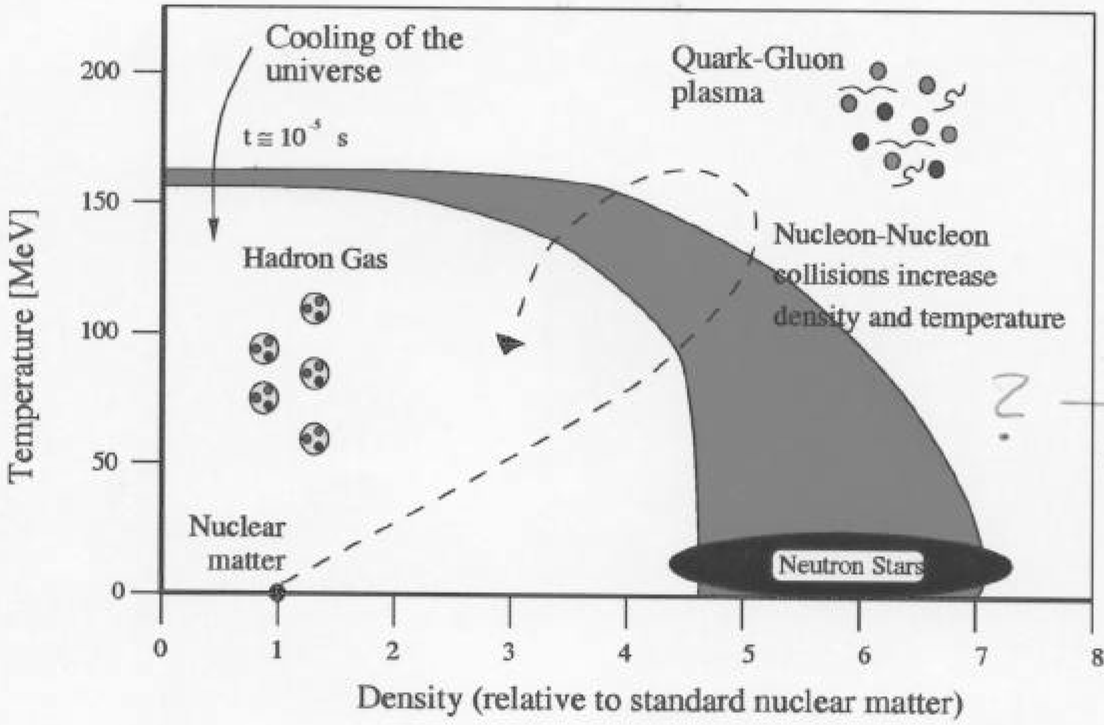
DESY workshop
Sept. 2002

LATTICE STUDIES OF QCD THERMODYNAMICS

- Introduction
- Phase transition
- Equation of State
- Thermal masses

Introduction



? → Rajagopal

estimates based on e.g. ideal gas and counting d.o.f.s

- $T_c \simeq 200$ MeV
- $\varepsilon(T_c) \simeq 1\ \mathrm{GeV/fm^3} \simeq 5 \times \varepsilon_{\text{nucleon}}$

→ early universe $t_c \simeq 10^{-5}$ sec

→ (core of) neutron stars

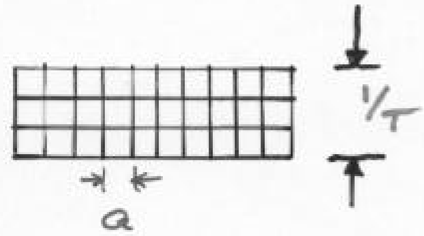
→ heavy ion collisions at RHIC, LHC
→ Gyulassy

non-perturbative

lattice : as in previous lattice talks

plus

- $T \neq 0$



$$\boxed{T = \frac{1}{N_\tau a}}$$

read also as $aT = \frac{1}{N_\tau}$
(discretization error)

$$Z(T, V, \mu) = \int_{\text{periodic}} \mathcal{D}\phi \; \exp\left\{ - \int_0^{1/T} d\tau \int_V d^3x \; \mathcal{L}_E(g, m, \mu) \right\}$$

- chemical potential $\mu \neq 0$ eventually

$$\mathcal{L}^{\text{fermion}} = \bar{\psi} \not{D} \psi + m \bar{\psi}\psi + \mu \psi^+ \psi$$

$\Rightarrow$ thermodynamics

$f = -\frac{T}{V} \ln Z$ free energy density

$p = T \frac{\partial \ln Z}{\partial V} \Big|_{T, \mu/T}$ pressure

$\varepsilon = \frac{T^2}{V} \frac{\partial \ln Z}{\partial T} \Big|_{V, \mu/T}$ energy density

$n = + \frac{1}{V} \frac{\partial \ln Z}{\partial (\mu/T)} \Big|_{V, T}$ number density to detect and study phase transitions:

order parameters $\langle \bar{\psi}\psi \rangle \sim \frac{\partial \ln Z}{\partial m}$ chiral condensate

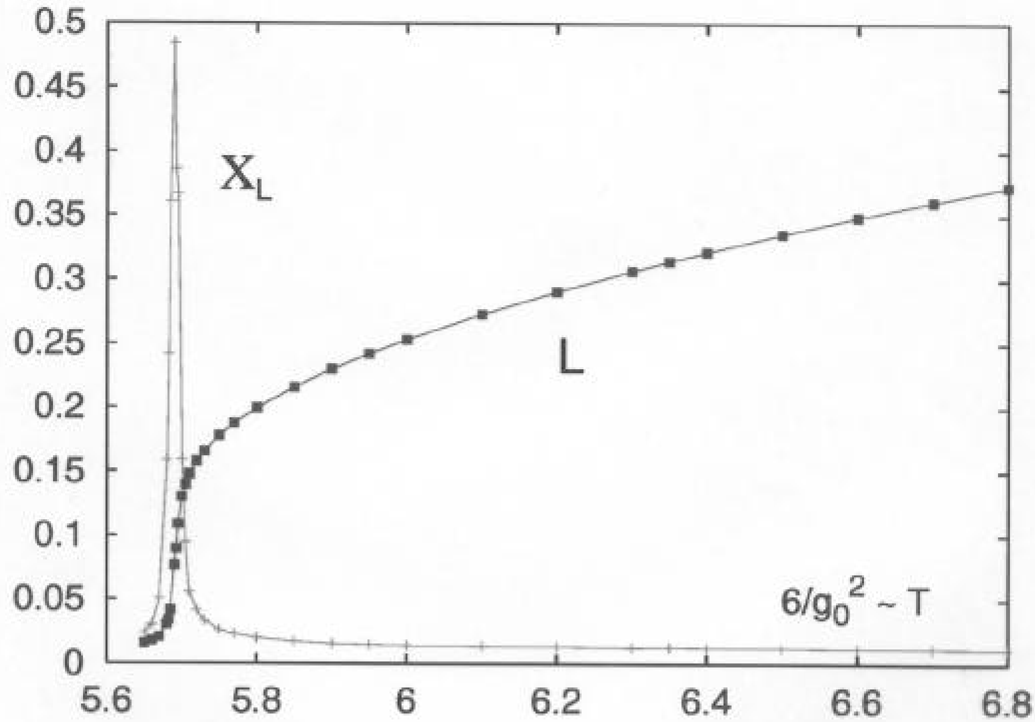
$\langle L \rangle \sim e^{-F_q/T}$ Polyakov loop

response functions $\chi_m \sim \frac{\partial^2 \ln Z}{\partial m^2}$

$\chi_L \sim \langle L^2 \rangle - \langle L \rangle^2$

close to (2nd order) phase transition

e.g. $\chi_L \sim |T-T_c|^{-\gamma}$

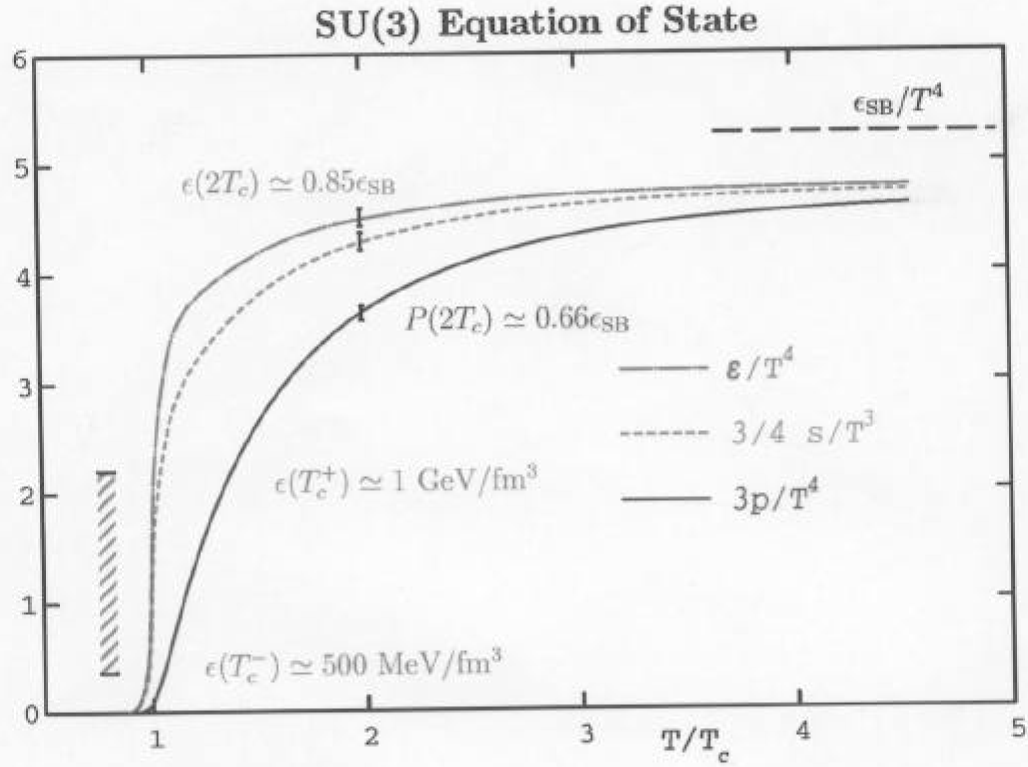


divergence described by critical exponents, e.g. γ

depending on long distance properties
(global symmetries, dimension)

universality classes

pure gauge: bulk thermodynamics solved

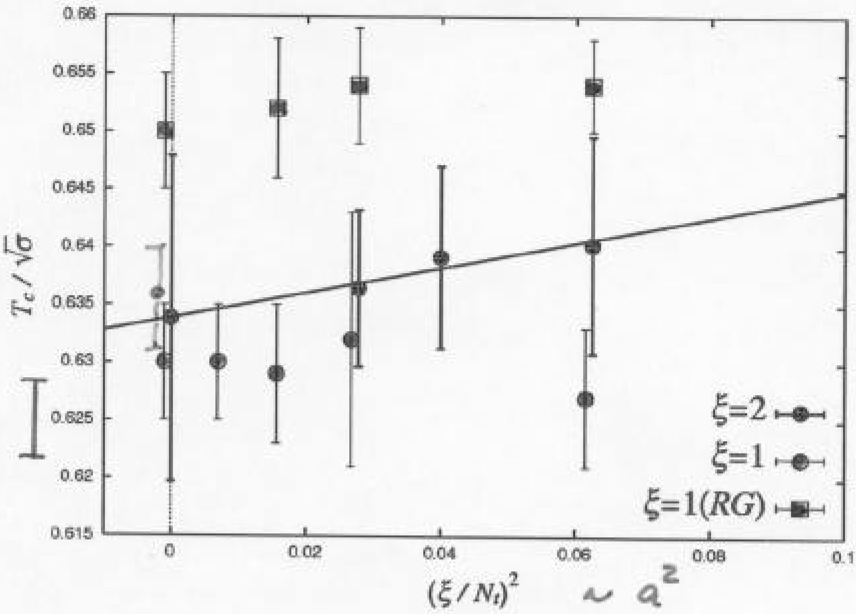


- 1^{st} order
- $T_c = 270(5)$ MeV
- $p(T)$, $\varepsilon(T)$ known

} in continuum limi

EoS:
- considerable deviations from ideal gas
- high T behavior can be reproduced in
  - HTL resummed pert. th. [Blaizot et al]
  - quasi-particle models [Levai; Heinz; Pesh

T_c



1%

$\sim a^2$

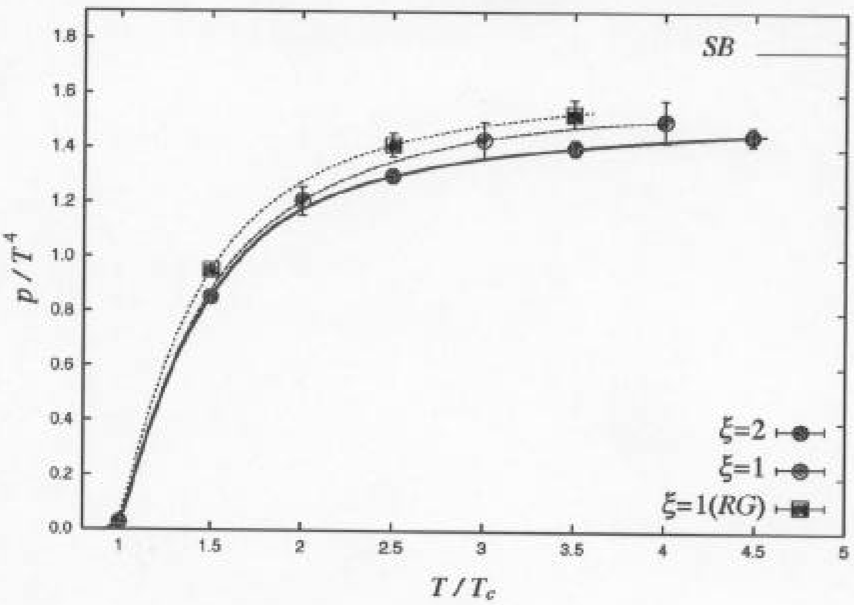
•	cp-pacs	standard, anisotropic
■	Iwasaki et al.	RG improved
•	Bielefeld group	standard

+ various Symanzik improved actions not shown

see also Necco, LAT 2002

$$T_c r_0 = 0.750(5) \quad \hat{=} \quad \frac{T_c}{\sqrt{\sigma}} = 0.636(4)$$

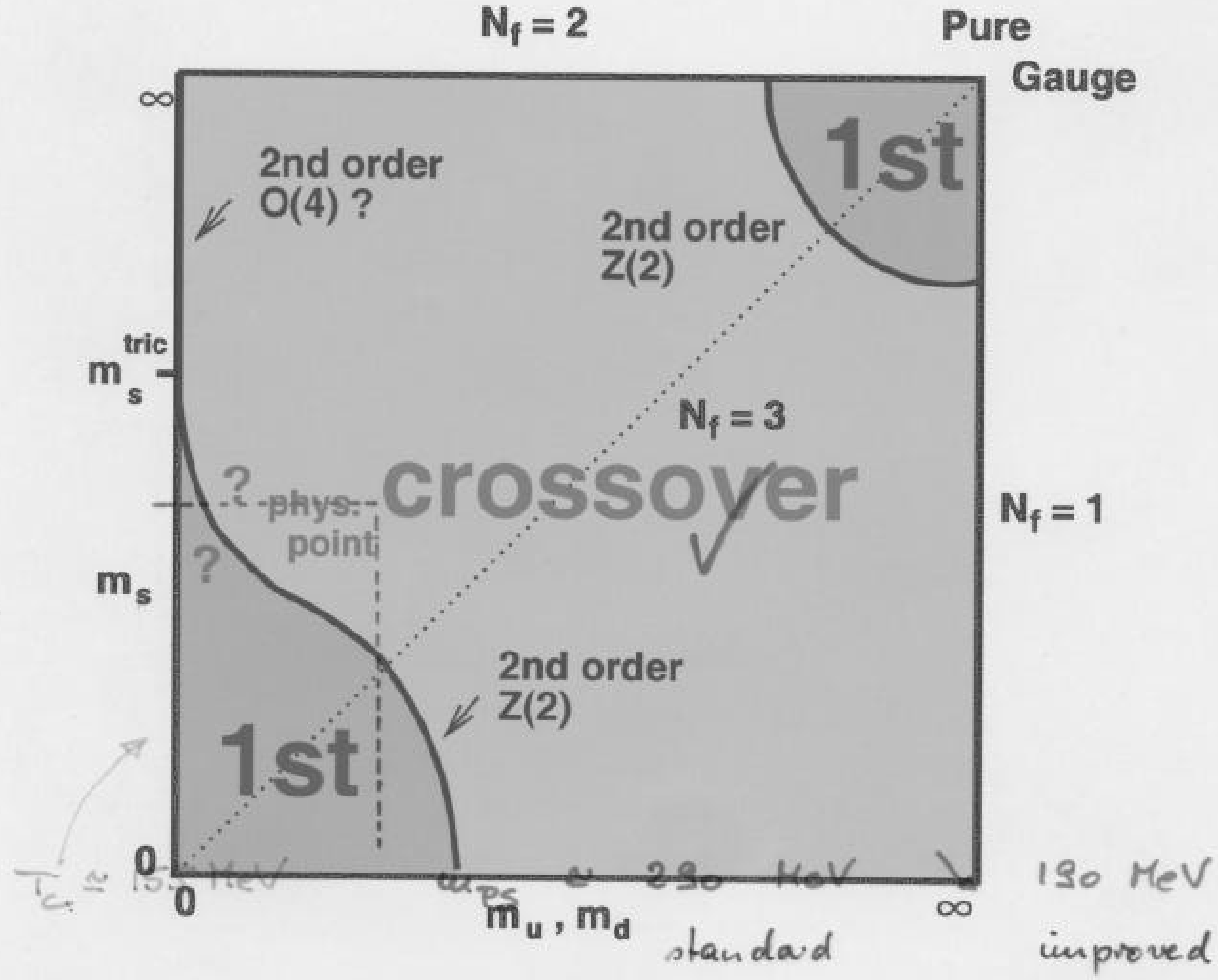
P/T^4 continuum limit !



- cp-pacs — standard, anisotropic $\frac{a_\sigma}{a_\tau} \neq 1$
- cp-pacs — RG improved
- Bielefeld — standard

$T_c \simeq 175$ MeV

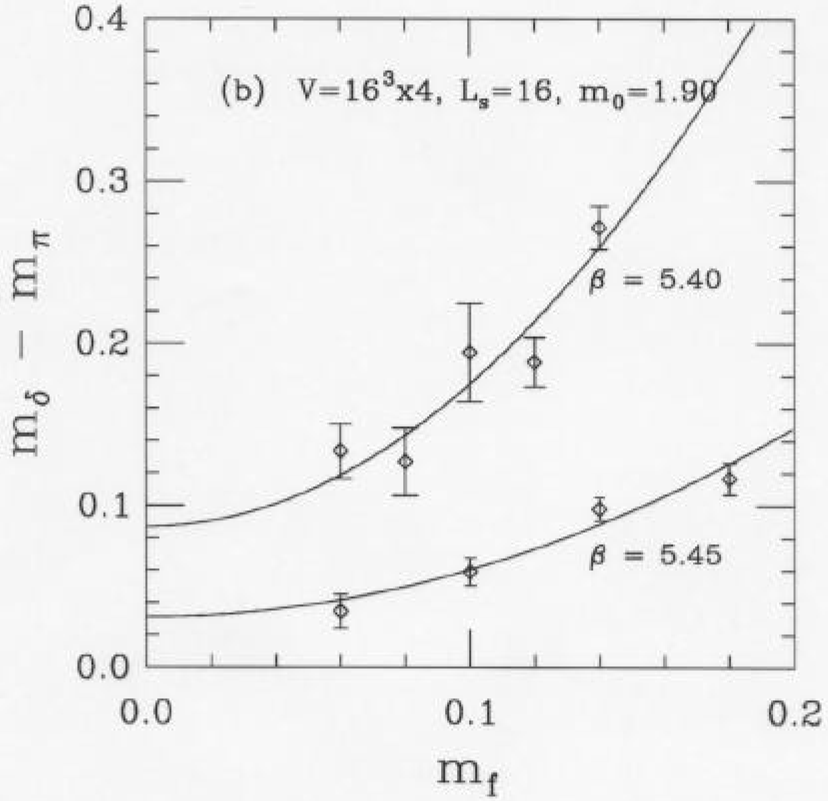
Expected phase diagram in the $m_{u,d} - m_s$ plane



screening masses with domain wall fermions

$$m_{a_0} - m_\pi = c_0 + c_2\, m_q^2$$

in accord with theoretical expectations



[Vranas et al., hep-lat/9903024]

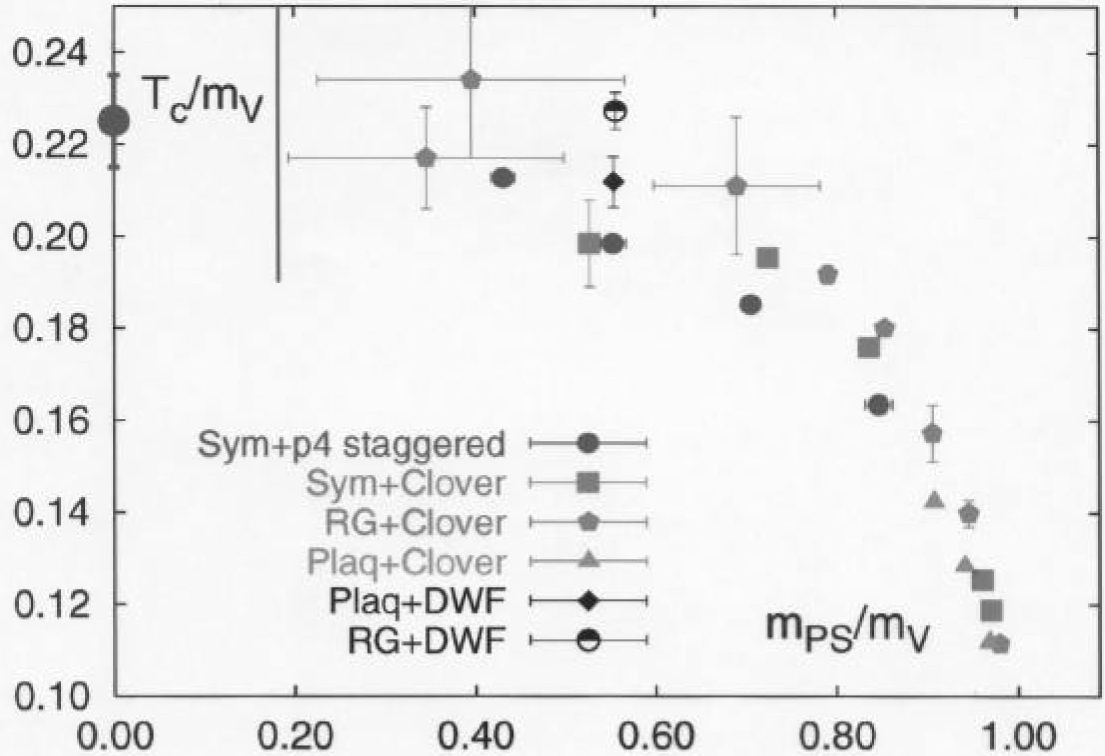
at $\beta = 5.40$ $(T \simeq 1.2\, T_c)$: c_0 small but non-vanishing

$\Rightarrow U_A(1)$ remains (weakly) broken

[Vranas et

$N_F = 2$

- improved actions mandatory



$\Rightarrow \quad T_c\,(m_q \to 0) \;=\; 173\,(8)\,(..)\;\text{MeV}$

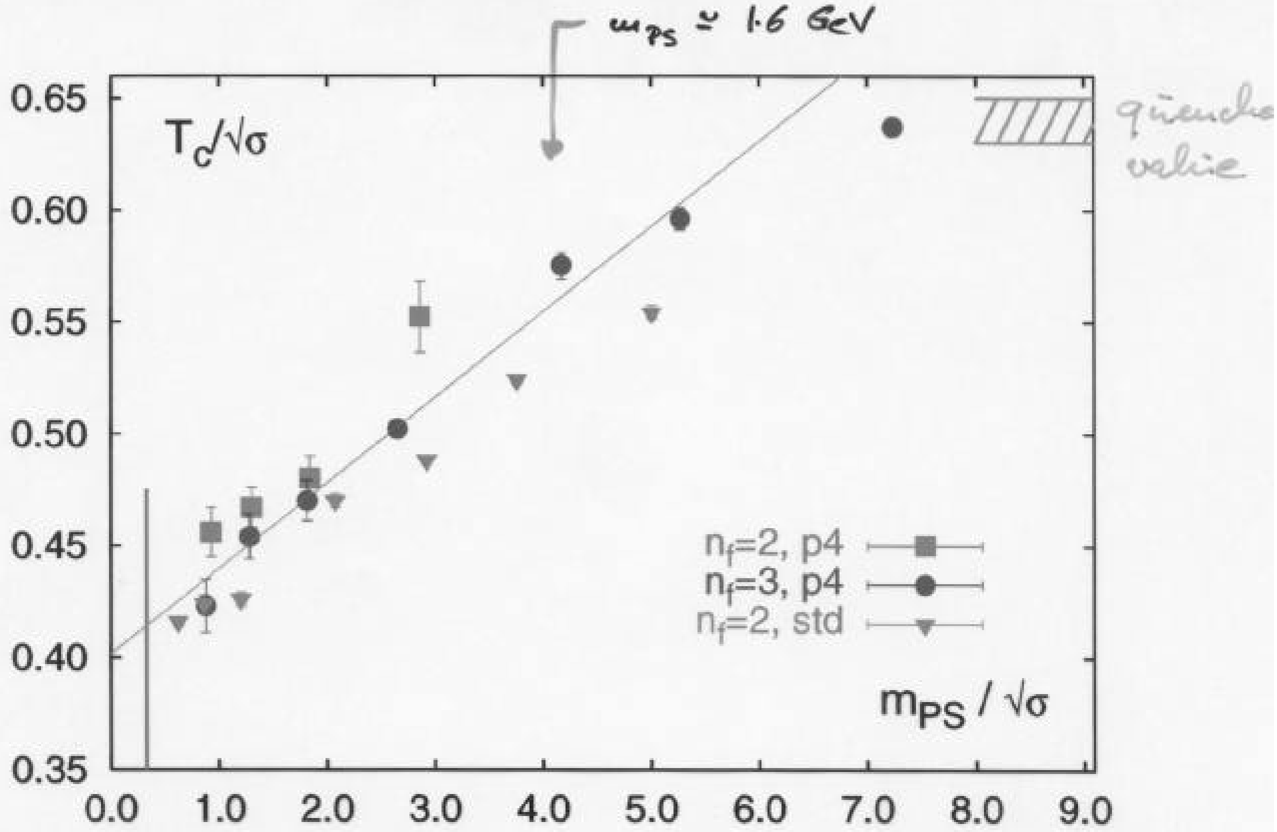
systematic / discretization error not yet known

- $O(4)$ scaling $\quad T_c\,(m_\pi) = T_c\,(0) + m_\pi^{1.1} * c_t$

$$m_V\,(m_\pi) = m_\rho + c_s \times m_\pi^2$$

$$\Rightarrow \quad \frac{T_c(m_\pi)}{m_V(m_\pi)} \simeq \frac{T_c(0)}{m_\rho} + c_t\, m_\pi^{1.1} - c_s\, m_\pi^2$$

N_F, m_q dependence of T_c:



[Peikert et al. hep-lat/0012023]

- what to use for setting the scale?
  $\frac{\sqrt{\sigma}}{m_q}$ in the valence quark mass limit → 0
  fairly unaffected by m_q^{sea}, N_F ⇒ $\sqrt{\sigma}$

→ moderate m_q dependence

→ weak N_F dependence $T_c^{N_F=2} - T_c^{N_F=3} \simeq 20$ MeV

→ in chiral limit:

$N_F = 2$ $T_c \simeq 175 \pm 10$ MeV imp. staggered [Bielefeld]

$N_F = 3$ $T_c \simeq 155 \pm 10$ MeV imp. Wilson [CP-PACS]

± systematic errors: no continuum limit yet!

locating the critical endpoint of

1^{st} order $N_F = 3$ PT line: m_c

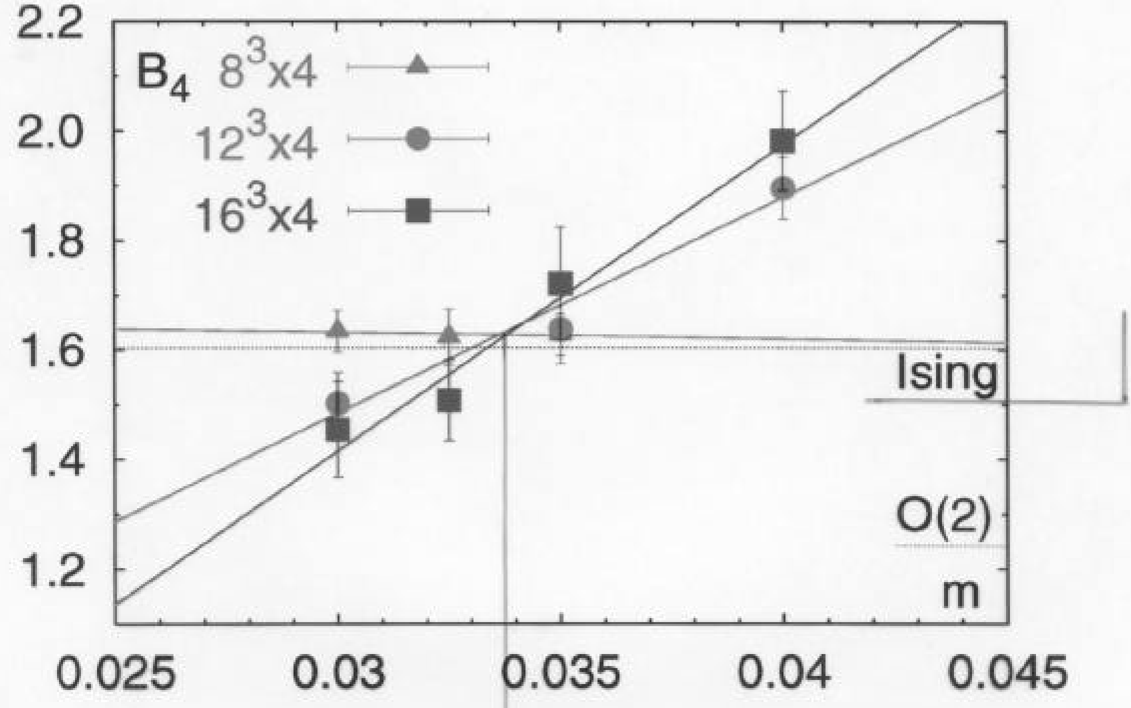
Binder cumulant

$$B_4 = \frac{\langle \delta M^4 \rangle}{\langle \delta M^2 \rangle^2}$$

$\longrightarrow$ universal value at m_c

here: M magnetization $\approx \bar{\psi}\psi$

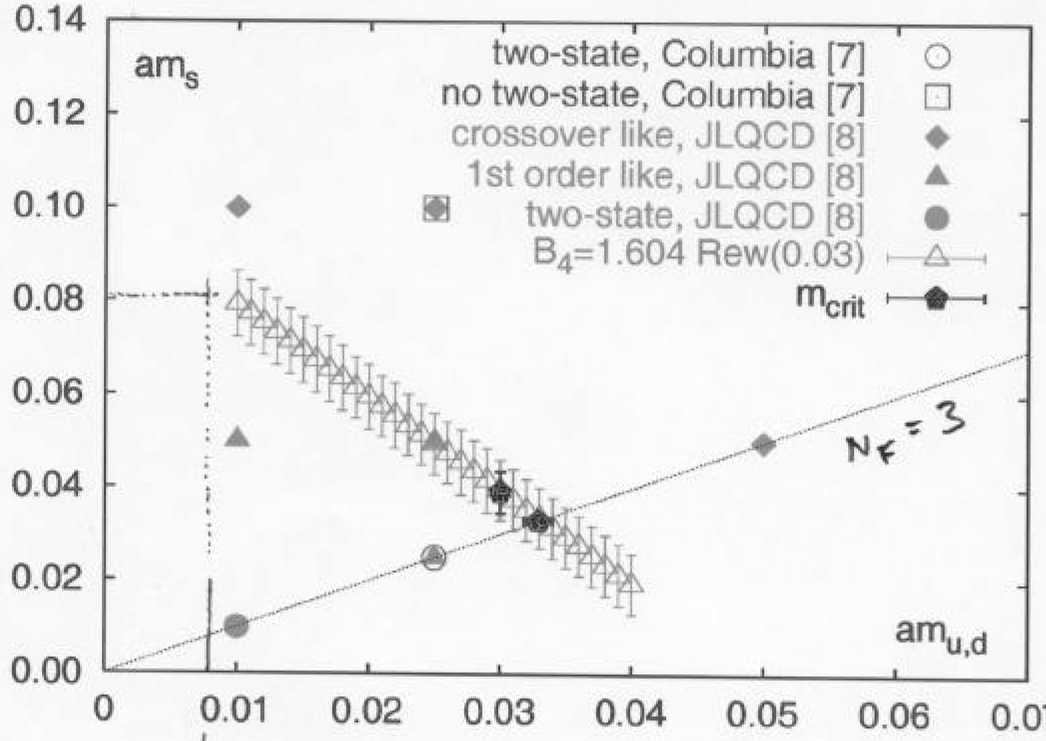
$$\delta M = M - \langle M \rangle$$



m_c

[Ch. Schmidt et al.]

results in the $m_{u,d} - m_s$ plane



standard staggered

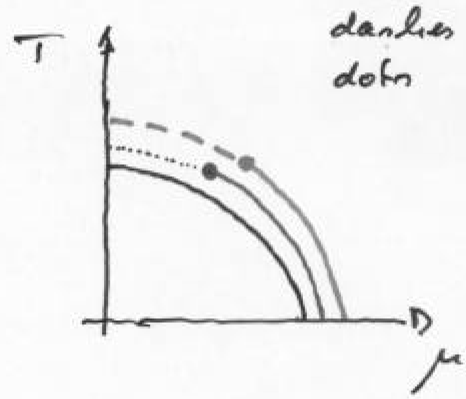
estimated physical $m_{u,d}$

$$\leadsto \quad \frac{m_s^{crit}}{m_{u,d}^{phys}} \simeq 10 \qquad \text{compare to} \quad \frac{m_s^{phys}}{m_{u,d}^{phys}} \simeq 20$$

if this holds then 1st order at physical point unlikely

$\mu \neq 0$:

solid 1st
dashes 2nd
dots crossover



• expectations:

$N_F = 3$

$N_F = 2+1$

$N_F = 2$

• technical difficulty:

$$Z(\mu, \beta) = \int \mathcal{D}U \, e^{-S_G(\beta)} \det M(\mu)$$

$\beta = 6/g_0^2$ — fermion determinant

$\det M(\mu \neq 0)$ complex $\Rightarrow$ not a statistical weight in a Monte Carlo

rewrite $\det M(\mu) = |\det M(\mu)| * e^{i\theta}$

and regard phase as (part of) observable

$$\langle O \rangle_{\det M} = \langle O e^{i\theta} \rangle_{|\det M|} \Big/ \langle e^{i\theta} \rangle_{|\det M|}$$

but $\langle e^{i\theta} \rangle \sim e^{-V}$ sign problem

reweighting: [Glasgow]

simulate at parameters p' and reweight to p

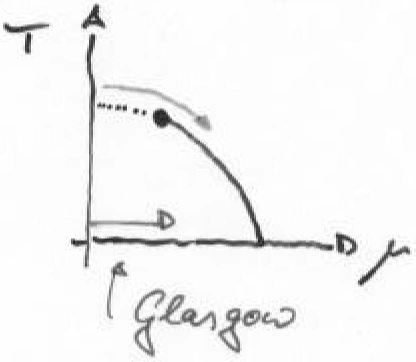
$$\mathcal{D}U \, e^{-S_G(p)} \det M(p)$$

$$= \underbrace{\mathcal{D}U' \, e^{-S_G(p')} \det M(p')}_{\text{simulation}} * \underbrace{e^{-[S_G(p) - S_G(p')]} \frac{\det M(p)}{\det M(p')}}_{\text{correction factor}}$$

need large overlap of confs U' at p' with confs U at p

Fodor, Katz:

reweighting
along transition line

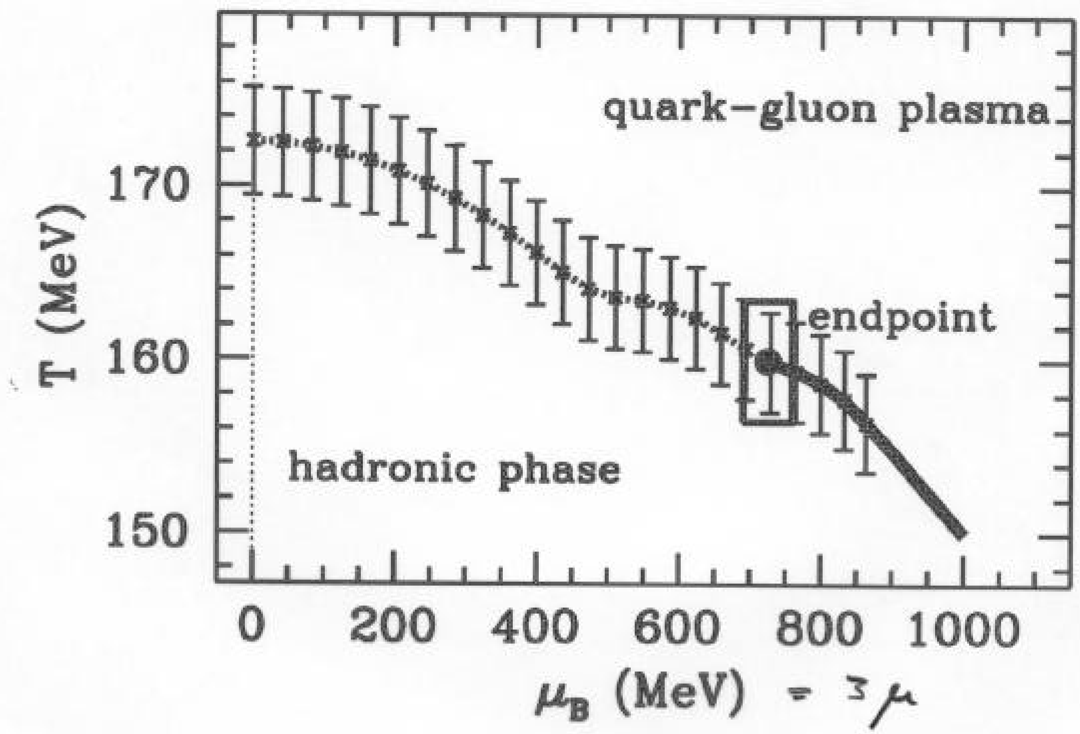


⇒ much better overlap

$N_F = 2+1$ $\quad m_{u,d} \simeq 4\, m_{u,d}^{phys}$, $m_s/m_u = 8$

$4 * (4,6,8)^3$

standard staggered



endpoint : $T_c(\mu^E) = 160 \pm 3.5$ MeV

$\mu_B^E = 725 \pm 35$ MeV

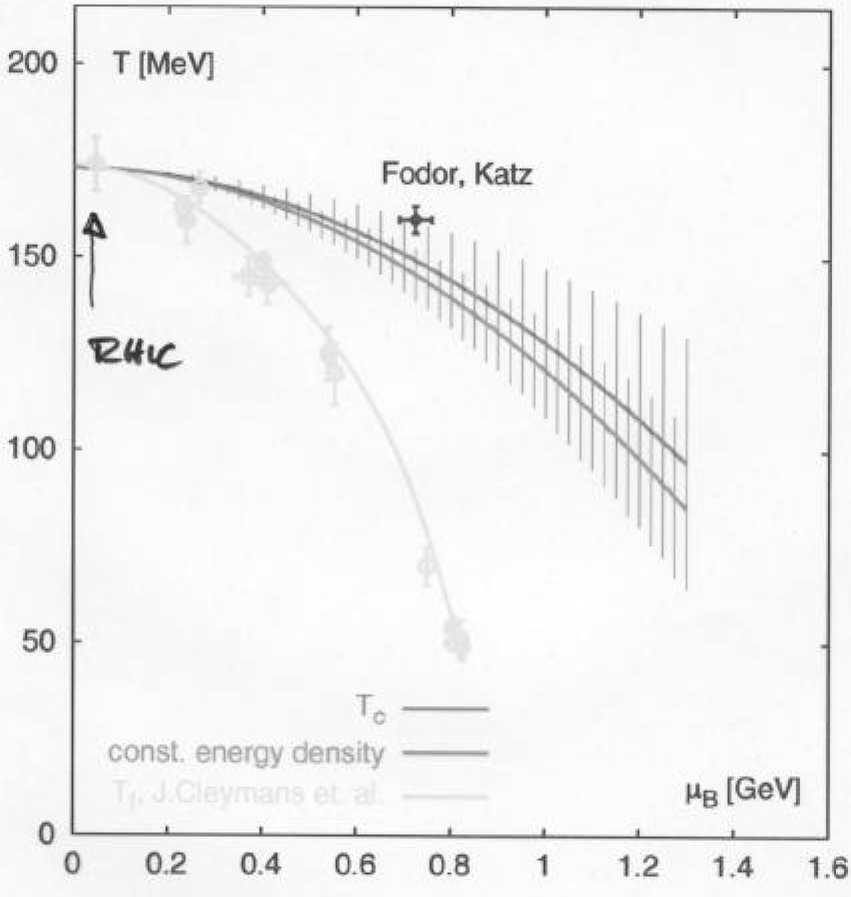
• Swansea – Bielefeld   Taylor expansion in μ/T

at RHIC: $\mu/T_c \simeq 0.1$

$$\ln \frac{\det M(\mu)}{\det M(0)} = \sum_n \frac{1}{n!} \left. \frac{\partial^n \ln \det M}{\partial \mu^n} \right|_{\mu=0} \mu^n$$

$N_F = 2$ $m_\pi / m_\rho \geq 0.7$

4×16^3 improved staggered

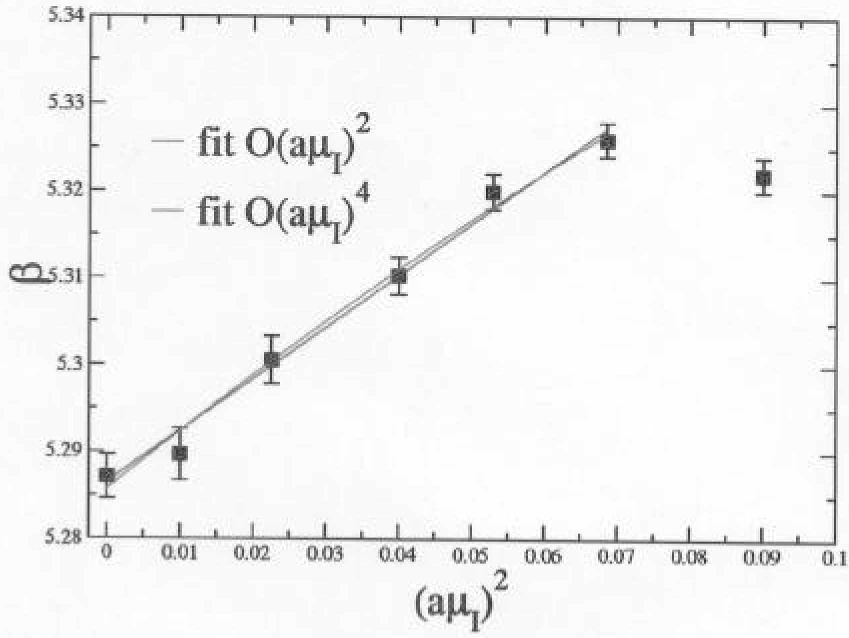


$$\frac{T_c(\mu)}{T_c(0)} = 1 - 0.0078 \left(\frac{\mu_B}{T}\right)^2$$

- imaginary $\mu_I = i\mu$ $\det M(i\mu)$ real

de Forcrand, Philipsen   see also d'Elia, Lombardo
Hart, Laine, Philipsen

- fit non-pert. results at μ_I by polynomial in μ_I/T
- continue to real μ

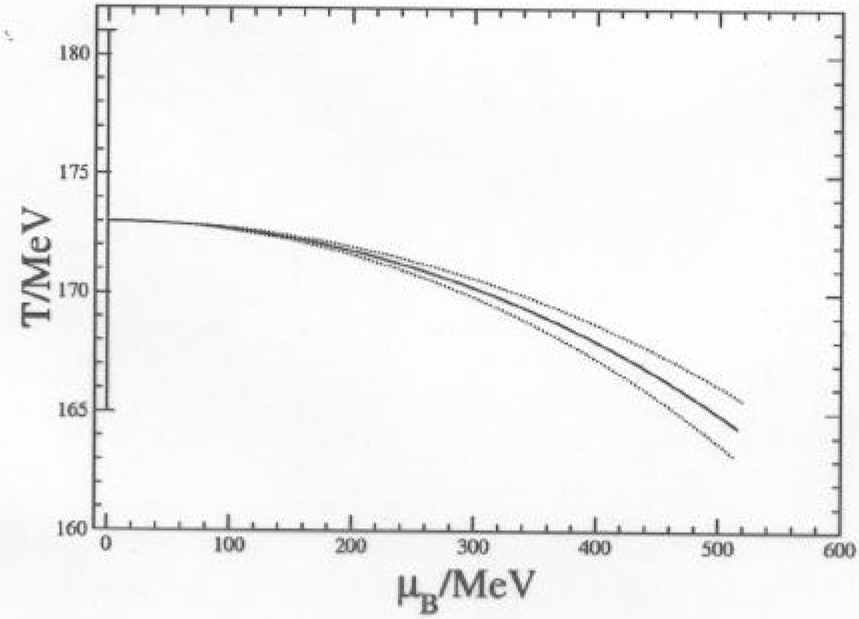


$N_F = 2$

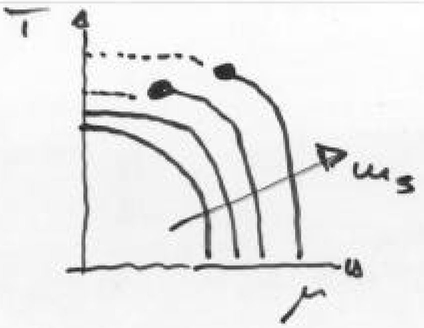
$m/T = 0.1$

standard staggered

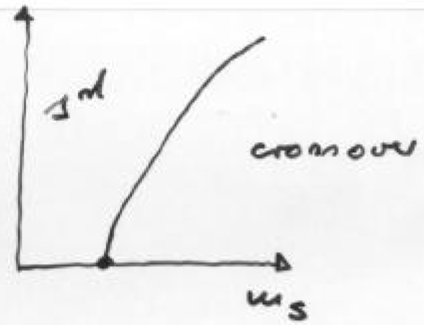
$4 \times (6,8)^3$



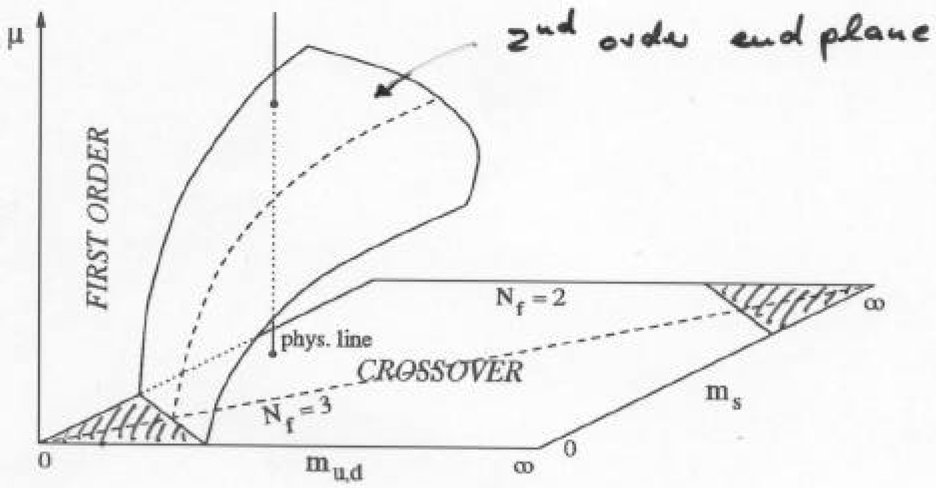
$$\frac{T_c(\mu)}{T_c(0)} = 1 - 0.0056(4)\left(\frac{\mu_B}{T}\right)^2$$



$\Rightarrow$

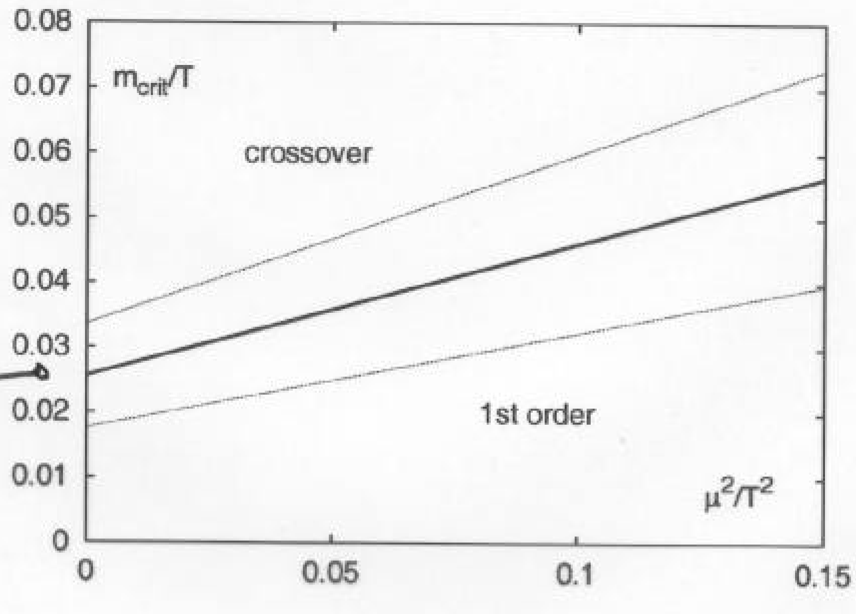


- plane diagram in $m_{u,d}$ – m_s – μ



Allton et al. $N_F = 3$

improved staggered 4×12^3



m_{PS}
$\simeq 200$ MeV

$$\frac{m_{crit}}{T_c}(\mu) = \frac{m_{crit}}{T_c}(0) + 0.21(6)\left(\frac{\mu}{T_c}\right)^2$$

Equation of State

- $\varepsilon \sim T^4 \;\Leftrightarrow\;$ relevant momenta $p \sim T$
- distorted by discretization effects $p_{max} = \frac{\pi}{a}$

(quenched) <u>Stefan-Boltzmann</u> (ideal gluon gas)

$$\frac{\varepsilon_{SB}^{LAT}}{T^4} = \frac{\varepsilon_{SB}^{cont}}{T^4}\left\{1 + \frac{10}{21}(\pi T a)^2 + \frac{2}{5}(\pi T a)^4 + \dots\right\}$$

$$\quad = \frac{\pi}{N_\tau} \Rightarrow \underline{N_\tau \text{ large}}$$

- on the other hand

use $\quad p = -f = \frac{T}{V}\ln Z$

$$\frac{\partial}{\partial(6/g^2)}\ln Z = \langle S \rangle$$

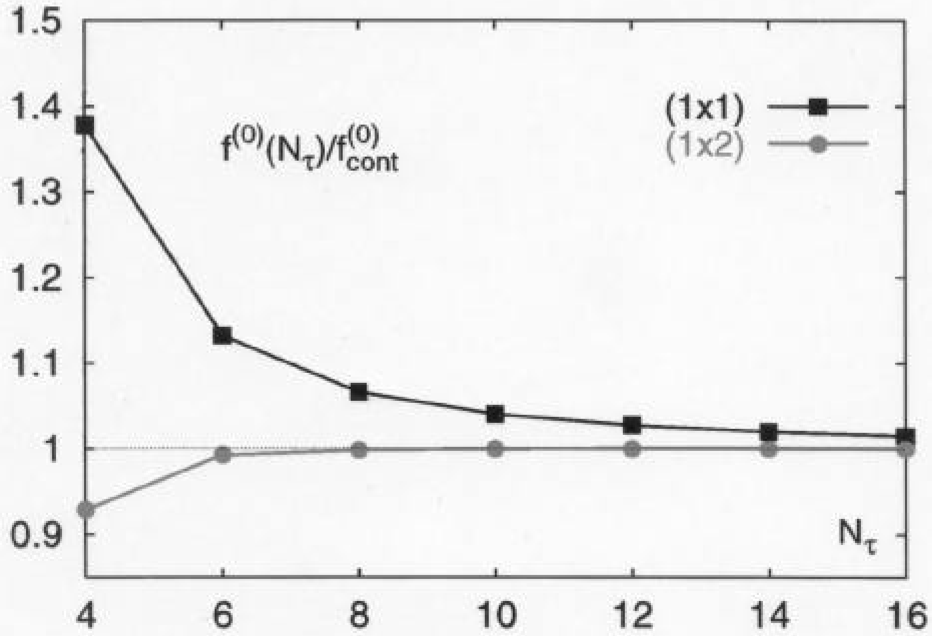
$$\Rightarrow \ln Z = \int d(6/g^2)\,\langle S \rangle$$

more precisely

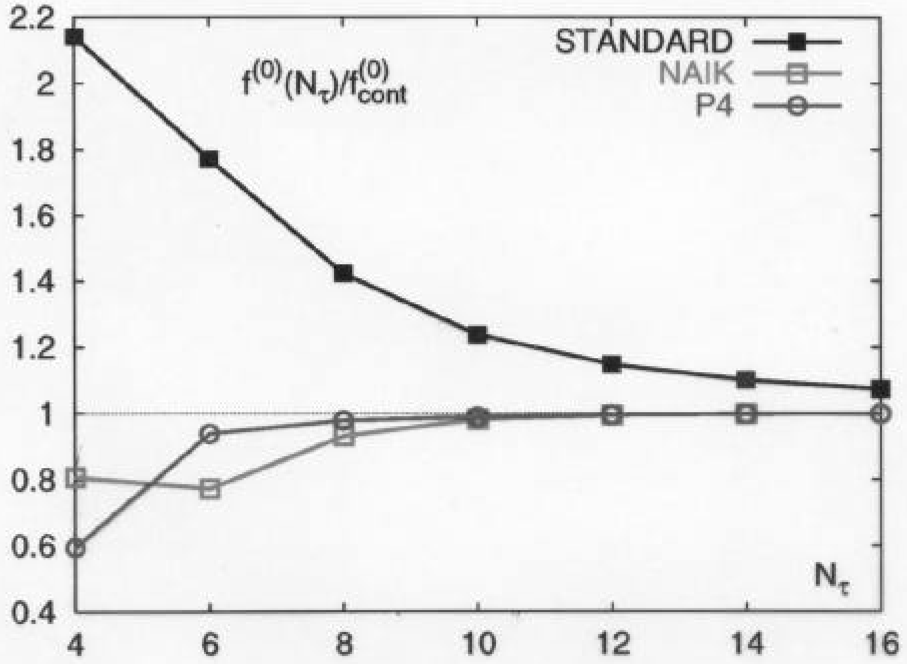
$$\frac{p}{T^4} = N_\tau^4 \underbrace{\int_0^{6/g^2} d(6/g^2)\left\{\langle S(T)\rangle - \langle S(T{=}0)\rangle\right\}}_{\text{numerical signal} \sim \frac{1}{N_\tau^4}}$$

$\Rightarrow$ <u>improved actions</u> <u>mandatory</u> for <u>full QCD</u>

Ideal gas limit at finite $aT = 1/N_\tau$



gluon part

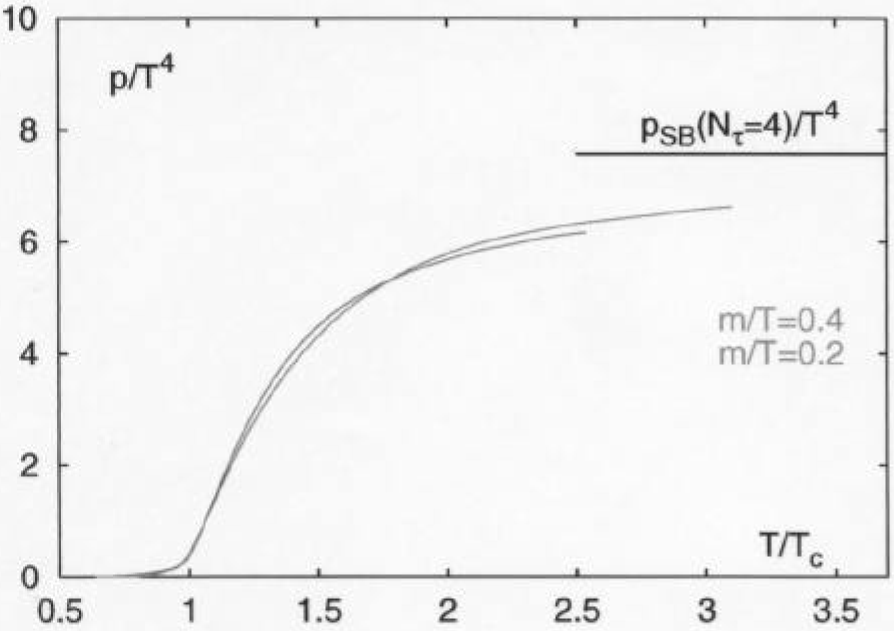


fermion part

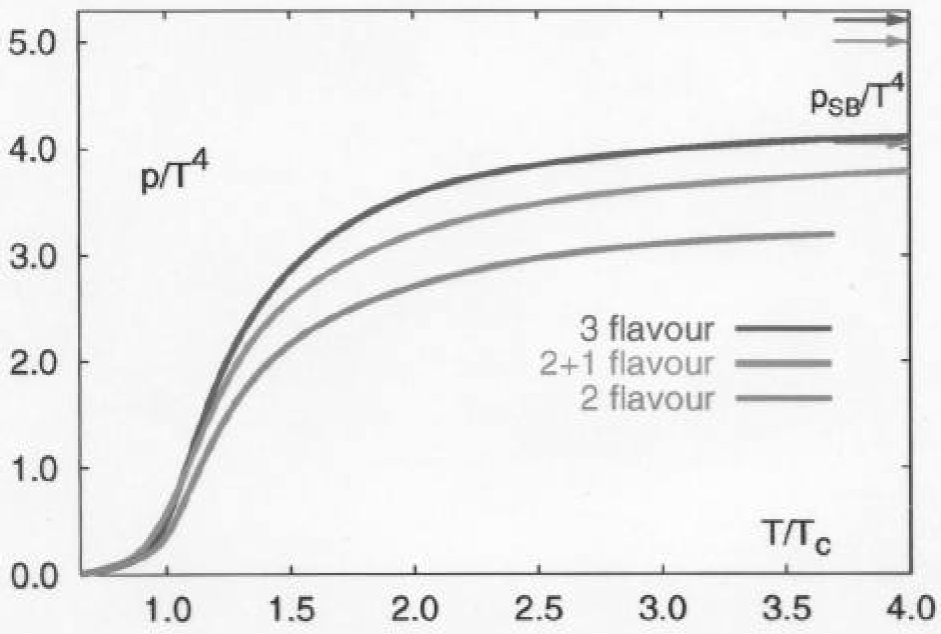
staggered

[Peikert et al., NP PS 83 (2000) 390]

MC results at $T \gtrsim T_c$, full QCD



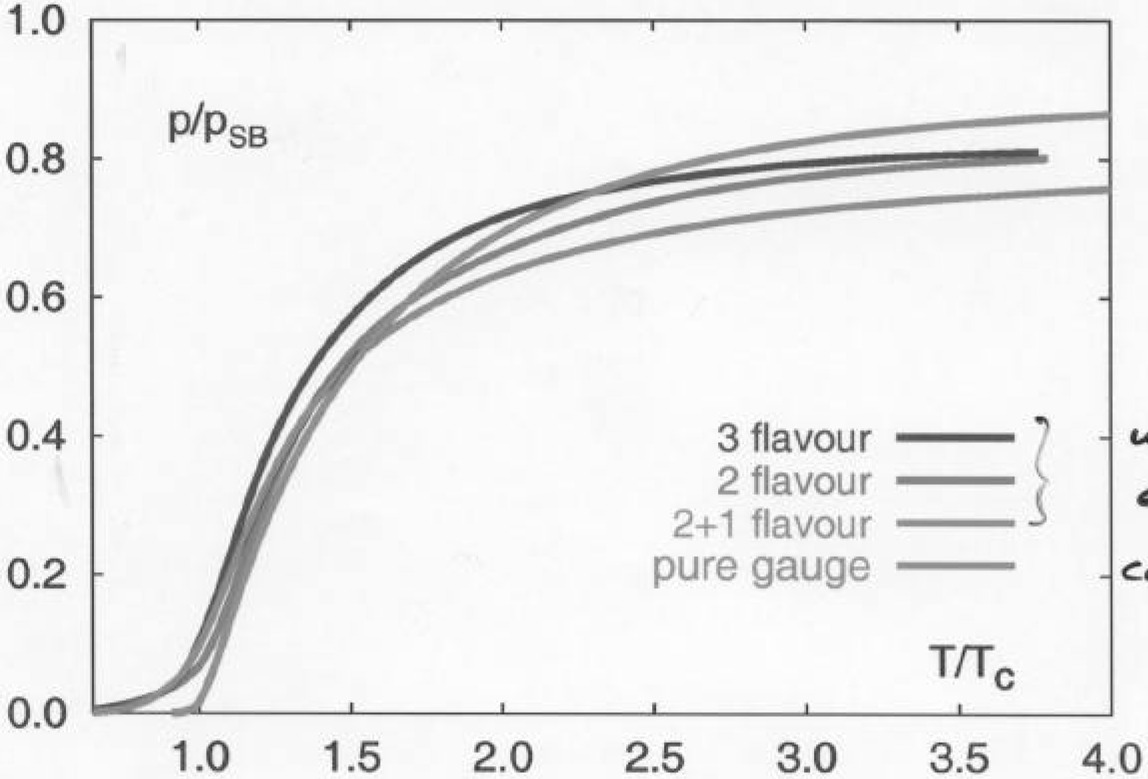
Naik action
$N_F = 4$
[Engels et al., Phys. Lett. B396 (1997) 219]



P4 action
[Peikert et al., PL B478 (2000) 447]

- weak m_q dependence
- marked N_F dependence (in absolute numbers)

normalized to ideal gas behavior —



improved actions

cont. limit

- flavor dependence as in ideal gases

$$\frac{P(N_F, T)}{T^4} \cong \left(16 + \frac{21}{2} N_F\right) \frac{\pi^2}{90} * f(T/T_c)$$

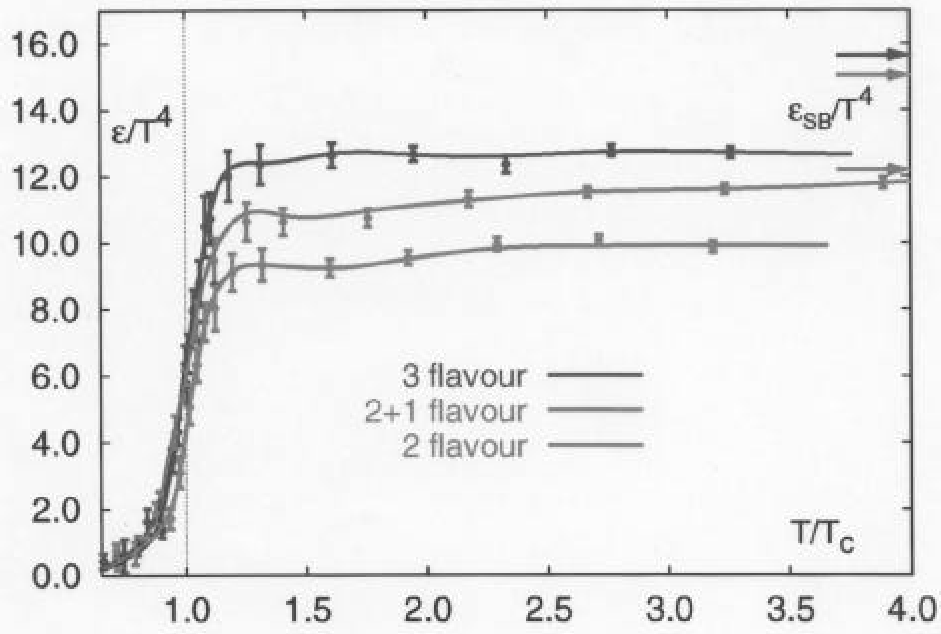
- for $T < 4T_c$: substantial deviations from ideal gas $f(T/T_c) \neq 1$

- strange contribution somewhat suppressed relative to massive ideal gas

- quark masses not yet realistic, $m_{PS}/m_V \lesssim 0.7$ yet, m_q dependence weak

- no continuum limit

energy density



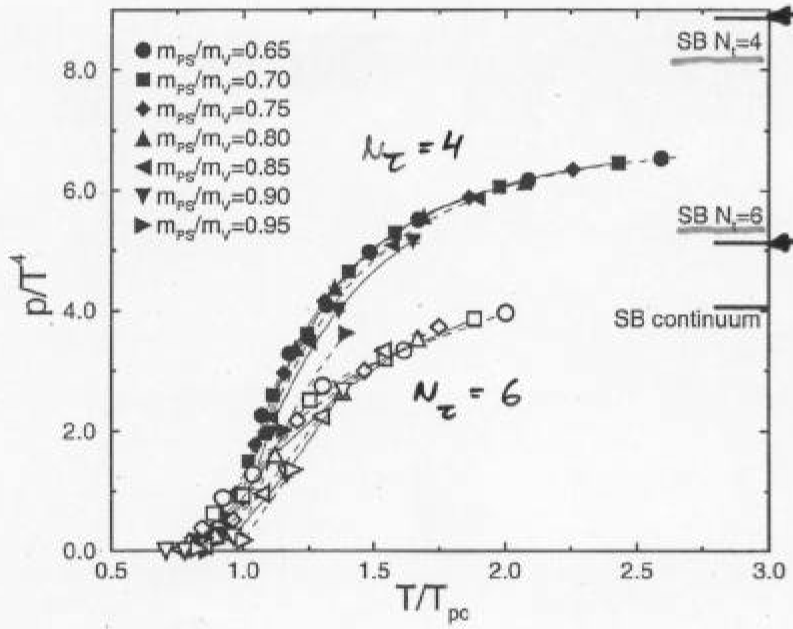
"partially quenched"

$$\frac{\varepsilon}{T^4} \sim \frac{\partial g^2}{\partial \ln a} \langle \Delta S_G \rangle + \frac{\partial (a m_q)}{\partial \ln a} \langle \Delta \bar{\psi}\psi \rangle$$

$\hookrightarrow \sim m_q \to 0$

critical energy density $\quad \varepsilon_c \simeq (6 \pm 2)\, T_c^4$

EoS for Wilson fermions   cp-pacs, hep-lat/0103028

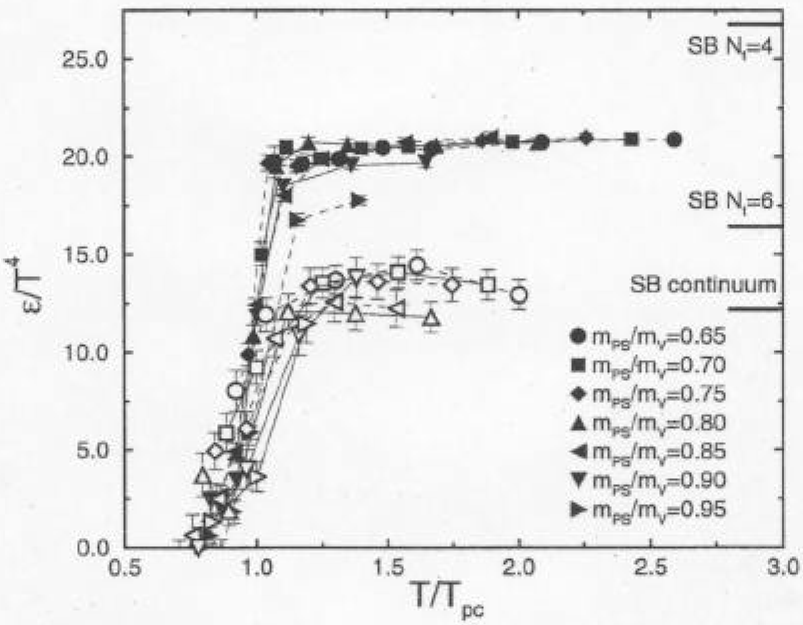


SB limit

⇓

strong cut-off dependence despite improved action

- weak m_q dependence



- weak cut-off dependence at $T \simeq T_c$

$$\frac{\varepsilon_c}{T^4} \simeq 5 \div 10$$

EoS at $\mu \neq 0$:

Fodor, Katz & co. $N_F = 2+1$

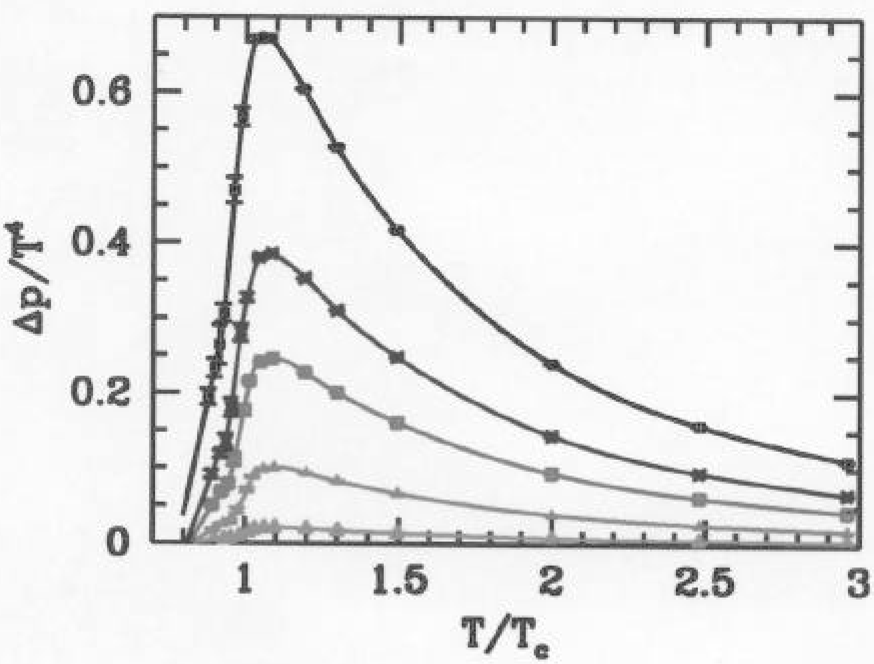
$m_{u,d}^{bare} \simeq 65\,MeV$

$m_s^{bare} \simeq 135\,MeV$

$4 \times (8,10,12)^3$

standard staggered

$$\Delta p = p(\mu \neq 0) - p(\mu = 0)$$



$\mu_B =$

530 MeV
410 MeV
330 MeV
210 MeV
100 MeV

Allton et al.

$$\Delta p, \Delta \varepsilon / T^4 \simeq + 1\%$$

Thermal masses

- thermal excitations modify correlation fcts
- coded in spectral density

$$G(\tau, \underline{x}) = T \sum_n \int_{\underline{p}} e^{-i(\tau\omega_n - \underline{x}\,\underline{p})}\, G(\omega_n, \underline{p})$$

ω_n Matsubara frequencies $2\pi T n$ (meso

$$G(\omega_n, \underline{p}) = \int d\omega\, \frac{\sigma(\omega, \underline{p})}{i\omega_n - \omega}$$

example: $\sigma(\omega, \underline{p}) \sim \delta(\omega^2 - \omega^2(\underline{p})) + \ldots$

with: $\omega^2(\underline{p}) = m^2 + \underline{p}^2 + \Pi_T(\underline{p})$

$$\simeq m^2(T) + \underbrace{a(T)}_{=1 \text{ at } T=0}\, \underline{p}^2$$

$\Rightarrow$ temporal correlator:

$$G(\tau, \underline{p}=0) \xrightarrow[\tau \text{ large}]{} \exp\left\{ - m(T) * \tau \right\}$$

pole mass

$\Rightarrow$ spatial correlator:

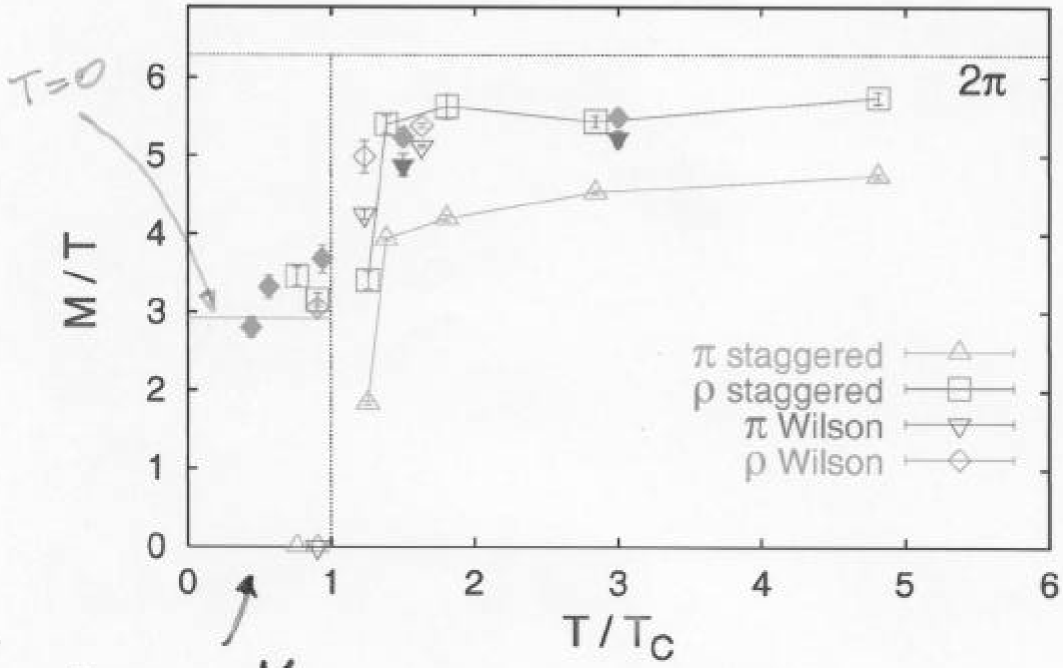
$$G(z, \omega_n = p_\perp = 0) \xrightarrow[z \text{ large}]{} \exp\left\{ - \frac{m(T)}{\sqrt{a(T)}} * z \right\}$$

screening mass

recall: $T \neq 0 \Rightarrow \tau \in \left[0, \frac{1}{T}\right]$

i.e. limited distances

Screening masses:



$T < T_c$: $\frac{M}{T_c}$

$T \to \infty$: free quarks ?

$$G(z) \sim \sum_{\ell=-\infty}^{+\infty} \exp\left\{ -2\sqrt{\underbrace{(2\ell+1)^2 (\pi T)^2}_{\text{quark Matsubara freq.}} + m_q^2} \, * z \right\}$$

$$\longrightarrow \exp\left\{ -2\pi T z \right\}$$

aside: chiral symmetry restoration observed in degeneracy V-AV, PS-S at $T \gtrsim 1.2\, T_c$

temporal correlators:

- limited physical distances
- anisotropic lattices ? ⇒ more points
- still, rely on fit ansatz

full information about masses, decay widths in $\sigma(\omega)$

$$G(\tau) = \int d\omega\, \sigma(\omega)\, \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\omega/2T)}$$

→ reconstruction of $\sigma(\omega)$ from temporal correlation fct.
ill posed problem

⇝ Maximum Entropy Method

$$P[\sigma|\text{data, prior knowledge}] \sim e^{\alpha S - L}$$

with likelihood $L = \frac{1}{2}\chi^2$

and entropy $S = \int d\omega \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \log \frac{\sigma(\omega)}{m(\omega)}\right]$

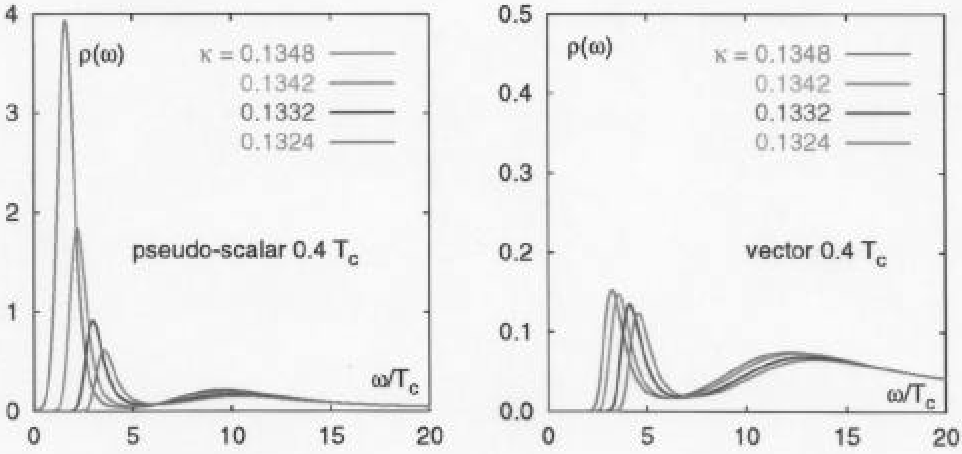
model density $m(\omega) = m_0\, \omega^2$

$T=0$ asymptotic ($\omega \to \infty$) perturbative behavior

* successfully applied in various fields
and at $T=0$
Arakawa et al.
cp-pacs

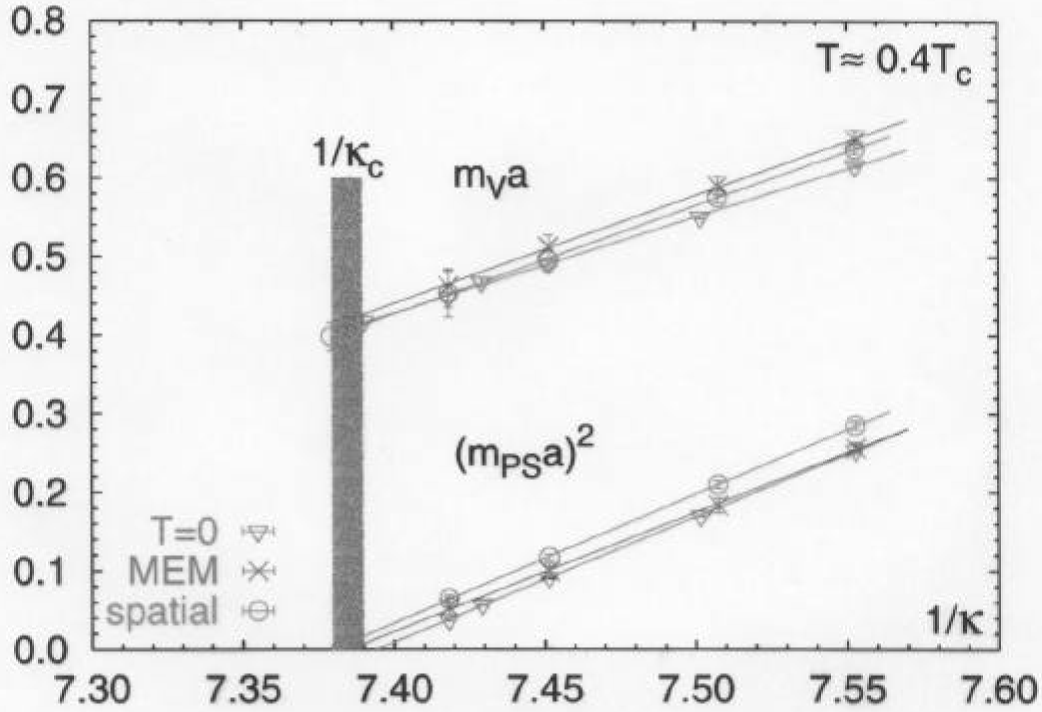
* at $T \neq 0$: Nomura et al.
Arakawa et al.
Wetzorke, Karsch; + EL, Petreczky, Stickan

MEM results $\quad 0.4\,T_c$



$\rho = \frac{\sigma}{\omega^2}$

comparison with $T=0$ masses and screening masses

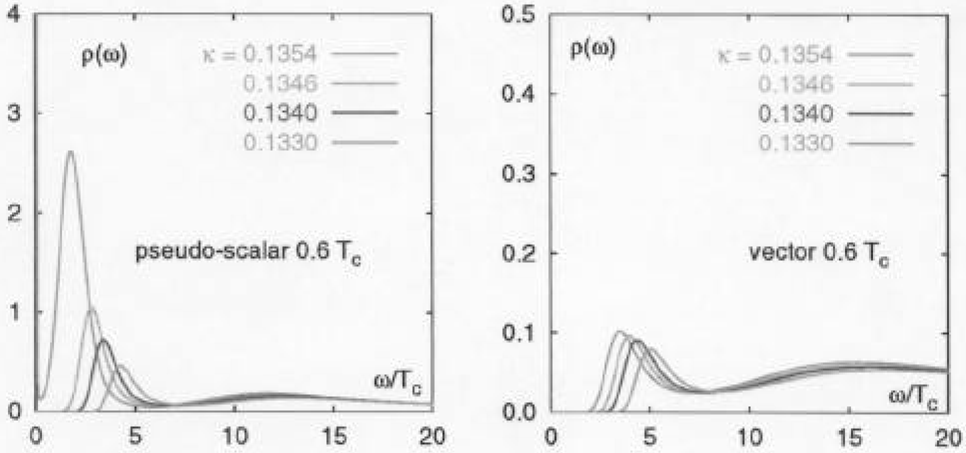


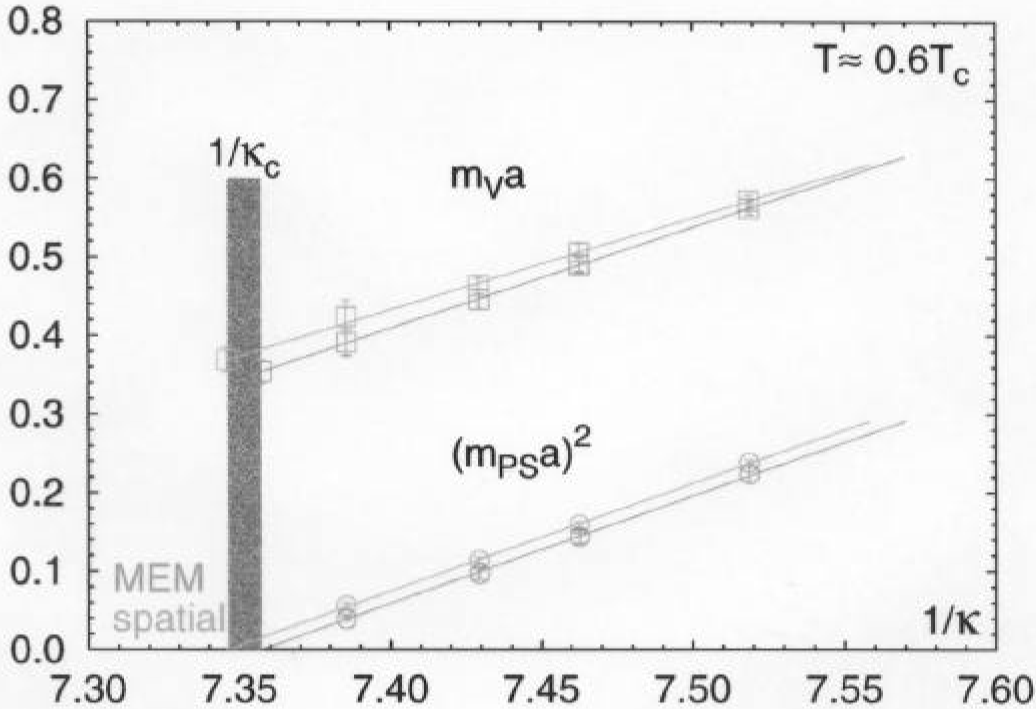
$\sim m_q$

[T=0: Göckeler et al.]

$m\,(\mathrm{MEM}) = m_{\mathrm{screen}}\,(\mathrm{spatial}) = m\,(T=0)$

the same at $0.6\,T_c$





$m\,(\mathrm{MEM}) \simeq m_{\mathrm{screen}}\,(\mathrm{spatial})$

action: Plaq + NP Clover

above T_c

* free quarks $G_V(\omega) = 2\,\frac{N_c}{8\pi^2}\,\omega^2 \tanh\left(\frac{\omega}{4T}\right)$
  massless

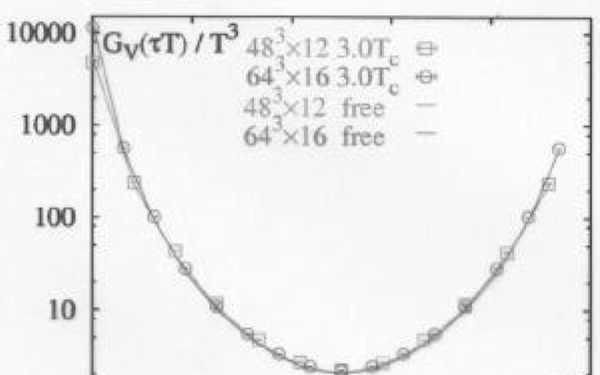
* note: $\tau = \frac{1}{2T}$ most sensitive to IR

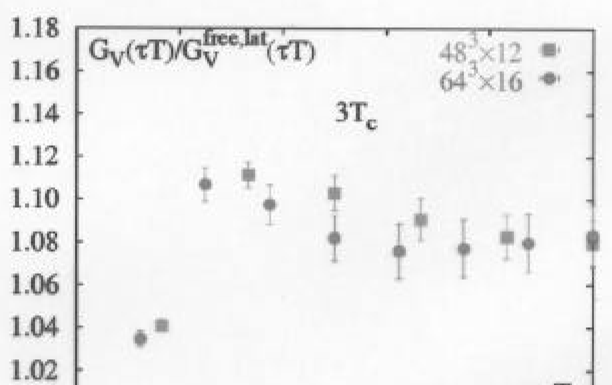
$$G\left(\tau = \frac{1}{2T}\right) = \int_0^\infty d\omega\, \frac{\sigma(\omega)}{\sinh(\omega/2T)}$$

- $\sigma_V(\omega) \sim \omega^{1-\varepsilon} \Rightarrow G_V\left(\tau = \frac{1}{2T}\right)$ diverging
  [as in HTL : $\varepsilon = 2$]

- free quarks $G_V^{free}\left(\tau = \frac{1}{2T}\right)/T^3 = 2$

- simple quasi-particle models $G_V < G_V^{free}$

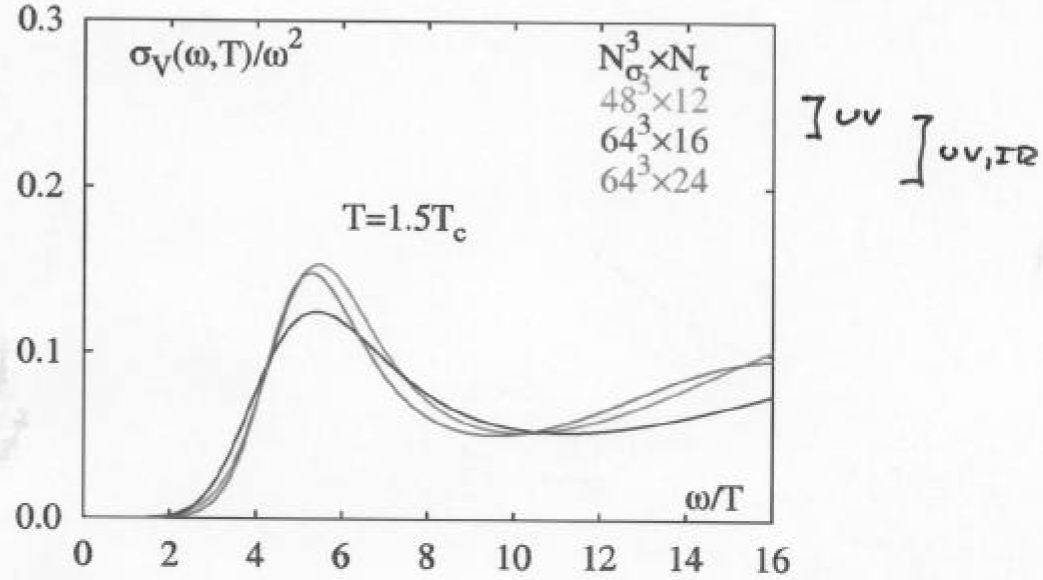


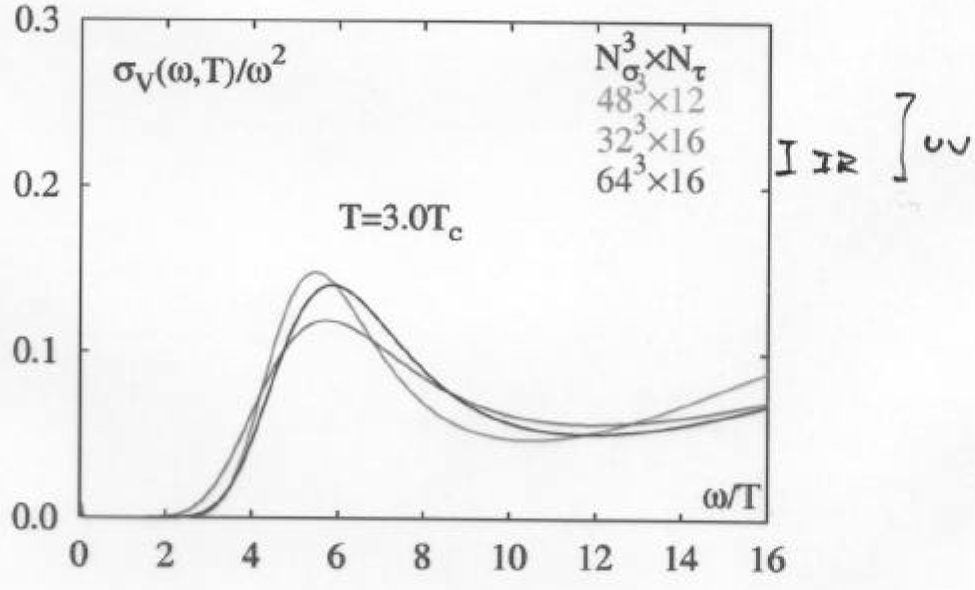


[$\kappa = \kappa_c(T=0) \;\hat{=}\; m_q \simeq 0$]

study sensitivity to

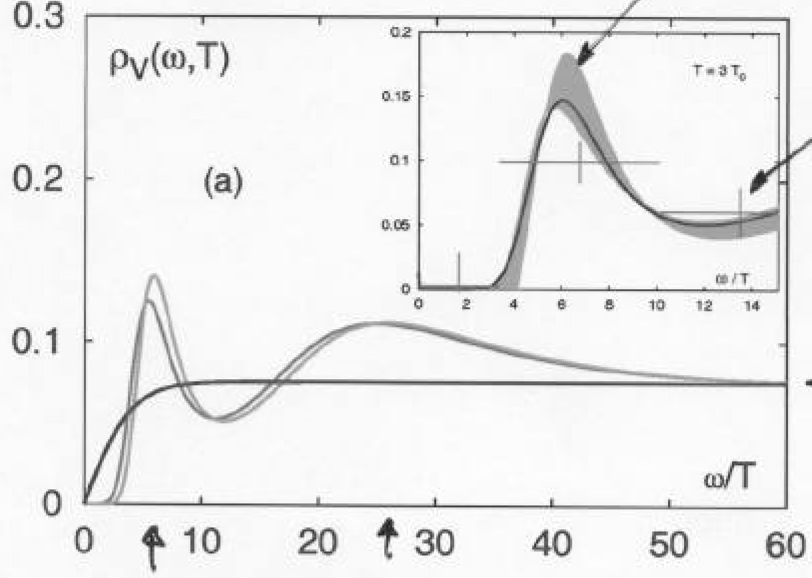
- UV effects $aT = \frac{1}{N_\tau}$
- IR effects $LT = \frac{N_\sigma}{N_\tau}$





$\varrho_V = \sigma_V / \omega^2$

statistical (jackknife) error



“reliability”

$\int d\omega\, d\omega'\, \mathrm{Cov}(\sigma(\omega)\sigma(\omega')$

$\tanh(\omega/4T)$

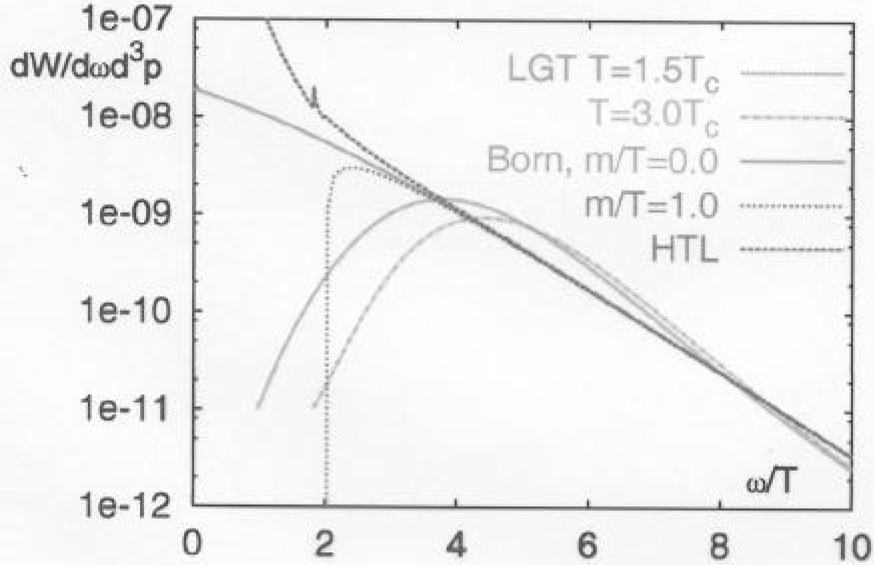
free quark cut

$64^3 \times 16$

broad structure scaling with T + “doublers”

$\Rightarrow$ dilepton rate:

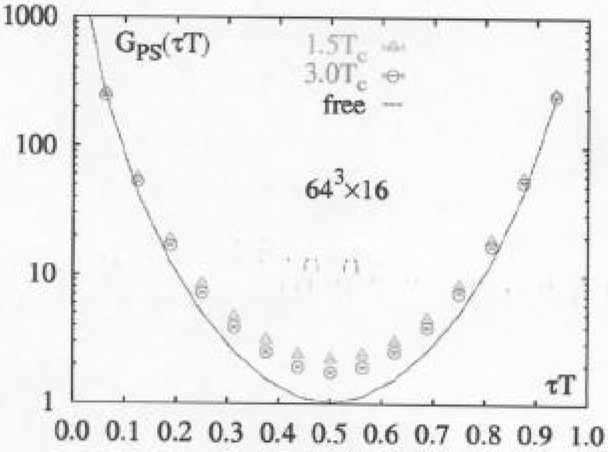
$$\left.\frac{dW}{d\omega\, d^3p}\right|_{p=0} \sim \frac{1}{\omega^2 (e^{\omega/T}-1)}\, \sigma_V(\omega, 0)$$

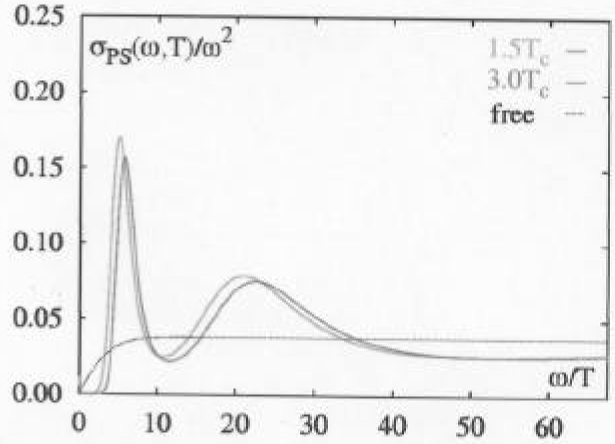


supports dilepton enhancement at $\omega \simeq 5T$

correlator & spectral density

for pseudoscalar channel





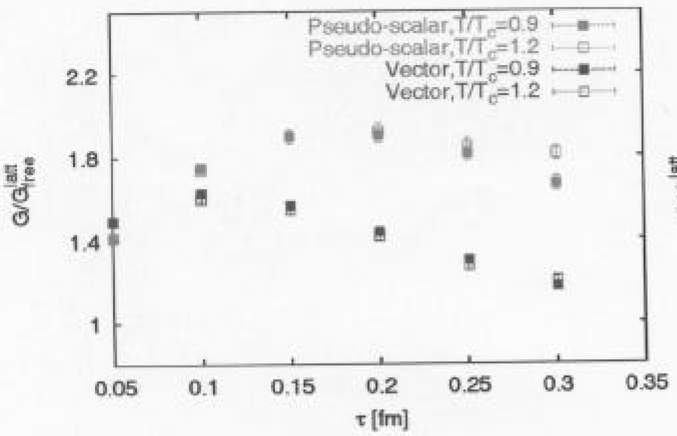
- sizeable differencies from
- slowly approach to free quark behavior

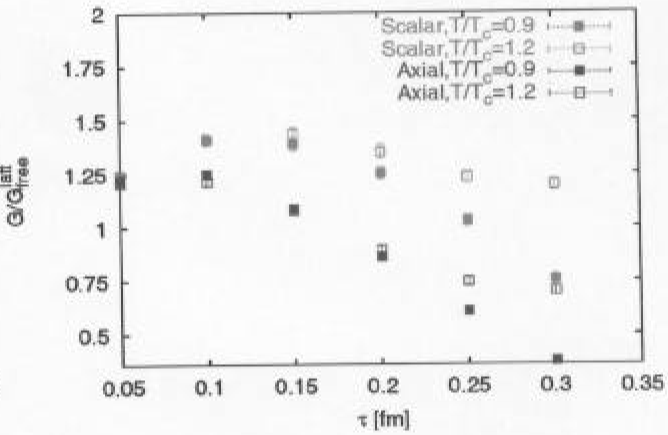
(close to) charmonium [Datta et al.] prelim.

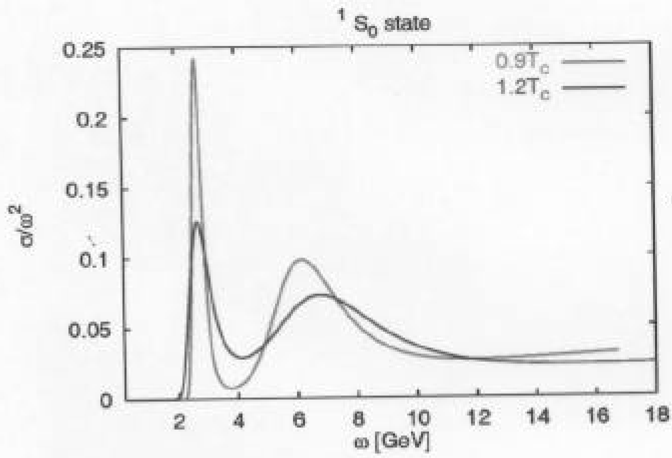
$\eta_c = 2.5$ GeV

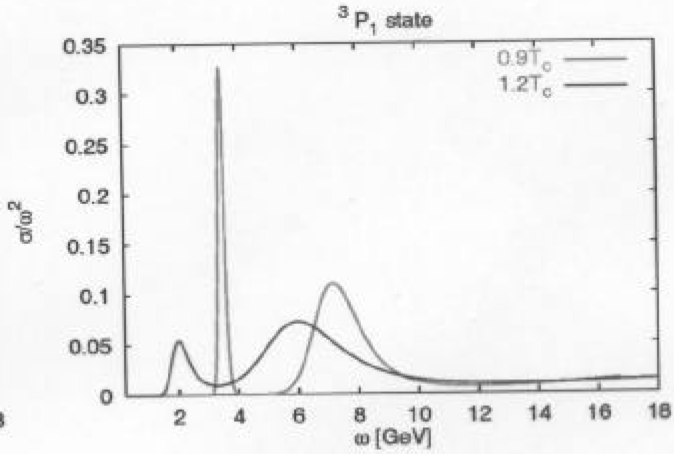
$48^3 \times 16$ $0.9\,T_c$

$48^3 \times 12$ $1.2\,T_c$









supports "melting" of $c\bar{c}$ in stages

<u>Summary</u>

- QCD (phase) transition weakly dependent on N_F, m_q

$N_F = 2$ $\quad T_c = 175(10)(\text{syst})$ MeV

3 $\quad 155(10)(\text{syst})$ MeV | chiral limit

- $Z(2)$ universality class for $N_F = 3$ endpoint established through finite size analysis

⇝ good starting point for incoming $N_F = 2+1$ results
 at present: 1st order unlikely

- find quantitative results for $\mu \neq 0$, small:
 at RHIC: differences from $\mu = 0$ small

- EOS: weakly dependent on N_F, m_q

$P/T^4 = \#\,\text{dof} * f(T/T_c)$

$\varepsilon(T_c) \simeq (6 \pm 2)\, T_c^4$ / no large discretization effec

- thermal masses:
 - no significant T-effects below T_c
 - non-perturbative structures at $T = \text{few} * T_c$