POWER CORRECTIONS IN THE DEEP-INELASTIC SCATTERING*

S. ALEKHIN

Institute for High Energy Physics (IHEP),
Pobeda, 1, Protvino 142281 Russia
E-mail: alekhin@sirius.ihep.su

We outline impact of the power corrections on the study of deep-inelastic-scattering and describe determination of the power corrections from the existing deep-inelastic-scattering data.

One of the main ingredients of the high energy phenomenology of the hadrons interaction is factorization. It allows to greatly simplify theoretical calculations using the connection of different processes through convolution of the coefficient functions (i.e. parton cross sections) with the parton distributions functions (PDFs). However factorization is valid only for the hard interactions, i.e. in the limit of large masses, transverse momenta, etc. For the soft interactions it is broken due to parton correlations are non-negligible. The formalism for description of the factorization breakdown is given by the operator-product expansion (OPE)[1]. In terms of this expansion the factorized part the cross section correspond to the contribution of the leading-twist (LT) operators and the factorization-breaking part – to the higher-twist (HT) ones. The coefficient functions for the LT terms and dependence of the PDFs on the transverse momentum Q can be calculated in the pQCD-improved parton model. In accordance to these calculations the LT (twist-2) structure functions of deep-inelastic scattering (DIS) demonstrate logarithmic Q-dependence. The Q-dependence of the HT terms is mainly defined by the factor of $1/Q^{\tau-2}$, where τ is corresponding value of twist. Due to the $1/Q$ factors the HT terms rapidly fall with Q and in the limit of large Q the LT terms give the dominant contribution. However the coefficients at the power-like factors cannot be calculated using existing approaches and therefore the on-set scale of this asymptotic regime is a priori unclear. Importance of this problem was recognized just after first attempts to confront the pQCD predictions to

*Presented at the Ringberg Workshop: *New Trends in HERA Physics 2003*, Sep. 28 – Oct. 3, 2003, Ringberg Castle, Rottach-Egern, Germany.

the earliest available data since it was understood that the measured Q-dependence of the DIS structure functions can be equally well described both by the log-like and the power-like behavior[2]. With more precise measurements in the wide region of Q appeared later we can better separate the LT and the HT terms, however impact of the HT terms on different aspects of the DIS phenomenology is not understood completely. In this report we describe modern status of this problem and outline possible ways to resolve it.

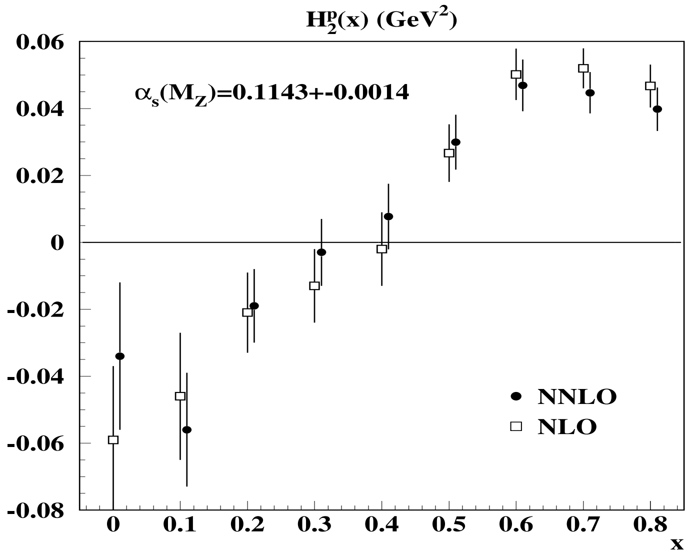

Figure 1. The x-dependence of twist-4 term in the proton structure function F_2 determined from the fit of Ref.[20] to the combined SLAC-BCDMS-NMC-H1-ZEUS data. The value of α_s given in the panel was obtained from the same fit.

For the DIS the OPE up to twist-4 terms reads

$$F_i(x,Q^2) = F_i^{\mathrm{LT,TMC}}(x,Q^2) + \frac{H_i(x,Q^2)}{Q^2} + O\left(1/Q^4\right), \qquad (1)$$

where i marks different structure functions and $F^{\mathrm{LT,TMC}}$ are the LT terms corrected for the target mass effects[3]. These target mass effects give contribution to the power-like terms of the OPE, but are regularly considered separately since they are just of kinematical origin and can be expressed as a convolution of the LT terms with the coefficient functions proportional to negative powers of Q (see Ref.[4]). The functions H give the dynamical HT contributions and correspond to the operators, which include correlations of the quark and gluon fields[5]. Calculation of matrix elements of these operators in the field theory is very difficult[6] and only very limited results has been obtained so far[7,8]. The predictions of most of the models for the HT terms contain essential free parameters and cannot be used without of fit to the data. For this reason the main source of information about the HT comes from the phenomenological studies. These studies are regularly based on the QCD extrapolation from the high-Q region to the low-Q one, when a discrepancy between the extrapolation and data is considered as a manifestation of the HT terms. This approach requires accurate treatment of all sources of theoretical uncertainties, which can have impact on the extrapolation, and therefore to bias the results or even generate a fake HT terms. Besides, since the the existing DIS data from any one single experiment do not cover the whole kinematic region necessary for the study of such kind, the combination of all available data is desirable. In this case the fake HT terms can be also generated due to incomparability of the separate data sets, especially if the kinematics of different experiments do not overlap.

The first firm evidence of the HT dynamical terms was found in Ref.[9]. In this analysis the twist-4 terms in the structure functions F_2 for proton and deuteron were determined from the next-to-leading order (NLO) QCD fit to the combined SLAC-BCDMS data with account of the HT terms parameterized in the factorized form:

$$F_2(x,Q^2) = F_2^{LT,TMC}(x,Q^2)\left[1+\frac{h_2(x)}{Q^2}\right]. \tag{2}$$

This form is equivalent to (1) modulo the impact of the anomalous dimensions of LT and HT operators. The latter must exist as follows from the theoretical expectations[10] and were indeed found in the phenomenological analysis of Ref.[11]. However at $x \leq 0.75$, which are relevant for the analysis of Ref.[9], the results of fits based on (1) and (2) are comparable within the experimental errors[12]. This indicates that the logarithmic corrections to the Q-dependence of the HT terms, which are generated by the anomalous dimensions, are unimportant in this region of x. The interpretation of the HT terms observed in the NLO analysis of Ref.[9] for a long time was questionable due the concern that they can be simulated by the missing higher-order QCD corrections. This concern is based on the observation

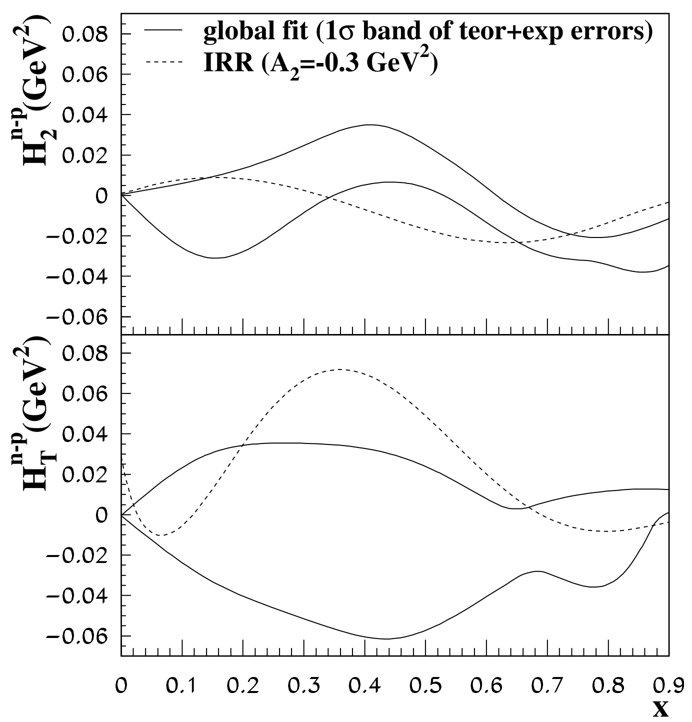

Figure 2. Comparison of the isospin asymmetry in the HT terms of the structure functions F_2 and $F_{\mathrm{T}} = F_2 + F_{\mathrm{L}}$ extracted from the global fit of Ref.[27] (solid curves) and calculated from the IRR model (dashes).

that the Q-dependence of the terms proportional to the large powers of strong coupling constant α_{s} can simulate power-like behavior in the limited range of Q (see Ref.[13]). However such simulation is possible only for the specific choice of powers of α_{s} and Q and/or insufficiently precise data. In DIS the next-to-next-to-leading (NNLO) QCD corrections do not simulate the power-like behavior within the precision of existing data and as a result the change of the dynamical twist-4 terms from the NLO to the NNLO fit does not exceed the experimental errors[14] (similar stability was observed earlier in Ref.[15], although conclusion about non-vanishing of the dynamical HTs was not drawn due to they were not separated from the target-mass effects). The same stability should be valid also for the higher-order corrections since the N^3LO correction is important at very high and very low x only as follows from the analyses based on resummation technique[16,17,18,19].

The x-dependence of twist-4 term in the proton structure function F_2 extracted from the data in assumption that functions $H(x,Q)$ do not depend on Q is given in Fig.1. This term is maximal at $x \sim 0.6$ and is determined in this region with the accuracy of $O(10\%)$ that allows to put a constraint on the some matrix elements of the twist-4 operators[21].

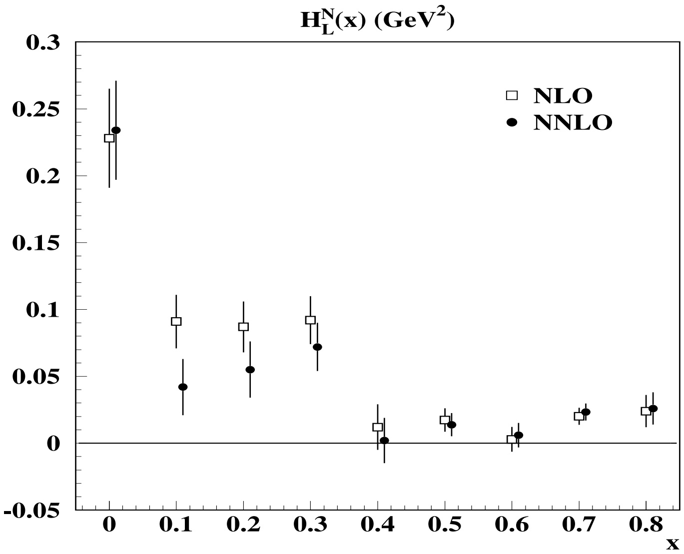

Figure 3. The HT terms in the nucleon structure function F_L determined from the global fit of Ref.[20].

Impact of the HT terms is important up to relatively large values of Q. E.g. at $x = 0.6$ the experimental error in F_2 averaged over global data set is less than the HT terms at $Q^2 \gtrsim 20$ GeV^2 only. In practice this means that inaccount of the HT terms can lead to the bias of the different quantities extracted from the DIS data. It is of particular importance for the extraction of α_s. Indeed, in the global fit with constraints $H = 0$ the value of $\alpha_s(M_Z) = 0.1215 + -0.0003$ (c.f. $\alpha_s(M_Z) = 0.1143 + -0.0014$ with the HT terms fitted)[22]. For the fit without account of the HT terms we observe the shift of the central value of and essential decrease of the error. Both effects are due to value of α_s is strongly anti-correlated with H_2. Essentially this means that for the global fit of existing data the error in α_s is determined by the uncertainty in HT terms and if these terms could be reliably estimated (e.g. from the lattice calculations) the error in α_s would be suppressed in several times. The cut of data with $Q^2 \lesssim 20$ GeV^2 aimed to decrease uncertainty in α_s due to the HT terms is ineffective since in this case the error in α_s rises due to loss of data corresponding to the fastest rate of the QCD evolution. The promising way in this direction is to collect new statistically significant sample of data at HERA.

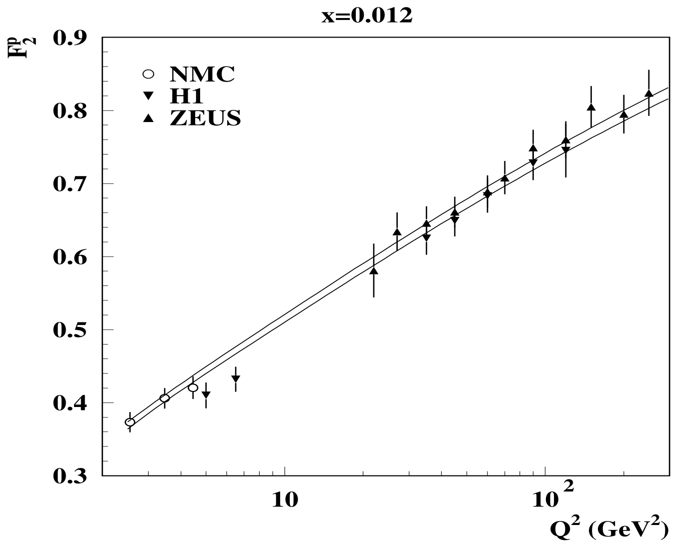

Figure 4. The 1σ band of the global fit (curves) compared to the data on F_2 used in the fit.

A good precision in determination of H_2 allows conclusive comparison to the theoretical models. One of the most predictive model of the HTs is based on the infrared renormalon (IRR) approach[23]. This model does not fix the general normalization of the HT terms, but puts a stringent constraint on their x-dependence. For the non-singlet terms the IRR model gives an especially simple predictions[24,25]

$$H_i^{\mathrm{NS}} = A \cdot F_2^{\mathrm{LT,NS}} \otimes C_i^{\mathrm{IRR}}, \tag{3}$$

where $\otimes$ denotes the Mellin convolution, C_i^{IRR} are the corresponding coefficient functions and A is the normalization factor. These predictions are in qualitative agreement to the data, however the subtle details are not reproduced. For example a study of the isospin asymmetry, which can be used for a test of the basic features of the IRR model since C_i^{IRR} are flavor-independent[26], demonstrates significant disagreement between the HT terms extracted from the data and the IRR predictions (see Fig.2).

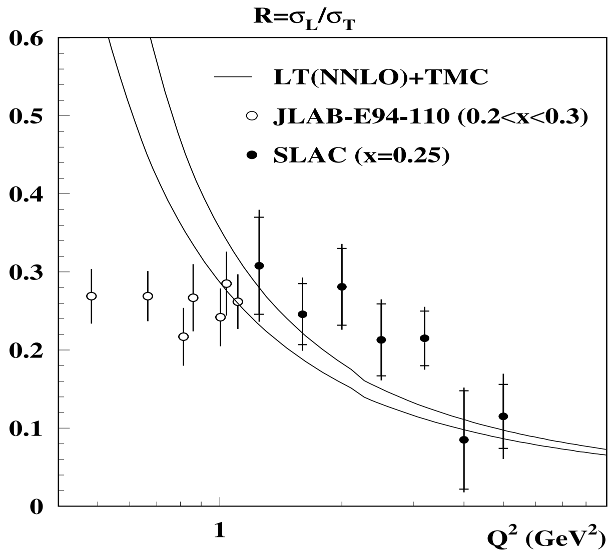

Figure 5. The JLAB and SLAC measurements of $R = \sigma_{\mathrm{L}}/\sigma_{\mathrm{T}}$ compared to 1σ band for $R^{\mathrm{LT,TMC}}$ extracted from the NNLO global fit of Ref.[20].

The HT term in F_L extracted from the global fit to the DIS data is given in Fig.3. The observed x-dependence of H_{L} is less regular than the one of H_2. It might be due to F_{L} extracted from the global fit is rather sensitive to disagreements in data. This point is illustrated in Fig.4, which contains the comparison of the results of global fit to the data on F_2 at $x \approx 0.012$. It is evident that due to the large gap between separate data sets any disagreement between the NMC data at low Q and/or the HERA ones at high Q can be compensated by the corresponding HT term in F_{L}, if one performs the fit to the data on cross section in order to extract H_{L}. Therefore the spike at small x might be driven by such disagreement and more data at small x and $Q^2 = 2 \div 30$ GeV^2 will be extremely useful for clarification of this point. An alternative explanation of this spike is effect of missing higher-order QCD corrections that is partially supported by the relative perturbative instability of $H_{\rm L}$, at least at $x \sim 0.2$. Indeed, the higher-order corrections at small x obtained using the $k_{\rm T}$ resummation technique are very large[28] and with the use of the approximate 3-loop coefficient function for $F_{\rm L}^{\rm LT}$ from Ref.[15] based on the calculations of Ref.[28,29] one probably should observe decrease of $H_{\rm L}$ extracted from the fit. However, one has to take into account that uncertainty in the 3-loop coefficient function of Ref.[15] due to missing terms in the 3-loop corrections is very large and the impact of the exact 3-loop correction might turn insufficient to explain the observed spike. The spike might be generated by a new phenomena such as the parton saturation[30] and, eventually, the OPE up to twist-4 might be insufficient[31]. In agreement with the latter the non-vanishing twist-6 contribution to $F_{\rm L}$ was found in the analysis of Ref.[11]. This analysis was based on the data from resonance region and therefore in order to obtain this result the duality hypothesis[32] was heavily used. However, the recent JLAB-E94-110 data[33] beyond the resonance region confirm this observation. In Fig.5 the JLAB measurements of $R = \sigma_{\rm L}/\sigma_{\rm T}$ are compared to the earlier SLAC data[34] and the LT term extracted from the global fit. Excess of the SLAC data over the LT term with account of the target mass correction at $Q^2 \sim 2$ GeV^2 is interpreted as a manifestation of the HT term in R and, correspondingly, in $F_{\rm L}$[35]. At small Q the relation between data and the LT term is opposite and the most natural interpretation within OPE is that effect of positive twist-4 contribution is overwhelmed by the negative twist-6 contribution. If so, the structure of the HT terms is very non-trivial, at least for $F_{\rm L}$. In summary, despite the predictions based on the HT terms given in Fig.3 are in good agreement with the independent measurements of $F_{\rm L}$ at small x and Q by H1 collaboration (see Ref.[36] for the details), the status of determination of $H_{\rm L}$ is rather unclear and for better understanding we need efforts both from theoretical and experimental side.

The dynamical HT terms in the polarized structure functions are even less known than for the unpolarized ones, although the potential impact of the HT terms is very important in this case due to relatively low typical Q of the data used in the analysis. The recent indication on the the HT terms in g_1[37] was not entirely confirmed in the latter study[38] and, evidently, more careful examination is necessary.

The HT terms in the DIS of neutrino are also poorly known because of low precision of the available data (see recent review[39] for more details). The essential progress in this area can be achieved only with the very intense neutrino beams (neutrino factories)[40]. These facilities would allow to constraint the dynamical HT terms in the neutrino-nucleon structure function at the level of precision comparable to the case of the charged leptons' DIS. However, due to neutrino experiments provide much wider set of constraints this additional input would allow to significantly improve disentangling the matrix element of different operators, which describe the partons' correlation.

In conclusion, our understanding of the power corrections in the DIS structure functions is rather limited. The kinematical component of the power correction (target mass effects) can be calculated with a reasonable accuracy, but the dynamical HT terms are essentially unknown for the most of structure functions. Meanwhile the further study of HT terms is necessary in order to outline the kinematical region, where the QCD factorization can be applied and the universality of PDFs can be proven. From practical point of view it is necessary in order to provide unbiased extrapolation of PDFs to the high-energy region for the searches of the effects of new physics. Eventually, study of the dynamical HT terms give information about the regime intermediate between asymptotic freedom and confinement. All this makes precise determination of the HT terms important and promising topic.

The research was supported in part by the RFBR grant 03-02-17177 and by the National Science Foundation under Grant No. PHY99-07949.

References

1. K. G. Wilson, Phys. Rev. **179** (1969) 1499.
2. L. F. Abbott and R. M. Barnett, Annals Phys. **125**, 276 (1980);
   L. F. Abbott, W. B. Atwood and R. M. Barnett, Phys. Rev. **D22**, 582 (1980).
3. O. Nachtmann, Nucl. Phys. B **63** (1973) 237.
4. H. Georgi and H. D. Politzer, Phys. Rev. D **14** (1976) 1829.
5. E. V. Shuryak and A. I. Vainshtein,Nucl. Phys. B **199** (1982) 451;
   E. V. Shuryak and A. I. Vainshtein,Nucl. Phys. B **201** (1982) 141;
   R. L. Jaffe and M. Soldate, Phys. Rev. D **26** (1982) 49.
6. R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B **212** (1983) 29.
7. V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B **283** (1987) 723.
8. B. Dressler, M. Maul and C. Weiss, Nucl. Phys. B **578** (2000) 293 [arXiv:hep-ph/9906444].
9. M. Virchaux and A. Milsztajn, Phys. Lett. B **274** (1992) 221.
10. A. P. Bukhvostov, E. A. Kuraev and L. N. Lipatov, Yad. Fiz. **38** (1983) 439.
11. G. Ricco, S. Simula and M. Battaglieri, Nucl. Phys. B **555** (1999) 306 [arXiv:hep-ph/9901360].
12. S. I. Alekhin, Eur. Phys. J. C **12** (2000) 587 [arXiv:hep-ph/9902241].
13. Y. L. Dokshitzer, arXiv:hep-ph/9911299.

14. S. I. Alekhin, arXiv:hep-ph/0212370.
15. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **18** (2000) 117 [arXiv:hep-ph/0007099].
16. G. Altarelli, R. D. Ball and S. Forte, Nucl. Phys. B **599** (2001) 383 [arXiv:hep-ph/0011270].
17. S. Schaefer, A. Schafer and M. Stratmann, Phys. Lett. B **514** (2001) 284 [arXiv:hep-ph/0105174].
18. E. Gardi and R. G. Roberts, Nucl. Phys. B **653** (2003) 227 [arXiv:hep-ph/0210429].
19. N. Bianchi, A. Fantoni and S. Liuti, Phys. Rev. D **69** (2004) 014505 [arXiv:hep-ph/0308057].
20. S. Alekhin, Phys. Rev. D **68** (2003) 014002 [arXiv:hep-ph/0211096].
21. S. Choi, T. Hatsuda, Y. Koike and S. H. Lee, Phys. Lett. B **312** (1993) 351 [arXiv:hep-ph/9303272].
22. S. I. Alekhin, JHEP **0302** (2003) 015 [arXiv:hep-ph/0211294].
23. M. Beneke, Phys. Rept. **317** (1999) 1 [arXiv:hep-ph/9807443].
24. E. Stein, M. Meyer-Hermann, L. Mankiewicz and A. Schafer, Phys. Lett. B **376** (1996) 177 [arXiv:hep-ph/9601356].
25. M. Dasgupta and B. R. Webber, Phys. Lett. B **382** (1996) 273 [arXiv:hep-ph/9604388].
26. S. J. Brodsky, arXiv:hep-ph/0006310.
27. S. I. Alekhin, S. A. Kulagin and S. Liuti, arXiv:hep-ph/0304210, to appear in PRD.
28. S. Catani and F. Hautmann, Nucl. Phys. B **427** (1994) 475 [arXiv:hep-ph/9405388].
29. S. A. Larin, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B **427** (1994) 41;
    S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B **492** (1997) 338 [arXiv:hep-ph/9605317];
    A. Retey and J. A. M. Vermaseren, Nucl. Phys. B **604** (2001) 281 [arXiv:hep-ph/0007294].
30. L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100** (1983) 1.
31. S. Liuti, R. Ent, C. E. Keppel and I. Niculescu, Phys. Rev. Lett. **89** (2002) 162001 [arXiv:hep-ph/0111063].
32. E. D. Bloom and F. J. Gilman, Phys. Rev. D **4** (1971) 2901.
33. Yongguang Liang, Thesis, American Univ., Washington DC (2002).
34. L. W. Whitlow, S. Rock, A. Bodek, E. M. Riordan and S. Dasu, Phys. Lett. B **250** (1990) 193.
35. J. L. Miramontes, M. A. Miramontes and J. Sanchez Guillen, Phys. Rev. D **40** (1989) 2184.
36. S. Alekhin, arXiv:hep-ph/0311184.
37. E. Leader, A. V. Sidorov and D. B. Stamenov, Phys. Rev. D **67** (2003) 074017 [arXiv:hep-ph/0212085].
38. J. Bartelski and S. Tatur, arXiv:hep-ph/0307374.
39. A. L. Kataev, arXiv:hep-ph/0107247.
40. M. L. Mangano *et al.*, arXiv:hep-ph/0105155.