DEEP-INELASTIC STRUCTURE FUNCTIONS AT NNLO AND BEYOND

S. MOCH

Deutsches Elektronensynchrotron DESY
Platanenallee 6, D–15738 Zeuthen, Germany

J.A.M. VERMASEREN AND A. VOGT

NIKHEF Theory Group
Kruislaan 409, 1098 SJ Amsterdam, The Netherlands

We have calculated the fermionic contributions to the flavour non-singlet structure functions in deep-inelastic scattering (DIS) at third order of massless perturbative QCD. We obtain complete results for the corresponding n_f-parts of the three-loop anomalous dimension and the three-loop coefficient functions for the structure functions F_2 and F_L. Our results agree with all partial and approximate results available in the literature. We discuss their implications for the threshold resummation at the next-to-next-to-leading logarithmic accuracy.

1. Introduction

The calculation of perturbative QCD corrections for deep-inelastic structure functions is an important task. The present and expected future experimental precision, for instance at HERA, calls for complete next-to-next-to-leading order (NNLO) predictions. These offer the possibility to determine the strong coupling constant α_s and to analyze the proton structure and its parton content with unprecedented precision. Knowledge of the latter is of particular importance for the analysis of hard scattering reactions at future LHC experiments.

At present, this level of accuracy is not yet possible, because the necessary anomalous dimensions governing the parton evolution at NNLO are not fully known. The two-loop coefficient functions of F_2, F_3 and F_L have been calculated some time ago [1,2,3,4,5], but for the corresponding three-loop anomalous dimensions $\gamma^{(2)}_{\mathrm{pp'}}$, only a finite number of fixed Mellin moments [6,7,8] are available. As a first step towards the complete calculation, we have computed the fermionic three-loop contributions to the flavour non-singlet (NS) structure functions F_2 and F_L in unpolarized electromagnetic deep-inelastic scattering [9]. Already these results have immediate consequences for threshold resummation of soft gluons which we will discuss in the following.

2. Results

We are considering only the non-singlet structure functions and the method for the calculation of their moments closely follows ref. [6]. Hence there is not much need to explain the physics of the method here again. Thus we will discuss only the differences introduced by the fact that we now compute all moments simultaneously as a function of the moment number N. Since N is not a fixed constant, we cannot resort to the techniques of ref. [6], where the Mincer program [10,11] was used as the tool to solve the integrals. Instead, we will have to introduce new techniques. However, we can give N a positive integer value at any point of the derivations and calculations, after which the Mincer program can be invoked to verify that the results are correct. From a practical point of view this is the most powerful feature of the Mellin-space approach, as it greatly simplifies the checking of all programs.

Similar to the fixed-N computations of refs. [7,8], the diagrams are generated automatically with a special version of the diagram generator QGRAF [12]. For all the symbolic manipulations of the formulae we use the latest version of the program FORM [13]. The calculation is performed in dimensional regularization [14,15,16,17] with $D = 4 - 2\epsilon$. Hence the unrenormalized Mellin-space results will be functions of ϵ, N, and the values $\zeta_{3,\ldots,5}$ of the Riemann ζ-function. The renormalization is carried out in the $\overline{\mathrm{MS}}$-scheme [18,19] as described in ref. [6].

The major difference to the fixed-moment calculations is now that we obtain much longer results due to the presence of the parameter N in the answer. We have checked the correctness of each individual diagram for several values of N, by comparing with the results of a Mincer calculation. In addition we have compared the complete renormalized results with the results in the literature for the available values of N.

The perturbative expansion of the non-singlet coefficient functions and anomalous dimensions can be written as

$$C_{i,\mathrm{ns}}(\alpha_s, N) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n c^{(n)}_{i,\mathrm{ns}}(N) \ , \tag{1}$$

$$\gamma_{\mathrm{ns}}(\alpha_s, N) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^{n+1} \gamma^{(n)}_{\mathrm{ns}}(N) \tag{2}$$

with $i = 2, L$ in eq. (1).

Our N-space results are expressed in terms of harmonic sums $S_{\vec{m}}(N)$. In the following all harmonic sums are understood to have the argument N, i.e., we employ the short-hand notation $S_{\vec{m}} \equiv S_{\vec{m}}(N)$. In addition we use operators $\mathbf{N}_{\pm}$ and $\mathbf{N}_{\pm\mathbf{i}}$ which shift the argument N of a given function by ± 1 or a larger integer i,

$$\mathbf{N}_{\pm}\, f(N) \; = \; f(N \pm 1) \ , \qquad \mathbf{N}_{\pm\mathbf{i}}\, f(N) \; = \; f(N \pm i) \ . \tag{3}$$

We normalize the trivial leading-order (LO) coefficient function and recover, of course, the well-known result for the LO anomalous dimension [20,21]

$$c^{(0)}_{2,\mathrm{ns}}(N) = 1 \ , \tag{4}$$

$$\gamma^{(0)}_{\mathrm{ns}}(N) = C_F\big(2(\mathbf{N}_- + \mathbf{N}_+)S_1 - 3\big) \ . \tag{5}$$

At next-to-next-to-leading order we have computed all fermionic contributions to the splitting function $\gamma_{\rm ns}^{(2)}$,

$$
\begin{aligned}
\gamma_{\rm ns}^{(2)}(N) = 16\, C_A C_F n_f \Big(& \frac{3}{2}\zeta_3 - \frac{5}{4} + \frac{10}{9}S_{-3} - \frac{10}{9}S_3 + \frac{4}{3}S_{1,-2} - \frac{2}{3}S_{-4} \\
& + 2S_{1,1} - \frac{25}{9}S_2 + \frac{257}{27}S_1 - \frac{2}{3}S_{-3,1} - \mathbf{N}_+\Big[S_{2,1} - \frac{2}{3}S_{3,1} \\
& - \frac{2}{3}S_4\Big] + (1-\mathbf{N}_+)\Big[\frac{23}{18}S_3 - S_2\Big] - (\mathbf{N}_- + \mathbf{N}_+)\Big[S_{1,1} \\
& + \frac{1237}{216}S_1 + \frac{11}{18}S_3 - \frac{317}{108}S_2 + \frac{16}{9}S_{1,-2} - \frac{2}{3}S_{1,-2,1} - \frac{1}{3}S_{1,-3} \\
& - \frac{1}{2}S_{1,3} - \frac{1}{2}S_{2,1} - \frac{1}{3}S_{2,-2} + S_1\zeta_3 + \frac{1}{2}S_{3,1}\Big]\Big) + 16\, C_F {n_f}^2 \Big(\frac{17}{144} \\
& - \frac{13}{27}S_1 + \frac{2}{9}S_2 + (\mathbf{N}_- + \mathbf{N}_+)\Big[\frac{2}{9}S_1 - \frac{11}{54}S_2 + \frac{1}{18}S_3\Big]\Big) \\
& + 16\, {C_F}^2 n_f \Big(\frac{23}{16} - \frac{3}{2}\zeta_3 + \frac{4}{3}S_{-3,1} - \frac{59}{36}S_2 + \frac{4}{3}S_{-4} - \frac{20}{9}S_{-3} \\
& + \frac{20}{9}S_1 - \frac{8}{3}S_{1,-2} - \frac{8}{3}S_{1,1} - \frac{4}{3}S_{1,2} + \mathbf{N}_+\Big[\frac{25}{9}S_3 - \frac{4}{3}S_{3,1} - \frac{1}{3}S_4\Big] \\
& + (1-\mathbf{N}_+)\Big[\frac{67}{36}S_2 - \frac{4}{3}S_{2,1} + \frac{4}{3}S_3\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[S_1\zeta_3 \\
& - \frac{325}{144}S_1 - \frac{2}{3}S_{1,-3} + \frac{32}{9}S_{1,-2} - \frac{4}{3}S_{1,-2,1} + \frac{4}{3}S_{1,1} + \frac{16}{9}S_{1,2} \\
& - \frac{4}{3}S_{1,3} + \frac{11}{18}S_2 - \frac{2}{3}S_{2,-2} + \frac{10}{9}S_{2,1} + \frac{1}{2}S_4 - \frac{2}{3}S_{2,2} - \frac{8}{9}S_3\Big]\Big) \\
& .
\end{aligned}
\tag{6}
$$

We have subjected our results to a number of checks. First of all, we have calculated some lower even moments in an arbitrary covariant gauge with the Mincer program [10,11], keeping the gauge parameter ξ in the gluon propagator. All dependence on ξ does cancel in the final results. Secondly the n_f^2-contribution to $\gamma_{\rm ns}^{(2)}$ is known from the work of Gracey [22] and we agree with his result. Furthermore the coefficients of $\ln^k N$, $k = 3, \ldots 5$, of $c_{2,\rm ns}^{(3)}(N)$ agree with the prediction of the soft-gluon resummation [23]. Finally, we have checked the result of each individual diagram for several integer values of N by comparing with the results of a Mincer calculation. Thus, as the strongest check, our results reproduce the fixed even moments $N = 2, \ldots, 14$ computed in refs. [6,7,8].

3. Threshold resummation

It is well known that perturbative QCD corrections to structure functions receive large logarithmic corrections, which originate from the emission of soft gluons. These corrections are relevant at large values of the scaling variable x (in Mellin space at large values of the Mellin moment N) and can be resummed to all orders in perturbation theory. It is interesting to investigate the implications of our three-loop results [9] for the threshold exponentiation [24,25,26] at next-to-next-to-leading logarithmic (NNLL) accuracy [27].

Here the quark coefficient function for F_2 can, up to terms which vanish for $N \to \infty$, be written as

$$C_2(\alpha_s, N) \;=\; (1 + a_s\, g_{01} + a_s^2 g_{02} + \ldots)\, \exp\left[\, G^N(Q^2)\,\right], \qquad (7)$$

where the resummation exponent G^N can be expanded as

$$G^N(Q^2) \;=\; L\, g_1(a_s\, L) + g_2(a_s\, L) + a_s\, g_3(a_s\, L) + \ldots \qquad (8)$$

with $a_s = \alpha_s/(4\pi)$ and $L = \ln N$. The functions g_l depend on universal coefficients $A_{i \leq l}$ and $B_{i \leq l-1}$ and process-dependent parameters $D^{\rm DIS}_{i \leq l-1}$ (see e.g. ref. [27] for the precise definitions of the functions $g_{1,2,3}$).

In a physical picture, the resummation of the perturbative expansion for C_2 rests upon the refactorization of C_2 (valid in the threshold region of phase space) into separate functions of the jet-like, soft, and off-shell quanta that contribute to its quantum corrections. Each of the functions organizes large corrections corresponding to a particular region of phase space [28].

To NNLL accuracy, the function g_3 involves the new coefficients A_3, B_2 and $D_2^{\rm DIS}$. These coefficients can be fixed by expanding eq. (7) in powers of α_s and comparing to the result of the full fixed-order calculation [9]. In the $\overline{\rm MS}$ scheme, the parameter A_3 is simply the coefficient of $\ln N$ in $\gamma_{\rm ns}^{(2)}(N)$ or, equivalently, of $1/(1-x)_+$ in $P_{\rm ns}^{(2)}(x)$. It reads

$$\begin{aligned} A_3 \;=\; & (1178.8 \pm 11.5) \\ & + C_A C_F n_f \left[-\frac{836}{27} + \frac{160}{9}\,\zeta_2 - \frac{112}{3}\,\zeta_3 \right] + C_F^2 n_f \left[-\frac{110}{3} + 32\,\zeta_3 \right] \\ & + C_F n_f^2 \left[-\frac{16}{27} \right] \;, \end{aligned} \qquad (9)$$

where $C_A = 3$, $C_F = 4/3$ and n_f is the number of light (massless) flavours. The estimate of the non-fermionic part [27] is based on the approximations of $P_{\rm ns}^{(2)}(x)$ constructed in ref. [29] using the first six even-integer moments [8] and its small-x limit [30]. The exact fermionic part has been obtained independently in refs. [9,31], while the n_f^2 contribution is already known from ref. [22].

The complete results for B_2 and $D_2^{\rm DIS}$ can be inferred from fermionic result of the three-loop coefficient function $c^{(3)}_{2,\rm ns}$ in ref. [9], yielding

$$B_2 = C_F^2 \left[-\frac{3}{2} - 24\,\zeta_3 + 12\,\zeta_2 \right] + C_F C_A \left[-\frac{3155}{54} + 40\,\zeta_3 + \frac{44}{3}\,\zeta_2 \right] + C_F n_f \left[\frac{247}{27} - \frac{8}{3}\,\zeta_2 \right] , \tag{10}$$

$$D_2^{\rm DIS} = 0 \; . \tag{11}$$

As a matter of fact, the contribution to $c^{(3)}_{2,\rm ns}$ involves only a linear combination, $\beta_0(B_2 + 2\,D_2^{\rm DIS})$, with β_0 being the one-loop coefficient of the QCD β-function. However, the different combination $B_2 + D_2^{\rm DIS}$ has been determined in ref.[27] by comparing the expansion of eq. (7) to the two-loop coefficient function $c^{(2)}_{2,\rm ns}$ of ref. [32]. Thus, B_2 and $D_2^{\rm DIS}$ can be disentangled. It is interesting to observe the vanishing of $D_1^{\rm DIS}$ and $D_2^{\rm DIS}$, for which an all-order generalization has been proposed in ref. [33,34]. This is in contrast to the Drell-Yan process, where the functions $D_l^{\rm DY}$ are generally different from zero. For instance, $D_2^{\rm DY}$ has been derived in ref. [27].

Let us briefly illustrate numerically for large N the improvement due to the NNLL corrections for the soft gluon exponent G^N of deep-inelastic scattering. In fig. 1 on the left, we show the resummation exponent $G^N(Q^2)$ of eq. (8). Here, we choose $\mu_r^2 = \mu_f^2 = Q^2$, $n_f = 4$ and $\alpha_{\rm s} = 0.2$, which corresponds to scales between about 25 and 50 GeV^2, depending on the precise value of $\alpha_{\rm s}(M_Z^2)$. In fig. 1 on the right, we display the convolution with a schematic, but typical input evaluated with the so-called 'minimal-prescription' contour [35]. It is obvious from both figures that knowledge of the leading logarithmic (LL) and next-to-leading logarithmic (NLL) terms [24,25,26] alone, i.e., those enhanced by factors $\ln N(\alpha_s \ln N)^n$ and $(\alpha_s \ln N)^n$, is not sufficient for reliably determining the function G^N and its impact after convolution even for rather large values of N and x.

The NNLL corrections discussed here are rather small over a wide range. This indicates that the soft-gluon exponent $G^N(Q^2)$ stabilizes and that the soft-gluon effects on the $\overline{\rm MS}$ quark coefficient function can be reliably estimated. Recall, however, that the NNLL corrections are large for the 'physical kernel' governing the scaling violations of the non-singlet structure function $F_{2,\rm ns}$ [36].

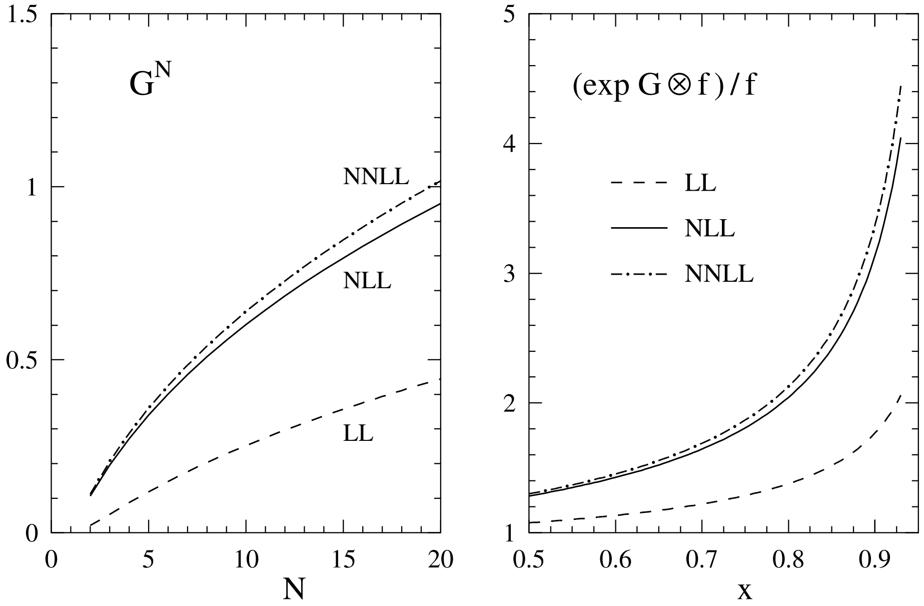

Figure 1. Left: The LL, NLL and NNLL approximations for the resummation exponent $G^N(Q^2)$ in eq. (8) at $\mu_r^2 = \mu_f^2 = Q^2$ for $\alpha_s(Q^2) = 0.2$ and four flavours. Right: These results convoluted with a typical input shape $xf = x^{1/2}(1-x)^3$.

References

1. W. L. van Neerven and E. B. Zijlstra, Phys. Lett. **B272**, 127 (1991).
2. E. B. Zijlstra and W. L. van Neerven, Phys. Lett. **B273**, 476 (1991).
3. E. B. Zijlstra and W. L. van Neerven, Phys. Lett. **B297**, 377 (1992).
4. E. B. Zijlstra and W. L. van Neerven, Nucl. Phys. **B383**, 525 (1992).
5. S. Moch and J. A. M. Vermaseren, Nucl. Phys. **B573**, 853 (2000), hep-ph/9912355.
6. S. A. Larin, T. van Ritbergen, and J. A. M. Vermaseren, Nucl. Phys. **B427**, 41 (1994).
7. S. A. Larin, P. Nogueira, T. van Ritbergen, and J. A. M. Vermaseren, Nucl. Phys. **B492**, 338 (1997), hep-ph/9605317.
8. A. Retey and J. A. M. Vermaseren, Nucl. Phys. **B604**, 281 (2001), hep-ph/0007294.
9. S. Moch, J. A. M. Vermaseren, and A. Vogt, Nucl. Phys. **B646**, 181 (2002), hep-ph/0209100.
10. S. G. Gorishnii, S. A. Larin, L. R. Surguladze, and F. V. Tkachev, Comput. Phys. Commun. **55**, 381 (1989).
11. S. A. Larin, F. V. Tkachev, and J. A. M. Vermaseren, NIKHEF-H-91-18.

12. P. Nogueira, J. Comput. Phys. **105**, 279 (1993).
13. J. A. M. Vermaseren, (2000), math-ph/0010025.
14. G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).
15. C. G. Bollini and J. J. Giambiagi, Nuovo Cim. **12B**, 20 (1972).
16. J. F. Ashmore, Lett. Nuovo Cim. **4**, 289 (1972).
17. G. M. Cicuta and E. Montaldi, Nuovo Cim. Lett. **4**, 329 (1972).
18. G. 't Hooft, Nucl. Phys. **B61**, 455 (1973).
19. W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. **D18**, 3998 (1978).
20. D. J. Gross and F. Wilczek, Phys. Rev. **D8**, 3633 (1973).
21. H. Georgi and H. D. Politzer, Phys. Rev. **D9**, 416 (1974).
22. J. A. Gracey, Phys. Lett. **B322**, 141 (1994), hep-ph/9401214.
23. A. Vogt, Phys. Lett. **B471**, 97 (1999), hep-ph/9910545.
24. G. Sterman, Nucl. Phys. **B281**, 310 (1987).
25. S. Catani and L. Trentadue, Nucl. Phys. **B327**, 323 (1989).
26. S. Catani and L. Trentadue, Nucl. Phys. **B353**, 183 (1991).
27. A. Vogt, Phys. Lett. **B497**, 228 (2001), hep-ph/0010146.
28. H. Contopanagos, E. Laenen, and G. Sterman, Nucl. Phys. **B484**, 303 (1997), hep-ph/9604313.
29. W. L. van Neerven and A. Vogt, Phys. Lett. **B490**, 111 (2000), hep-ph/0007362.
30. J. Blumlein and A. Vogt, Phys. Lett. **B370**, 149 (1996), hep-ph/9510410.
31. C. F. Berger, Phys. Rev. **D66**, 116002 (2002), hep-ph/0209107.
32. T. Matsuura, S. C. van der Marck, and W. L. van Neerven, Nucl. Phys. **B319**, 570 (1989).
33. S. Forte and G. Ridolfi, Nucl. Phys. **B650**, 229 (2003), hep-ph/0209154.
34. E. Gardi and R. G. Roberts, Nucl. Phys. **B653**, 227 (2003), hep-ph/0210429.
35. S. Catani, M. L. Mangano, P. Nason, and L. Trentadue, Nucl. Phys. **B478**, 273 (1996), hep-ph/9604351.
36. W. L. van Neerven and A. Vogt, Nucl. Phys. **B603**, 42 (2001), hep-ph/0103123.