OPEN HEAVY-FLAVOUR PHOTOPRODUCTION AT NLO

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I review theoretical approaches to open heavy flavour photoproduction and discuss theoretical aspects of a new calculation in a massive theory with $\overline{\rm MS}$ subtraction based on the factorization theorem of John Collins with heavy quarks.

1. Introduction

Heavy quarks are those with masses $m_h \gg \Lambda_{\rm QCD}$ such that $\alpha_s(m_h^2) \propto \ln^{-1}(\frac{m_h^2}{\Lambda^2_{\rm QCD}}) \ll 1$ and hence, according to this definition, the charm, bottom and top quarks (c, b, t) are heavy whereas the up, down and strange quarks (u, d, s) are light. A lot of reasons can be found in the literature why heavy quark production is interesting. Most importantly, such processes are fundamental elementary particle processes which take place with substantial rates at high energy colliders. Therefore a good phenomenological understanding of heavy quark production is very important. Moreover, there are more theoretical reasons: The heavy quark mass m_h acts as a physical long-distance cut-off such that heavy quark production is an ideal testing ground of perturbative QCD. Very often in addition to m_h another large scale is involved in the hard scattering process such that one has to deal with multi-scale processes which complicate the perturbative analysis.

In this talk I will restrict to open charm photoproduction, $\gamma p \to D^\star X$, and describe theoretical aspects of a new calculation in a massive variable flavour number scheme (VFNS) based on the factorization theorem of John Collins including heavy quark masses [1]. This work is along the lines of previous studies of the process $\gamma\gamma \to D^\star X$ for the direct contribution [2] and the single-resolved part [3]. Note also that the latter constitutes the direct contribution to $\gamma p \to D^\star X$ [4] considered here. A calculation of the resolved contribution is in progress [5] which will allow to perform a complete NLO

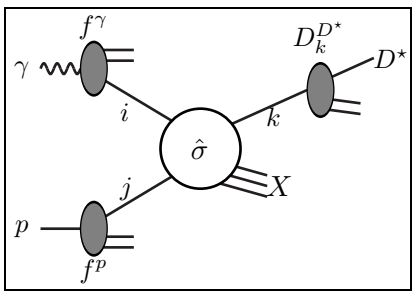

Figure 1. Open charm photoproduction, $\gamma p \to D^\star X$, according to Eq. (1).

analysis of inclusive photoproduction of $D^\star$ mesons in a massive VFNS which extends the successful massless calculation [6] by terms of the order $O(m_c^2/p_T^2)$ where p_T is the transverse momentum of the $D^\star$ meson. A different approach to construct a massive VFNS has been adopted in Refs. 7, 8 ('FONLL').

2. Factorization Formula

Generically, the theoretical description of open charm photoproduction is based on the following factorization formula [see Fig. 1]

$$\mathrm{d}\sigma(\gamma p \to D^\star X) = \sum_{i,j,k} \int dx_1\ dx_2\ dz\ f_i^\gamma(x_1,\mu_f)\ f_j^p(x_2,\mu_f)\ \times$$
$$\mathrm{d}\hat{\sigma}(ij \to kX)\ D_k^{D^\star}(z,\mu_f') + O(\alpha_s^{n+1}, \tfrac{\Lambda^2}{p_T^2}),\ (1)$$

where Λ is a typical hadronic scale of a few hundred MeV, p_T is the transverse momentum of the $D^\star$ meson and where $f_i^\gamma(x_1,\mu_f)$ and $f_j^p(x_2,\mu_f)$ are (universal) parton distribution functions (PDFs) of the photon and the proton, respectively. The partons of the proton are gluons and quarks, $j = g, q, \ldots$ ($q = u, d, s$) while the partons of the photon are $i = g, q, \ldots$ ('resolved contribution') and $i = \gamma$ ('direct contribution'[a]). The ellipses indicate that the question whether heavy quarks are included as partons depends on the factorization scale μ_f and the heavy flavour scheme to be discussed in the next section. Furthermore, the fragmentation of the final state parton $k = g, q, \ldots$ into the $D^\star$ meson is described by (universal) fragmentation functions (FFs) $D_k^{D^\star}(z,\mu_f')$. The ellipses again refer to

[a] $f_\gamma^\gamma(x,\mu_f) = \delta(1-x)$

the heavy quark treatment. [b] Finally, the hard scattering cross sections $\mathrm{d}\hat{\sigma}(ij \to kX)(\mu_f, \mu'_f, \alpha_s(\mu), [\frac{m_c}{p_T}])$ are pertubatively calculable and depend on the factorization scales μ_f and μ'_f and the renormalization scale μ. As has been proved to all orders of perturbation theory in Ref. 1 heavy quark mass effects can be consistently included in the hard part. The square bracket around $\frac{m_c}{p_T}$ indicates that, depending on the heavy flavour scheme, these terms are included or neglected. Of course, in order to obtain a physically meaningful (scheme independent) result the hard part and the PDFs and FFs have to be employed in the same factorization scheme.

Before turning to a discussion of heavy flavour schemes it is instructive to consider the theoretical uncertainties of predictions based on Eq. (1). The sources are:

- Uncertainties in the PDFs and FFs.
- The error of the factorization formula which is of the order $O(\frac{\Lambda^2}{p_T^2})$.
- The order of the perturbative calculation of the hard part. An NLO calculation includes terms of the order $O(\alpha_s^2)$ ($n = 2$ in Eq. (1)). Usually the error due to higher order terms is estimated by varying the factorization and renormalization scales.

3. Heavy Flavour Schemes

In this section I will give a basic overview over the main heavy flavour schemes used in the literature. For a more detailed review see, e.g., Ref. 9. References to NLO calculations of $\gamma p \to D^\star X$ based on Eq. (1) and using a given heavy flavour scheme can be found in the corresponding section headlines.

3.1. *Conventional massless Parton Model: ZM-VFNS* [6]

The conventional (massless) parton model is also known as 'zero mass variable flavour number scheme' (ZM-VFNS). In this approach the charm mass m_c is set to zero in the calculation of the hard scattering cross section. The number of partons is variable depending on the factorization scale with the

[b] It should be noted that in any case charm quarks (heavy quarks) will be produced in the final state of the partonic subprocess ($k = c$). However, if the final state charm quark is not treated as a parton it is nevertheless phenomenologically necessary to include a scale-independent fragmentation function $D_c^{D^\star}(z)$. Such a function is in principle not related to the factorization formula in Eq. (1) and it is not clear whether it has a universal meaning. See also Sec. 3.

following finite transformations (valid up to NLO) at the so-called transition scale $Q_0 = m_c$: [c]

$$\begin{aligned} n_f = 3 &\to n_f = 4 \\ \alpha_s^{(3)} &\to \alpha_s^{(4)} = \alpha_s^{(3)} + O(\alpha_s^3) \\ f_i^{(3)} &\to f_i^{(4)} = f_i^{(3)} + O(\alpha_s^2) \end{aligned} \tag{2}$$

and the perturbative boundary condition for the charm quark

$$f_c^{(4)}(x, Q_0^2 = m_c^2) = 0 \; . \tag{3}$$

The advantage of this approach is that it is simple and large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ are resummed to all orders in perturbation theory in the evolved distributions $f_c(x, \mu^2)$ and $D_c^{D^\star}(x, \mu^2)$. On the other hand, charm mass terms $\frac{m_c^2}{p_T^2}$ are neglected in the hard part, such that theoretical predictions are good only for $p_T^2 \gg m_c^2$.

3.2. *($n_f = 3$)-Fixed Flavour Number Scheme (3-FFNS)* [11,12]

In this approach $m_c \neq 0$ and the number of flavours is fixed to $n_f = 3$. The partonic subprocesses involve gluons and light quarks as partons in the initial state whereas no charm parton distribution is taken into account ($f_c(x, \mu_f) = 0$). On the other hand charm quarks are produced in the partonic subprocesses and hence appear in the final state. Here it is phenomenologically unavoidable to include a non-perturbative function, for example of Peterson type, describing the transition from the charm quark to the observable $D^\star$ meson. It should be noted that this fragmentation function is *not* related to the fragmentation functions in the factorization theorem in Eq. (1) which are universal and factorization scale dependent.

In the fixed flavour scheme perturbative calculations are more involved due to $m_c \neq 0$. The advantage is that $\frac{m_c^2}{p_T^2}$ terms are included and a correct threshold behaviour is guaranteed which is especially important in deep inelastic scattering. On the other hand, the fixed order logarithms $\ln \frac{p_T^2}{m_c^2}$ can become large for $p_T^2 \gg m_c^2$ such that a resummation of these large logarithms is necessary. Therefore, calculations within the FFNS are ex-

[c] For a discussion of the general case $Q_0 = O(m_c)$ see, e.g., Ref. 10. Therein, also references to the literature of the matching conditions of the PDFs and α_s can be found.

pected to be good mainly in the kinematical region $p_T^2 \simeq m_c^2$. [d] A major drawback is that the charm fragmentation function $D_c^{D^\star}(z)$ is not based on the factorization theorem and hence not universal. Therefore there is no deep theoretical reason to fix this function in one experiment and to use it making predictions for other experiments. Finally, it is noteworthy that it is not possible to include a non-perturbative charm component in the proton in this scheme.

3.3. *Massive VFNS (GM-VFNS)* [7,8,5]

The massive VFNS (GM-VFNS) is a variable flavour number scheme as introduced in Sec. 3.1 which includes heavy quark mass effects in the hard scattering cross sections/coefficients. Besides the usual gluon and light quark parton distributions a charm quark parton distribution is taken into account. In order to avoid double counting collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ are subtracted from the partonic subprocesses and are resummed to all orders with help of the DGLAP evolution equations ($\rightarrow f_c \neq 0$). The same is true for the final state, where collinear logarithms are subtracted from the hard part and are resummed to all orders by introducing a charm quark fragmentation function $D_c^{D^\star}(z, {\mu'_f}^2)$.

This scheme is most ambitious aiming at combining the virtues of both the massless VFNS and the FFNS and therefore it is technically more involved. A calculation requires to consider all subprocesses occurring already in the FFNS with $m_c \neq 0$ and to subtract the collinear parts in order to obtain 'IR-safe' hard parts. Technically this is equivalent to mass factorization within a massive regularization scheme. In addition the subprocesses with charm quarks in the initial state (occurring also in the massless VFNS) have to be included. However, all desired features are fulfilled in this approach: The large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ are resummed in terms of evolved charm quark PDFs and FFs $f_c(x,\mu^2)$ and $D_c^{D^\star}(x,\mu^2)$, respectively, and charm quark mass terms $O(\frac{m_c^2}{p_T^2})$ are included in the hard parts. Therefore, calculations in this scheme are expected to be valid for both $p_T^2 \simeq m_c^2$ and $p_T^2 \gg m_c^2$ such that for $p_T^2 \lesssim m_c^2$ the transition to the 3-FFNS is achieved and asymptotically, for $p_T^2 \gg m_c^2$, the ZM-VFNS is reproduced. Again it should be noted that as in the massless VFNS a non-perturbative charm component in the proton can be included.

[d] Of course it is a matter of debate when conditions like $p_T^2 \gg m_c^2$ are actually satisfied and the answer certainly also depends on the process under consideration.

4. Theoretical Aspects of Open Charm Photoproduction in a massive VFNS

In this section some theoretical issues relevant for open charm photoproduction in a massive VFNS will be addressed. One of the key ingredients in establishing such a scheme is the removal of collinear would-be singularities in order to avoid double counting. As already mentioned, this is technically nothing else but mass factorization within a massive regularization scheme and will be discussed in Sec. 4.1. Then, in Sec. 4.2 the residual implementation freedom concerning finite mass terms will be discussed which is the main reason for the different approaches to massive VFNS existing in the literature. Our procedure adopted for establishing a massive VFNS which fixes these ambiguities and which is applicable to all kinds of processes will be described.

4.1. *Mass Factorization*

Before turning to mass factorization in a massively regularized theory it is helpful to recap some facts about $\overline{\rm MS}$ mass factorization in a dimensionally regularized theory. As is well-known, mass divergences ($\equiv$ 'collinear divergences') occur when an internal propagator gets on its mass-shell due to a *collinear* splitting process as exemplified in Fig. 2. The associated

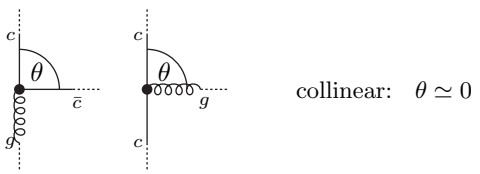

Figure 2. Collinear splitting processes $g \to c\bar{c}$ and $c \to cg$.

regularized collinear 'divergences' are subtracted from the (partonic) cross sections and are absorbed into PDFs and FFs. Along with the genuine (regularized) poles *finite terms* can be subtracted leading to different factorization schemes; among them the $\overline{\rm MS}$ scheme considered here. It should be noted that even for the same factorization scheme the *subtraction terms* do depend on the *regularization procedure*. Usually the $\overline{\rm MS}$ (factorization/renormalization) scheme is introduced or defined within *dimensional regularization* ($m_c = 0$) such that collinear divergences occur as $\frac{1}{\epsilon}$ poles.

As an example consider Fig. 3. The collinear part to be subtracted via

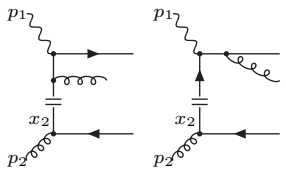

Figure 3. Diagrammatical representation of the subtraction term in Eq. (4).

mass factorization is given by

$$\mathrm{d}\sigma_{\mathrm{SUB}}(\mu_f{}^2,\mu^2) = \int_0^1 \mathrm{d}x_2\; G^1_{cg}(x_2)\mathrm{d}\hat{\sigma}^{(0)}_{\gamma c\to cg}(p_1,x_2p_2) \equiv G^1_{cg}\otimes \mathrm{d}\hat{\sigma}^{(0)}_{\gamma c\to cg} \quad (4)$$

with the universal function $[n = 4-2\epsilon]$

$$G^1_{cg}(x,\mu_f{}^2,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\Big[P^{(0)}_{cg}(x)(-\tfrac{1}{\epsilon}+\gamma_E-\ln 4\pi)(\tfrac{\mu^2}{\mu_f{}^2})^\epsilon + \underbrace{g^1_{cg}(x)}_{\overline{\mathrm{MS}}=0}\Big] \quad (5)$$

containing the $\frac{1}{\epsilon}$ pole and the finite part $g^1_{cg}(x)$ being zero in the $\overline{\mathrm{MS}}$ scheme. A few remarks are in order: (i) The variable x_2 in Eq. (4) can be calculated in terms of the external Mandelstam variables, $x_2 = x_2(s,t,u)$, since the Born cross section is proportional to the Delta-function $\delta(\hat{s}+\hat{t}+\hat{u})$ where $\hat{s},\hat{t},\hat{u}$ are obtained in the usual way from the 4-momenta p_1, x_2p_2 entering the Born subprocess. (ii) The Born cross section in Eq. (4) is usually taken to be n dimensional which generates finite terms when multiplied with the $\frac{1}{\epsilon}$ pole in the function G^1_{cg}. (iii) Finally it is interesting to keep track of the logarithms involved in the mass factorization procedure. The collinear part of the unsubtracted partonic cross section is proportional to $-\frac{1}{\epsilon}(\mu^2/s)^\epsilon = -\frac{1}{\epsilon}+\ln\frac{s}{\mu^2}$ from which $-\frac{1}{\epsilon}(\mu^2/\mu_f{}^2)^\epsilon = -\frac{1}{\epsilon}+\ln\frac{\mu_f{}^2}{\mu^2}$ is subtracted [see Eq. (5)] generating a logarithm $\ln\frac{s}{\mu_f{}^2}$ in the IR-safe hard part which depends on the factorization scale.

The above example can be systematically generalized using universal functions of the form

$$G^1_{ij}(x,\mu_f{}^2,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\Big[P^{(0)}_{ij}(x)(-\tfrac{1}{\epsilon}+\gamma_E-\ln 4\pi)(\tfrac{\mu^2}{\mu_f{}^2})^\epsilon + \underbrace{g^1_{ij}(x)}_{\overline{\mathrm{MS}}=0}\Big]\,. \quad (6)$$

In the case of heavy quark production it is natural to use the heavy quark mass m as collinear regulator such that the collinear poles appear as logarithms $\ln\frac{s}{m^2}$. The *massive* $\overline{\mathrm{MS}}$ subtraction terms $\mathrm{d}\sigma_{\mathrm{SUB}}(\mu_f,m)$ are identified by the condition

$$\lim_{m\to 0}\left(\mathrm{d}\sigma(m) - \mathrm{d}\sigma_{\mathrm{SUB}}(m)\right) \stackrel{!}{=} \mathrm{d}\hat{\sigma}(\overline{\mathrm{MS}}) \quad (7)$$

As before the subtraction terms can be factorized into universal functions $G^1_{ij}(x,{\mu_f}^2,\mu^2)$ and $D^1_{ij}(x,{\mu_f}^2,\mu^2)$ for the initial and final state, respectively, and Born cross sections: $\mathrm{d}\sigma^1_{\mathrm{SUB}}({\mu_f}^2,\mu^2) = G^1(x) \otimes \mathrm{d}\hat{\sigma}^{(0)}$, $\mathrm{d}\hat{\sigma}^{(0)} \otimes D^1(z)$. The hard (IR safe) cross sections entering the factorization formula in Eq. (1) are then obtained by removing the mass singular parts from the partonic cross section

$$\mathrm{d}\hat{\sigma}^{(1)} = \mathrm{d}\sigma^1 - \mathrm{d}\sigma^1_{\mathrm{SUB}} \ . \tag{8}$$

Setting $\mu_f = \mu$, the massively regularized functions G^1_{ij} (initial state) are given in the $\overline{\mathrm{MS}}$ scheme by

$$\begin{aligned} G^1_{cg}(x,\mu^2) &= \tfrac{\alpha_s(\mu)}{2\pi} P^{(0)}_{cg}(x) \ln \tfrac{\mu^2}{m^2} \ , \\ G^1_{cc}(x,\mu^2) &= \tfrac{\alpha_s(\mu)}{2\pi} P^{(0)}_{cc}(x) \ln \tfrac{\mu^2}{m^2} \ , \\ G^1_{gg}(x,\mu^2) &= \tfrac{\alpha_s(\mu)}{2\pi} \ln \tfrac{\mu^2}{m^2} \delta(1-x) \ , \end{aligned} \tag{9}$$

and the functions D^1_{ij} (final state) read

$$\begin{aligned} D^1_{cg}(z,\mu^2) &= \tfrac{\alpha_s(\mu)}{2\pi} P^{(0)}_{cg}(z) \ln \tfrac{\mu^2}{m^2} \ , \\ D^1_{cc}(z,\mu^2) &= C_F \tfrac{\alpha_s(\mu)}{2\pi} \left[\tfrac{1+z^2}{1-z} (\ln \tfrac{\mu^2}{m^2} - 2\ln(1-z) - 1) \right]_+ \ . \end{aligned} \tag{10}$$

The function $D^1_{cc}(z,\mu^2)$ has been derived for the first time from the process $\gamma^\star \to c\bar{c}g$ with a timelike photon [13] and has also been found by taking the massless limit of the process $\gamma^\star c \to cg$ with a spacelike photon [14] and also in Ref. 2 for the process $\gamma\gamma \to c\bar{c}g$ demonstrating the universality of this function. All other functions not listed here are zero to this order in the coupling constant α_s. Furthermore, analogous results for processes involving photon splittings can be found by some obvious replacements ($g \to \gamma$, $\alpha_s \to \alpha$ and appropriate modifications of colour factors) in Eqs. (9) and (10).

4.2. *Implementation Freedom and Adopted Procedure*

Even fixing the factorization scheme to $\overline{\mathrm{MS}}$ leaves some freedom in the implementation of a massive VFNS as has been discussed for the case of deep inelastic scattering (DIS) in Ref. 9. This can be most easily seen from the condition in Eq. (7) which fixes the subtraction term $\mathrm{d}\sigma_{\mathrm{SUB}}(\frac{\mu}{m}, \frac{m}{p_T})$ only up to terms $\frac{m}{p_T}$ vanishing in the limit $m \to 0$. The precise treatment of such terms $\frac{m}{p_T}$ is not prescribed by factorization.

In our approach to open charm photoproduction the following procedure has been adopted [2,3]:

- Derive the $m \to 0$ limit of the massive 3-FFNS calculation only keeping m as regulator in logarithms $\ln \frac{m^2}{s}$. Here special care is required in order to recover certain distributions ($\delta(1-w)$, $(\frac{1}{1-w})_+$, $(\frac{\ln(1-w)}{1-w})_+$) occurring in the massless $\overline{\rm MS}$ calculation.
- Compare the massless limit with the corresponding contribution in the (massless) $\overline{\rm MS}$ calculation in order to identify the subtraction terms by the unique prescription

$$d\sigma_{\rm SUB} = (\lim_{m\to 0} d\sigma(m)) - d\hat{\sigma}(\overline{\rm MS}) \ . \tag{11}$$

  By this procedure no mass terms are removed from the partonic cross section apart from the collinear logarithms $\ln \frac{m^2}{s}$.
- Include contributions with charm quarks in the *initial state* in the massless approach [15]. This rule is of great practical importance and also has been employed in the recent work [16] of the CTEQ collaboration.

5. Summary

In this talk I have discussed theoretical issues relevant for heavy flavour photoproduction, $\gamma p \to D^\star X$, in a massive VFNS. A first study including mass effects for the direct contribution has been performed in Ref. 4. These results indicate an improved description of HERA data at small transverse momenta of the $D^\star$ meson. A calculation of the resolved contribution including mass effects is in progress allowing for a complete NLO analysis of open heavy flavour production in a massive VFNS rigorously based on the factorization theorem with massive partons by John Collins [1] which is the theoretical basis to incorporate information on the FFs into $D^\star$ mesons from other reactions, as for example $e^+e^- \to D^\star X$.

Acknowledgments

I am grateful to the organizers for the kind invitation to this very well organized workshop at such an inspiring location and to B. A. Kniehl, G. Kramer and H. Spiesberger for their collaboration.

References

1. J. C. Collins, Phys. Rev. **D58**, 094002 (1998).
2. G. Kramer and H. Spiesberger, Eur. Phys. J. **C22**, 289 (2001).
3. G. Kramer and H. Spiesberger, Eur. Phys. J. **C28**, 495 (2003).

4. G. Kramer and H. Spiesberger, *Inclusive photoproduction of D* mesons with massive charm quarks*, hep-ph/0311062.
5. B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, work in progress.
6. J. Binnewies, B. A. Kniehl, and G. Kramer, Phys. Rev. **D58**, 014014 (1998), and references therein.
7. M. Cacciari, S. Frixione, and P. Nason, JHEP **03**, 006 (2001).
8. S. Frixione and P. Nason, JHEP **03**, 053 (2002).
9. W.-K. Tung, S. Kretzer, and C. Schmidt, J. Phys. **G28**, 983 (2002).
10. S. Kretzer, Phys. Lett. **B471**, 227 (1999).
11. S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, Phys. Lett. **B348**, 633 (1995).
12. S. Frixione, P. Nason, and G. Ridolfi, Nucl. Phys. **B454**, 3 (1995).
13. B. Mele and P. Nason, Nucl. Phys. **B361**, 626 (1991).
14. S. Kretzer and I. Schienbein, Phys. Rev. **D59**, 054004 (1999).
15. M. Krämer, F. I. Olness, and D. E. Soper, Phys. Rev. **D62**, 096007 (2000).
16. S. Kretzer, H. L. Lai, F. I. Olness, and W. K. Tung, *CTEQ6 parton distributions with heavy quark mass effects*, hep-ph/0307022.