms in the splitting functions and of quark links in the evolution chains. Some results are shown in Fig. 1[9 12]. Among the LDC fits the result denoted *standard* contains both quark links and non-singular splitting terms, *gluonic* contains only gluon links, and *leading* only singular terms. (In the fit *gluonic-2*, refered to in the next section, the power of $(1-x)$ in the input gluon density is changed to 7 instead of its value 4 in *gluonic*.) In all cases the input distributions for $k_\perp^2 = Q_0^2$ are adjusted to give good fits to experimental F_2 data. The CCFM result J2003 set 1 contains only gluons and only the $1/z$ pole in the splitting function. We see that the effect of quarks and non-singular terms become larger for larger $k_\perp$. The sensitivity to the scale $\bar{q}$ is illustrated in Fig. 2a, which shows the gluon density $\mathcal{F}(x, k_\perp^2, \bar{q})$ as function of $\bar{q}/k_\perp$ for fixed $k_\perp$. As discussed above, in the CCFM formalism the density is very sensitive to the $\bar{q}$ scale, and varies strongly for $\bar{q}/k_\perp$ between 1 and 2.

It is, however, interesting to note that observable cross sections differ much less than the parton densities, as seen in the next section. The reason is that the distributions differ in particular for $\bar{q} < k_\perp$, while in a hard subcollision the dominant contributions are obtained for $k_\perp^2 \approx \bar{q}^2/4$ [9]. In Fig. 2b we see that the results are indeed not so different for $\bar{q} = 2k_\perp$. In this figure also the single scale result from KMS and the derivative of the GRV result (denoted dGRV) are included.

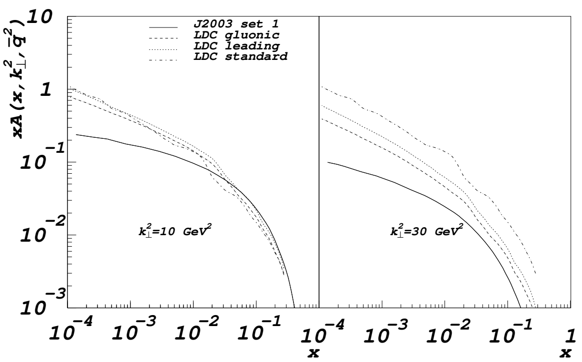

Figure 1. Comparison of different sets of unintegrated gluon densities at scale $\bar{q} = 10$ GeV. For the notations, see the main text.

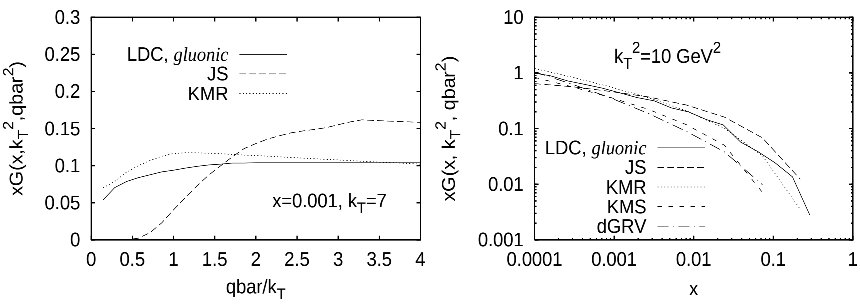

Figure 2. The LDC *gluonic* unintegrated gluon density compared to the results of JS and KMR. *Left*: As function of $\bar{q}/k_\perp$ for fixed x and $k_\perp$. *Right*: As function of x for $k_\perp^2 = 10$ GeV^2 and $\bar{q} = 2k_\perp$. Also shown are here the single scale results from KMS and the derivative of the GRV fit.

4. Applications

4.1. *Heavy quark production*

Applications of the $k_\perp$-factorization formalism to b quark production at the Tevatron are shown in Fig. 3. We see that for both the LDC and the CCFM models the fits with only the leading term reproduce the data best, while the other fits, which should be expected to be more accurate, although being significantly better than the NLO QCD result, do not give equally good fits. We should note, however, that b production is also well reproduced by collinear factorization plus parton showers, as implemented in the PYTHIA event generator[17 18].

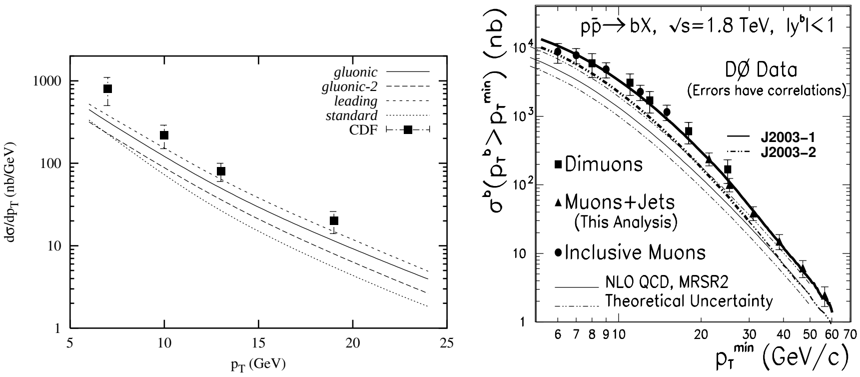

Figure 3. *Left*: Production of b-quarks at CDF[13] compared to results from the LDC model[14]. *Right*: D0 data[15] compared with CCFM results[16]. Here also a NLO QCD result is included. For notations, see the main text.

4.2. *Forward jets*

Results for forward jet production at HERA are shown in Fig. 4. Here we have a comparatively large separation in rapidity, and we see that LO and NLO dijet calculations are far below the data. The CCFM model gives a much better description, but also here we see that the best fit is obtained including only the singular terms in the splitting function. The reason for this is still not understood; is there some dynamical mechanism which somehow compensates the effect of the non-singular terms?

4.3. *Minimum bias and underlying events in pp collisions*

In hadron-hadron collisions collinear factorization works well for calculations of high-$p_\perp$ jets. However, in this formalism the minijet cross section diverges with $\sigma_{jet} \sim 1/p_\perp^4$, which implies that also the total $E_\perp$ diverges. This implies the need for a soft cutoff, and in PYTHIA fits to experimental data give a cutoff $p_{\perp 0} \sim 2$ GeV. This cutoff is also growing with energy, which makes it difficult to extrapolate safely to the high energies at LHC.

The symmetry between the two ends of the parton chain implies that the LDC formalism also is applicable to hadron-hadron collisions[20]. In the

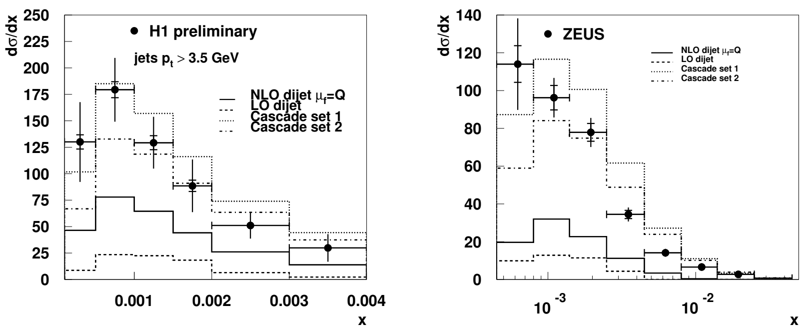

Figure 4. Forward jet data from H1 and ZEUS compared to the CASCADE MC and LO and NLO dijet calculations[19].

$k_\perp$-factorization formalism the off-shell matrix element does not blow up when the exchanged transverse momentum $k_\perp \to 0$, and we have therefore a dynamical cutoff for soft minijets. The cross section for a *chain* in *pp* collisions (which possibly may contain more than one hard subcollision) can thus be obtained from the fit to DIS data. An important point is here that the result is insensitive to the soft cutoff, Q_0, in the evolution. DIS data can be fitted with different values for Q_0, if the input distribution $f_0(x, Q_0^2)$ is adjusted accordingly. If Q_0 is increased, the number of hard chains decreases, but at the same time the number of soft chains (for which all emissions have $q_\perp < Q_0$) increases, so that the total number of chains is approximately unchanged.

There are two sources for *multiple interactions*: It is possible to have two hard scatterings in the same chain, and there may be more than one chain in a single event. The LDC model, when applied to *pp* collisions, can predict the correlations between hard scatterings within one chain, and also the average number of chains in a single event. The experimentally observed "pedestal effect" indicates that the hard subcollisions are highly correlated, so that central collisions have many minijets, while peripheral collisions have fewer minijets. In PYTHIA comparisons with data favour a distribution in the number of subcollisions, which is very close to a geometric distribution[21].

Some preliminary results from the LDC model are shown in fig. 5. Here a geometric distribution is assumed for the number of chains in one event. Fig. 5a shows the number of minijets in the "minimum azimuth region" $60° < \phi < 120°$ at $\sqrt{s} = 1.8$ TeV. The two LDC curves are obtained for soft cut-off values 0.99 and 1.3 GeV, showing the insensitivity to this cut-off. The two PYTHIA curves correspond to default parameter values, and parameters tuned to CDF data[22]. We note that the LDC result agrees very well with the tuned PYTHIA result. Fig. 5b shows corresponding results for LHC. Also here the two curves correspond to different cut-off values, and for comparison the result for 1.8 TeV is also indicated. We see that the activity increases by a little more than a factor of 2 between the two energies.

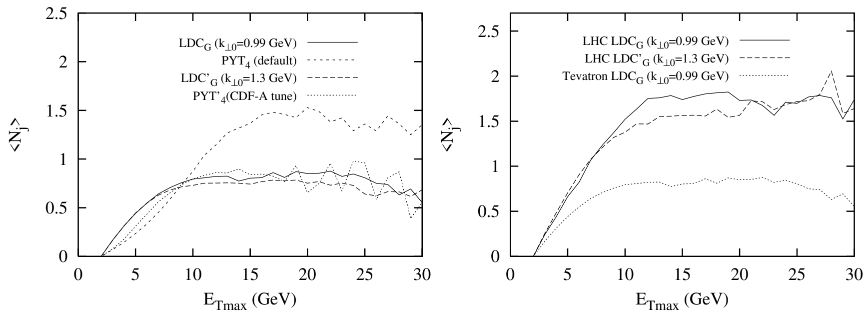

Figure 5. The average number of minijets in the "minimum azimuth region" for $|\eta| < 2.5$ *vs.* $E_\perp$ for the hardest jet. *Left*: For $\sqrt{s} = 1.8$ TeV. *Right*: For 14 TeV.

The symmetry of the formalism implies that the chains join at one end at the same rate as they multiply at the other. The chain cross section grows like s^λ, and therefore the average chain multiplicity satisfies $<n_{\rm chain}> \propto s^\lambda/\sigma_{\rm tot}$. Thus the results also may have implications for unitarization, saturation and diffraction. Work in these areas is in progress.

5. Conclusions

Our main conclusions can be summarized as follows:

- Unintegrated parton densities are not observables, and their properties depend strongly on the definitions.

- Different formalisms give similar results for $\mathcal{F}(x, k_\perp^2, \bar{q}^2 = 4k_\perp^2)$. This also implies that the predictions for observable quantities often are similar.

- The rôle of the non-singular terms in the splitting functions is still a problem.

- Observables without a large rapidity separation are often well described by higher order matrix elements plus DGLAP evolution.

- There is a close relation between DIS and high energy *pp* collisions. The properties of the underlying event and minimum bias events can be predicted from DIS data.

Acknowledgements

I am indebted to Leif Lönnblad and Hannes Jung for help in the preparation of this talk.

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