SINGLE HADRON PRODUCTION IN DEEP INELASTIC SCATTERING

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The NLO-QCD correction to single hadron production in deep inelastic scattering is calculated. We require the final state meson to carry a non-vanishing transversal momentum, thus being sensitive to perturbative QCD effects. Factorization allows us to convolute the hard scattering process with parton densities and fragmentation functions. The predictions are directly comparable to experimental results at the HERA collider at DESY. The results are sensitive to the gluon density in the proton and allow us to test universality of fragmentation functions.

1. Introduction

The predictive power of QCD lies in the factorization theorem. In deep inelastic scattering (DIS) factorization in short and long distance parts allows us to describe the observed hadrons as a convolution of the partonic processes with non-perturbative parton densities and fragmentation functions [1]. Single meson production in electron proton scattering

$$e^-(k) + P(p_a) \to e^-(k') + h(p_b) + X \tag{1}$$

occurs partonically already in the absence of strong interactions ($\mathcal{O}(\alpha_S^0)$), where one parton of the proton (a quark) interacts with the leptonic current and fragments into a meson (h) (naive parton model).
Since we are interested in perturbative QCD effects we require the meson to carry a non-vanishing transversal momentum with respect to the centre-of-mass frame of virtual vector boson coming from the electron and initial proton ($p_{b\perp} > 0$). Thus at partonic level at least two final state partons are required to balance the transversal momentum. The leading order processes with non-vanishing transversal momentum of a fragmenting parton into a meson ($\mathcal{O}(\alpha_S)$) are

(a) $\gamma^* + q \to q + g$

(b) $\gamma^* + g \to q + \overline{q}$,

where the virtual photon originates from the electron current. If the virtuality of the photon $Q^2 := -q^2$ ($q := k - k'$) is not too large compared to the squared Z-mass, the contribution from Z boson exchange is suppressed and may be neglected. Dealing then with C-invariant processes we do not have to calculate separately the processes with interchanged quarks and antiquarks. Factorization, proven generally for DIS processes [1], describes the events as a convolution of the hard scattering processes with parton densities and fragmentation functions.

The investigation of single hadron production is interesting for several reasons: First of all it is a test of perturbative QCD and factorization. The predictions depend beside the perturbative partonic calculation essentially on universal parton densities and fragmentation functions. Especially fragmentation functions, fitted to electron positron annihilation data, may be tested, in particular their universality.

Further, the predictions allow for a direct comparison with experimental data, in particular there is no need to use any kind of Monte Carlo procedure. Thus we may expect very meaningful results.

The predictions are due to process (b) directly sensitive to the gluon density in the proton and may allow us to draw conclusions concerning the gluon density in the proton. From the experimental side precise data are available from the HERA collider at DESY. For instance π mesons were measured in the forward region (with small angles with respect to the proton remnant) based on events detected in the H1 detector [2] and charged hadrons were measured at the ZEUS experiment [3].

In 1978 the process (1) with non-vanishing transversal momentum of the hadron ($p_{b\perp}$) was calculated by Méndez [4] at tree level accuracy ($\mathcal{O}(\alpha_S)$). Since QCD corrections are typically large and we are confronted with precise experimental data it is desirable to compare these data with predictions of at least next-to-leading order (NLO) ($\mathcal{O}(\alpha_S^2)$) accuracy. Also NLO-QCD predictions were computed with the assumption of purely transversal photons, neglecting the longitudinal degrees of freedom of the exchanged virtual photon [5].

Here we present a NLO-QCD calculation based on the dipole subtraction formalism [6]. In contrast to the more conventional phase space slicing method there is no need to introduce any unphysical parameter to cut the phase space in soft, respectively collinear regions. Also all cancellations of infrared singularities occur before any numerical phase space integration is

performed. Thus we may present numerically very stable predictions.

2. Calculation

The differential cross section for process (1) reads as a convolution with the parton densities and fragmentation functions as

$$\frac{d\sigma^h}{d\overline{x}dyd\overline{z}d\phi} = \sum_{ab} \int\limits_{\overline{x}}^{1} \frac{dx}{x} \int\limits_{\overline{z}}^{1} \frac{dz}{z}\, f_a(\frac{\overline{x}}{x}, Q^2)\, \frac{d\sigma^{ab}}{dxdydzd\phi}\, D_b^h(\frac{\overline{z}}{z}, Q^2), \qquad (2)$$

where, as usual, the variables x, y, and z are defined as $x = \frac{Q^2}{2p_a q}$, $y = \frac{p_a q}{p_a k}$, $z = \frac{p_a p_b}{p_a q}$ with respect to the partonic momenta and the bar quantities $\bar{x}$, $\bar{y}$, $\bar{z}$ with respect to the momenta of the hadrons (with $\bar{y} = y$). The angle ϕ denotes the azimuthal angle between the planes defined on one hand by the directions of the leptons and on the other hand by the momenta of final state hadron and virtual vector boson in the centre-of-mass frame of vector boson and initial parton. The sum is over the different initial (a) and final (b) state partons and f_a and D_b^h denote the corresponding parton density respectively fragmentation function. The hard scattering process may be written as a contraction of a lepton tensor ($l^{\mu\nu}$) with a hadron tensor ($H_{\mu\nu}^{ab}$):

$$\frac{d\sigma^{ab}}{d\overline{x}dyd\overline{z}d\phi} = \frac{\alpha^2}{16\pi^2} y \frac{1}{Q^4} l^{\mu\nu} H_{\mu\nu}^{ab}. \qquad (3)$$

If we consider the centre-of-mass system of virtual photon and initial parton, both unpolarized, there cannot be any dependence on the azimuthal angle ϕ. Integrating out this angle dependence we find a decomposition into a transversal and a longitudinal part of the virtual photon:

$$\frac{d\sigma^{ab}}{d\overline{x}dyd\overline{z}d\phi} = \frac{\alpha^2}{8\pi} y \frac{1}{Q^4} \left\{ Q^2 \frac{2y - y^2 - 2}{2y^2} g^{\mu\nu} + 2Q^4 \frac{y^2 - 6y + 6}{\hat{s}^2 y^4} p_a^\mu p_a^\nu \right\} H_{\mu\nu}^{ab}. \qquad (4)$$

The computation of the correction to process (1) were carried out in the subtraction formalism [6]. The general idea of the subtraction formalism is to subtract from the real correction an artificial counterterm which has the same pointwise singular behaviour in $D = 4 - 2\epsilon$ dimensions as the real correction itself. Thus the limit $\epsilon \to 0$ can be performed and the real phase space integral can be evaluated numerically. The artificial counterterm is constructed in a way that it also can be integrated over the one-parton subspace analytically leading to ϵ poles. Adding these terms to the virtual part of the correction these poles cancel all singularities in the virtual part

analytically and the remaining integration over the phase space can be carried out numerically. The advantage compared to phase space slicing methods is that all singularities cancel *before* any numerical integration is performed. Also there is no need to introduce an unphysical cut parameter which in phase space slicing methods separates soft, respectively collinear phase space regions from the remaining hard region.
In our case of a convolution of a partonic cross section with distribution functions additional kinematic constraints have to be taken into account. The momenta of the partons which enter the convolution have to be kept fixed (called *identified* partons). This leads to modified artificial counterterms. Its analytical integration gives collinear singularities which cancel the singularities of the non-perturbative distribution functions yielding scheme and scale dependent convergent parts.

The whole calculation was done with the help of the algebra package Form [8]. At $\mathcal{O}(\alpha_S^2)$ the real correction is given by the squared matrix elements of the diagrams

(a) $\gamma^* + q \to q + g + g$
(b) $\gamma^* + g \to q + \overline{q} + g$
(c) $\gamma^* + q \to q + q + \overline{q}$,

where in turn each final state parton serves as an observed hadron. In process (c) we have to consider two flavours in the fermion traces. This yields 4 diagrams for every pair of different flavours in contrast to 8 diagrams in the case of one uniform flavour. Care must be taken to adjust statistical factors properly. The artificial counterterm was constructed as described by the subtraction formalism. The phase space integral over the 3-particle final state can be performed yielding a finite real contribution. The virtual contribution of the correction, i.e. the interference term of the Born matrix elements and the one-loop matrix elements is computed. Here we encounter 2-point, 3-point, and 4-point tensor integral contributions which were reduced to scalar integrals via tensor reduction [9]. The scalar integrals, containing ultraviolet and infrared singularities, were computed analytically in dimensional regularization. The analytic expressions were compared with the literature [7]. The virtual contribution was renormalized in a mixed scheme, where the wave functions were renormalized on-shell and the strong coupling constant in the $\overline{MS}$ scheme, yielding an ultraviolet finite virtual contribution. The infrared singularities cancel exactly the contributions given by the integrated artificial counterterm in the subtraction formalism.

From the subtraction of remaining singularities into the parton densities and the fragmentation functions we obtain also finite remainders which depend on the factorization scheme and on the factorization scale. In this context we choose the $\overline{MS}$ scheme.

Thus we end up with three contributions, the real part, the virtual part, and the part related to identified partons. All these finite parts have to be integrated over the 2- respectively 3- particle final state phase space. To this purpose a C-routine was written to perform these integrations numerically.

3. Results

In the convolution (2) we use the parton densities published by the CTEQ collaboration [10]. Herein the parton distribution set, called CTEQ5M, where M denotes the $\overline{MS}$ scheme, matches our conditions with the assumption of 5 light quarks.

We adopt the KKP fragmentation functions to our calculation [11]. Since these fragmentation functions are fitted to e^+e^- data with high accuracy and applied here to a DIS process the comparison with experimental data will serve as a good check of universality.

We use the value of the strong coupling constant at the Z-scale as given by the Particle Data Group [12] $\alpha_S(M_Z) = 0.118$ and evolve this value with the NLO evolution equation. In the LO approximation we evolve this value with the LO evolution equation for consistency reasons.

In Fig. 1 we show the differential cross section $\frac{d\sigma^h}{d\bar{x}dyd\bar{z}}$ as a function of $p_{b\perp}$ for fixed values of $\bar{x} = 0.1$, $y = 0.1$, $\bar{z} = 0.5$, and $Q^2 = 360$ GeV^2, where $p_{b\perp}$ is defined in the centre-of-mass frame of vector boson and initial parton. We set the renormalization scale (μ_R) equal to both the initial (μ_{F_i}) and the final (μ_{F_f}) fragmentation scales $\mu := \mu_R = \mu_{F_i} = \mu_{F_f}$, where the initial scale is related to the parton densities and the final scale to the fragmentation functions.

The steep fall of the cross section over several orders of magnitude makes it hard to resolve the NLO contribution (full line) in contrast to the LO prediction (dashed line) graphically. The lower part of the figure shows the ratio of the NLO correction over the LO result and displays more clearly the higher order effects. The correction changes from +10 % for small $p_{b\perp}$ to −10 % for large $p_{b\perp}$. For $p_{b\perp} \approx 1$ GeV we get an even larger correction due to collinear configurations.

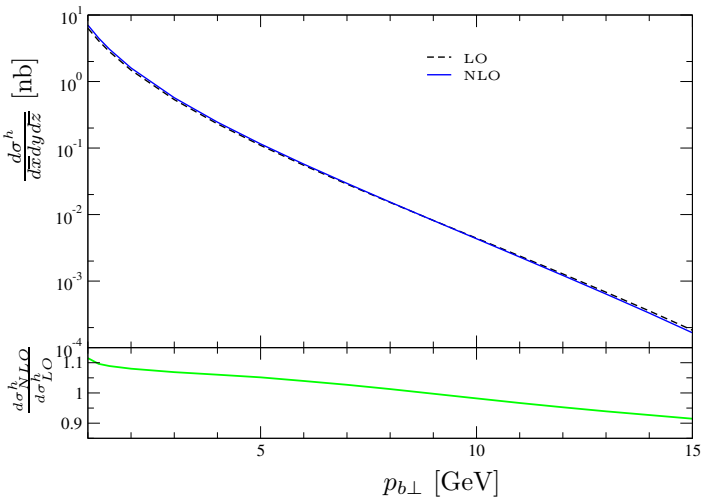

Figure 1. Differential cross section versus $p_{b\perp}$ for $\bar{x} = 0.1$, $y = 0.1$, and $\bar{z} = 0.5$ with $Q^2 = 360$ GeV^2. The lower part shows the corresponding K-factor $\frac{d\sigma^h_{NLO}}{d\sigma^h_{LO}}$ of the differential cross sections at NLO and LO.

The theoretical uncertainty of the predictions of a fixed order calculation is mainly given by the renormalization and fragmentation scale dependence. The theoretical uncertainties due to the parton densities and fragmentation functions are not considered here. Another source of uncertainty lies in the factorization theorem itself since it predicts correct results with an error of $\mathcal{O}(\Lambda^2_{QCD}/p^2_{b\perp})$ which may be become large for very low $p_{b\perp}$.

We check the scale dependence of the cross section for the kinematics already depicted in context with Fig. 1 but with a fixed transversal hadron momentum of $p_{b\perp} = 5$ GeV. In Fig. 2 the differential cross section is shown where we vary the common scale μ^2 over two orders of magnitude with respect to the reference scale $\mu_0^2 = Q^2$. The scale dependence varies in this range about +33 % (+27 %) for very low scales to -20 % (-18 %) for rather high scales at LO (NLO). Thus there is only an unexpectedly slight scale dependence reduction at NLO compared to LO.

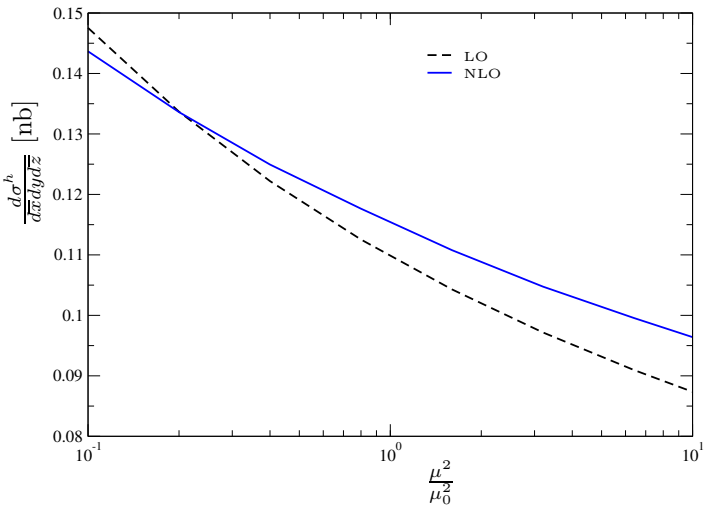

Figure 2. Scale dependence of the differential cross section for $\bar{x} = 0.1$, $y = 0.1$, and $\bar{z} = 0.5$ with $Q^2 = 360$ GeV^2 and a fixed $p_{b\perp} = 5$ GeV. The scale $\mu^2 := \mu_R^2 = \mu_{F_i}^2 = \mu_{F_f}^2$ is varied with respect to the reference scale $\mu_0^2 = 360$ GeV^2.

4. Conclusion

The calculation of single hadron production in deep inelastic scattering to $\mathcal{O}(\alpha_S^2)$ was presented. Owing to the factorization theorem this calculation was performed as a convolution of the hard scattering process with universal and process independent parton densities and fragmentation functions. This allows for meaningful predictions, directly comparable to experimental data.

Results were shown for specific kinematic values. A correction of the order of several percent is predicted depending on the transversal momentum of the observed hadron. The scale dependence of the correction gives slightly reduced theoretical uncertainties compared to LO predictions. Varying the renormalization and factorization scales over two orders of magnitude yields theoretical uncertainties of about ± 25 % at NLO.

An extensive study of NLO effects will be published elsewhere which in particular will include a detailed comparison of predictions with experimental data from the HERA experiments H1 and ZEUS.

Note added

Just after finishing the calculation, a preprint appeared on NLO calculations of hadron production with non-vanishing transversal momentum [13]. In this paper matrix elements were adopted from the DISENT program package and the phase space slicing method was applied to handle singularities.

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