# Unquenched Lattice Calculations: Results and Perspectives

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This is NOT a general review !

# Quenched vs. full QCD (Present status)

## 1. Quenched QCD

CP-PACS collaboration
PRL84(2000)238, hep-lat/0206009

- Plaquette gauge action
- Wilson quark action
- $a^{-1} \simeq 2$, 2.5, 3, 4 GeV
- $L \simeq 3$ fm
- $m_\pi/m_\rho = 0.4$,0.5,0.6,0.7, 0.75

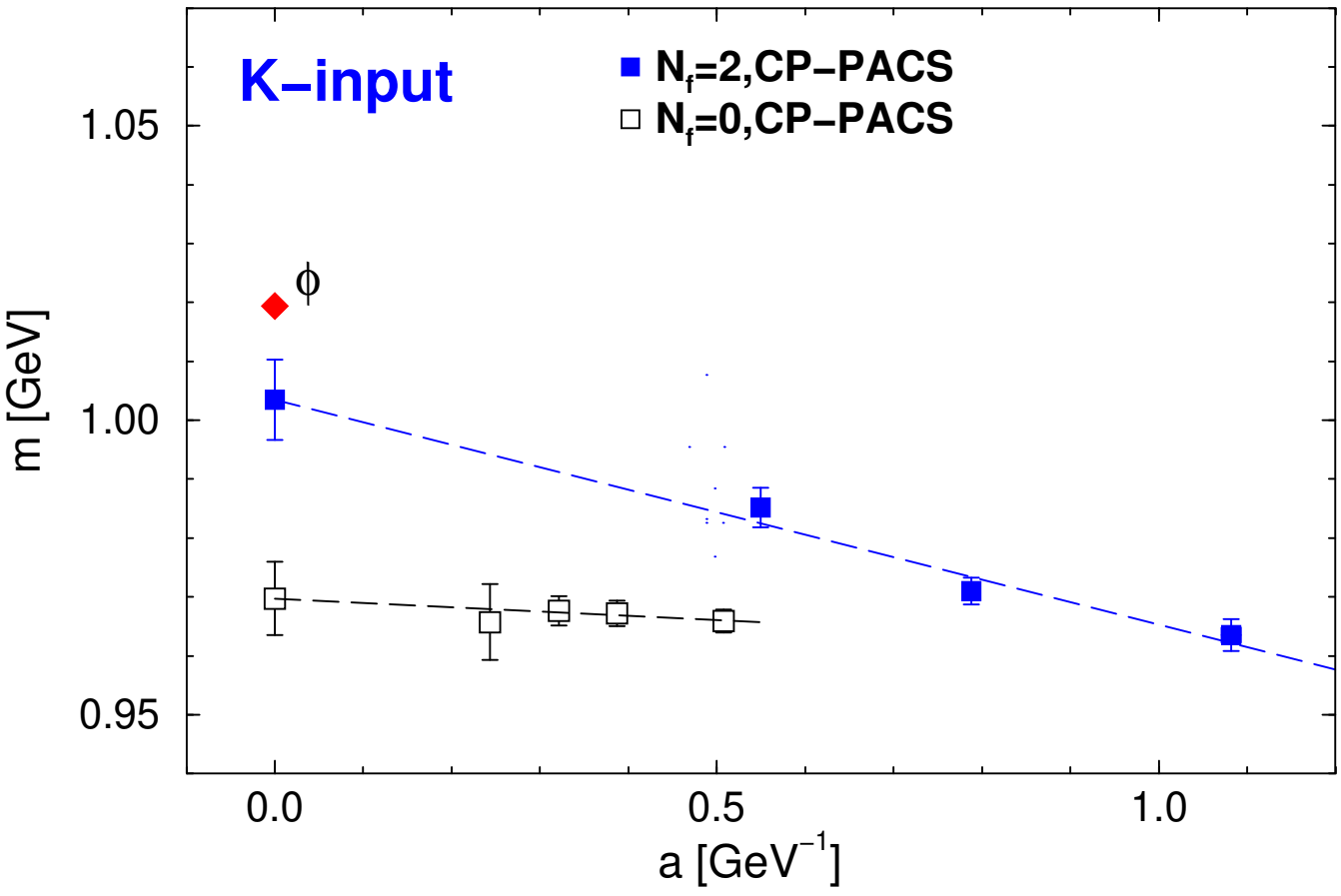
## 2. Full QCD ($N_f = 2$)

CP-PACS collaboration
PRL85(2000)4674, PRD65(2002)054505

- Renormalization Group(RG) improved gauge action,
- $O(a)$ improved Wilson quark action with MF improved $c_{\rm SW}$
- $a^{-1} \simeq$1, 1.5, 2 GeV
- $L \simeq 2.4$ fm
- $m_\pi/m_\rho = 0.6$, 0.7, 0.75, 0.8

## $\phi$ Meson mass

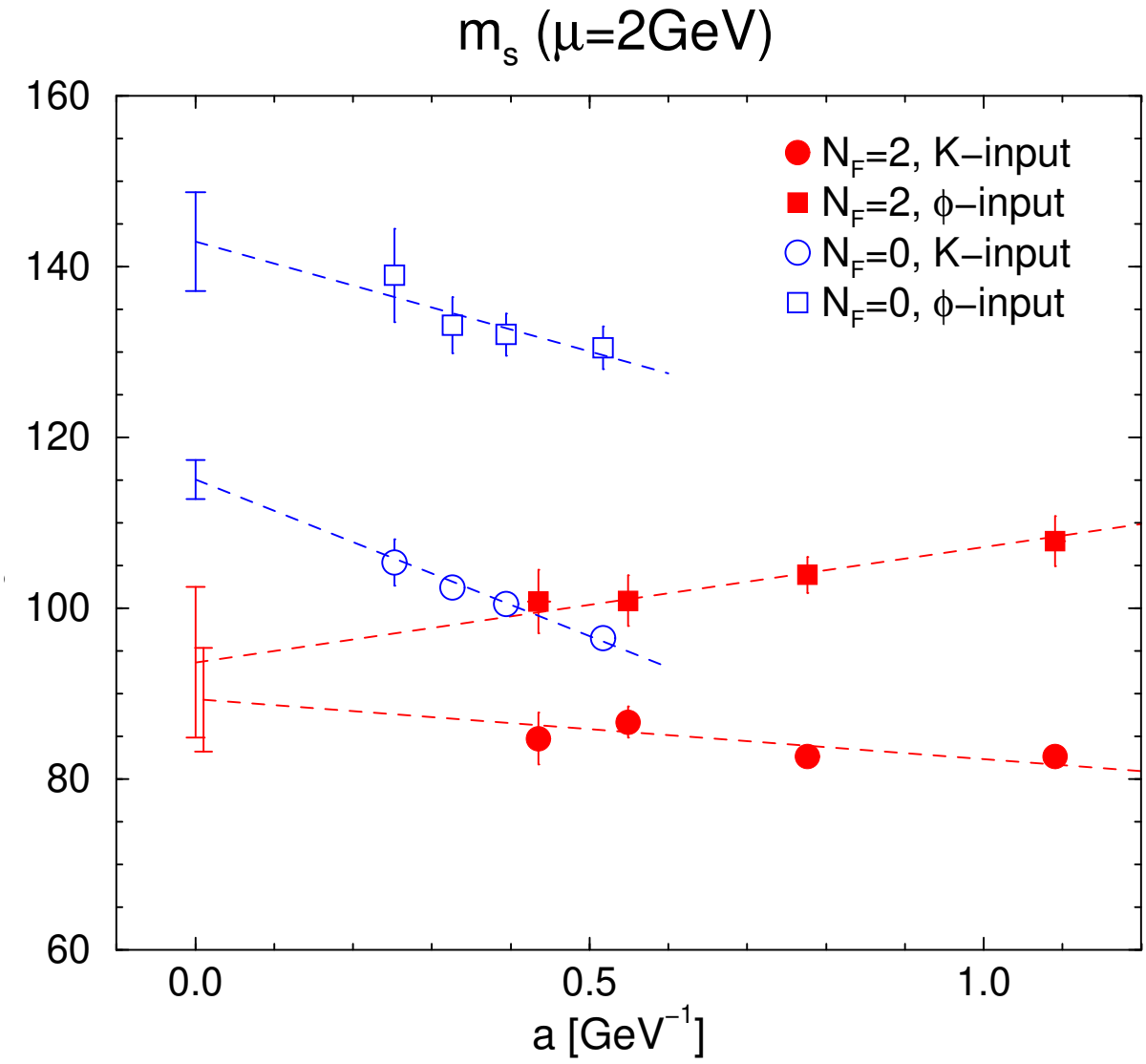
- $m_\rho \to a^{-1}$
- $m_\pi \to m_l = (m_u + m_d)/2$
- $m_K \to m_s$ (K-input), $m_\phi \to m_s$ ($\phi$-input)




In the continuum limit

- $m_\phi$ disagrees with the experiment (quenched QCD)
- $m_\phi$ becomes much closer to the experiment ($N_f = 2$ full QCD)

## strange quark mass




In the continuum limit

- $m_s$ (K-input) and $m_s(\phi$-input) disagree (quenched QCD)
- $m_s$ (K-input) and $m_s(\phi$-input) almost agree ($N_f = 2$ full QCD)
- $m_s \simeq 90$ MeV (full QCD) $\ll m_s \simeq 120 - 140$ MeV (quenched QCD)

While $N_f = 2$ full QCD simulations by CP-PACS almost cures the pathology in the quenched approximation, the following points are unsatisfactory and must be improved.

1. finite size effect ?
2. chiral extrapolation ( $m_q \to 0$ ) ?
3. continuum extrapolation ( $a \to 0$ ) ?
4. (perturbative) renormalization for $m_s$ ?
5. quenched strange quark ?

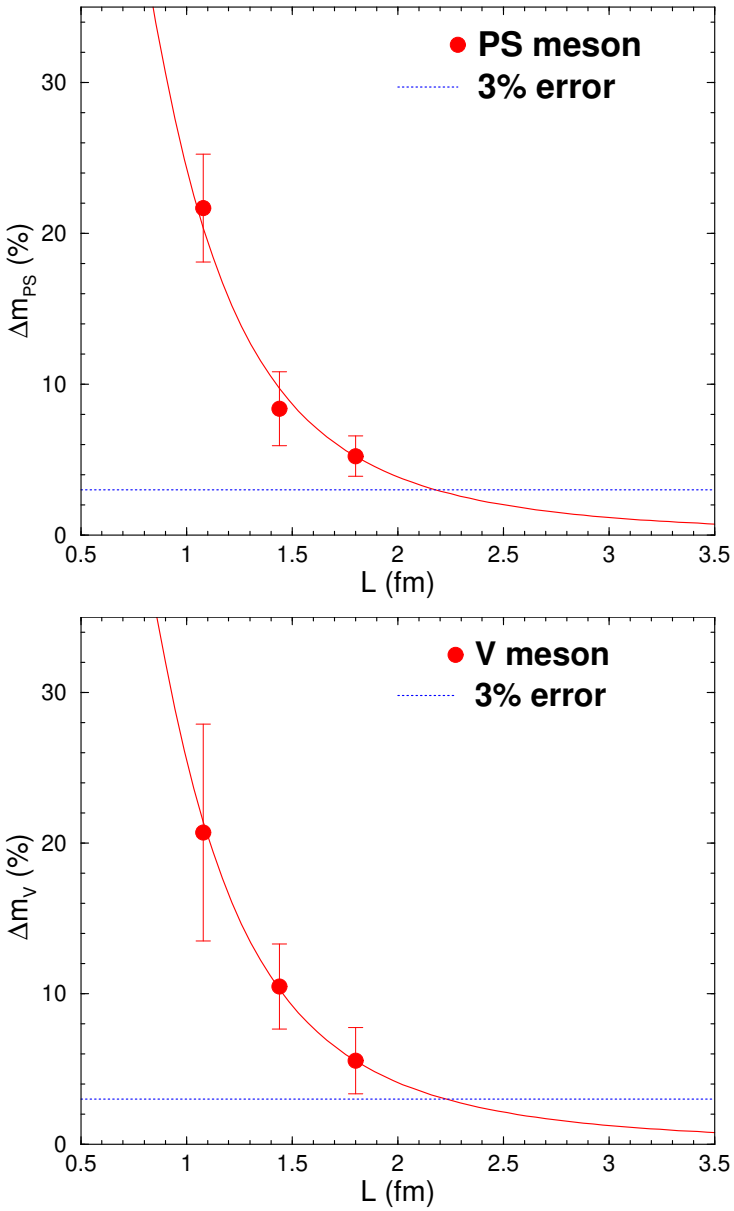
# Finite size effect in full QCD

JLQCD collaboration

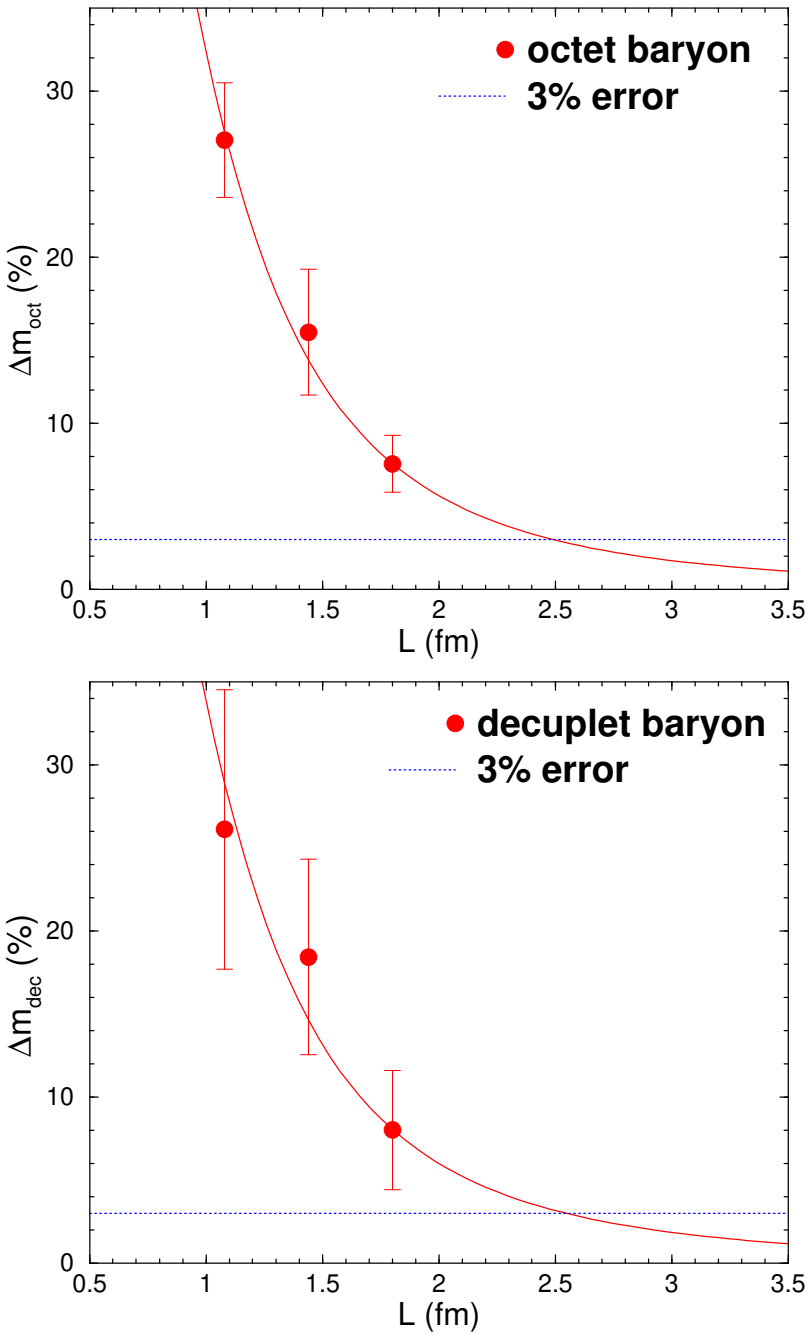
- Plaquette + non-perturbatively (NP) $O(a)$ improved Wilson
- $a^{-1} \simeq 2$ GeV, $m_\pi/m_\rho = 0.60$

$$\Delta m_H(L) \equiv \frac{m_H(L) - m_H(\infty)}{m_H(\infty)} = cL^{-3}$$

$\Delta m_H$ vs. $L$ (Mesons)



## $\Delta m_H$ vs. $L$ (Baryons)



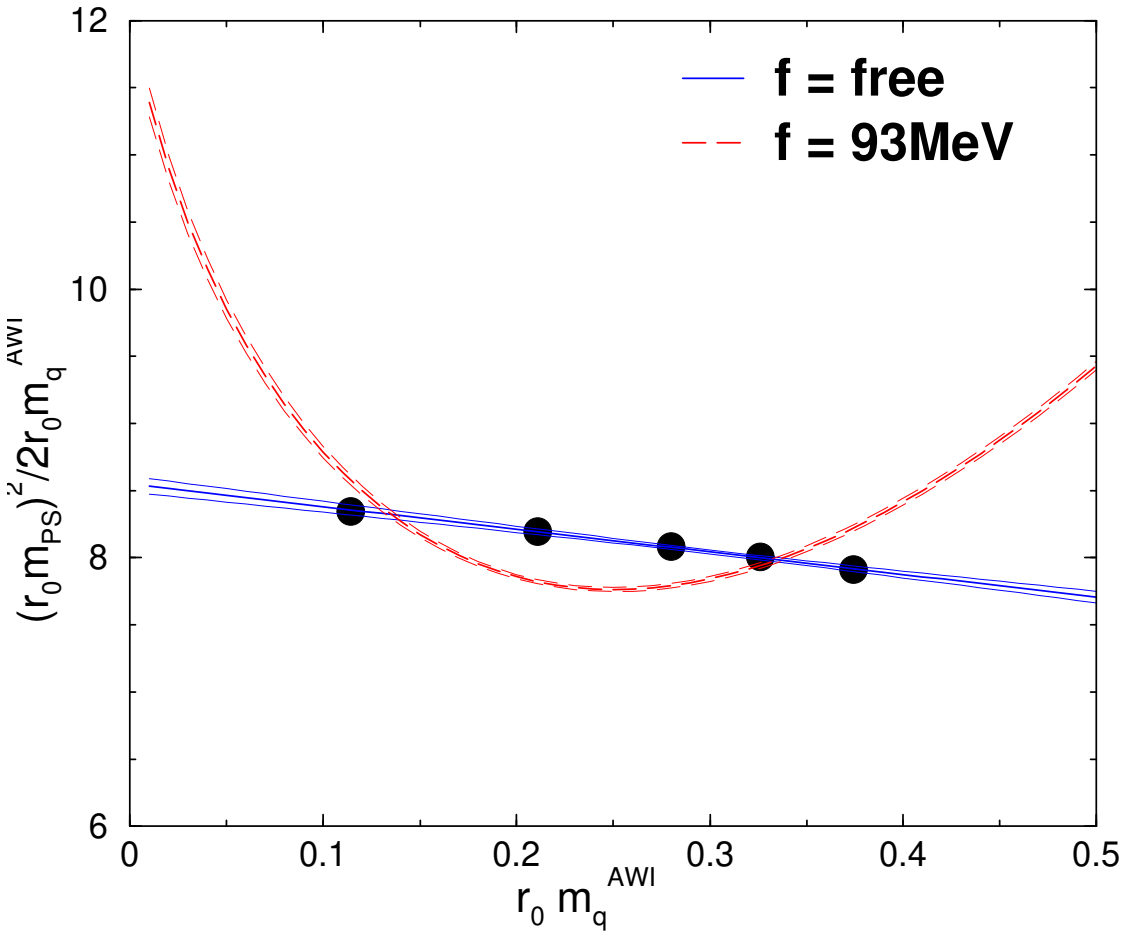

$\Delta m_H(L) \leq 3\%$

- $La \geq 2.2$ fm for mesons $\Rightarrow$ satisfied in simulations by CP-PACS
- $La \geq 2.6$ fm for baryons

# Chiral extrapolation

Comparison with the chiral perturbation theory($\chi$PT) at 1-loop

$$\frac{m_\pi^2}{2B_0 m_q} = 1 + \frac{1}{N_f} y \log(y) + Ay, \qquad y = \frac{2B_0 m_q}{(4\pi f)^2}$$



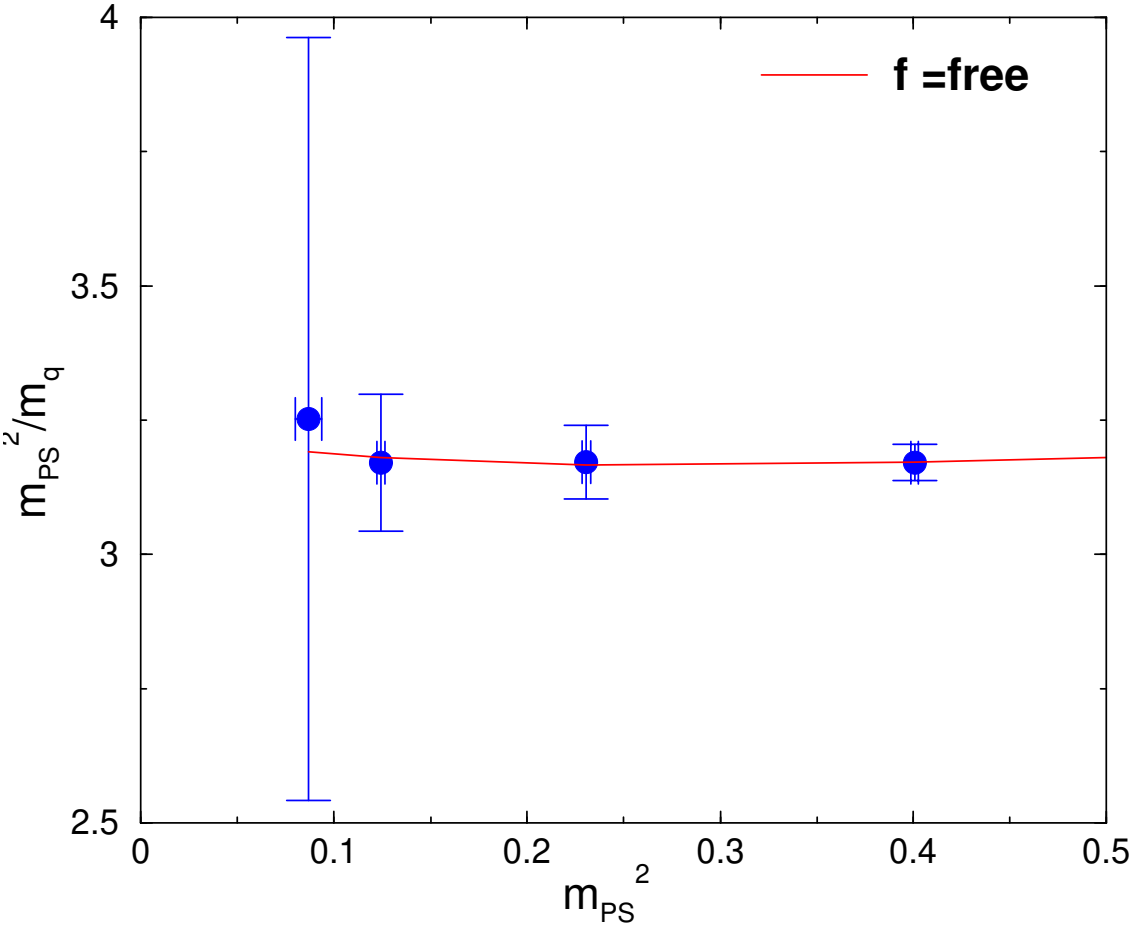

Lattice data disagree with the prediction of $\chi$PT !

Panel discussion in lattice 2002 (hep-lat/0209086)

$m_\pi/m_\rho \geq 0.6$ is too large for the 1-loop $\chi$PT ?

## CP-PACS preliminary

- RG + $O(a)$ (MF) improved Wilson
- $a^{-1} = 1$ GeV, $L \simeq 2.4$ fm
- $m_\pi/m_\rho = 0.35,\ 0.4,\ 0.5,\ 0.6$




no sign for log at $m_\pi/m_\rho \geq 0.4$

( error is too large at $m_\pi/m_\rho \simeq 0.35$)

- the explicit breaking of the chiral symmetry for the Wilson-type quark actions ?

  $\longrightarrow$ KS, domain-wall, overlap quark actions

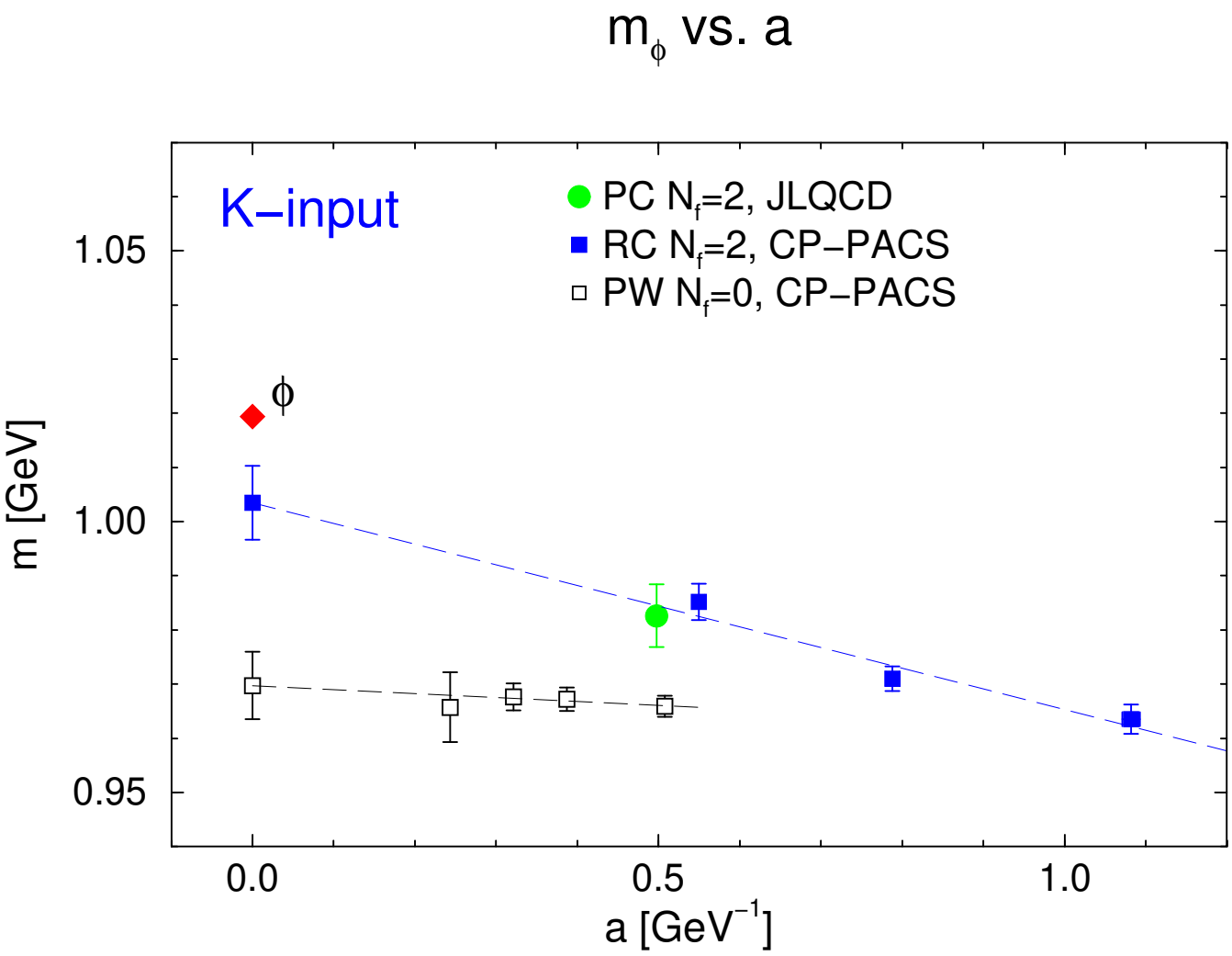
- better algorithm and/or action toward lighter quark masses

  $\rightarrow$ Two-Step Multi-Boson (TSMB), twisted mass QCD

# Continuum extrapolation

## $N_f = 2$ full QCD simulation by JLQCD collaboration

- Plaquette gauge + NP $O(a)$ improved Wilson quark
- $a^{-1} \simeq 2$ GeV, $L \simeq 1.8$ fm
- $m_\pi / m_\rho = 0.60$–$0.80$



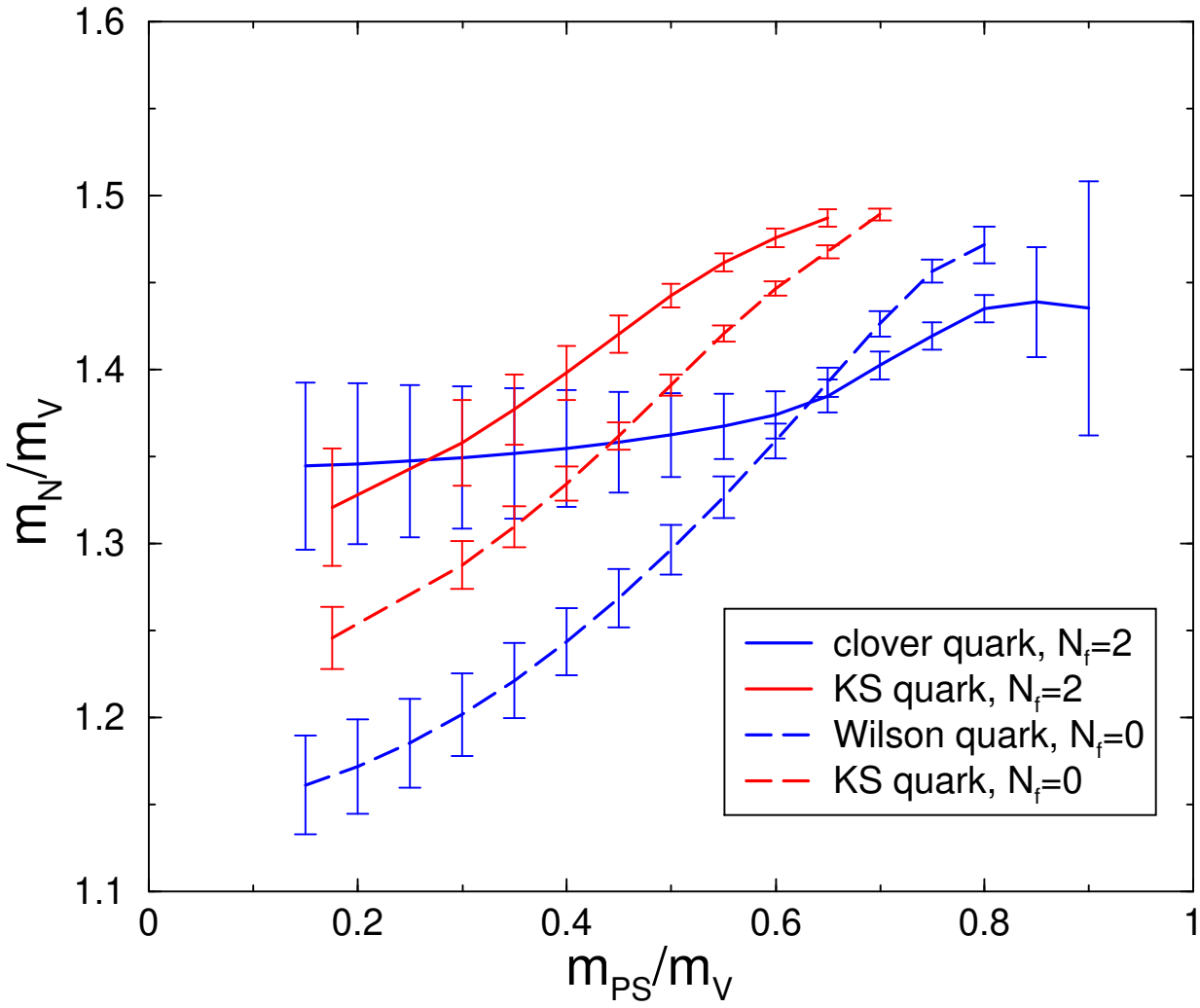

- consistent with $m_\phi$ from CP-PACS
- continuum extrapolation is necessary for the definite conclusion

## A test for universality

S.A., hep-lat/0011074

$m_N/m_\rho$ vs. $m_\pi/m_\rho$

CP-PACS, MILC




Wilson-type quarks $\neq$ KS quarks !?

- even in the continuum limit
- already for the quenched QCD
- even at relatively large quark masses

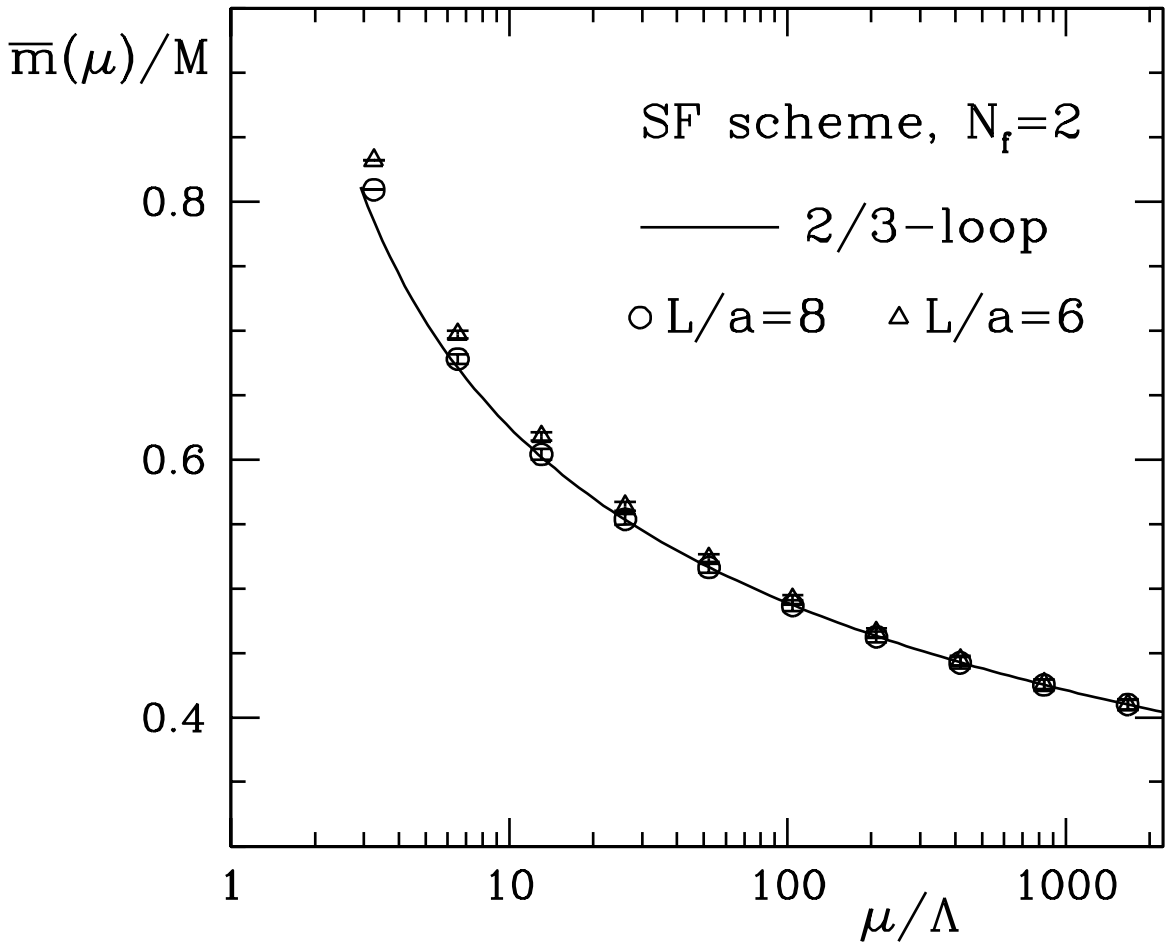
$\Rightarrow$ should be understood !

## Renormalization

Non-perturbtaive renormalization for quark mass

Schrödinger functional scheme
ALPHA Collaboration, hep-lat/0209025

Running quark mass $\overline{m}(\mu)/M$ ($N_f = 2$, preliminary)




$M$: renormalization group invariant quark mass

- continuum extrapolation is still necessary
- comparison with the perturbation theory
- should apply this to $m_s$ from CP-PACS
  $\Rightarrow$ need more work

# Dynamical strange quark

## Algorithm for $N_f = 2 + 1$ full QCD simulations

Polynomial Hybrid Monte Carlo (PHMC)

$$Z = \int \mathcal{D}U \, (\det D_1)^{2N_1} \, (\det D_2)^{N_2} \, e^{-S_G(U)}$$

with $D_{1,2}^\dagger = \gamma_5 D_{1,2} \gamma_5$.

even flavors $2N_1$ (HMC)

$$(\det D_1)^{2N_1} = \int \mathcal{D}\phi_1^\dagger \mathcal{D}\phi_1 \exp\left[ - \left| D_1^{-N_1} \phi_1 \right|^2 \right]$$

odd flavor $N_2$ (PHMC)

$$\begin{aligned} (\det D_2)^{N_2} &= \left[ \frac{\det[D_2 P_{N_{poly}}[D_2]]}{\det P_{N_{poly}}[D_2]} \right]^{N_2} \\ &= \left[ \det[D_2 P_{N_{poly}}[D_2]] \right]^{N_2} \int \mathcal{D}\phi_2^\dagger \mathcal{D}\phi_1 \exp\left[ - \left| \left( T_{N_{poly}}[D_2] \right)^{N_2} \phi_2 \right|^2 \right] \end{aligned}$$

where

$$\begin{aligned} \det[P_{N_{poly}}[D_2]] &= c \prod_{j=1}^{N_{poly}/2} \det[D_2 - z_j]^\dagger \det[D_2 - z_j] \\ &= \det\left[ T_{N_{poly}}[D_2]^\dagger T_{N_{poly}}[D_2] \right], \\ D_2 P_{N_{poly}}[D_2] &\simeq 1. \end{aligned}$$

Here a Metropolis test is needed for the correction factor

$$\det W \equiv \left[ \det[D_2 P_{N_{poly}}[D_2]] \right]^{N_2}.$$

## A preliminary test for $N_f = 3$

JLQCD collaboration, hep-lat/0110088

Plaquette gauge + $O(a)$ (MF) improved Wilson quark

$\Downarrow$

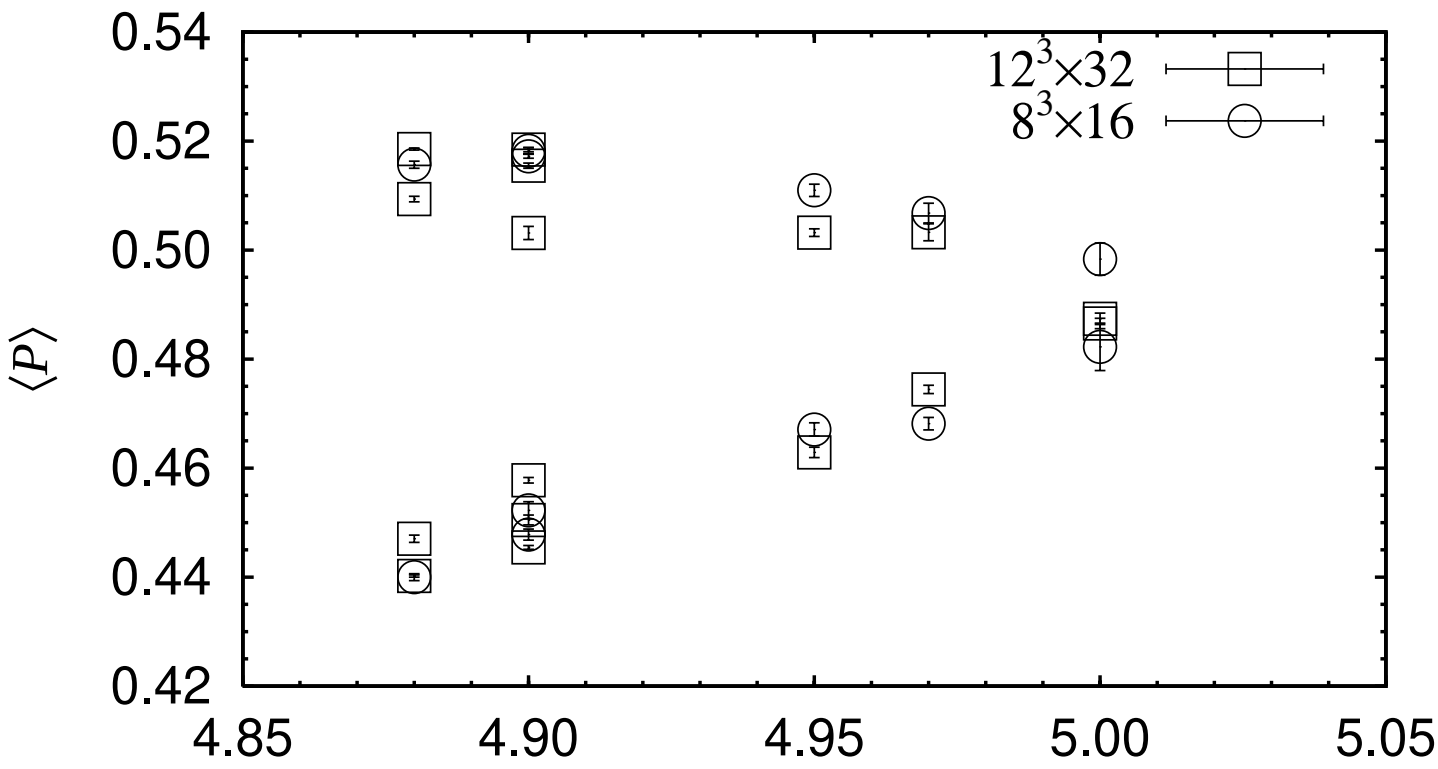
1st order phase transition (lattice artifact)

at $\beta \leq 5.0$ ($a^{-1} \leq 2.6$ GeV)

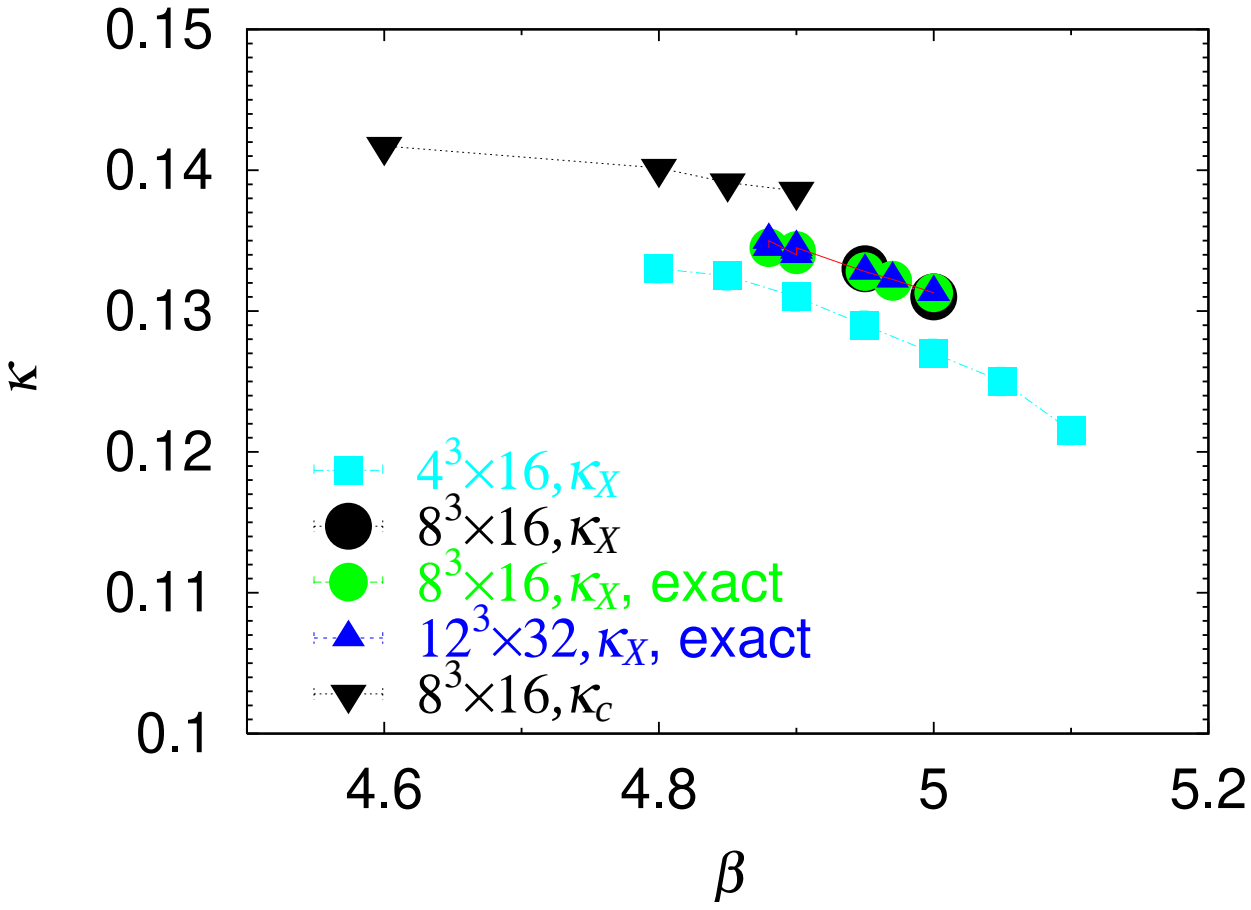
$\Downarrow$

continuum extrapolation from $a^{-1} \simeq 2$ GeV is impossible !

$P$ vs. $\beta$ at the transition point



Phase structure




This lattice artifact disappears for the RG gauge action.

JLQCD

$\Downarrow$

RG gauge action + $O(a)$ improved Wilson quark action

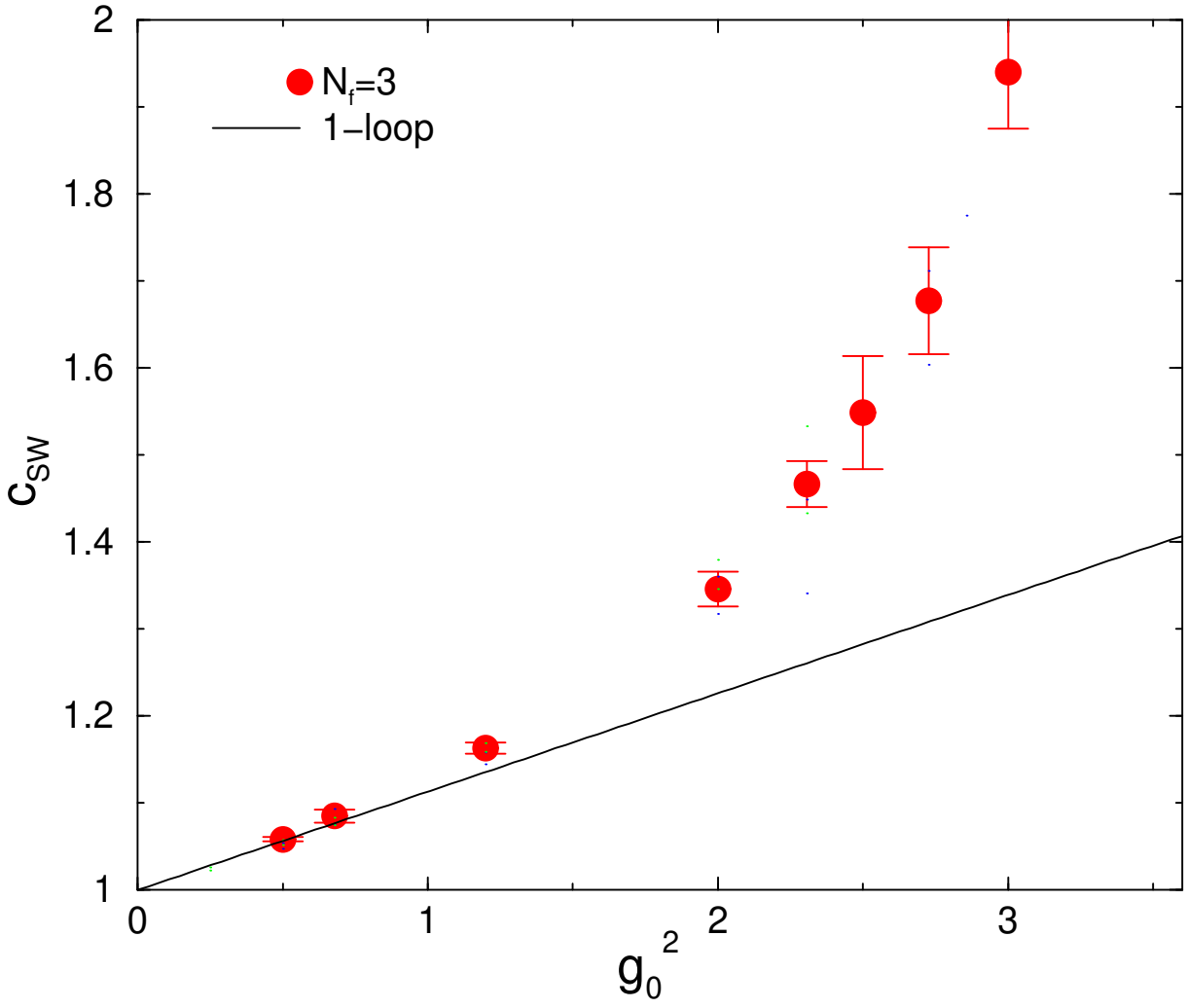
CP-PACS+JLQCD collaborations

## Non-perturbative determination of $c_{\rm SW}$

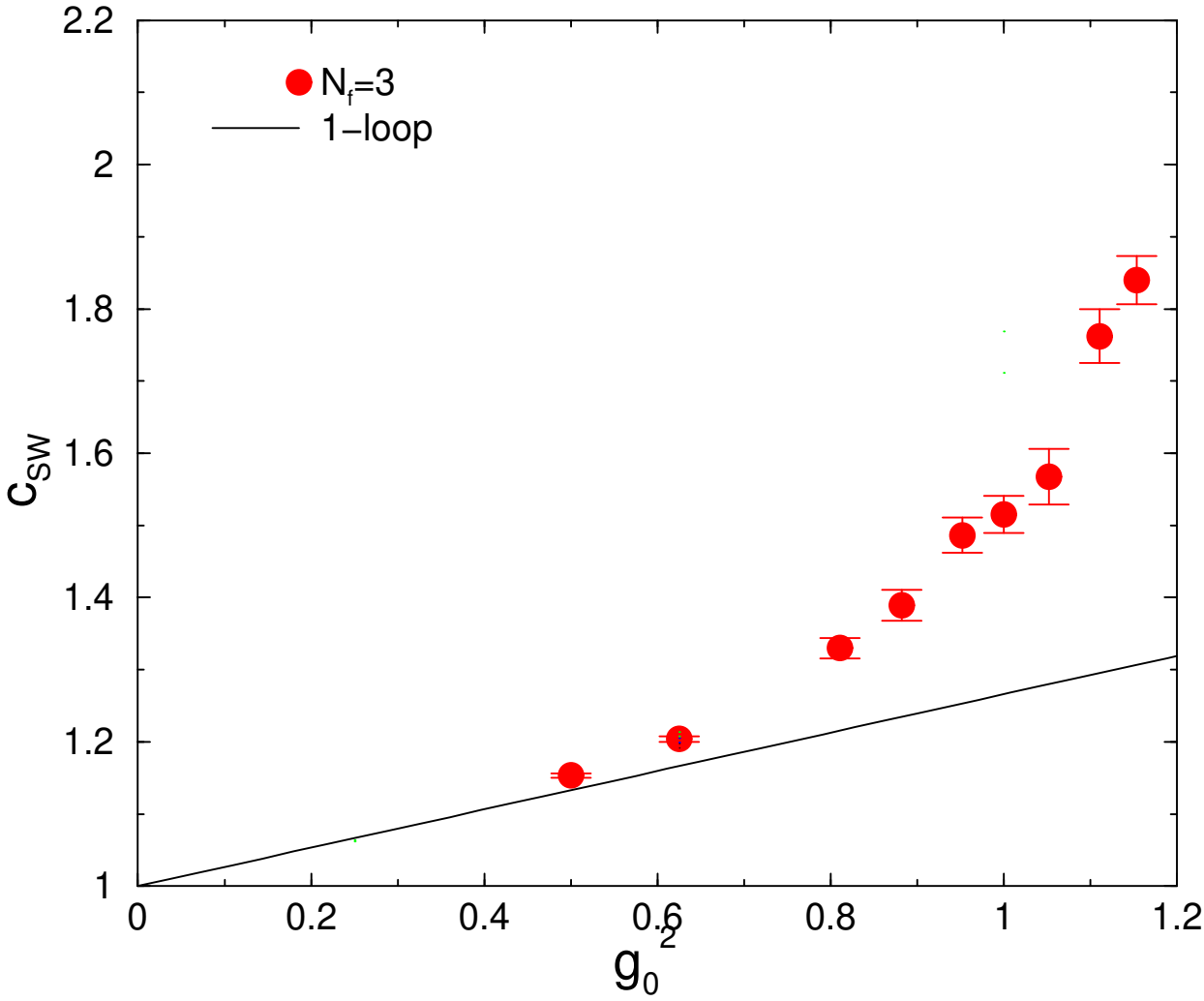
- Schrödinger functional, $8^3 \times 16$
- RG action, $N_f = 3$

$c_{\rm SW}$ vs. $g_0^2$

CP-PACS+JLQCD, RG action, Preliminary



CP-PACS+JLQCD, Plaquette action, Preliminary



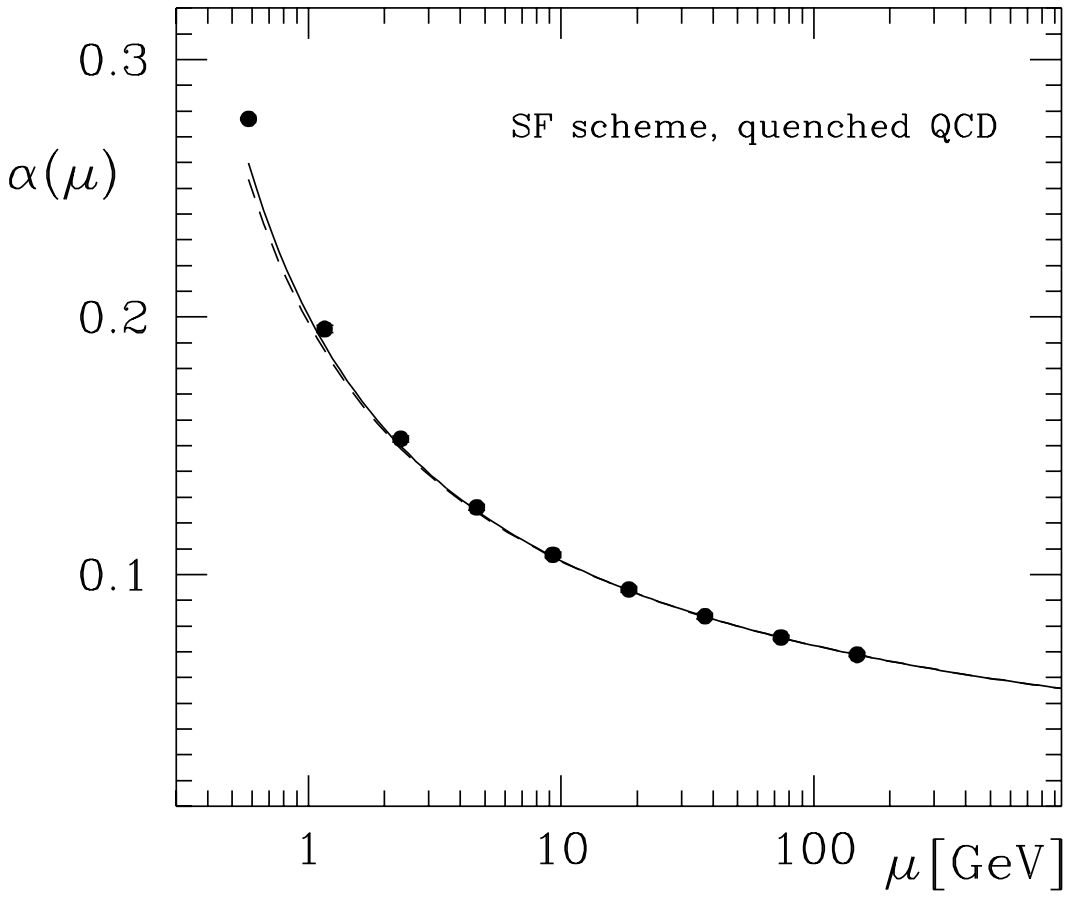
# Running coupling in QCD

## Method

- finite box, Schrödinger functional scheme
- step scaling function → large range of energy scale
- non-perturbative, continuum extrapolation
- scale → $\Lambda_{\overline{\rm MS}}$

## quenched QCD ($N_f = 0$)

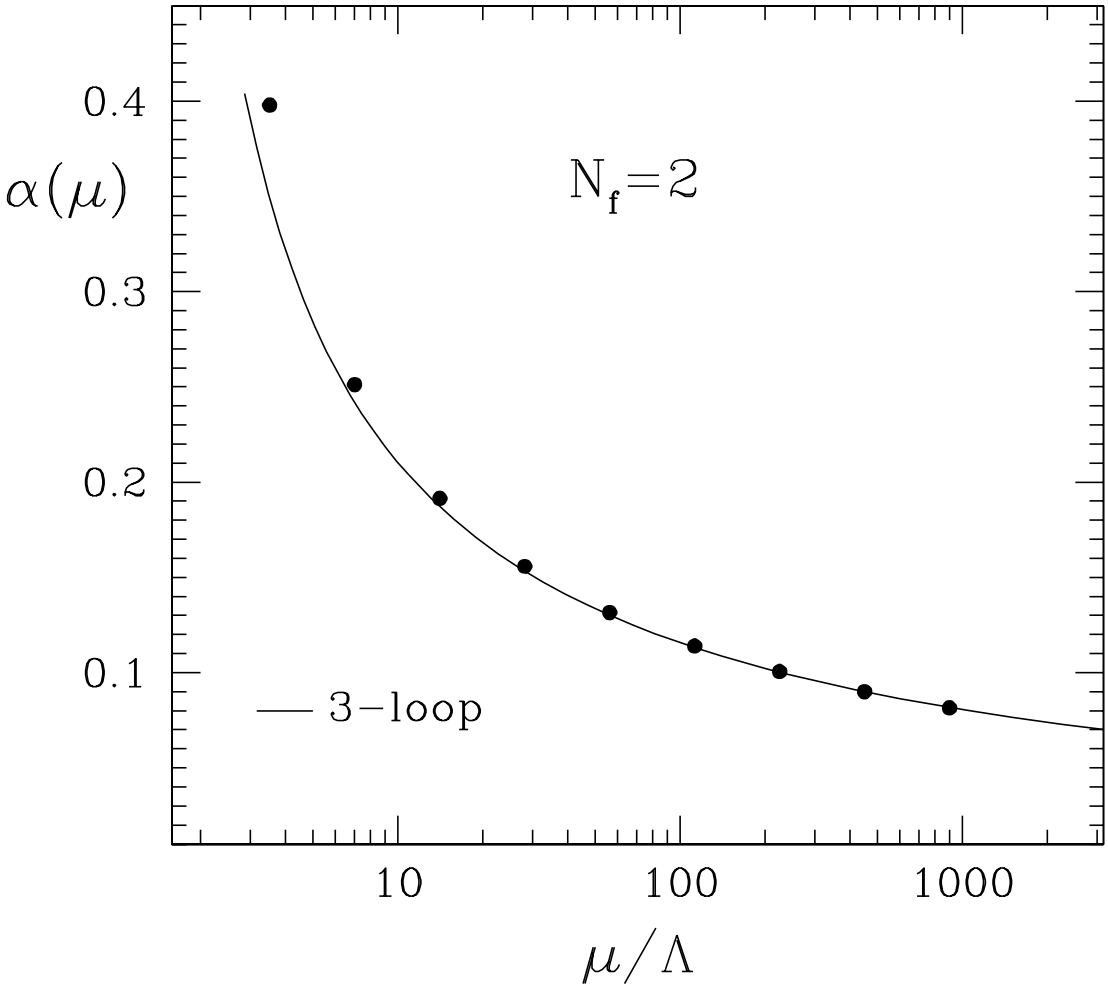
ALPHA collaboration, hep-lat/9309005




$$\Lambda_{\overline{\rm MS}}^{(0)} = 251 \pm 21 \quad \text{MeV}$$

## full QCD ($N_f = 2$)

ALPHA collaboration, hep-lat/0105003,0209023




- comparison with the perturbation theory
- the scale determination will be needed for $\Lambda^{(2)}_{\overline{\rm MS}}$

## Conclusions and Discussions

1. Plaquette + NP $O(a)$ improved Wilson actions
   - continuum extrapolation to check $N_f = 2$ result from CP-PACS

2. dynamical quark simulations with lighter quark masses
   - better algorithms and/or better actions
   - comparison with the $\chi$PT
   - new theoretical considerations

3. Universality
   - Wilson vs. KS
   - results from Domain-wall/overlap quarks may help

**4. dynamical quark simulations of $N_f = 2+1$ QCD**

- non-perturbative $c_{SW}$
- meson masses
- strange quark mass ($\leftarrow$ non-perturbative renormalization)
- running coupling