Hadronic vacuum polarization effects in $\alpha_{\mathrm{em}}(M_Z)$ and $(g-2)_\mu$

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① Introduction

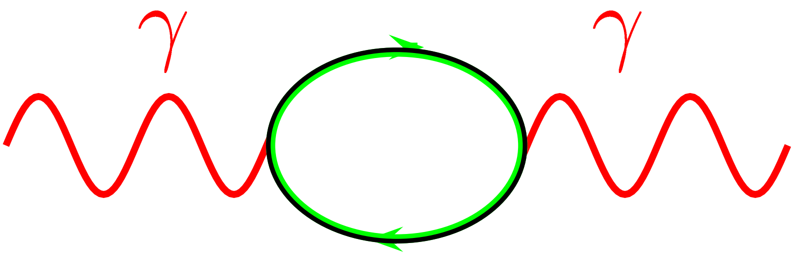
Fundamental "constant" α is a function of the energy scale $\alpha \to \alpha(E)$ of a process: charge screening.

effect due to vacuum polarization in the photon propagator

$$e^2 D_{\mu\nu}(q) = \frac{g_{\mu\nu}\, e^2\, Z}{q^2\ (1 + \Pi(q^2))} + \text{gauge terms} \quad .$$

$$\Downarrow$$

$$e^2 \to e^2(q^2) = \frac{e^2 Z}{1 + \Pi(q^2)}$$

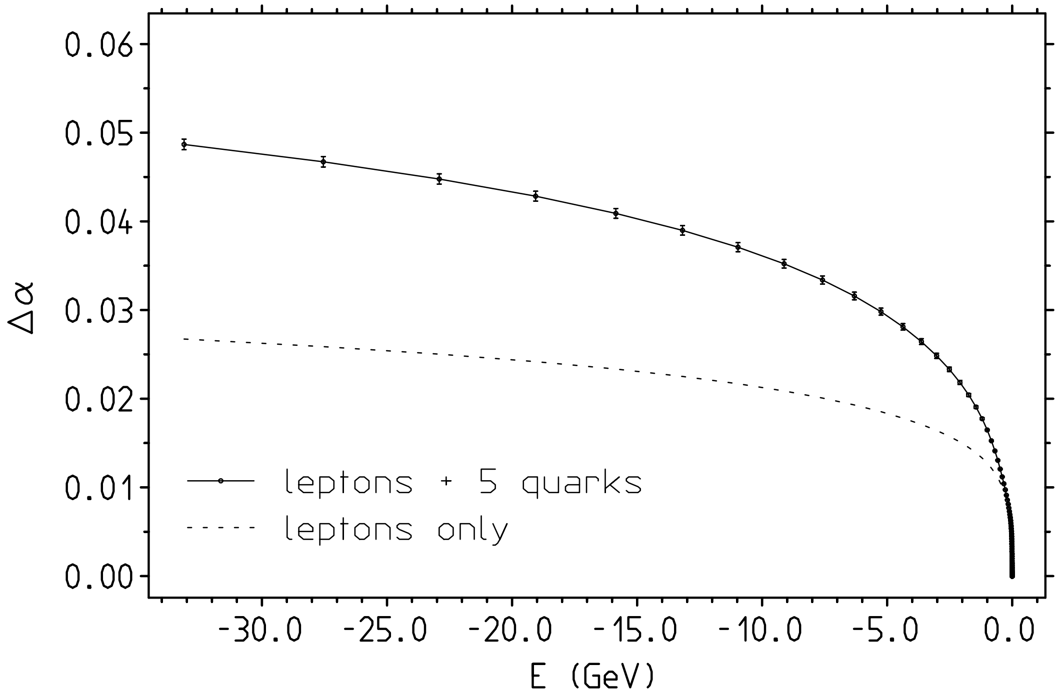


virtual pairs

$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots \to \gamma^*$$

Effective finestructure “constant” $\alpha(E)$

appears in many places in physics! large EWRC !



Of particular interest:

$\alpha(M_Z)$ and $a_\mu \equiv (g-2)_\mu/2$

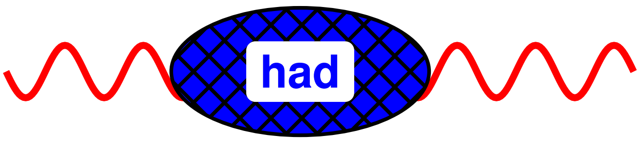
- electroweak effects (leptons etc.) calculable in perturbation theory
- strong interaction effects (hadrons/quarks etc.) perturbation theory fails

What do we need for calculating $\alpha(s)$ and $a_\mu^{\rm had}$?

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha} \quad ; \quad \Delta\alpha = -e^2 \mathrm{Re}\ \left(\Pi'(s) - \Pi'(0)\right)$$

$$\Delta\alpha = \Delta\alpha_{\rm lep} + \Delta^{(5)}\alpha_{\rm had} + \Delta\alpha_{\rm top} + \cdots$$

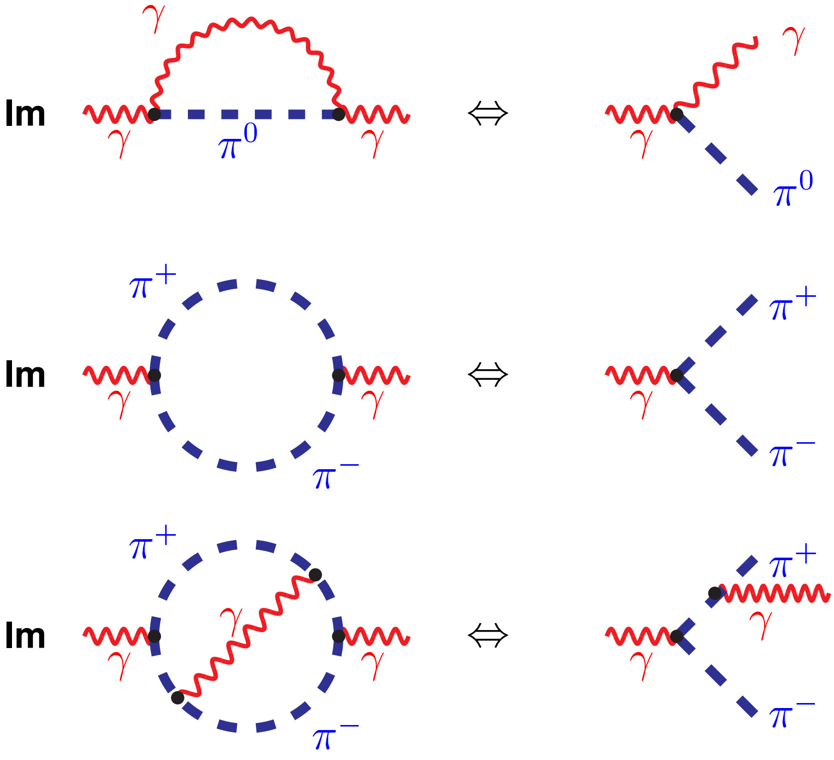
$$\Pi'_{\rm had}(q^2) \quad =$$



1pi blob

1pi blob: $\pi^0\gamma, \rho, \pi\pi, \pi\pi\gamma, \pi\pi Z, \cdots, \pi\pi H, \cdots$ **at least one hadron plus any strong, electromagnetic or weak interaction**

At low energies:



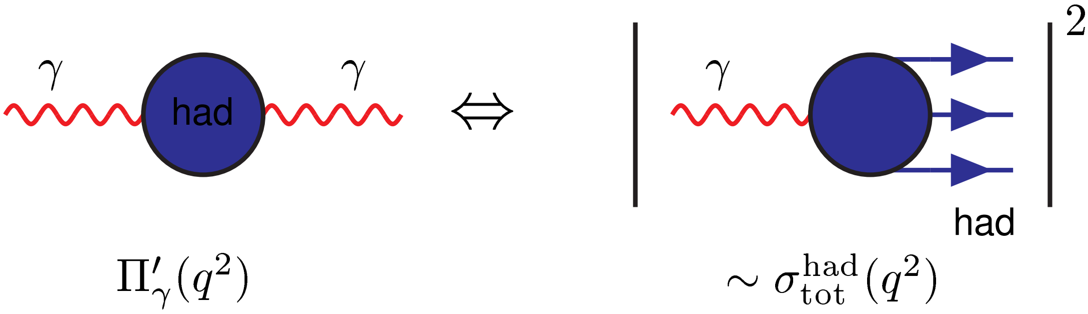
❑ The way out:

non–perturbative approach by dispersion relation

The total cross section

$$\sigma(e^+e^- \to hadrons)$$

for hadro-production in e^+e^-– annihilation directly determines the hadronic contribution to the photon vacuum polarization function $\Pi'_\gamma(q^2)$



causality (analyticity) + unitarity (optical theorem)

$$\Delta\alpha(s) \equiv -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

$$\Delta\alpha^{(5)}_{\mathrm{had}}(s) = -\frac{\alpha s}{3\pi}\left(\quad \mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\mathrm{cut}}} ds' \frac{R^{\mathrm{data}}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\mathrm{cut}}}^{\infty} ds' \frac{R^{\mathrm{QCD}}_\gamma(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

Encoded in R ratio:

$$R(s) \equiv \frac{\sigma_{\rm had}(s)}{\sigma_{\mu\mu}^{(m_\mu=0)}(s)} = \frac{\sigma_{\rm had}^{(0)}(s)}{\sigma_{\mu\mu}^{(0,m_\mu=0)}(s)}$$

$$\begin{array}{cc} || & || \\ \dfrac{4\pi\alpha^2(s)}{3s} & \dfrac{4\pi\alpha^2}{3s} \end{array}$$

$$\sigma_i^{(0)}(s) = \sigma_i(s)\,(\alpha/\alpha(s))^2$$

cross section undressed from QED corrections. Note:

$$\frac{\sigma_{\rm had}(s)}{\sigma_{\mu\mu}^{(m_\mu=0)}(s)} \neq \frac{\sigma_{\rm had}^{\rm obs}(s)}{\sigma_{\mu\mu}^{\rm obs}(s)}$$

convolution with γ radiation does not factor out !

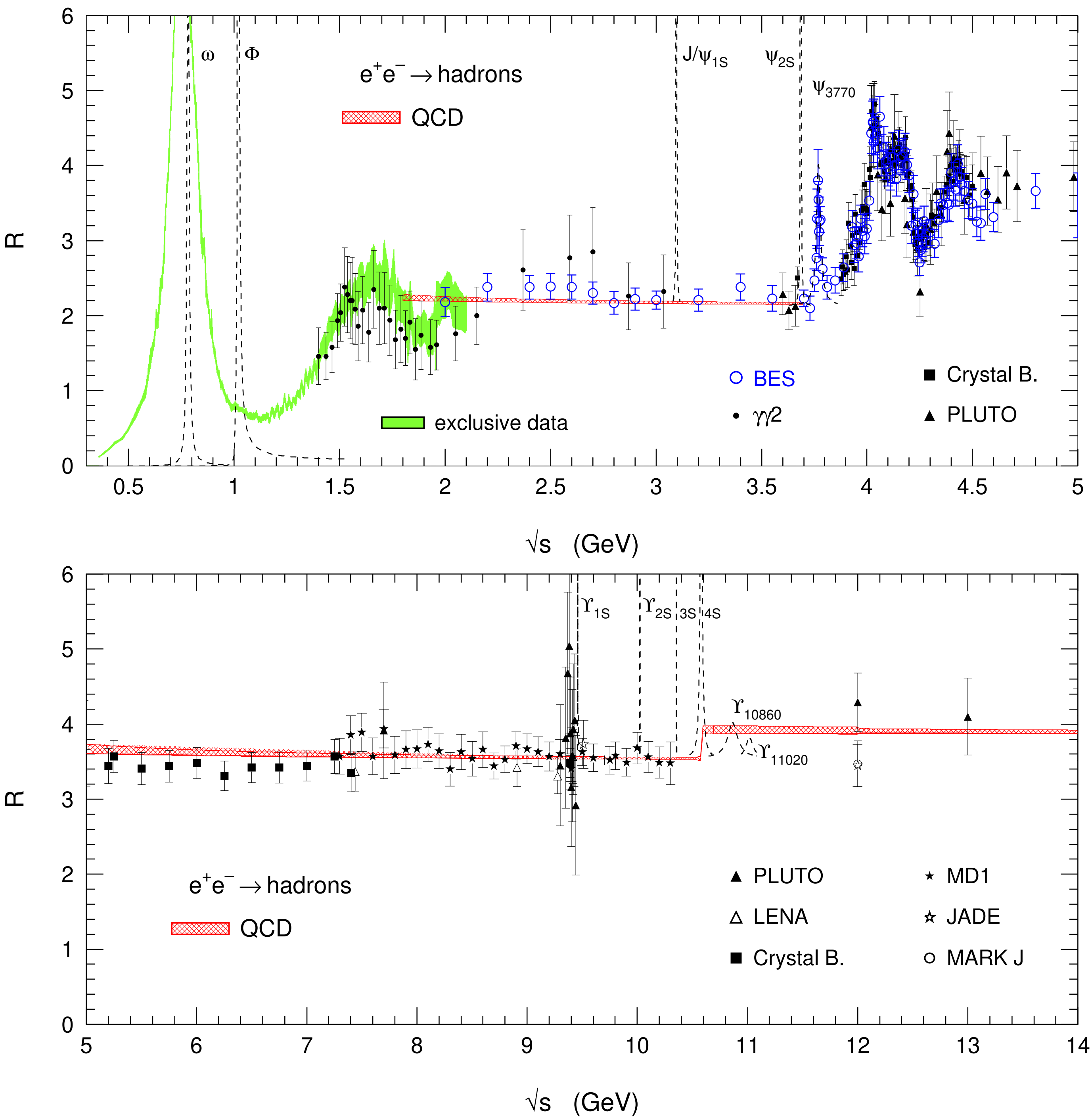
Theory of $R(s) \Rightarrow$ (M. Steinhauser's talk)

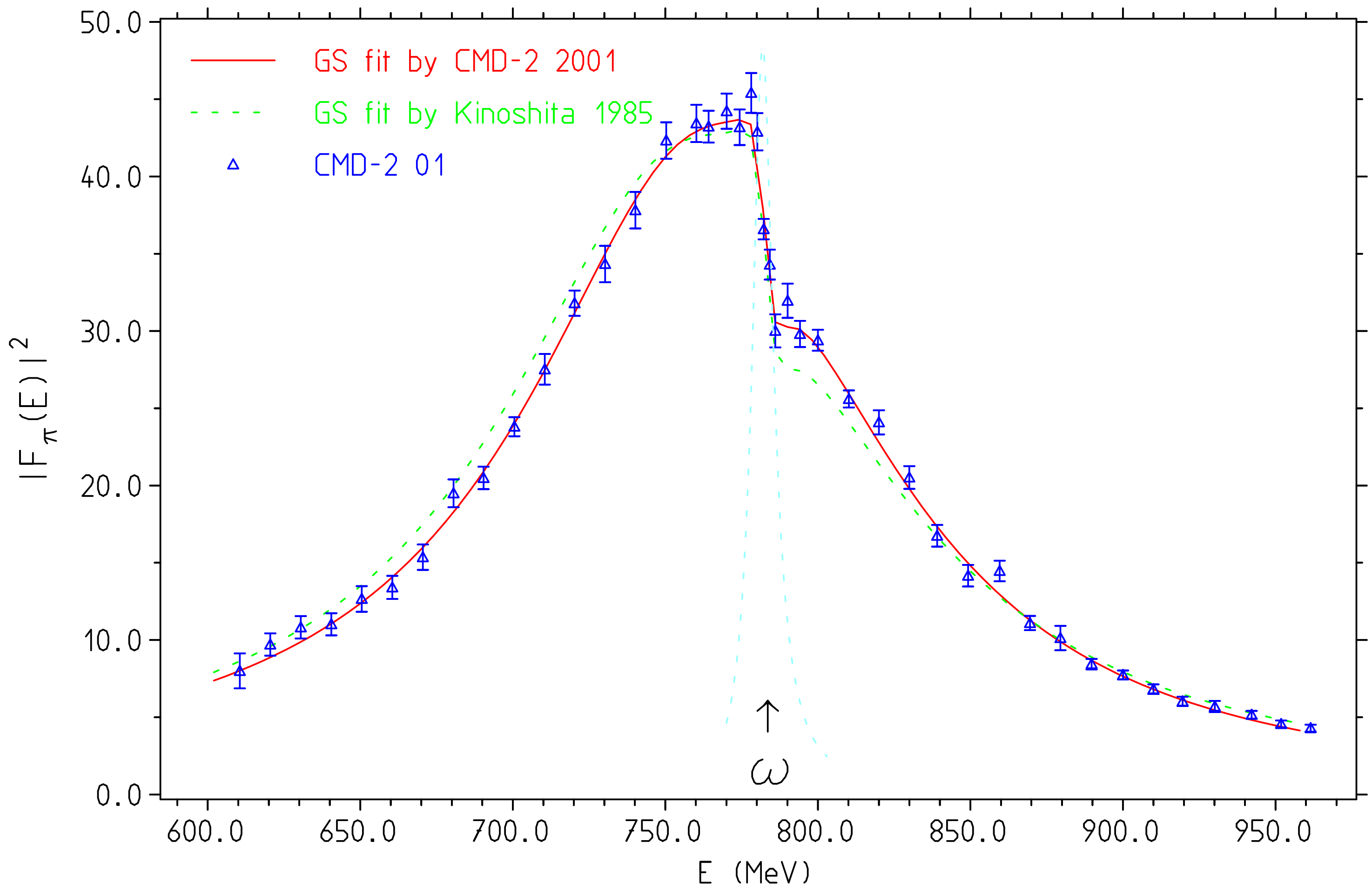
Recent progress:

- **2002: Final CMD-2 $\pi\pi$–data: syst error 1.4% $\to$ 0.6 % !**
  some other CMD-2, SND data at $E < 1.4$ GeV
  (Akhmetshin et al.)
- **2001: BES-II R–data: syst error 20% $\to$ 7% !**
  region 2.0 GeV to 5.0 GeV
  (Bai et al. 2000, 2002)
- **2000: τ–data**
  (ALEPH, OPAL, CLEO Collab., 1996-2002)
- **Iso-spin breaking effects τ–data vs. e^+e^-–data**
  (Cirigliano et al. 2002)
- **new situation: τ–data not compatible with e^+e^-–data at 10% level**

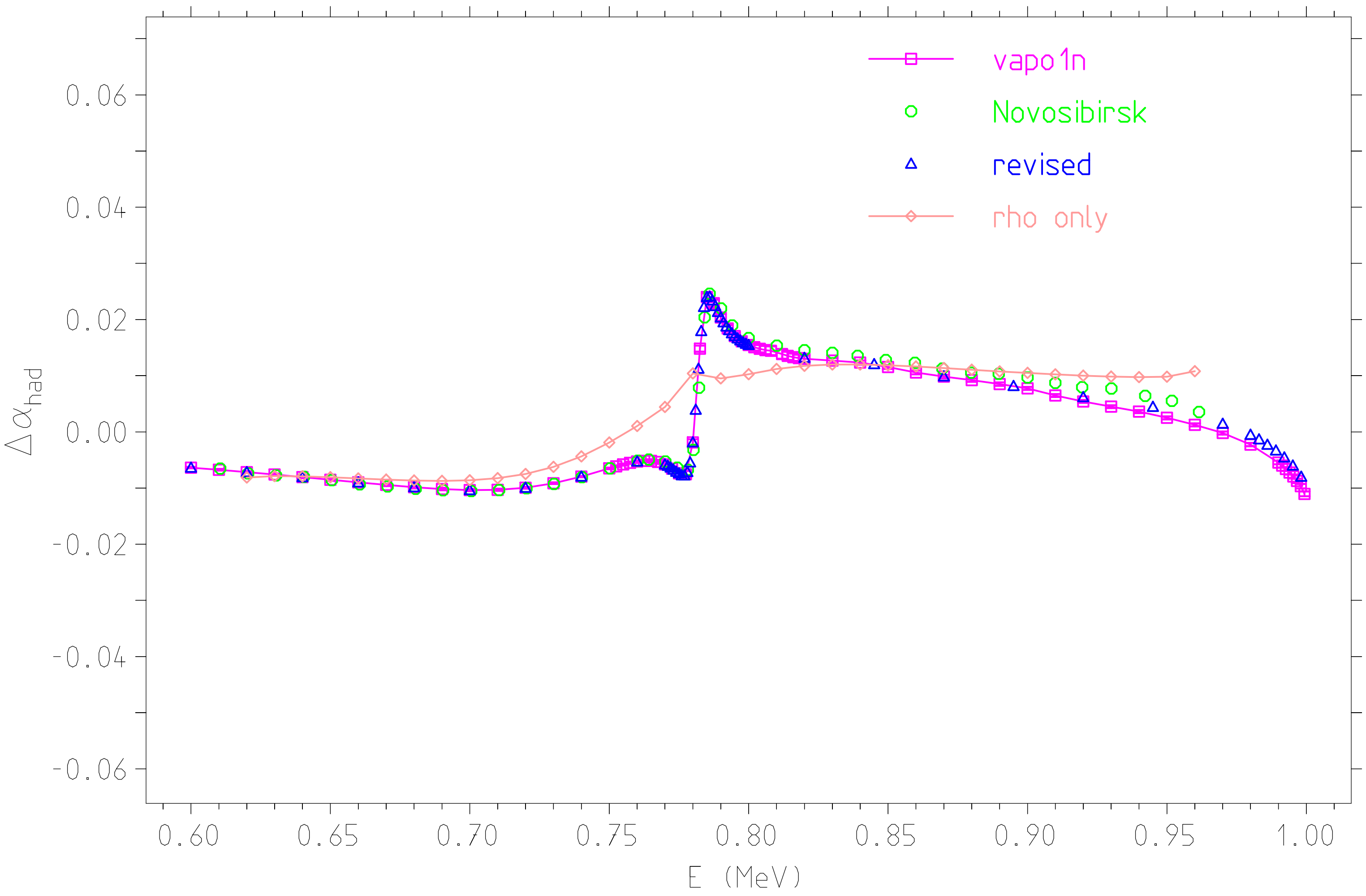
Compilation: Davier, Eidelman et al. 02

Theory = pQCD: Groshny et al. 91, Chetyrkin et al. 97



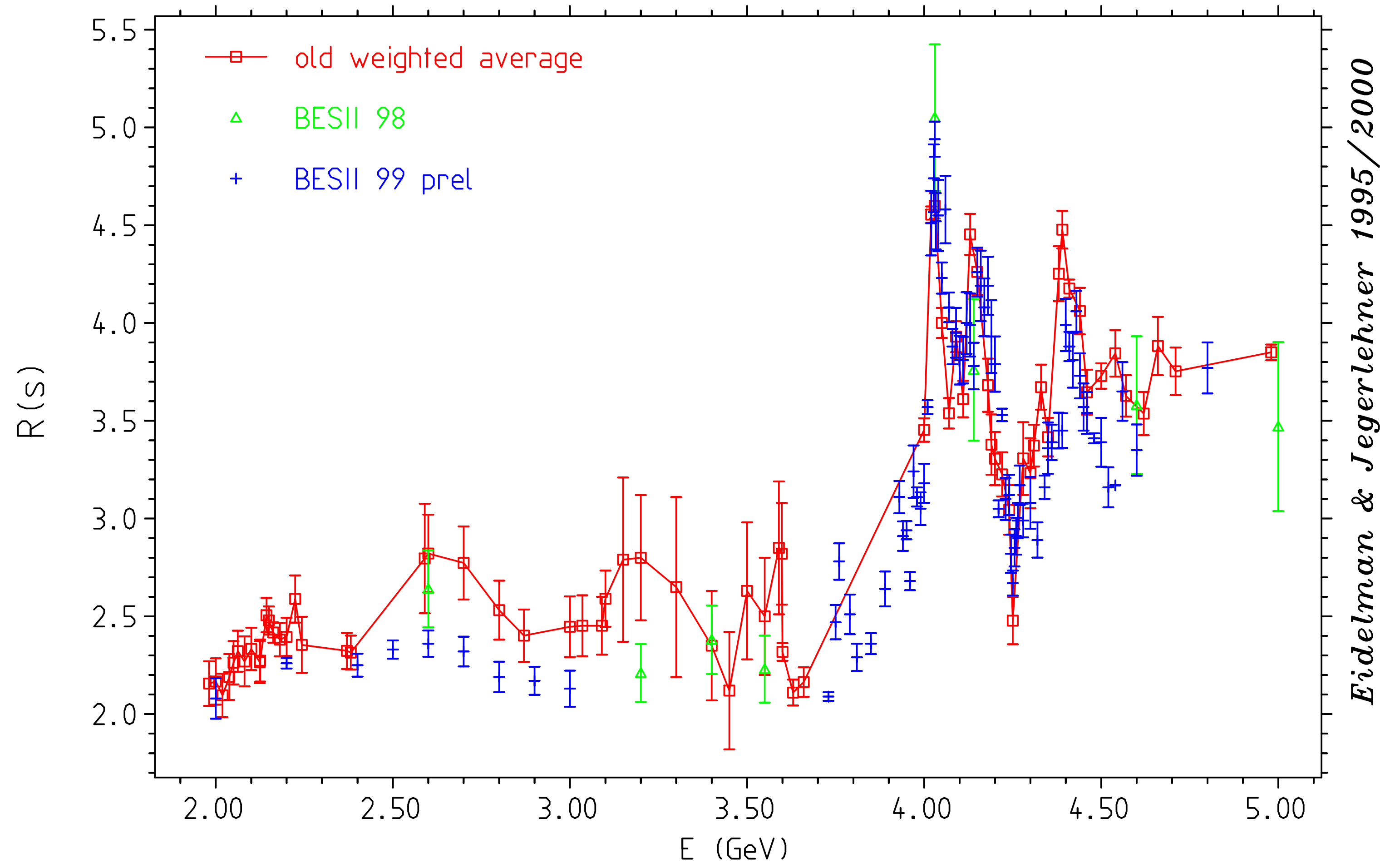

GS fit by CMD-2 2001
GS fit by Kinoshita 1985
CMD-2 01
$|F_\pi(E)|^2$
E (MeV)
ω
50.0
40.0
30.0
20.0
10.0
0.0
600.0
650.0
700.0
750.0
800.0
850.0
900.0
950.0

- **substantially lower than old e^+e^-–data !**



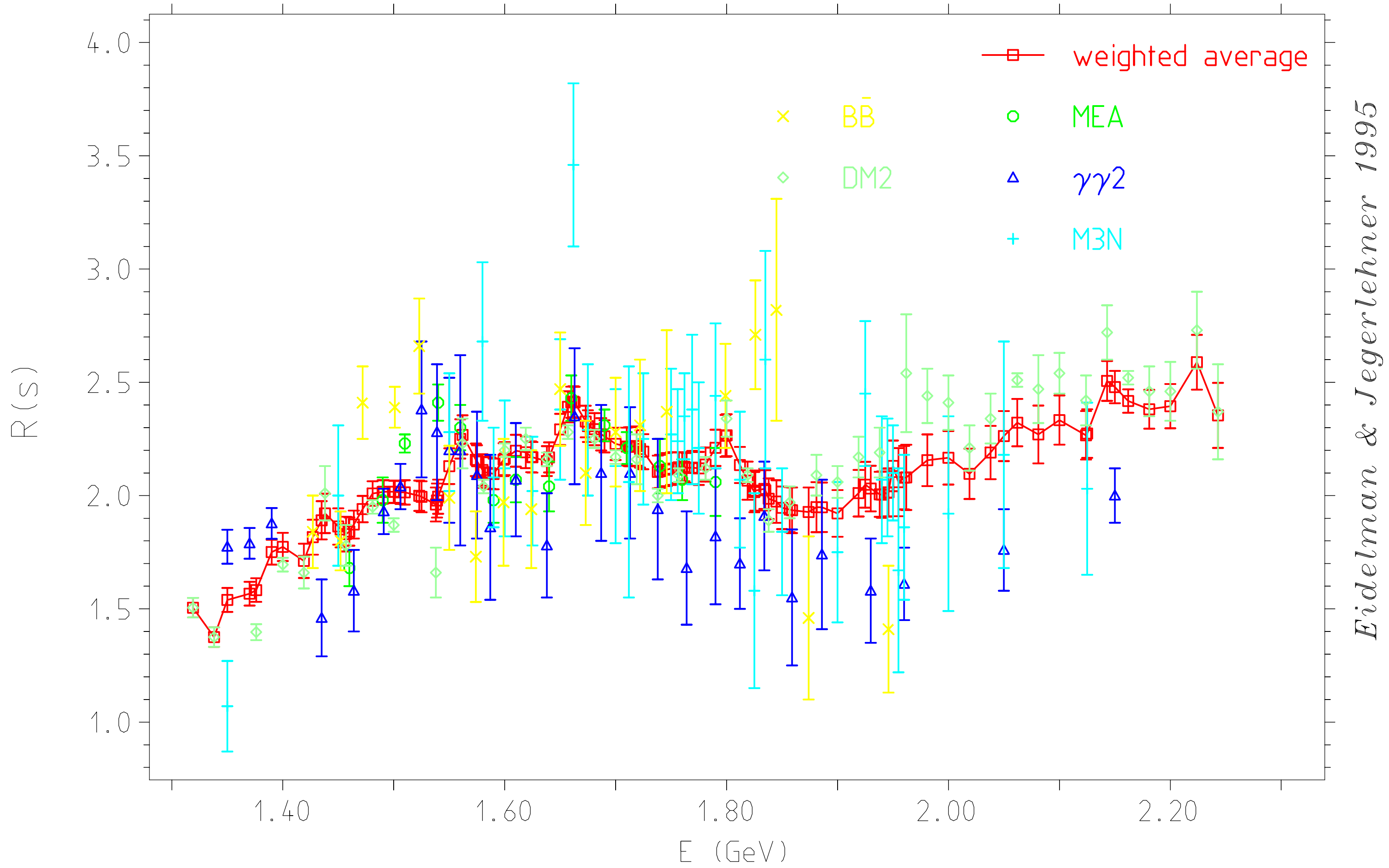
checking VP subtraction by CMD-2 group !

New BESII data (Feb. 2001)



data 20% syst error −− → 7% syst error !

- **Most problemetic region now 1.4-2.0 GeV**



special effort in interpretation of these data ! **(Hagiwara et al. 02)**

② $\alpha_{\rm em}(s)$ in Precision Physics

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

Precision physics limitations:

$\alpha\ ,\ G_\mu, M_Z$ **most precise input parameters**

$\Updownarrow$

partially non-perturbative relationship

precision predictions

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

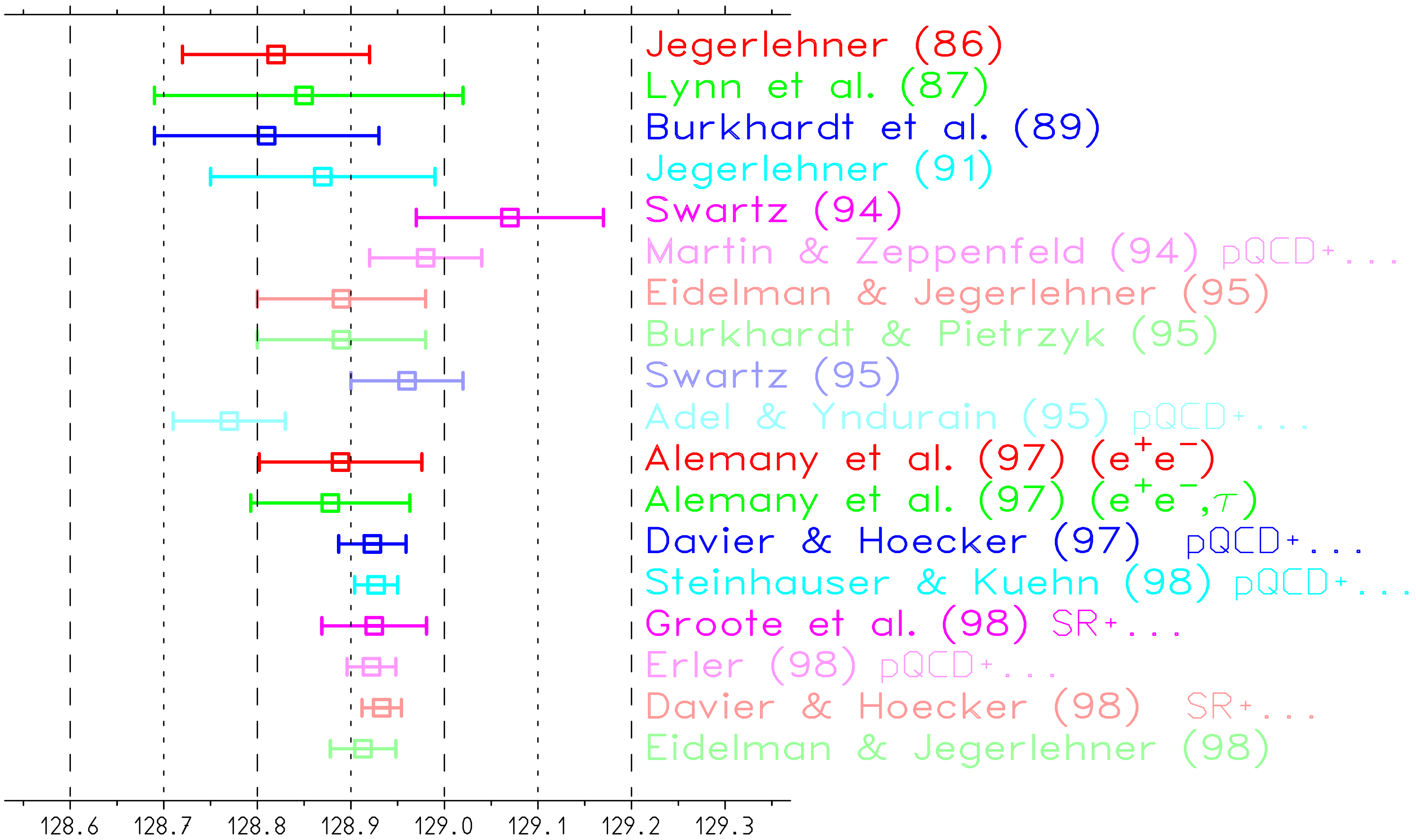
$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$
\begin{array}{cccc}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times & 10^{-4}
\end{array}
$$

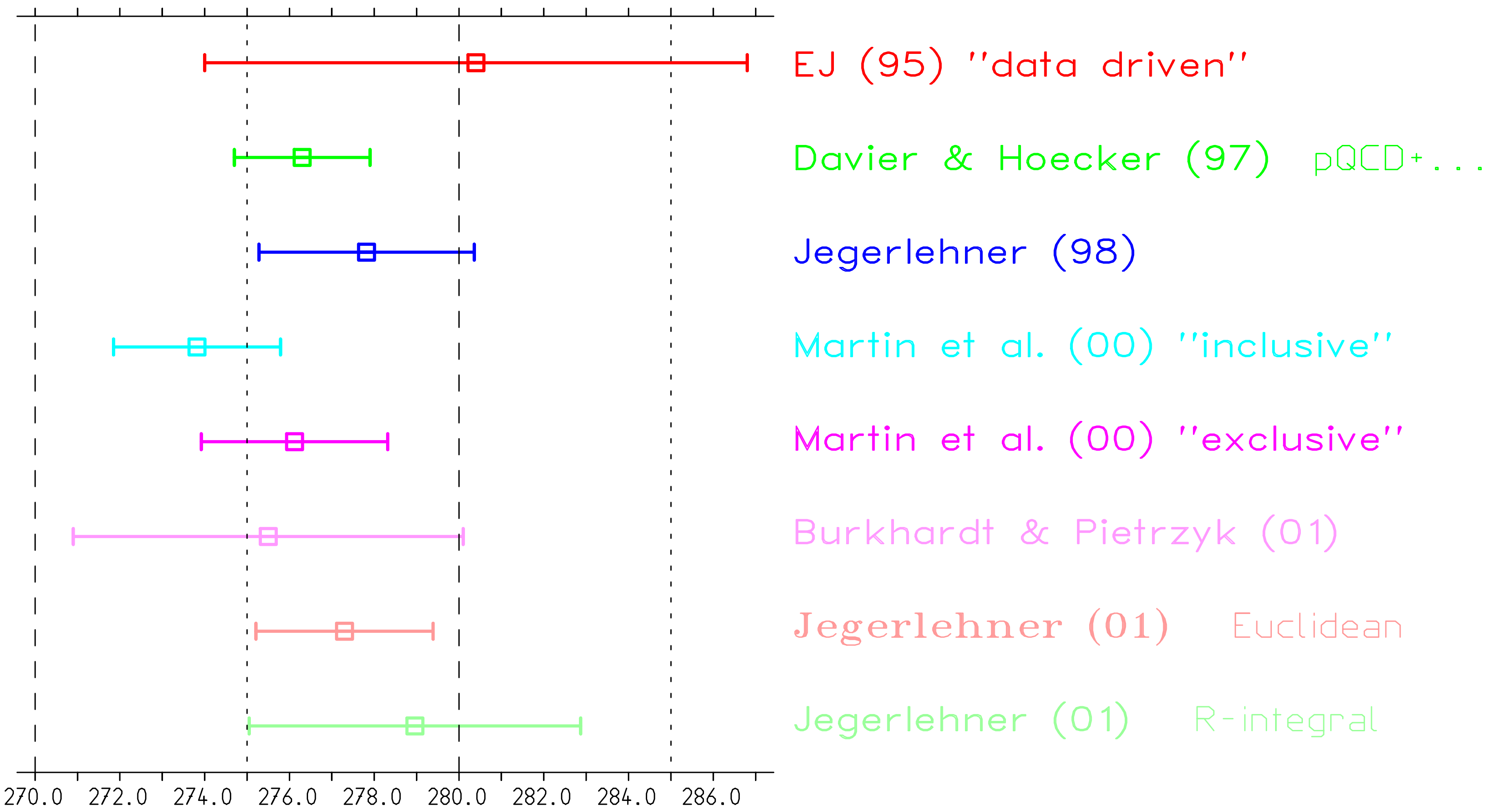
$\Rightarrow$ now a limiting factor for precision physics

Evaluations of $\alpha(M_Z)$ by different authors (before 2000)

All values have been rescaled to $M_Z = 91.1887$ GeV. The small top quark and the W contributions are not included.



Recent evaluations of $\Delta\alpha(M_Z)$



$\Delta\alpha$ in electroweak precision physics

Calculate $\sin^2\Theta_i$ **from** α, G_μ **and** M_Z **:**

$$\boxed{\sqrt{2}\,G_\mu\,M_Z^2\;\sin^2\Theta_i\;\cos^2\Theta_i = \pi\alpha\,\frac{1}{1-\Delta r_i}}$$ **Sirlin 1980**

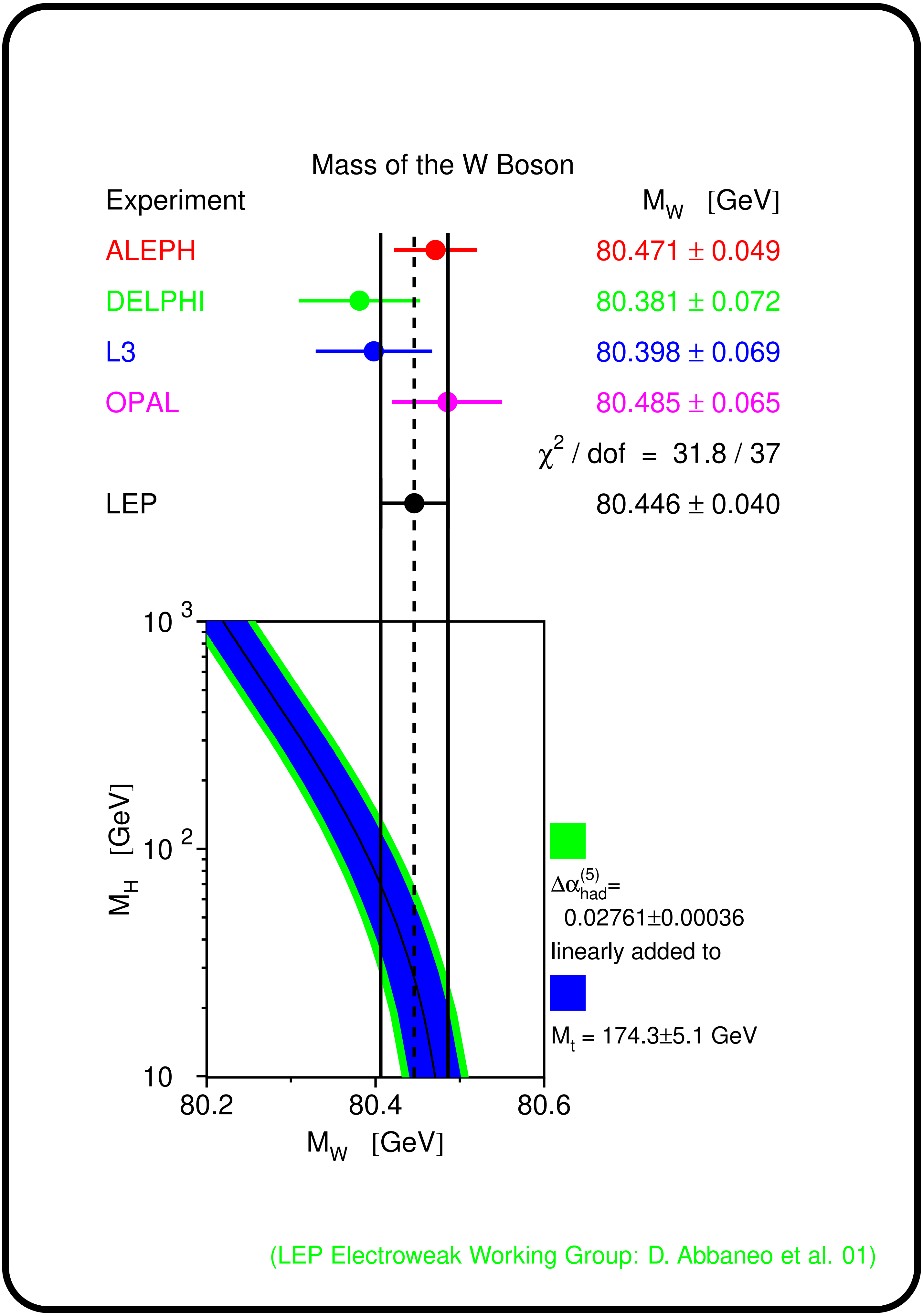
Various definitions of $\sin^2\Theta_i$**, which coincide at tree level: From weak gauge boson masses, from electroweak gauge couplings and from the neutral current coupling of the charged fermions:**

$$
\begin{aligned}
\sin^2\Theta_W &= 1-\frac{M_W^2}{M_Z^2}\\
\sin^2\Theta_g &= e^2/g^2\\
\sin^2\Theta_f &= \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\;,\quad f\neq\nu
\end{aligned}
$$

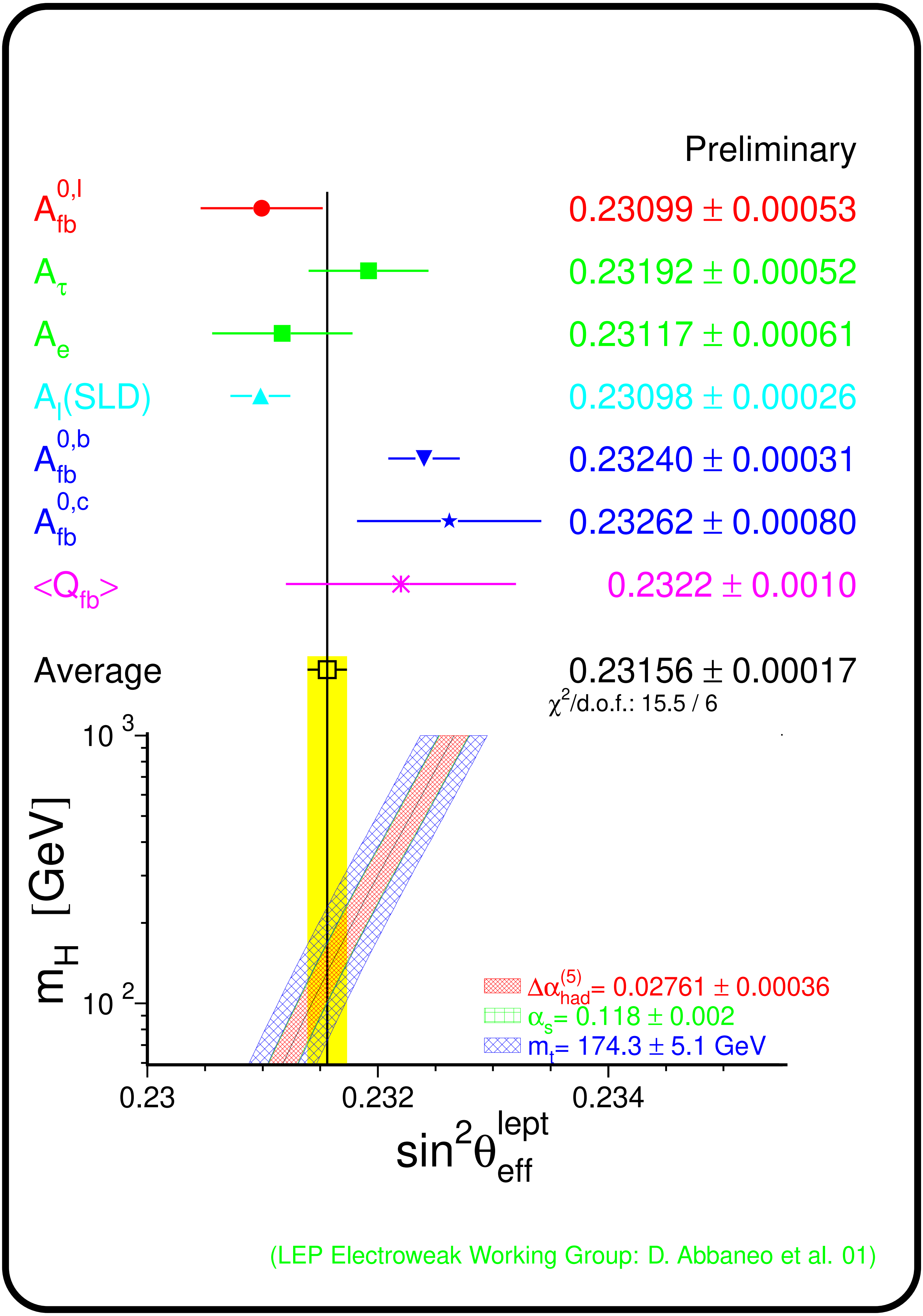
$$
\begin{aligned}
\Delta r_i &= \Delta r_i(\alpha,\,G_\mu,\,M_Z,m_H,\,m_{f\neq t},\,m_t)\\
\Delta r_i &= \underbrace{\Delta\alpha}_{\text{universal}} - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\mathrm{rem}}
\end{aligned}
$$

$\rightarrow$ **predictions for** M_W, A_{LR}, A_{FB}^f, Γ_f, $\cdots$

$$\delta\Delta\alpha \rightarrow \delta M_W\;,\quad \delta\sin^2\Theta_i\;,\;\cdots$$



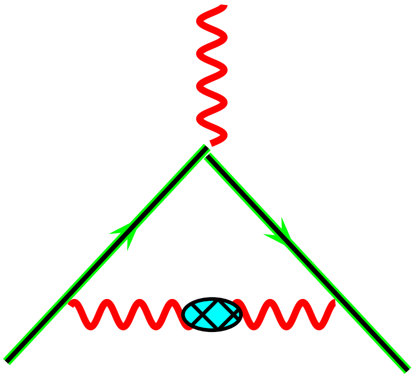
Mass of the W Boson
Experiment
M_W [GeV]
ALEPH 80.471 ± 0.049
DELPHI 80.381 ± 0.072
L3 80.398 ± 0.069
OPAL 80.485 ± 0.065
χ² / dof = 31.8 / 37
LEP 80.446 ± 0.040
10³
10²
10
M_H [GeV]
80.2
80.4
80.6
M_W [GeV]
Δα(5)had=
0.02761±0.00036
linearly added to
M_t = 174.3±5.1 GeV
(LEP Electroweak Working Group: D. Abbaneo et al. 01)



Preliminary
$A_{fb}^{0,l}$ 0.23099 ± 0.00053
A_τ 0.23192 ± 0.00052
A_e 0.23117 ± 0.00061
A_l(SLD) 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23240 ± 0.00031
$A_{fb}^{0,c}$ 0.23262 ± 0.00080
$\langle Q_{fb} \rangle$ 0.2322 ± 0.0010
Average 0.23156 ± 0.00017
χ^2/d.o.f.: 15.5 / 6
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$= 0.02761 ± 0.00036
α_s= 0.118 ± 0.002
m_t= 174.3 ± 5.1 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$
(LEP Electroweak Working Group: D. Abbaneo et al. 01)

③ Hadronic contribution to $(g-2)$ of the muon

❑ **Leading order hadronic contributions**



$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} ds \, \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int_{E^2_{\rm cut}}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$

- **Note the extra $1/s$–enhancement of contributions from low energies in a_μ as compared to $\Delta\alpha$.**

$$a_\mu^{\rm had} = (688.9 \pm 5.8) \times 10^{-10} \qquad \textbf{(FJ 02)}$$
$$a_\mu^{\rm had} = (684.7 \pm 6.0_{\rm exp} \pm 3.6_{\rm rad}) \times 10^{-10} \qquad \textbf{(DEHZ 02)}$$
$$a_\mu^{\rm had} = (701.9 \pm 4.7_{\rm exp} \pm 1.2_{\rm rad} \pm 3.8_{\rm iso}) \times 10^{-10} \qquad *$$
$$a_\mu^{\rm had} = (683.1 \pm 5.9_{\rm exp} \pm 2.0_{\rm rad}) \times 10^{-10} \qquad \textbf{(HMNT 02)}$$
$$a_\mu^{\rm exp} = (\ \cdots \ \pm\ 4.0\ ) \times 10^{-10} \qquad \textbf{(BNL E821 (0?))}$$

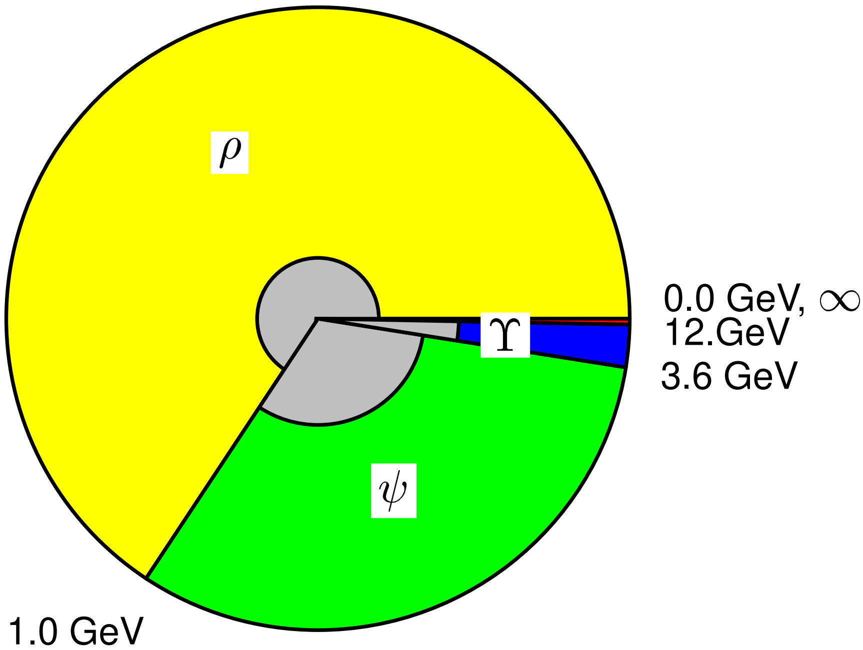
* **using τ–data**

others use e^+e^-–data only

Distribution of hadronic contributions to $a_\mu^{\rm had}$

e^+e^- data based approach:

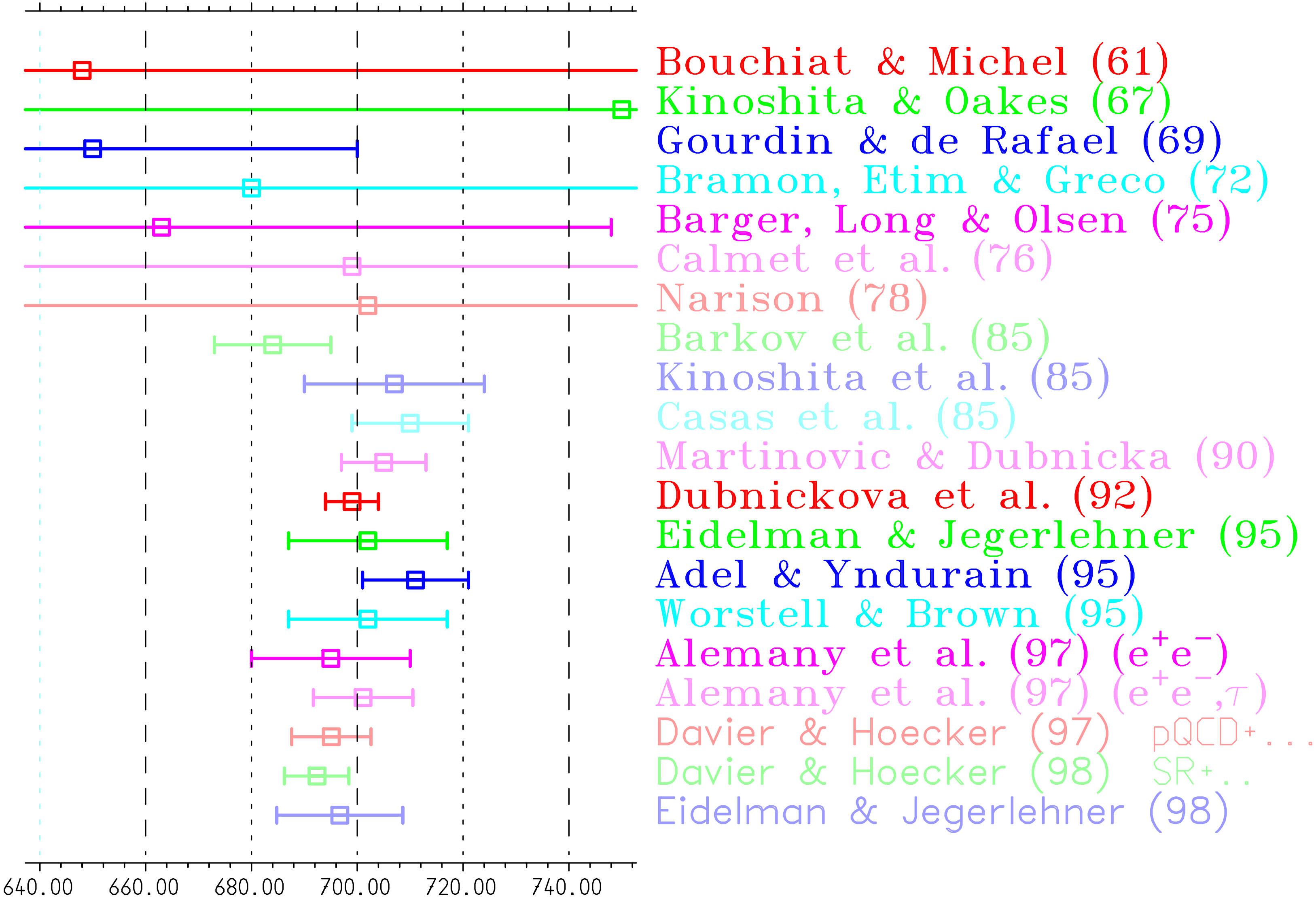
Based on available data up to 12 GeV (FJ 00)



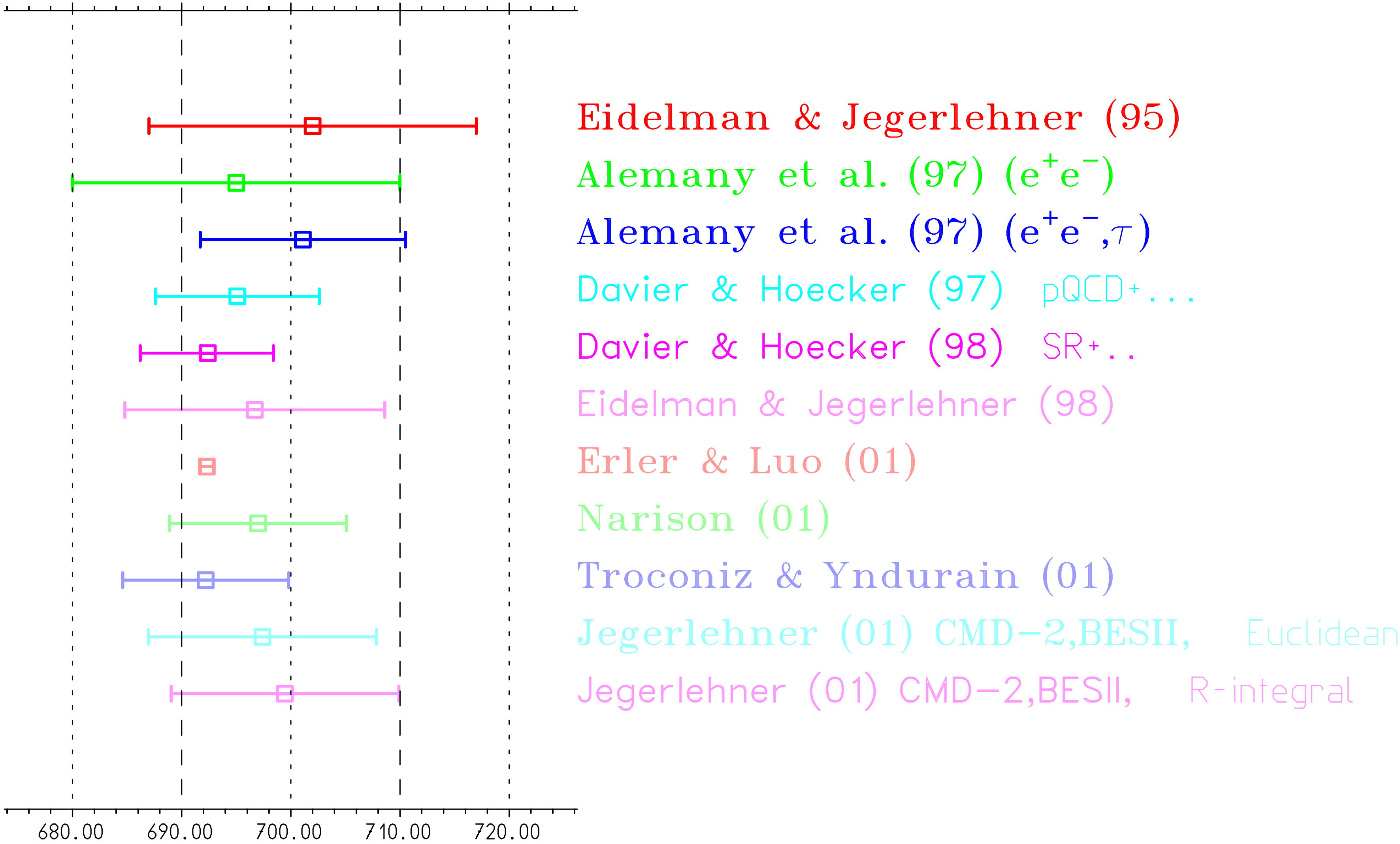
Shaded area: 10 $\times$ error (scaled up)

Evaluations of $a_\mu^{\mathrm{hadrons}} \times 10^{10}$ by different authors (before 2000)

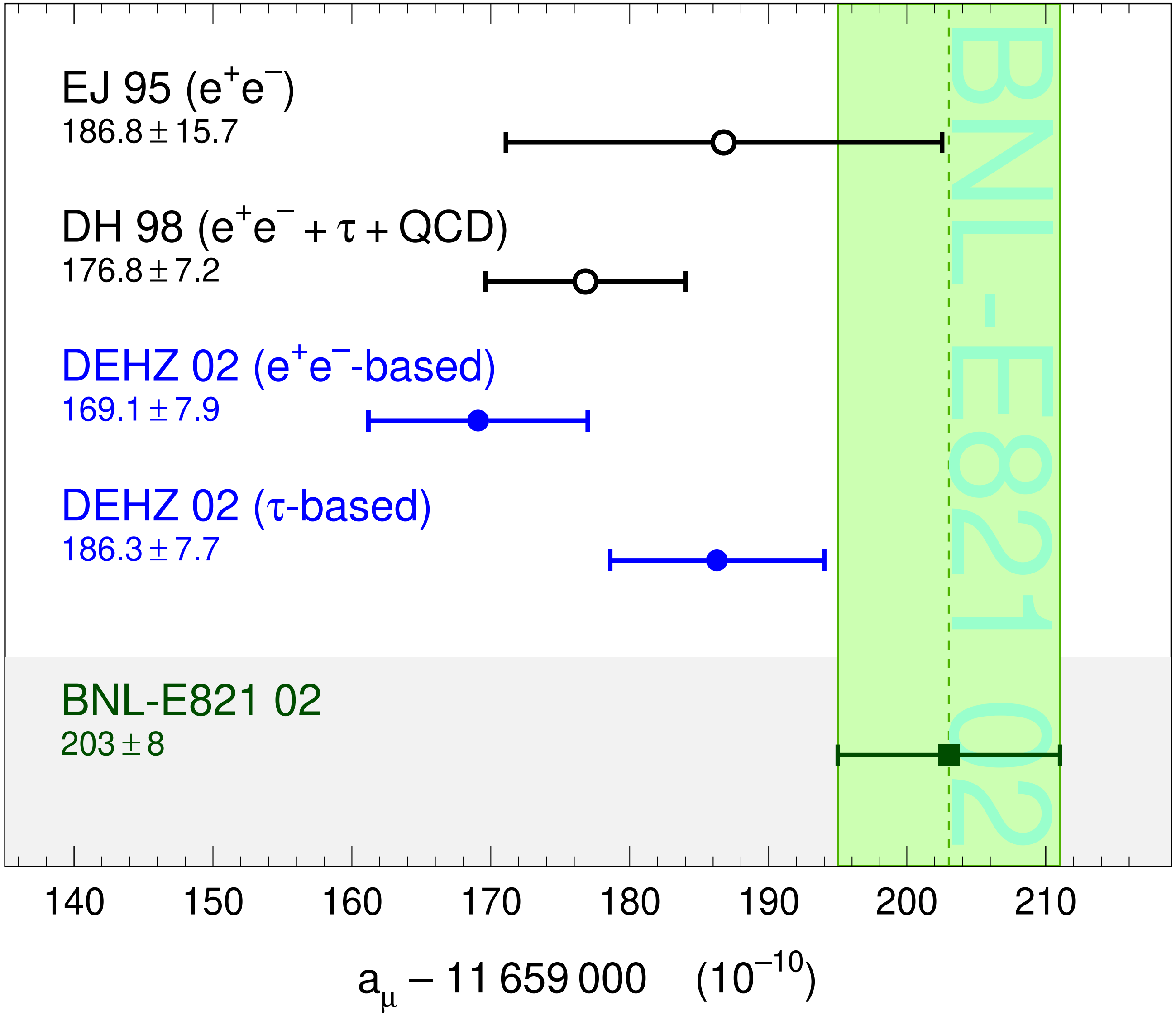
Only the leading hadronic vacuum polarization diagram is included.



Recent evaluations of a_μ^{had}

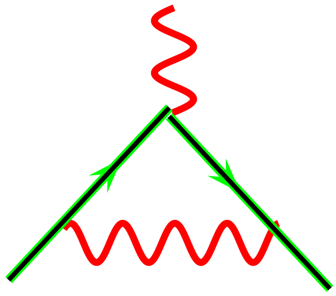


DEHZ evaluation of a_μ^{had}



Other contributions: (see next talk)

- QED



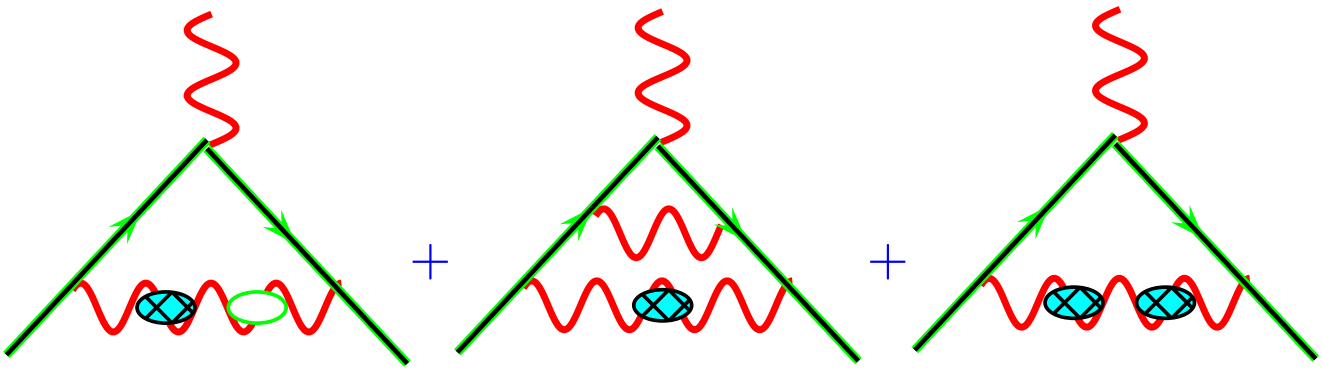
$$a_\mu^{\mathrm{QED}(1)} = (116140981 \pm 6) \times 10^{-11}$$

$$\vdots$$

$$a_\mu^{\mathrm{QED}(5)} = (116584706 \pm 3) \times 10^{-11}$$

(Schwinger 48, $\cdots$, Kinoshita 93, $\cdots$, Laporta, Remiddi 96)

- Higher order hadronic contributions



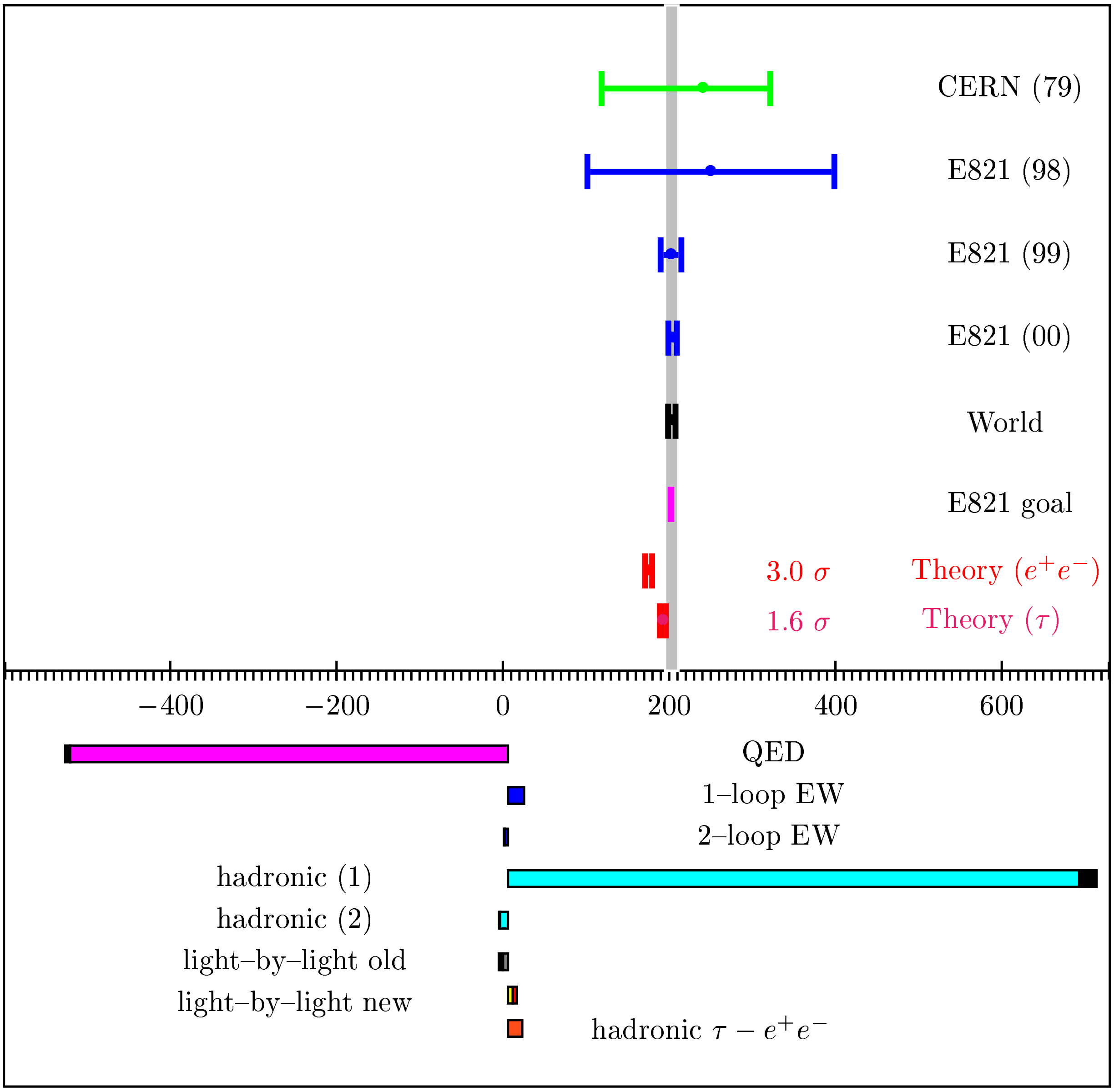
$$a_\mu^{\mathrm{had}(2)} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K^{(2)}(s)\, \sigma_{\mathrm{had}}(s)$$

$$a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$$

(Krause 96)

a_μ Summary:

a_μ-11659000 $\times 10^{-10}$: theory vs. experiment in the year 2002 for (g-2) of the muon. The new E821 expriment at Brookhaven revield a 1.6/3.0 σ deviation from the theory. The various types of SM contributions are shown in the lower part of the figure.



Details: $\Rightarrow$ **(E. de Rafael's talk)**

④ The τ vs. e^+e^- problem

① **Additional data: τ–data $+$ CVC**

ALEPH–Coll., (OPAL, CLEO), Alemany, Davier, Höcker 96

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+ e \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

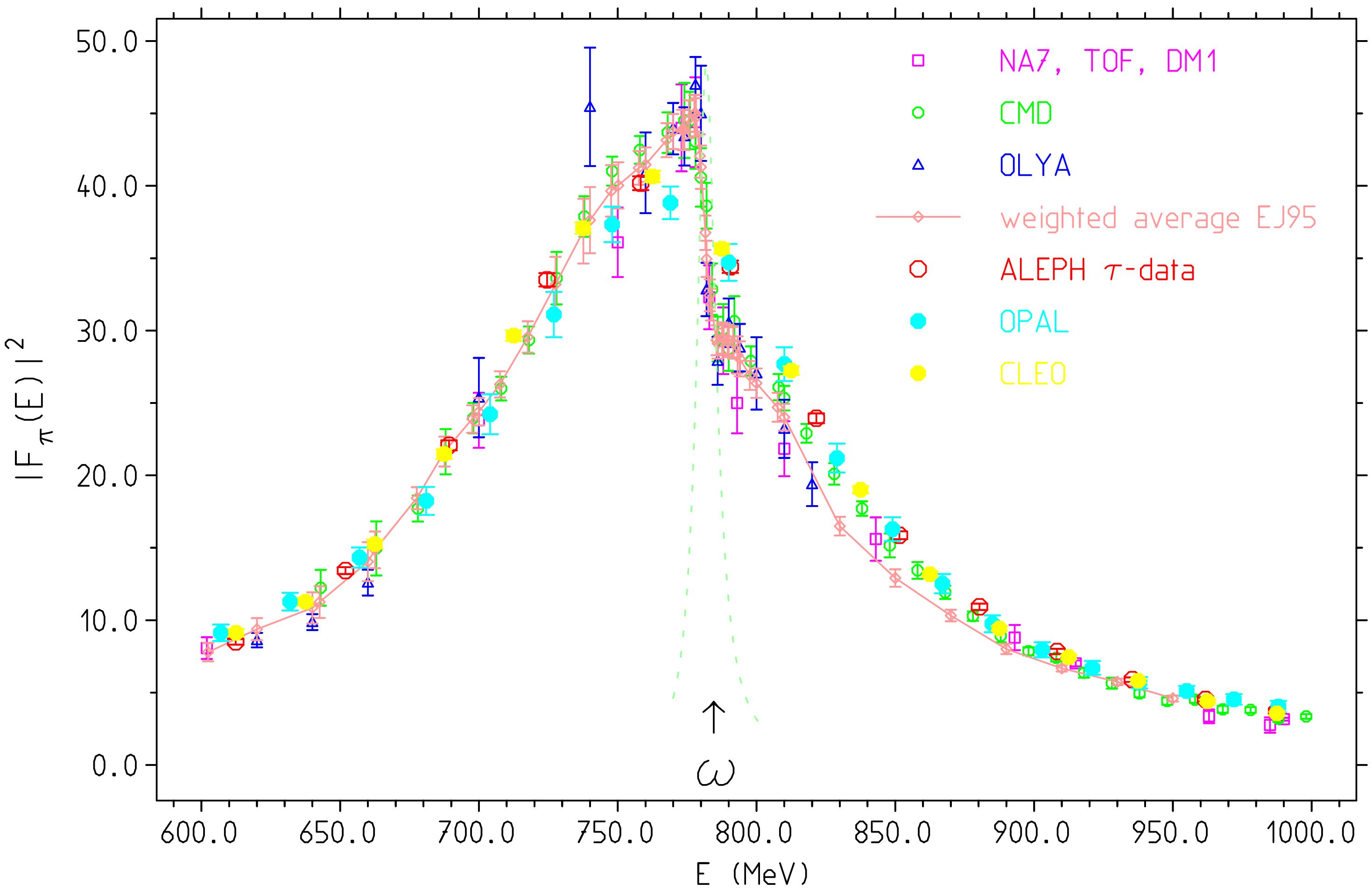
in terms of the τ spectral function v_1.

- **mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)**
- **which is dominating in $a_\mu^{\rm had}$ (72%)**

$$I = 1 \sim 75\% \, ; \; I = 0 \sim 25\%$$

τ–data cannot replace e^+e^-–data

$$\begin{array}{lclcl} \delta a_\mu & : & 15.6 \times 10^{-10} & \to & 10.2 \times 10^{-10} \\ \delta \Delta\alpha & : & 0.00067 & \to & 0.00065 \end{array}$$



systematic deviations: can they be understood?

e^+e^-–data*= data corrected for isospin violations:

$$|F(s)|^2 \text{ = (} |F(s)|^2 \text{–data) / } \left| \left(1 + \frac{\epsilon s}{(s_\omega - s)}\right) \right|^2$$

with

$$s_\omega = (M_\omega - \tfrac{i}{2}\Gamma_\omega)^2$$

ϵ determined by fit to the data: $\epsilon = 0.00172$

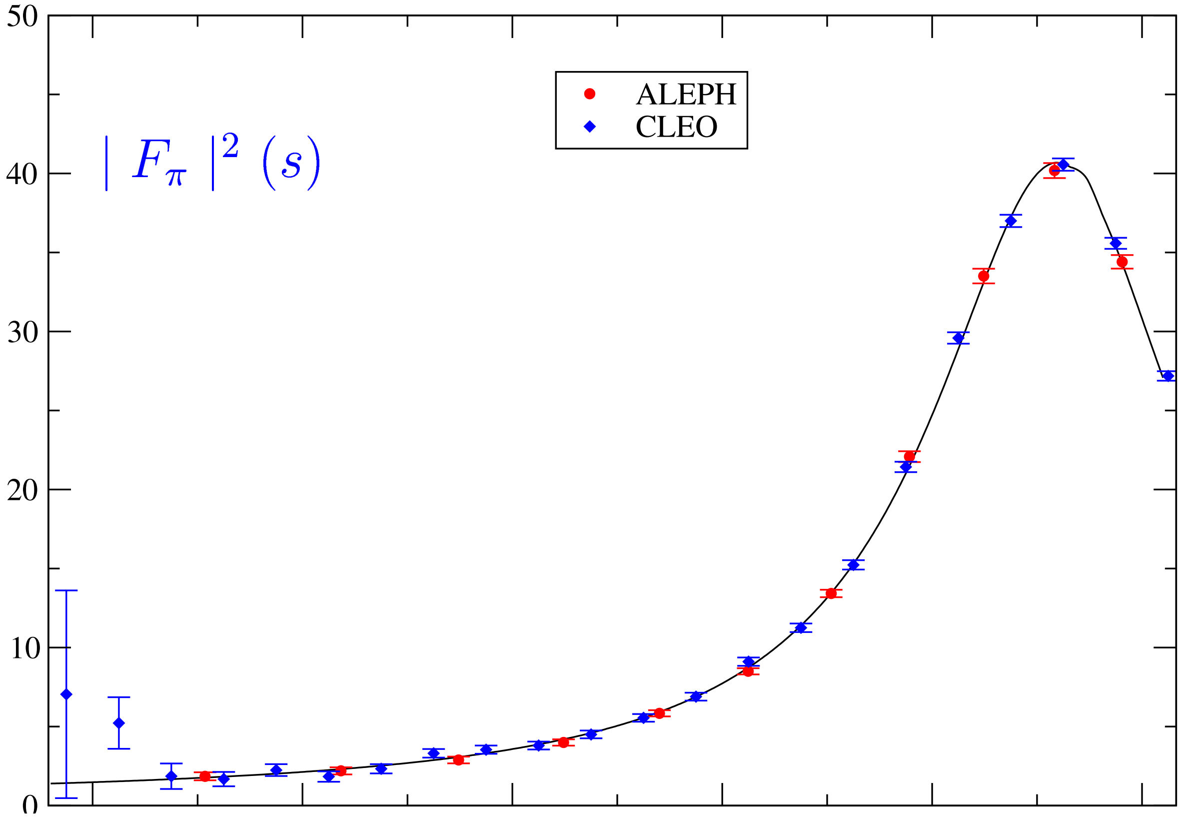
(Leutwyler 00)

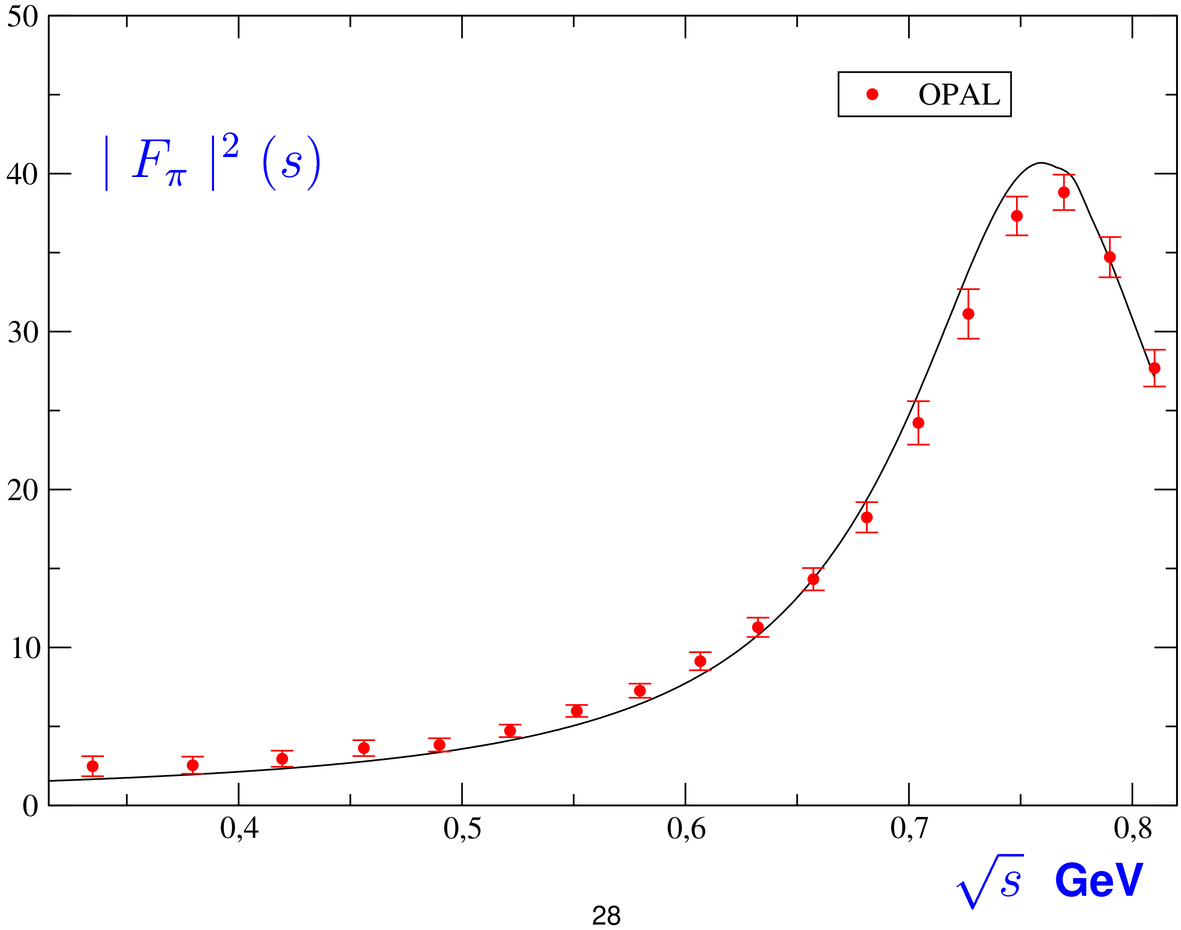
After correction and shifts of normalizations no systematic deviations between CLEO/ALEPH and DM1/OLYA/CDM1/CDM2 data beyond fluctuations of e^+e^- data, while OPAL data show substantial deviations below 800 MeV:

work in progress

I. Caprini, G. Colangelo, J. Gasser, F. J., H. Leutwyler

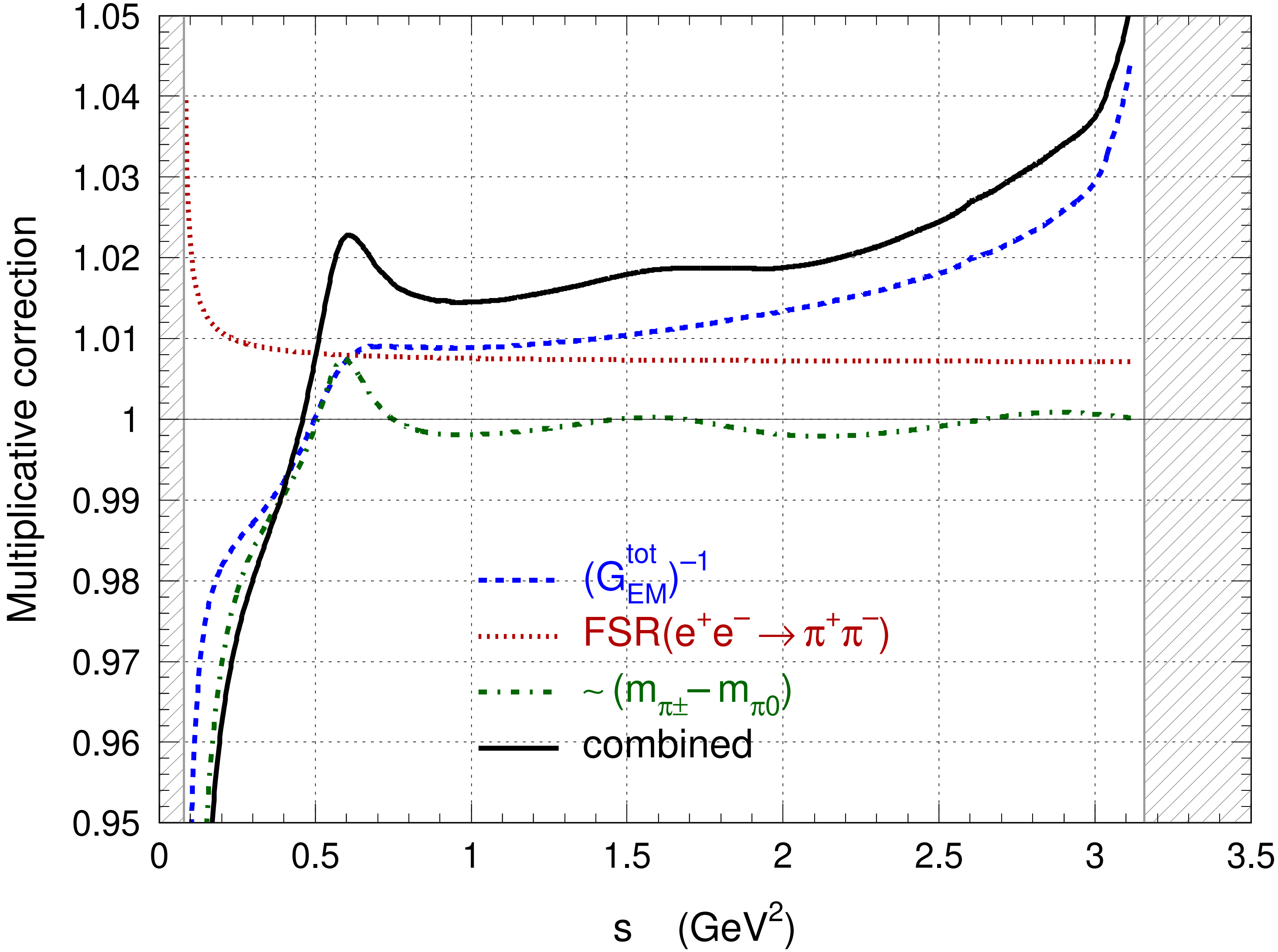
NA7+CMD-2 e^+e^-–data analytized+unitarized+chiral limit: solid line vs. τ –data



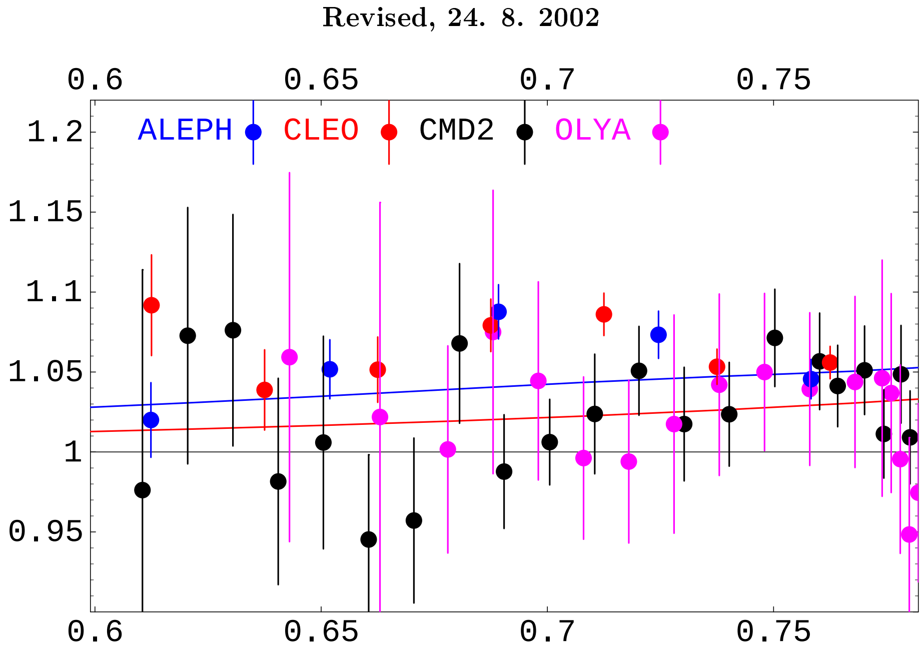


Isospin breaking effects: (Cirigliano et al.)

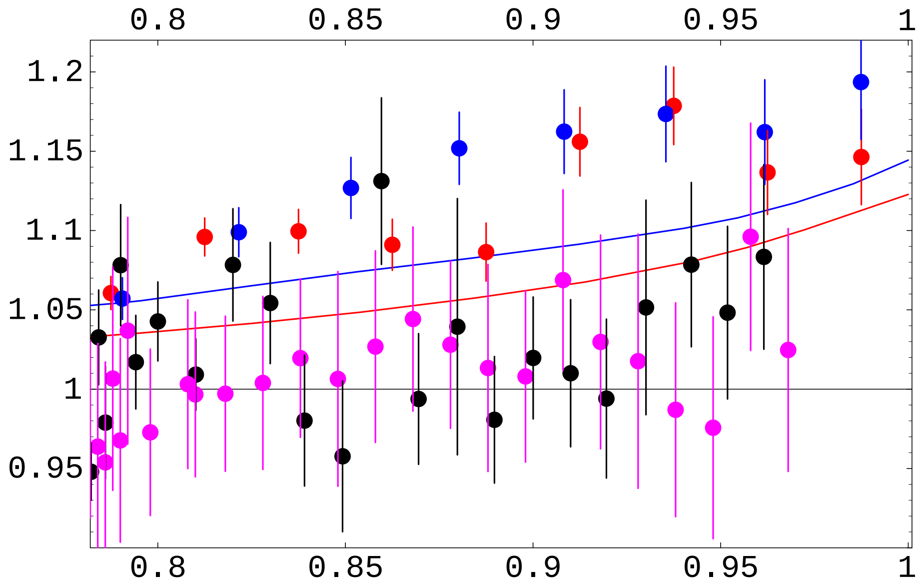
Surprisingly, after corrections at larger energies 10% deviations !



- τ **vs.** e^+e^-



Region below the ω $(0.6 < E < 782$ **MeV**$)$



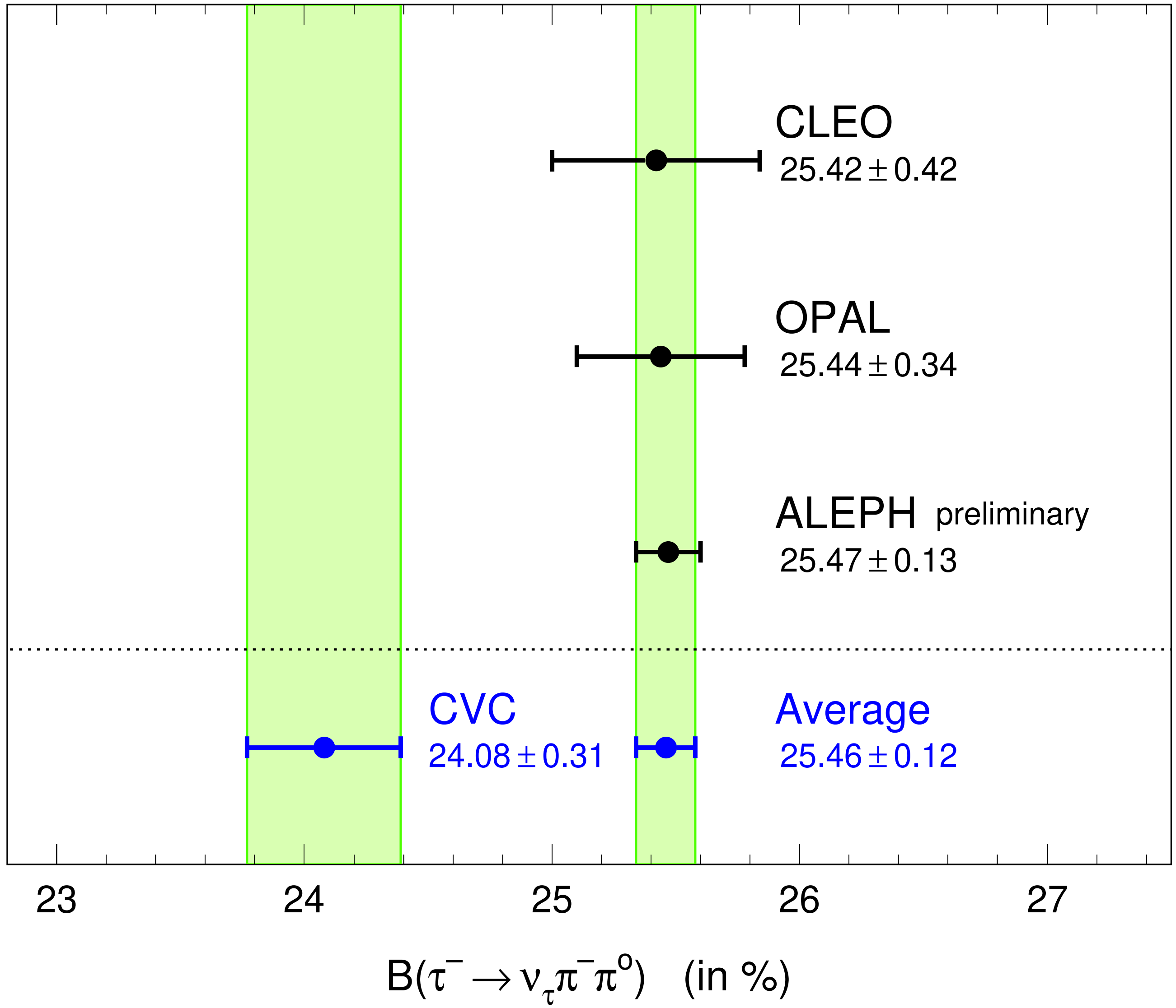
Region above the ω $(782\,$**MeV** $< E < 1$ **GeV**$)$

The plots show the square of the form factor divided by the Omnes factor. In the case of the e^+e^- data, the contributions from ω exchange and vacuum polarization are removed and only the statistical errors are shown. In the case of the τ data, the error bars represent the diagonal elements of the error matrix.

cannot be understood within SM !

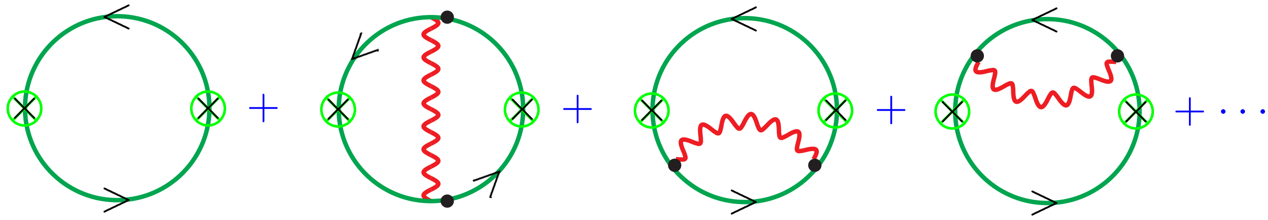
likely an experimental problem

The measured branching ratios for $\tau \to \nu_\tau \pi^- \pi^0$ compared to the prediction from the $e^+e^- \to \pi^+\pi^-$ after applying the isospin breaking correction



⑤ Controlling pQCD via the Adler function

❶ **pQCD calculations of vacuum polarization amplitudes**



up to 4–loops massless	Groshny, Kataev, Larin 91
up to 3–loops massive	Chetyrkin, Kühn et al. 97
up to 2–loops massive BF–MOM RG	F. J., Tarasov 98

❷ **use old idea: testing non–perturbative effects with help of the Adler function**

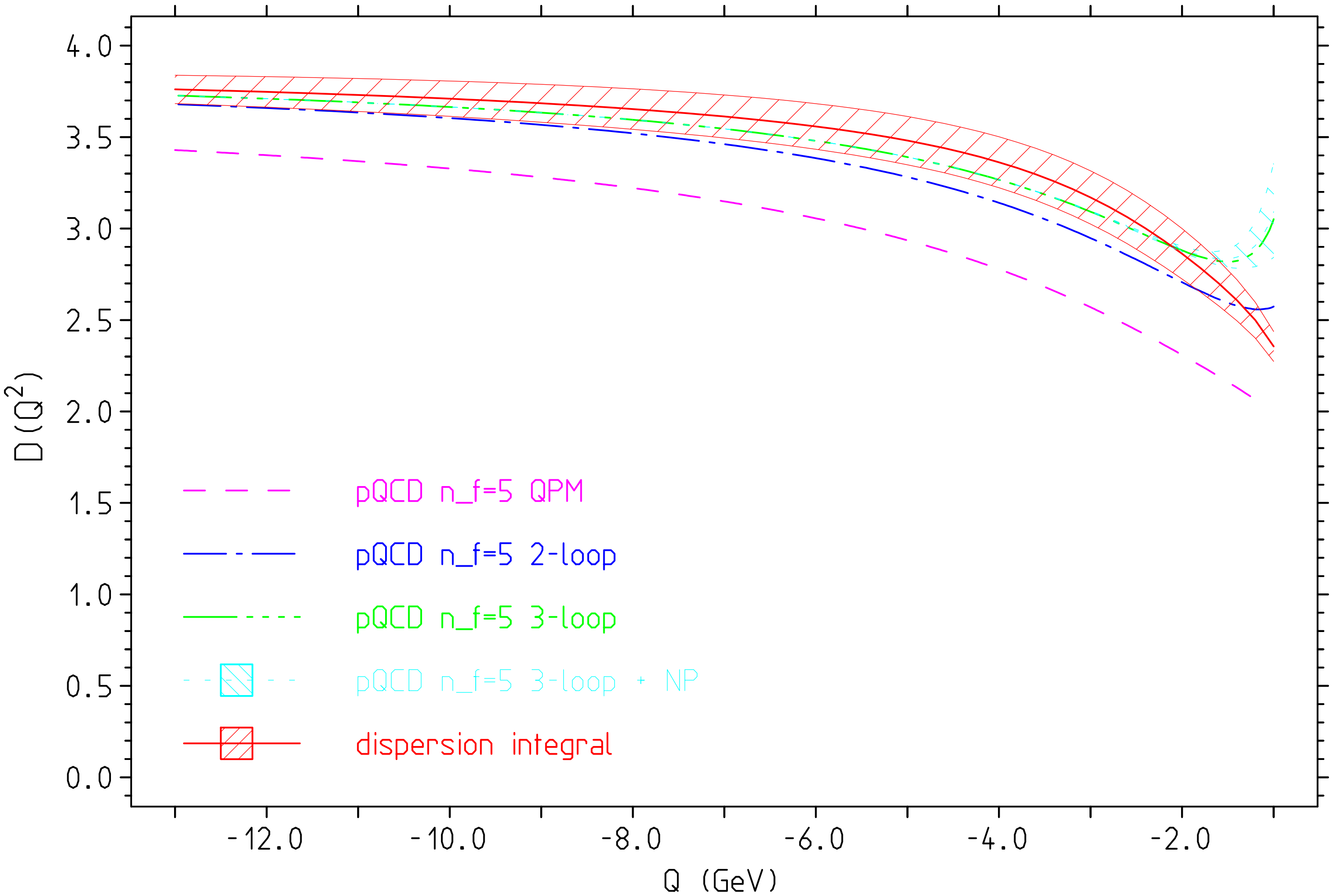
Eidelman, F. J., Kataev, Veretin 98

$$
\begin{aligned}
D(-s) &\doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds} \Delta\alpha_{\mathrm{had}}(s) \\
&= -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}
\end{aligned}
$$

$$
\Rightarrow \quad D(Q^2) = Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}
$$

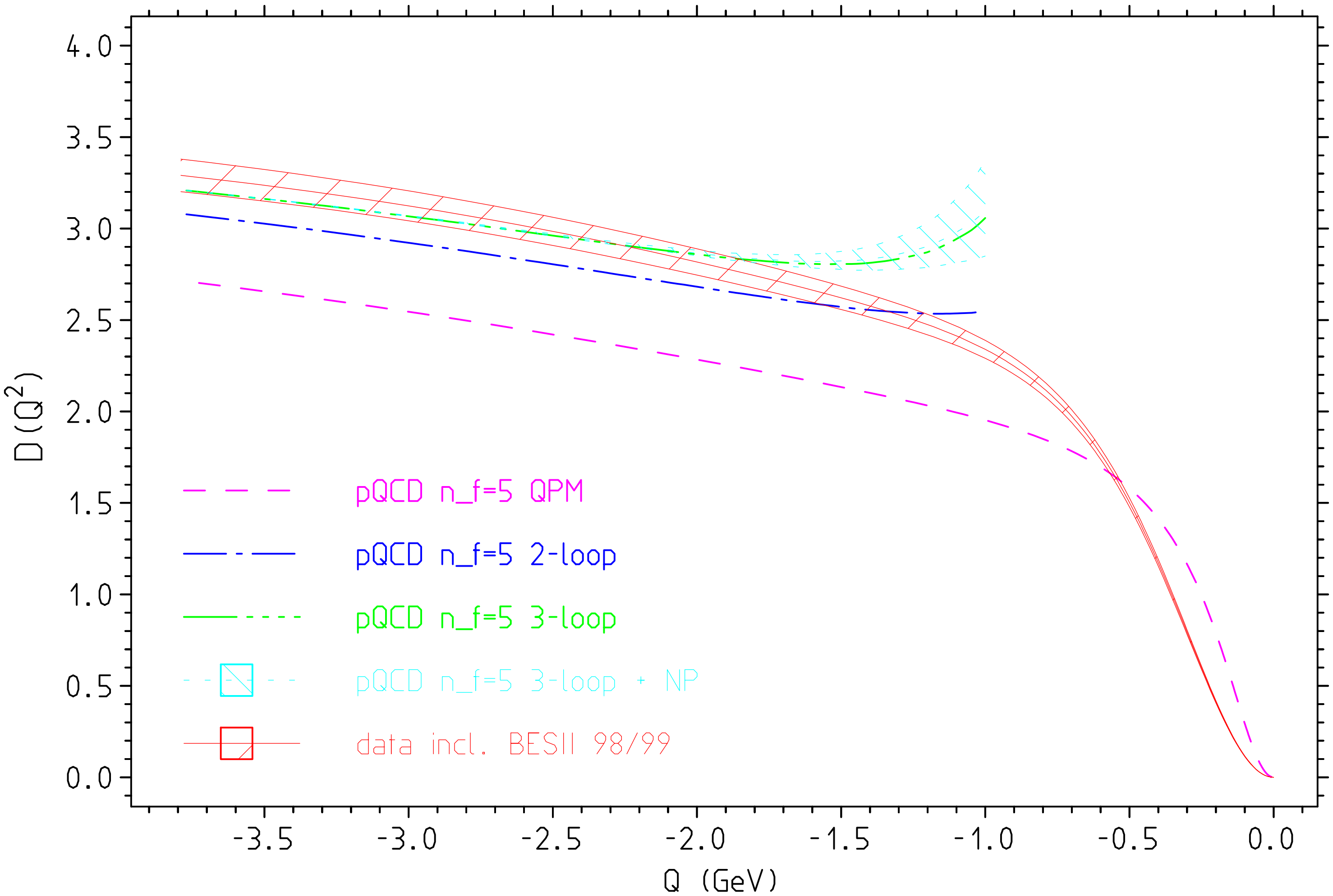
pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

"Experimental" Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98)

“Experimental” Adler–function versus theory (pQCD + NP)



(Update 2000: EJKV98 + BESII 98/99 prel.)

$\Rightarrow$ **pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\text{GeV})^2$ (not down to m_τ ! however); use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi} \int dQ'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\boxed{\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) \;=\; & \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} \\ & + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\end{aligned}}$$

and obtain, for $s_0 = (2.5\,\text{GeV})^2$: **(FJ 98)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007541 \pm 0.000254$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027737 \pm 0.000254$$

$$\alpha^{-1}(-M_Z^2) = 128.917 \pm 0.035$$

The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

The second error comes from the variation of the pQCD parameters. In square brackets the error if we assume the uncertainties from different parameters to be uncorrelated. The uncertainties coming from individual parameters are listed in the following table (masses are the pole masses):

parameter	range	pQCD uncertainty	total error
α_s	0.117 ... 0.123	0.000051	0.000155
m_c	1.550 ... 1.750	0.000087	0.000170
m_b	4.600 ... 4.800	0.000011	0.000146
m_t	170.0 ... 180.0	0.000000	0.000146
all correlated		0.000149	0.000209
all uncorrelated		0.000101	0.000178

The largest uncertainty is due to the poor knowledge of the charm mass. I have taken errors to be 100% correlated. The uncorrelated error is also given in the table.

Remaining problems are the following:

a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002$$

which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$: Including this term yields

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027782 \pm 0.000254$$

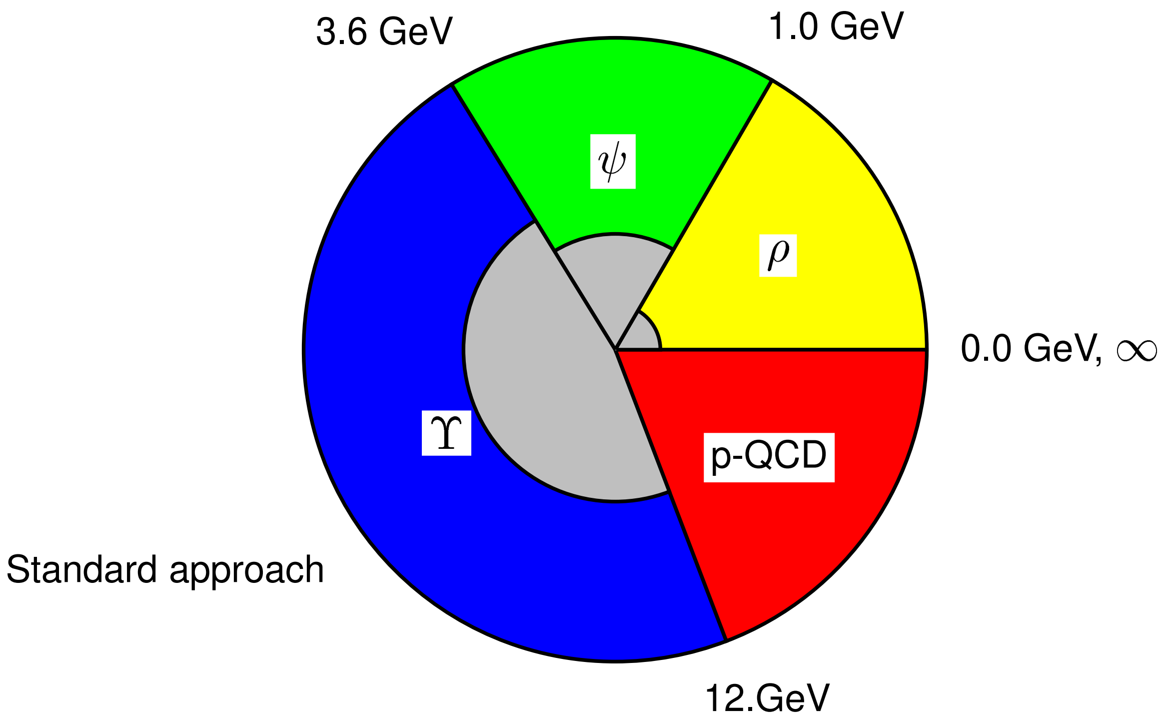
$$\alpha^{-1}(M_Z^2) = 128.913 \pm 0.035$$

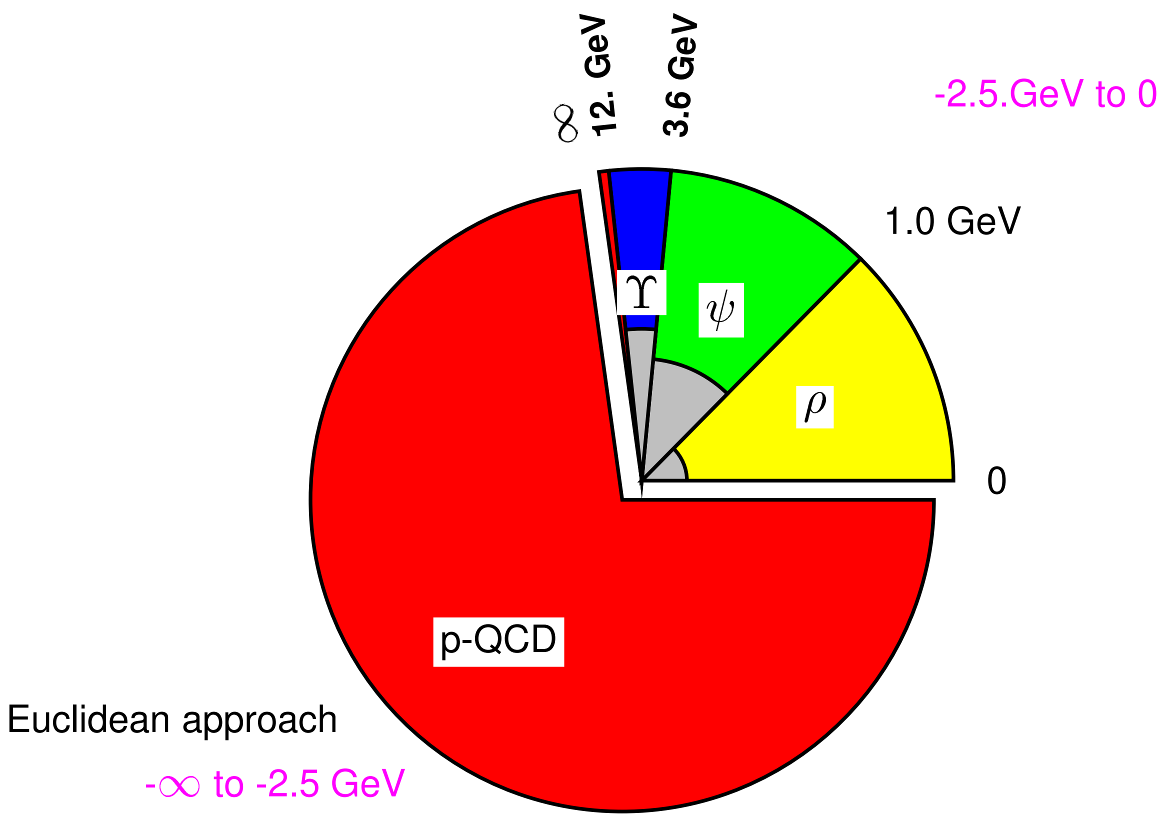
One may ask the question whether terms like Δ should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. Note that the Dyson resummation in general is not a systematic resummation of leading, sub-leading etc. terms as the RG resummation is.

Distribution of hadronic contributions to $\Delta\alpha^{\mathrm{had}}$

e^+e^- data based approach:

Based on available data up to 12 GeV (FJ 00)





Shaded area: 10 $\times$ error (scaled up)

$a_\mu^{\rm had}$ via the Adler function

t–channel or Euclidean field theory approach

non–perturbative regime directly accessible by lattice QCD !

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int\limits_0^\infty \frac{ds}{s}\int\limits_0^1 dx\,\frac{x^2\,(1-x)}{x^2+(1-x)s/m_\mu^2}\;e^2\frac{1}{\pi}{\rm Im}\Pi'_{\gamma\;{\rm had}}$$

interchange of integrations yields:

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int\limits_0^1 dx\;(1-x)\;\Delta\alpha^{\rm had}\left(-Q^2(x)\right) \qquad (1)$$

where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ **is the space like square momentum transfer or**

$$x = \frac{Q^2}{2m_\mu^2}\left(\sqrt{1+\frac{4m_\mu^2}{Q^2}}-1\right)$$

1) calculate the Adler function from the e^+e^---data and pQCD tail above 11 GeV

2) calculate the corresponding shift $\Delta\alpha^{\rm had}$ with or without t--channel cut at 2.5 GeV

3) calculate $a_\mu^{\rm had}$

By writing $(1-x) = -\frac{1}{2}\frac{d}{dx}(1-x)^2$ and performing a partial integration we obtain

$$a_\mu^{\rm had} = \frac{\alpha}{\pi} m_\mu^2 \int\limits_0^1 dx\, x\,(2-x)\, \left(D(Q^2(x))/Q^2(x)\right) \qquad (2)$$

the number of integrations is reduced by one.

The evaluation in both forms provides a good stability test of the numerical integrations involved.

Utilizing the 2000 compilation of the e^+e^-–data we obtain the following results: (as an illustration; CMD-2 2001 data not yet included)

$a_\mu^{\rm had}$	method
6994.7 (104.3)	e^+e^--data ,time-like, pQCD above 11 GeV
6974.0 (104.4)	Eq. (1)
6974.0 (104.7)	Eq. (1) pQCD above 2.5 GeV
6974.0 (104.7)	Eq. (2) pQCD above 2.5 GeV

$$\boxed{6974 \pm 105}$$

τ–data: central value $\nearrow$, uncertainty $\searrow$

⑥ Status and Outlook

$\boxed{a_\mu^{\mathrm{had}}}$

Maximum confusion ! Likely only new experiments can settle the problems

$\Rightarrow$ **Photon tagging experinents at KLOE, BARAR**

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$:

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha$$

$\delta\Delta\alpha$	δM_W **(MeV)**	**Ref.**	**Method**
0.00065	**12.0**	[EJ95]	data $<$ $12.$ GeV
0.00016	**3.0**	[DH98]	data $<$ 1.7 GeV
0.00025	**4.6**	[FJ98]	Euclidean $>$ 2.5 GeV
0.000197	**3.6**	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	**4.0**	[MOR00]	same but “exclusive”
0.000460	**8.5**	[BP00]	data
0.00007	**1.3**	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	**0.9**	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	**22.0**	Osaka 2000	
MegaW	**6.0**	TESLA	

$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha$$

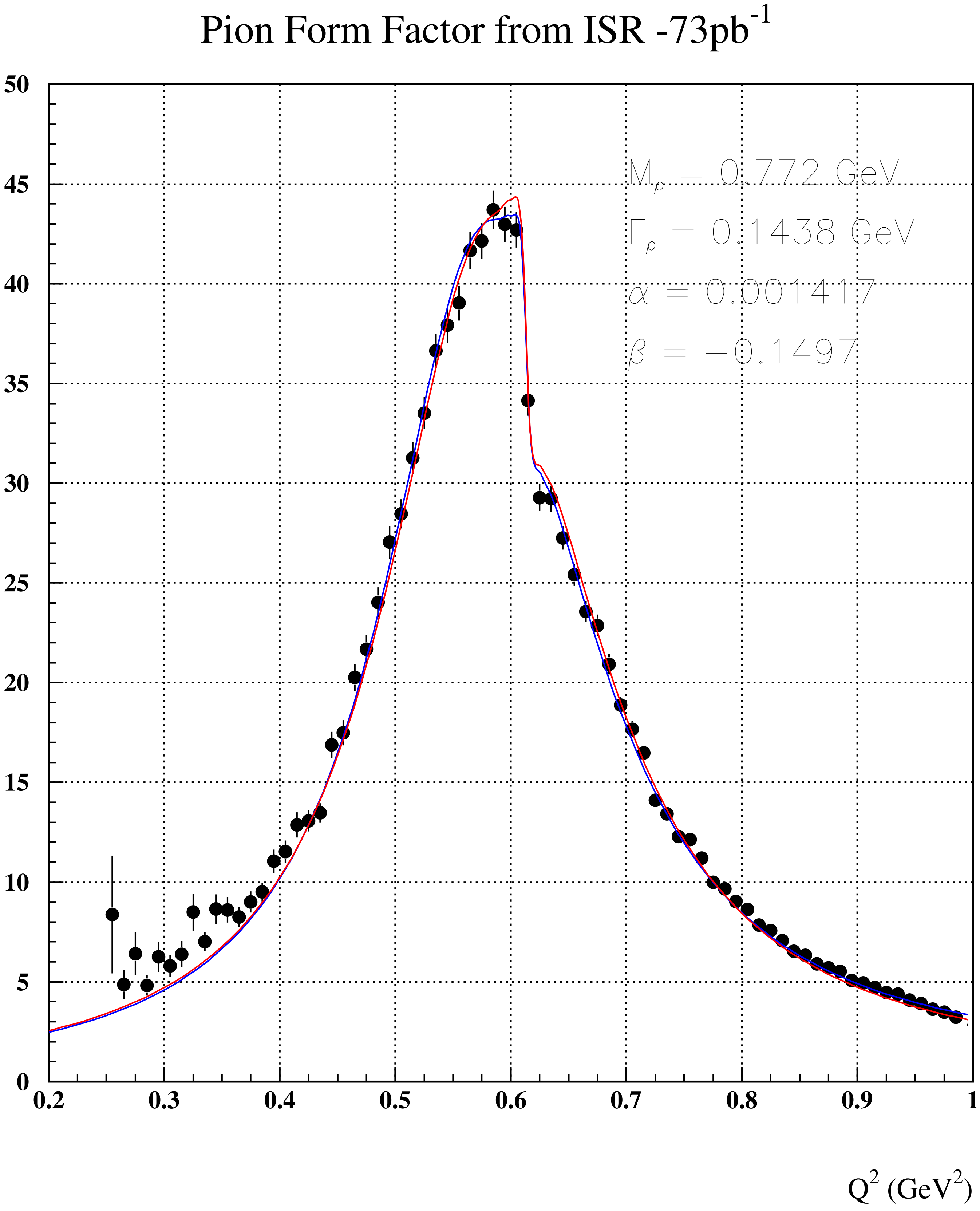
$\delta\Delta\alpha$	$\delta\sin^2\theta$	Ref.	Method
0.00065	**0.000232**	**[EJ95]**	**data** $< 12.$ **GeV**
0.00016	**0.000057**	**[DH98]**	**data** < 1.7 **GeV**
0.00025	**0.000089**	**[FJ98]**	**Euclidean** > 2.5 **GeV**
0.000197	**0.000070**	**[MOR00]**	**rescaled data, pQCD 2.8-3.7, 5-**∞
0.000220	**0.000078**	**[MOR00]**	**same but "exclusive"**
0.000460	**0.000164**	**[BP00]**	**data**
0.00007	**0.000025**	$\delta\sigma \lesssim 1\%$ **up to** J/ψ	
0.00005	**0.000018**	$\delta\sigma \lesssim 1\%$ **up to** Υ	
WA	**0.000160**	**LEP+SLD**	
GigaZ	**0.000013**	**TESLA**	

Future:

- **KLOE (DAΦNE Frascati):** expect a 1% measurement for $\sigma(e^+e^- \to hadrons)$ at $\sqrt{s} \lesssim$ 1 GeV
  - photon tagging method — theory: $\sim$ 2%
  - $\pi^+\pi^-$ inv. mass spectrum — theory: $\sim$ 0.5%
  - ✗ very important new masurement with different systematics (cross check of Novosibirsk measurements)
  - ✗ still a lot of theory efforts necessary !
    - large angle Bhabha ?
    - RC to photon tagging ?
    - photon radiation from hadrons ?
  - ✗ eagerly awaiting for fist release of KLOE data

    big attention for new input for $g-2$ now !
- **CMD-2, SND (VEPP-2M Novosibirsk):** CMD-2 final data analysis 0.6% systematics ? upgrade up to 2 GeV
- **BES (BEPC Beijing):** upgrade, higher luminosity, improved detector 2 to 5 GeV region
- **SLAC/BABAR:** $R(s)$ measurement in range $M_\Phi < E_{\rm cm} < M_{J/\psi}$

  Also: radiative return $\pi\pi$ inv. mass spectrum at B factories (BABAR/SLAC, BELLE/KEK)



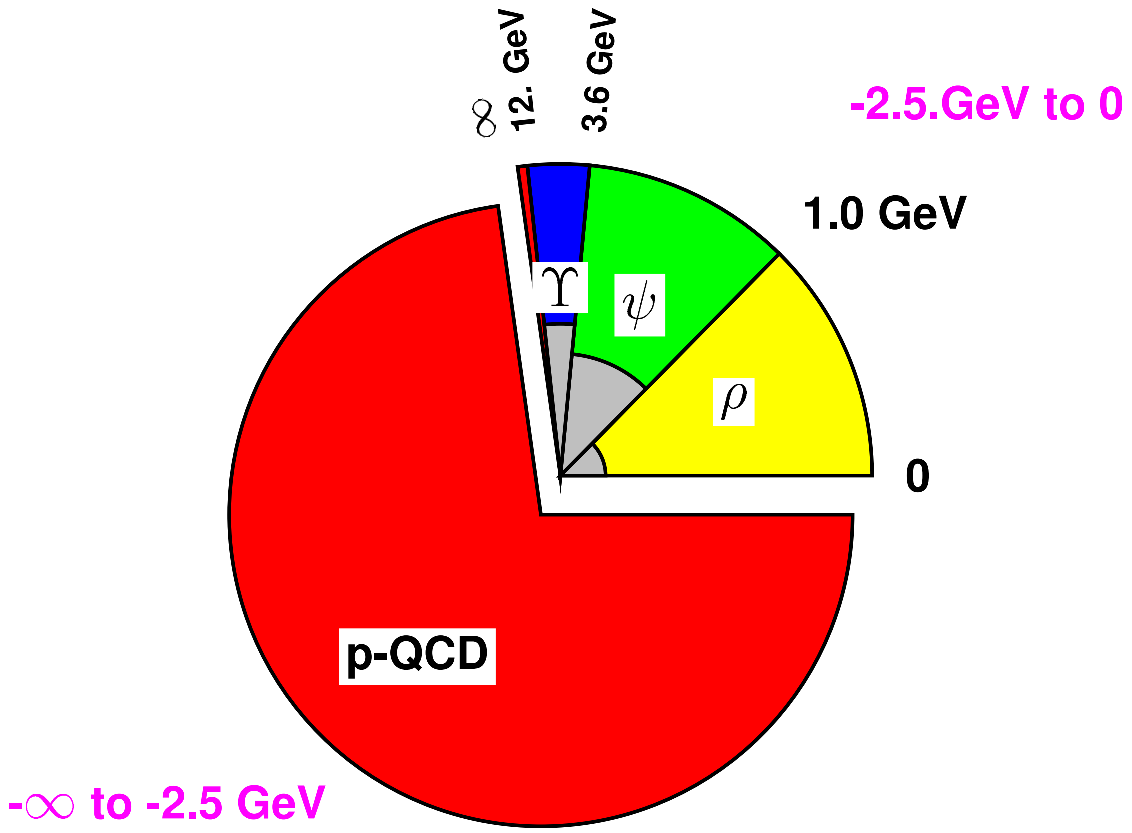
Data: KLOE Collaboration (A. Aloisio et al.)

Fits: KLOE/Frascati; CMD-2/Novosibirsk

Long term: need $\sigma(e^+e^- \to hadrons) \lesssim$ 1% up to 3.6 GeV both for $a_\mu^{\rm had}$ and for $\Delta\alpha^{\rm had}$:

Distribution of hadronic contributions to $\Delta\alpha^{\rm had}$

"Adler function based" approach (EJKV99,FJ 00):



Shaded area: 10 $\times$ error (scaled up)

- **τ–charm factory**

  able to perform an energy scan between 2 and 3.6 GeV

  would satisfy requirements of future precision experiments

  g-2, GigaZ,...

- Last but not least: need further progress in theory
  - ✗ more careful study of isospin breaking in τ–data vs. e^+e^-–data
  - ✗ constraints to $F_\pi(s)$ from χPT, unitarity and analyticity below the ρ
  - ✗ still a theoretical challenge:
    the hadronic light–by–light scattering contribution !

Concerning τ–data vs. e^+e^-–data discrepancy: need more careful check of radiative corrections which have been applied !