Finite Temperature Field Theory

Dietrich Bödeker, Universität Bielefeld

1. Thermodynamics (better: thermo-statics)
   (a) Imaginary time formalism
   (b) free energy: scalar particles, resummation
      i. pedestrian way
      ii. dimensional reduction
   (c) free energy: QCD

2. Dynamics
   (a) analytic continuation
   (b) hard thermal loop resummation
   (c) applications

Finite temperature field theory

goal: describe macroscopic systems of relativistic particles in or close to thermal equilibrium

applications: early universe, relativistic heavy ion collisions

finite temperature field theory = quantum field theory + stat. Mech.

What one is interested in

Thermodynamics (-statics): equilibrium properties of relativistic matter

- pressure (T)
- screening length
- susceptibilities

Dynamics

- particle production rates
- diffusion constants
- transport coefficients

determined by unequal time correlation functions, i.e., temporal correlations of fluctuations

Imaginary time formalism

natural approach for thermodynamics, can be used for real time as well, requires analytic continuation

also possible: real time formalism

in my talk: only imaginary time formalism

partition function:

$$Z = e^{-\beta F} = tr\left(e^{-\beta H}\right) = \sum_n \langle n|e^{-\beta H}|n\rangle$$

$$\beta \equiv \frac{1}{T} \qquad F = \text{free energy}, \qquad H = \text{Hamiltonian}$$

time evolution operator from time $t = 0$ to $t = t_f$: $\qquad e^{-iHt_f}$

$t = 0$ to $t_f = -i\beta$: $\qquad e^{-\beta H}$

Imaginary time formalism (cont'd)

transition amplitude: Feynman path integral

$$\langle \phi_f | e^{-iHt_f} | \phi_i \rangle = N \int [d\phi \, d\pi] \exp i \underbrace{\int_0^{t_f} dt \int d^3x \, [\pi(x)\partial_t \phi(x) - \mathcal{H}]}_{\text{classical action}}$$

boundary conditions $\phi(0) = \phi_i$, $\phi(t_f) = \phi_f$

$\mathcal{H}$: Hamilton-density

partition funtion: $t = -i\tau$, $0 < \tau < \beta$

$$Z = \int [d\phi_i] \langle \phi_i | e^{-\beta H} | \phi_i \rangle = N \int [d\phi \, d\pi] \, \exp \int_0^\beta d\tau \int d^3x \Big[i\pi(x)\partial_\tau \phi(x) - \mathcal{H} \Big]$$

boundary conditions: $\phi(0) = \phi(\beta) = \phi_i$ periodic

Fermions: anti-periodic

Imaginary time formalism (cont'd)

for bosonic fields: usually $\mathcal{H}$ is quadratic in the canonical momenta π, integration over π can be trivially performed

$$Z = N'(\beta) \int [d\phi] \exp \int_0^\beta d\tau \int d^3x \mathcal{L}$$

$\mathcal{L}$: Lagrange-density

normalization factor $N'(\beta)$ depends on temperature and volume, but not on the interaction strenght

one obtains a T-independent measure by rescaling the fields, $\phi = \sqrt{\beta}\bar{\phi}$

$$Z = N' \int [d\bar{\phi}] \exp \int_0^\beta d\tau \int d^3x \mathcal{L}(\sqrt{\beta}\bar{\phi})$$

Matsubara frequences

finite temperature QFT looks like $T = 0$ QFT with finite imaginary time

Fourier expansion of fields:

time interval finite, (anti-) periodicity $\Rightarrow$ frequences are discrete

$$\phi(\tau, \mathbf{x}) = T \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} e^{-i\omega_n \tau + i\mathbf{p}\cdot\mathbf{x}} \, \phi_n(\mathbf{p})$$

$$\omega_n = \pi T \times \left\{ \begin{array}{cl} 2n & \text{Bosons} \\ 2n+1 & \text{Fermions} \end{array} \right.$$

Feynman rules

like at $T = 0$ except: frequences are (i) imaginary (ii) discrete

$$p_0 = i\omega_n$$

frequency integrals $\rightarrow$ frequency sums

$$\int \frac{d^4p}{(2\pi)^4} f(p_0, \mathbf{p}) \rightarrow T \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} f(i\omega_n, \mathbf{p})$$

How to perform frequency sums

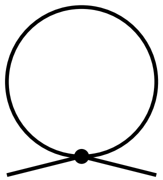
$$T \sum_{\omega_n} f(i\omega_n) = \int_{C_1} \frac{d\omega}{2\pi i} \Big[\pm n(\omega) + \frac{1}{2} \Big] f(\omega)$$

$\Big[\pm n(\omega) + \frac{1}{2} \Big]$ has poles at $\omega = i\omega_n$, residue T

$$n(\omega) = \begin{cases} \frac{1}{e^{\beta\omega}-1} & \text{Bose} \\ \frac{1}{e^{\beta\omega}+1} & \text{Fermi} \end{cases}$$

How to perform frequency sums (cont'd)

example: 1-loop self-energy in scalar φ^4 theory



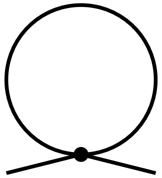
$$= \frac{\lambda}{2}\int\frac{d^3p}{(2\pi)^3}T\sum_{\omega_n}\underbrace{\frac{1}{\omega_n^2+\mathbf{p}^2+m^2}}_{f(i\omega_n)} = \int\frac{d^3p}{(2\pi)^3}\left[n(\omega_p)+\frac{1}{2}\right]\frac{1}{\omega_p}$$

$f(\omega)$: poles at $\omega = \pm\omega_p \equiv \sqrt{\mathbf{p}^2+m^2}$, residue $\frac{-1}{2\omega_p}$

$n(\omega_p) \sim e^{-\beta|\mathbf{p}|}$ for $|\mathbf{p}| \to \infty$ $\to$ ultraviolet finite contribution

ultraviolet divergence is temperature-independent, removed by $T = 0$ renormalization of m

renormalized selfenergy for $T \gg m$:



$$= \frac{\lambda}{2} \int \frac{d^3p}{(2\pi)^3}\, n_{\mathrm{B}}(|\mathbf{p}|)\, \frac{1}{|\mathbf{p}|} = \frac{\lambda}{2}\, \frac{T^2}{2\pi^2} \int\limits_0^\infty dx \frac{x}{e^x - 1} = \frac{\lambda}{2}\, \frac{T^2}{12}$$

Perturbative expansion of Z

massless scalar theory with

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi}_{\mathcal{L}_0} - \underbrace{\frac{\lambda}{4!}\varphi^4}_{\mathcal{L}_{int}} = -\frac{1}{2}(\partial_\tau\varphi)^2 - \frac{1}{2}(\nabla\varphi)^2 - \frac{\lambda}{4!}\varphi^4$$

expansion in λ: $Z = Z_0 + Z_1 + \cdots$

lowest order:

$$Z_0 \quad = \quad N'(\beta)\int[d\phi]\,\exp\left\{\int_0^\beta d\tau\int d^3x\mathcal{L}_0\right\}$$

Lowest order

result, taking into account normalization factor:

$$Z_0 = e^{-\beta F_0} = \exp\left(\frac{\pi^2}{90}T^3 V\right)$$

free energy: $$F_0 = -\frac{\pi^2}{90}T^4 V$$

pressure: $$P_0 = -\frac{\partial F_0}{\partial V} = \frac{\pi^2}{90}T^4$$

First correction (2-loop)

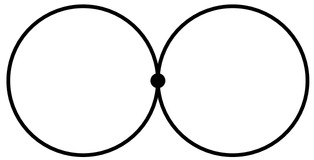
$$Z = N'(\beta)\int [d\phi]\, \exp\left\{ \int_0^\beta d\tau \int d^3x \mathcal{L}_0 \right\} \left[1 + \int_0^\beta d\tau \int d^3x \mathcal{L}_{int} + \cdots \right]$$

$$= Z_0 \left[1 - \frac{\lambda}{4!} \int_0^\beta d\tau \int d^3x \langle \phi^4(x) \rangle_0 + \cdots \right] = Z_0 \left[1 - \frac{\lambda}{4!} \beta V \langle \phi^4(x) \rangle_0 + \cdots \right]$$

with $\qquad \langle \phi^4(x) \rangle_0 = \langle \phi^4(0) \rangle_0 = \dfrac{N'(\beta)}{Z_0} \displaystyle\int [d\phi]\, \phi^4 \exp\left\{ \int_0^\beta d\tau \int d^3x \mathcal{L}_0 \right\}$

independent of the normalization factor $N'(\beta)$

2-loop contribution

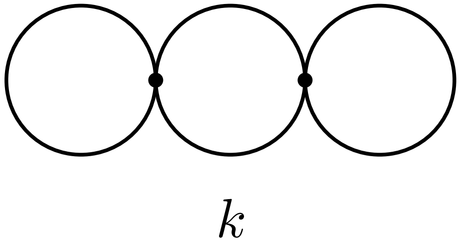
$$\frac{\lambda}{4!}\left\langle\phi^4\right\rangle_0 = \frac{\lambda}{4!}\,3$$  $$= \frac{\lambda}{8}\left(\frac{T^2}{12}\right)^2$$

$$Z = Z_0\left[1 - \beta V\frac{\lambda}{8}\left(\frac{T^2}{12}\right)^2\right]$$

$$\begin{aligned} P &= \frac{1}{\beta V}\log(Z) = \frac{1}{\beta V}\left[\log(Z_0) - \beta V\frac{\lambda}{8}\frac{T^4}{144}\right] \\ &= T^4\left(\frac{\pi^2}{90} - \frac{\lambda}{8\cdot 144}\right) \end{aligned}$$

Higher loop contributions

naively: 3-loop result for



is of order λ^2

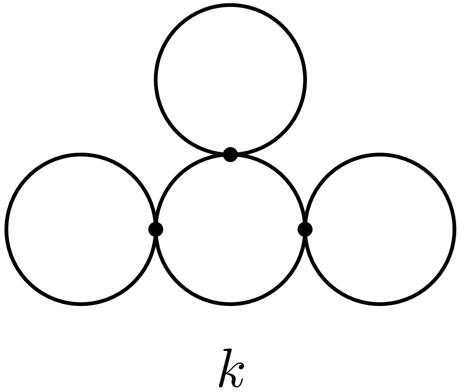
however: zero Matsubara frequency contribution $k_0 = 0$

$$\propto \int \frac{d^3k}{\mathbf{k}^4}$$

is infrared-divergent

Higher loop contributions (cont'd)

higher loop diagrams are even more divergent

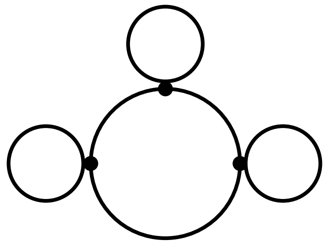


contains

$$\int \frac{d^3k}{\mathbf{k}^6}$$

Resummation of thermal mass insertions

sum infinite class of diagrams before doing the k-integration

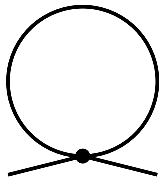


$\dots$

corresponds to the replacement

$$\frac{1}{\mathbf{k}^2} \to \frac{1}{\mathbf{k}^2 + m_{th}^2}$$ 'hard thermal loop resummation'

thermal mass

$m_{th}^2 =$  $= \frac{\lambda}{24} T^2$

Resummation of thermal mass insertions

resummed zero Matsubara frequency contribution

$$P_{res} = \frac{1}{\beta V} \log Z_{res} = -\frac{T}{2} \int \frac{d^3k}{(2\pi)^3} \log(\mathbf{k}^2 + m_{th}^2) = -\frac{T}{2}\frac{1}{2\pi^2} \underbrace{\int_0^\infty dk k^2 \log(k^2 + m_{th}^2)}_{= I}$$

only temperature dependence through m_{th}^2

$$\frac{\partial I}{\partial m_{th}^2} = \int_0^\infty dk \frac{k^2}{k^2 + m_{th}^2} = \int_0^\infty dk \left(1 - \frac{m_{th}^2}{k^2 + m_{th}^2}\right) \to -\int_0^\infty dk \frac{m_{th}^2}{k^2 + m_{th}^2} = -\frac{\pi}{2}\left[m_{th}^2\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad I = -\frac{\pi}{3}\left[m_{th}^2\right]^{\frac{3}{2}}$$

Resummation of thermal mass insertions

$$P_{res} = \frac{T m_{th}^3}{12\pi} = \frac{\lambda^{\frac{3}{2}} T^4}{96\pi \cdot 6^{\frac{3}{2}}}$$

result in non-analytic in the coupling constant λ

consequence of the infrared divergences in unresummed perturbation theory

perturbative expansion $=$ expansion in $\lambda^{\frac{1}{2}}$

higher orders also contain logarithms of λ through $\log \frac{m_{th}}{T}$

Dimensional reduction

infrared divergent contributions are due to low momenta, corresponding to large distances

idea: seperate high momentum ($\mathbf{k} \sim T$) and low momentum ($\mathbf{k} \ll T$) contributions

perform calculation step by step, starting by integrating out high momentum modes

$\rightarrow$ effective theory for low momentum modes, it contains only zero Matsubara frequency fields

effective theory $=$ 3-dimensional theory

write down most general 3-dimensional Lagrangian consistent with the symmetries

compute coefficients of 3-dimensional Lagrangian by low momentum matching Green functions in the full and the effective theory

matching calculation is simple because no resummation is necessary: full theory and effective theory are equivalent in the infrared

Dimensional reduction

3-dim. Lagrangian for $\quad \chi(\mathbf{x}) = \frac{1}{\sqrt{\beta}}\varphi_0(\mathbf{x})$

$$\mathcal{L}_3 = -f - \frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}m_3^2\chi^2 - \frac{\lambda_3}{4!}\chi^4 + \text{higher dimensional operators}$$

$\lambda_3 = T\lambda +$ higher order

partition function:

$$Z = e^{-\beta F} \quad = \quad \int^{(\Lambda)} [d\chi] \exp \int d^3x \mathcal{L}_3 = e^{-Vf} \int^{(\Lambda)} [d\chi] \exp \int d^3x \mathcal{L}_3(\chi)$$

f/β: high momentum ($\mathbf{k} \gg m_{th}$) contribution to free energy density

all parameters depend on ultraviolet cutoff Λ of the effective theory

use dimensional regularization

Determination of f

compute Z in the full and in the effective theory and match the results

no resummation

full theory:

$$\log(Z) = VT^3 \left[\frac{\pi^2}{90} - \frac{\lambda}{8 \cdot 144} + O(\lambda^2) \right]$$

effective theory: (treat mass term as perturbation)

$$\log(Z) = -Vf + \text{loop contributions}$$

all loop contributions in the effective theory vanish in dimensional regularization

result:

$$f = -T^3 \left[\frac{\pi^2}{90} - \frac{\lambda}{8 \cdot 144} + O(\lambda^2) \right]$$

Determination of m_3^2

compute propagator in full theory and effective theory

full theory

$$\Delta(\mathbf{p}) = \frac{1}{\mathbf{p}^2} - \frac{1}{\mathbf{p}^4}\frac{\lambda}{24}T^2 + O(\lambda^2)$$

effective theory (treat mass term as perturbation)

$$\Delta(\mathbf{p}) = \frac{1}{\mathbf{p}^2} - \frac{1}{\mathbf{p}^4}m_3^2 + O(\lambda^2)$$

result:

$$m_3^2 = \frac{\lambda}{24}T^2 + O(\lambda^2)$$

Second step: perturbation theory in 3-dim. theory

include the mass term in the non-perturbed Lagrangian

$$(\mathcal{L}_3)_0 = -\frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}m_3^2\chi^2$$

$$(\mathcal{L}_3)_{int} = -\frac{\lambda_3}{4!}\chi^4$$

do perturbative expansion in λ_3

λ_3 has mass dimension 1, dimensionless loop expansion parameter is

$$\frac{\lambda_3}{m_3} = O(\sqrt{\lambda})$$

Second step: perturbation theory in 3-dim. theory (cont'd)

lowest order in perturbation theory

$$\frac{1}{V}\log(Z_3) = -f - \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\log(\mathbf{p}^2 + m_3^2) = -f + \frac{m_3^3}{12\pi}$$

using this method the free energy has been calculated to order λ^3

QCD

partition function

$$Z = N'(\beta) \int [dAd\psi d\overline{\psi}] \exp \int_0^\beta \int d^3x \left\{ -\frac{1}{2} \mathrm{tr} F_{\mu\nu} F^{\mu\nu} + \overline{\psi} \gamma^\mu D_\mu \psi \right\}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \qquad D_\mu = \partial_\mu - igA_\mu$$

this expression is exact, valid at all temperatures

for $T \sim \Lambda_{QCD}$: strong coupling, no perturbative expansion possible ($\rightarrow$ lattice)

$T \gg \Lambda_{QCD}$: asymptotic freedom $\Rightarrow$

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} \ll 1$$

perturbative calculation of Z possible

Perturbation theory for QCD

lowest orders similar to $\lambda\varphi^4$-theory, g^2 takes the role of λ

at 2-loop order infrared divergences appear for $\omega_n = 0$ gluon fields

complicaton: only A_0 aquires thermal mass $m_{\mathrm{D}} \sim gT$

even after resummation infrared divergences from spatial gauge fields remain

dimensional reduction: Matsubara frequences always non-zero for fermions $\Rightarrow$

dim. reduced theory contains only gluon fields

Dimensional reduction for QCD

3-dimensional Lagrangian: symmetric under 3-dimensional (time independent) gauge transformations, rotations

3-dim. fields

$$\left[A_\mu(\mathbf{x})\right]_{3-d} = \frac{1}{\sqrt{\beta}} \int_0^\beta d\tau \left[A_\mu(\tau, \mathbf{x})\right]_{4-d}$$

$$\begin{aligned} \mathcal{L}_3 \quad = \quad & -f - \frac{1}{2}\mathrm{tr}F^{ij}F^{ij} - \frac{1}{2}(D_iA_0)^a(D_iA_0)^a - \frac{1}{2}m_\mathrm{D}^2 A_0^a A_0^a - \frac{\lambda_3}{8}(A_0^a A_0^a)^2 \\ & + \text{higher dimensional operators} \end{aligned}$$

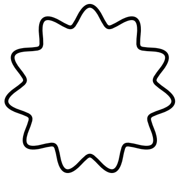
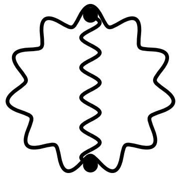
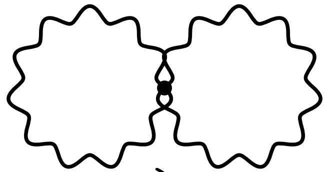
$D_i = \partial_i - ig_3A_i$ with 3-dim. gauge coupling

$$g_3 = \sqrt{T}g$$

gauge invariance does not allow a mass term for spatial gauge fields

Determination of f

full 4-dimensional theory

$\frac{1}{V}\log Z =$  $+$   $+$ fermion, ghosts $+ O(g^4)$

$$= -T^3 d_A \frac{\pi^2}{9}\left\{\frac{1}{5} - N_c \frac{g^2}{(4\pi)^2} + O(g^4)\right\} + \text{fermion contributions}$$

$N_c = 3$: number of colors, $d_A = N_c^2 - 1$: number of gluon fields

3-dim theory (dimensional regularization)

$$\frac{1}{V}\log Z = 0$$

Determination of m_D^2

determine pole of A^0-propagator which is at $\mathbf{p}^2 \sim g^2T^2$

4-dim. theory

$$\Pi_{00}(p=0) = \frac{1}{3}g^2T^2\left(N_c + \frac{N_f}{2}\right) + O(g^4)$$

3-dim. theory

$$\Pi_{00}(p=0) = 0$$

result:

$$m_\mathrm{D}^2 = \frac{1}{3}g^2T^2\left(N_c + \frac{N_f}{2}\right) + O(g^4)$$

Dimensional reduction for QCD

even in the 3-dim. theory infrared divergences appear due to loops with spatial gauge fields

A_0-loops are finite due do Debye mass $m_{\rm D} \sim gT$

second step in reduction: integrate out momentum scale gT

loop expansion parameter:

$$\frac{g_3^2}{m_{\rm D}} \sim g$$

result: effective theory for spatial gauge fields (magneto-static QCD)

$$\mathcal{L}_{mag} = -f_{mag} - \frac{1}{2}{\rm tr}F^{ij}F^{ij} + \text{higher dimensional operators}$$

Determination of f_{mag}

full 3-dim. theory:

$$\frac{1}{V}\log(Z) = -f - f_3$$

contribution from momentum scale $m_{\rm D} \sim gT$:

$$f_3 = -\frac{d_A}{12\pi} m_{\rm D}^3 + O(g^4)$$

magnetostatic QCD:

$$\frac{1}{V}\log(Z) = 0$$

Contribution from magnetostatic gluons

$$Z = \int^{\Lambda_3} [d\mathbf{A}] \exp \int d^3x \mathcal{L}_{mag}$$

rescale the gauge fields $\mathbf{A} \to \frac{1}{g_3}\mathbf{A} \Rightarrow$

$$\mathcal{L}_{mag} \quad = \quad -f_{mag} - \frac{1}{2g_3^2} \mathrm{tr} F^{ij} F^{ij} + \text{higher dimensional operators}$$

only parameter: $g_3^2 = g^2 T$, loop expansion parameter $\sim 1 \Rightarrow$ non-perturbative

contribution of magnetostatic gluons:

$$\log Z \sim V \left(g_3^2\right)^3, \qquad P = \frac{T}{V} \log Z \sim T \left(g_3^2\right)^3 \sim g^6 T^4$$

$$P_{mag} = \frac{1}{\beta V} \log(Z) = O\Big(T (g_3^2)^3\Big) = O(g^6 T^4)$$

Summary of QCD thermodynamics

perturbation series is an expansion in g, not g^2 due to infrared effect

pressure has been calculated to order g^5, convergence of perturbation series is poor

order g^6 contribution is non-perturbative

determined by magnetostatic QCD $=$ 3-dim. QCD

order g^6 contribution can be determined from 3-dim. lattice simulation

Dynamical quantities

objects of interest: real time correlation functions

$$C(t) = \langle O(t)O(0)\rangle = \frac{1}{Z}\mathrm{tr}\left[O(t)O(0)e^{-\beta H}\right]$$

unequal time correlations in thermalized system

much less is known than for thermostatics

more scales are involved, also for (real) frequences

Examples

1) photon production from a thermalized system of quarks and gluons

photons couple only through e.m. interactions

once produced, they escape from a finite system

probability

$$\frac{1}{tV}(2\pi)^3 2|\mathbf{k}|\frac{dP}{d^3k} = \epsilon_\mu \epsilon_\nu^* \int d^4x e^{ik\cdot x}\langle j^\mu(x) j^\nu(0)\rangle$$

j^μ: electromagnetic current

2) transport coefficients

Kubo formula for electric conductivity

$$\sigma = \frac{1}{6}\lim_{k_0\to 0}\frac{1}{k_0}\int d^4x e^{ik_0 t}\langle [j^i(x), j^i(0)]\rangle$$

similar formulas for viscosity, flavor diffusion coefficients

Imaginary time correlation functions

imaginary time 2-point function of bosonic operator O

$$\Delta_E(\tau) = \langle \mathrm{T}_E\{O(-i\tau)O(0)\}\rangle = \int [d\phi] O(\tau) O(0) \exp \int_0^\beta d\tau' \int d^3x \mathcal{L}$$

is periodic in τ,

$$\Delta_E(\tau) = \Delta_E(\tau + \beta)$$

Fourier expansion:

$$\Delta_E(\tau) = T \sum_n e^{-i\omega_n \tau} \Delta(i\omega_n)$$

defines Δ for all Matsubara frequences, i.e., for a discrete set of points

Analytic continuation

there is a unique analytic continuation $\Delta(\omega)$ of $\Delta(i\omega_n)$ into the complex ω-plane which

1. does not grow faster than a power for $\omega \to \infty$

2. that has singularities only on the real axis

all real time correlation functions can be obtained from this analytic continuation

example:

$$\int dt e^{i\omega t} \langle O(t) O(0) \rangle = i[1 + 2n_{\rm B}(\omega)][\Delta(\omega + i\epsilon) - \Delta(\omega - i\epsilon)]$$

same technique as for static quantities can be used, resummation becomes more complicated

Hard thermal loops

for propagators carrying 'static' soft momenta ($k^0 = 0$, $\mathbf{k} \sim gT$):

only A_0 propagator requires resummation

not so for propagators carrying 'dynamical' soft momenta ($k^0 \sim \mathbf{k} \sim gT$)

1-loop polarization tensor for soft momenta

$$\Pi_{\mu\nu}(k) \simeq m_{\mathrm{D}}^2 \left[-g_{\mu 0}\, g_{\nu 0} + k_0 \int_{\mathbf{v}} \frac{v_\mu v_\nu}{v \cdot k} \right]$$

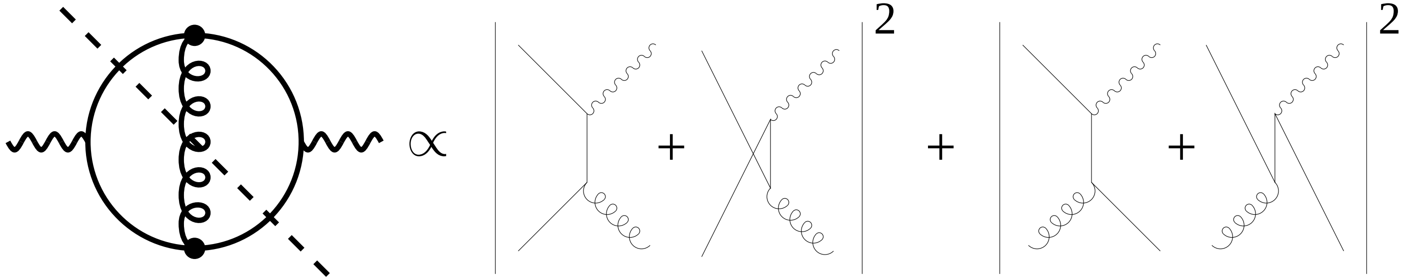
"hard thermal loop polarization tensor"

$g_{\mu\nu}$: metric tensor,

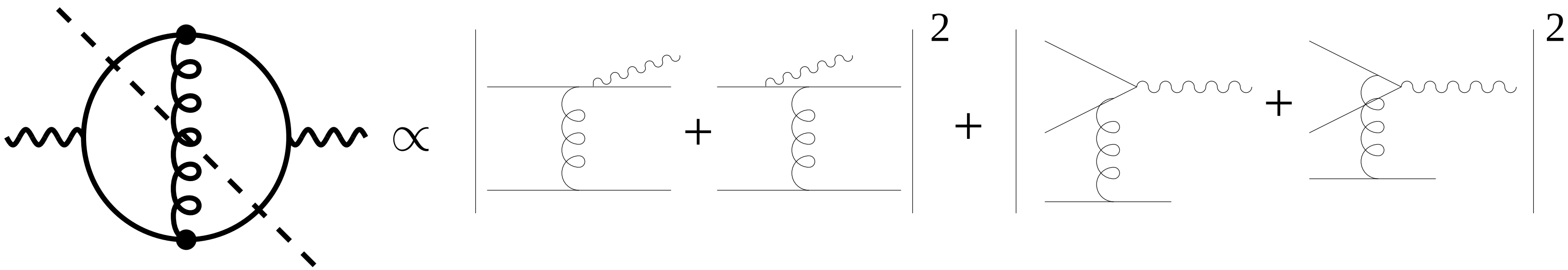
$\mathbf{v}$: velocity of particle in the loop, $\quad v^\mu = (1, \mathbf{v})$, $\quad \int_{\mathbf{v}} = \int \frac{d^2\Omega_{\mathrm{v}}}{4\pi}$

contributions to photon production rate

hard gluon ($k \sim T$)

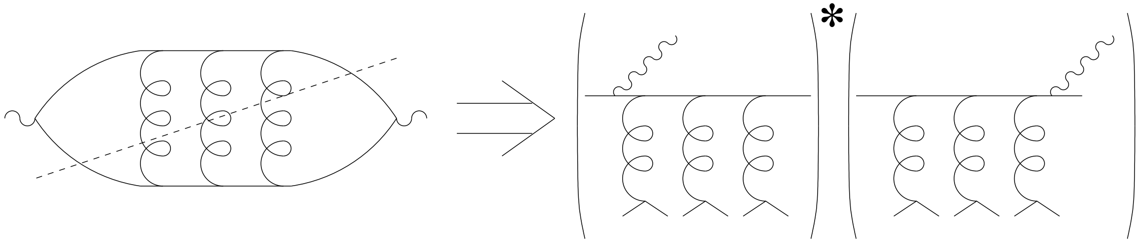


soft gluon (with hard thermal loop resummed propagator):



contributions to photon production rate

also multiple scattering contributes at leading order:



summation of infinite set of multiple scattering processes leads to integral equation

more on hard thermal loops

hard thermal loop vertices: same order as tree level vertices for external momenta of order $m_{\mathrm{D}} \sim gT$

there are HTL n-point functions for all n

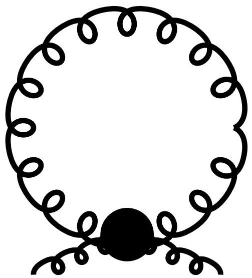
when some external momenta are of order of the magnetic scale:

HTL vertices give larger contribution than tree level vertices due to multiple factors

$$\frac{1}{v \cdot q} \sim \frac{1}{g^2 T}$$

polarization tensor for magnetic scale gluons:

$$\Pi_{\mu\nu}(q) = \Pi_{\mu\nu}^{\mathrm{HTL}}(q) +$$



$$+ \cdots$$

more on hard thermal loops

infinite set of diagrams with HTL vertices and loop momenta $\sim gT$ contributes at leading order

represent multiple scattering of hard ($p \sim T$) particles

resummation possible

result: effective theory for the dynamics of magnetic scale gluons

$$\mathbf{D} \times \mathbf{B} = \sigma \mathbf{E} + \boldsymbol{\zeta}$$

$\boldsymbol{\zeta}$: Gaussian white noise, $\langle \zeta(x)\zeta(x') \rangle = 2\sigma T \delta(x - x')$, $\sigma \sim T/\log(1/g)$

classical equation of motion, describes magnetic gluon up to corrections of order $(\log(1/g))^{-2}$

can be solved on the lattice (Moore) $\rightarrow$ electroweak baryon number violation rate

generalization of magnetostatic QCD: LL equation generates thermal ensemble of magnetic scale gluons (stochastic quantization of magnetostatic QCD)

Summary

- imaginary time formalism is suitable for computing both static and dynamics quantities
- perturbative expansion is expansion in g, not g^2 due to infrared effects
- infrared contribution require resummation of thermal masses
- convenient framework for resummation: dimensional reduction, infrared contributions are due to 3-dim. bosonic fields
- dim. reduction also solves problem of infrared divergences for magnetostatic gluons
- dynamical quantities more infrared sensitive
- no dim. reduction possible
- different observables require different types of resummation for infrared contributions
- most results only valid at leading log order, some at leading order in g