SU(N) gauge theories & the bosonic string

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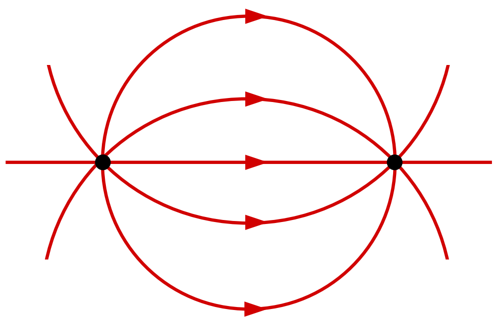
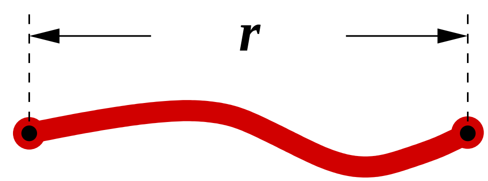
Flux distribution in the presence of static colour sources



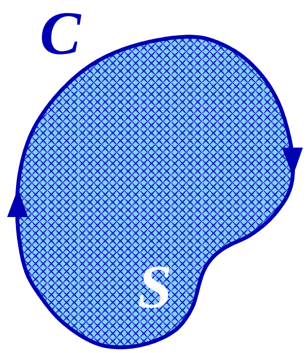
r < 0.1 fm



r >> 1 fm

DESY Theory Workshop, September 24–27, 2002

Low-energy effective string theory



$$\text{Wilson loop} \simeq \int_{\text{surfaces}} \mathrm{e}^{-\sigma A(S)}$$

Nambu 1979

* Expected to hold when C is large

* $\Rightarrow$ expansion in powers of $\sigma^{-1/2}$ about $S_{\min}$

M.L., Symanzik & Weisz 1980

Is this basically correct? If so . . .

- Exactly which string theory?
  - ◇ Alternative string actions ("rigid" string, etc.)
    Polyakov 1986, Savvidy & Savvidy 1993
  - ◇ String theories with fermionic modes
    Ramond 1971, Neveu & Schwartz 1971

- At which distances does string behaviour set in?

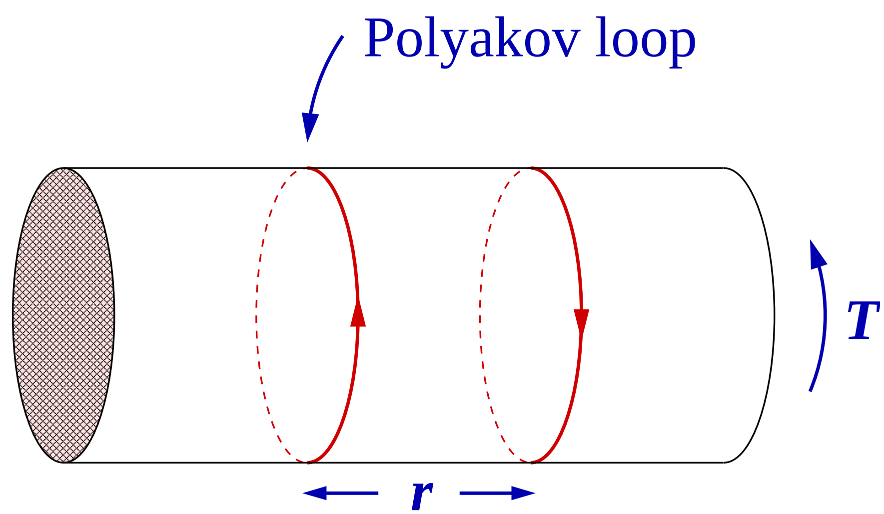
$\longrightarrow$ lattice gauge theory

Michael & Perantonis 1990, Juge, Kuti & Morningstar 1998ff

Caselle et al. 1997, Lucini & Teper 2001, Necco & Sommer 2002, M.L. & Weisz 2002

Polyakov loop correlation function

$$\langle P(r)^* P(0)\rangle = \sum_{n=0}^{\infty} w_n \mathrm{e}^{-E_n T}$$



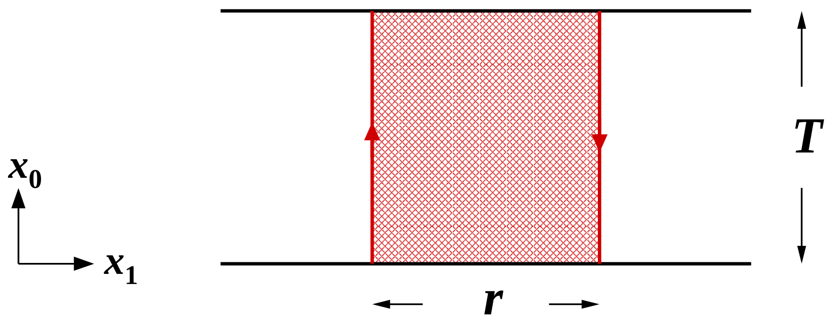
$E_0(r) \equiv V(r),\ w_0 = 1$

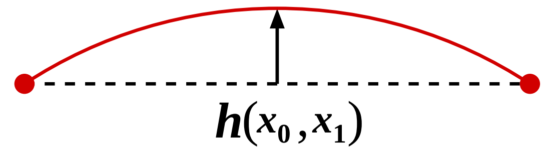
(static quark potential)

$E_n(r), n \geq 1,\ w_n \in \mathbb{Z}$

(excited states)

In the effective string theory





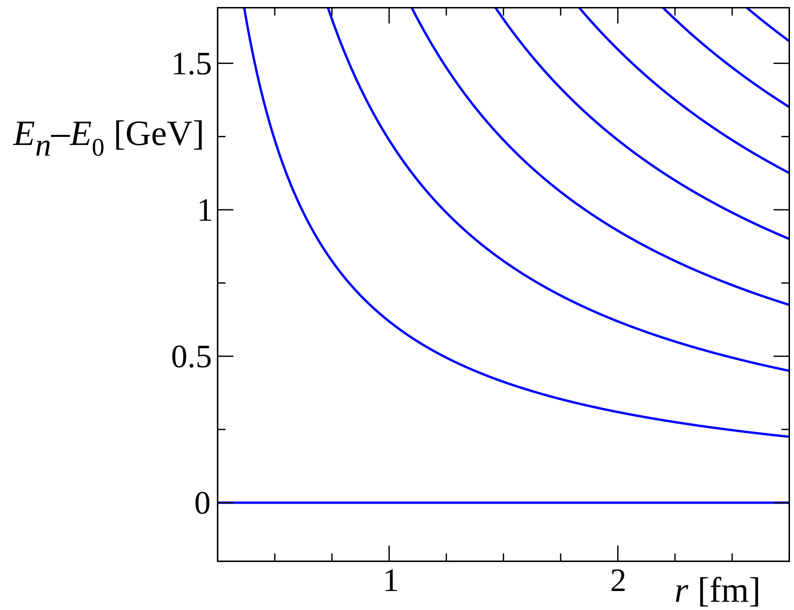
$$\langle P(r)^* P(0) \rangle = \mathrm{e}^{-\sigma r T - \mu T} \times \int_{\text{fluctuations } h} \mathrm{e}^{-S_{\text{eff}}}$$

$$S_{\text{eff}} = \int_0^T \int_0^r \mathrm{d}x_0 \mathrm{d}x_1 \left\{ \tfrac{1}{2} \left(\partial h\right)^2 + \tfrac{1}{4} c \left(\partial h\right)^2 \left(\partial h\right)^2 + \ldots \right\}$$

To leading order

$$E_0 = \sigma r + \mu - \frac{\pi}{24r}(d-2)\,, \qquad w_0 = 1 \qquad (d\text{: space-time dimension})$$

$$E_n = E_0 + \frac{n\pi}{r}, \qquad w_n \in \mathbb{Z}$$



Higher-order corrections

- Interactions are non-renormalizable
- $\Rightarrow$ expansion in powers of r^{-1}

$$E_0 = \sigma r + \mu - \frac{\pi}{24r}(d-2)\,(1 + b/r + \ldots)$$

$$E_1 = E_0 + \frac{\pi}{r}(1 + b/r + \ldots)$$

M.L. 1981, M.L. & Weisz 2002

Calculation of $\langle PP \rangle$ in LGT

The principal difficulties are

! The signal

$$\langle PP \rangle \propto \mathrm{e}^{-\sigma r T}$$

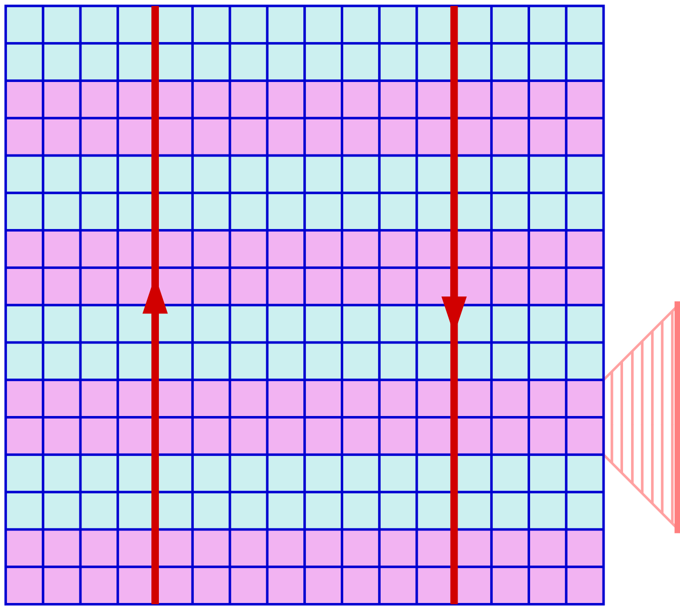
decreases exponentially ($\sim 10^{-25}$ at $a = 0.1\,\mathrm{fm}$, $rT = 5\,\mathrm{fm}^2$)

! The significance loss in

$$-\tfrac{1}{2} r^3 V''(r) = \frac{\pi}{24}(d-2) + \ldots$$

grows proportionally to $\sigma r^4 / a^2$

Multilevel algorithm



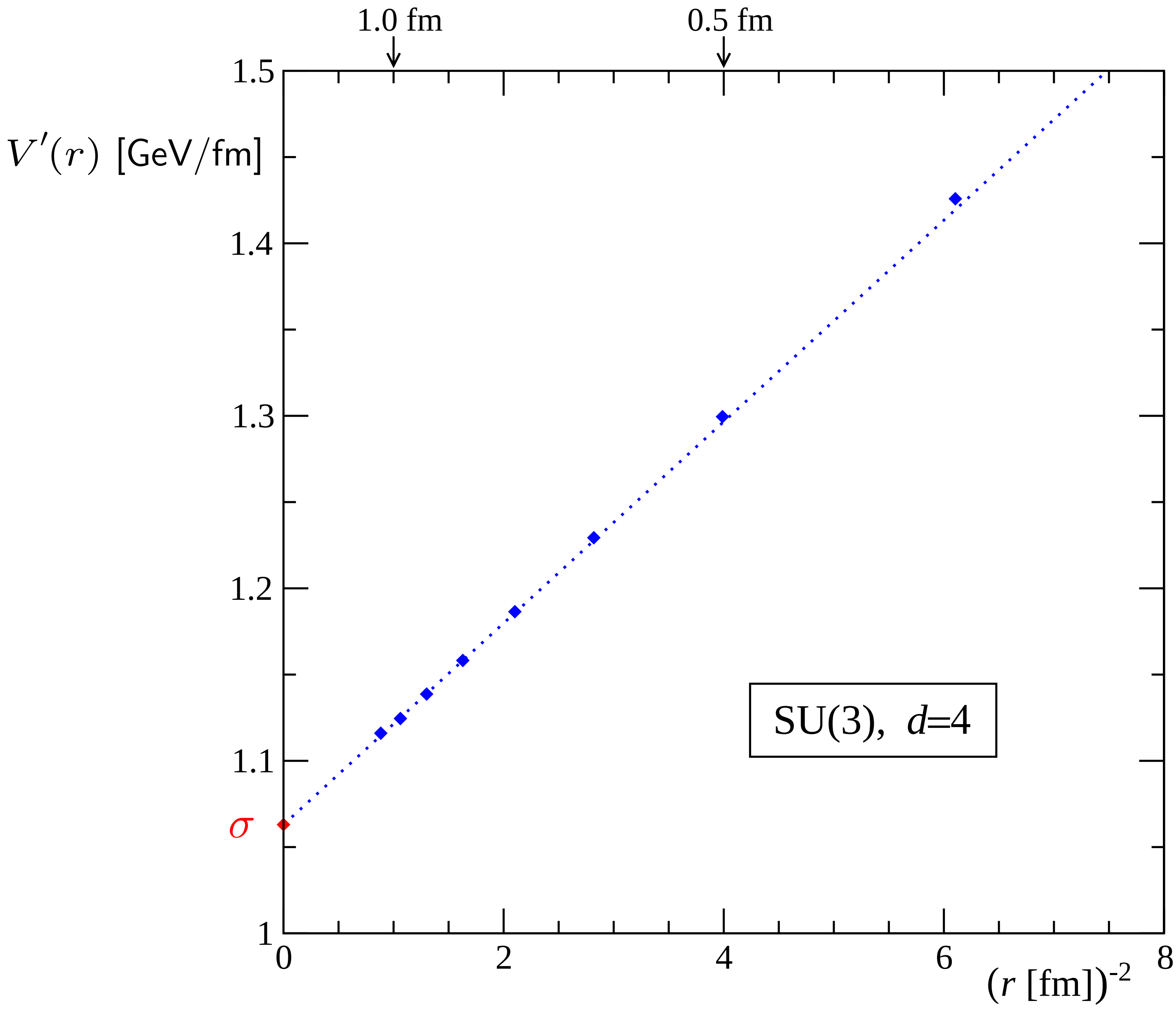
First average $U^* \otimes U$ here for fixed b.c. and then take product

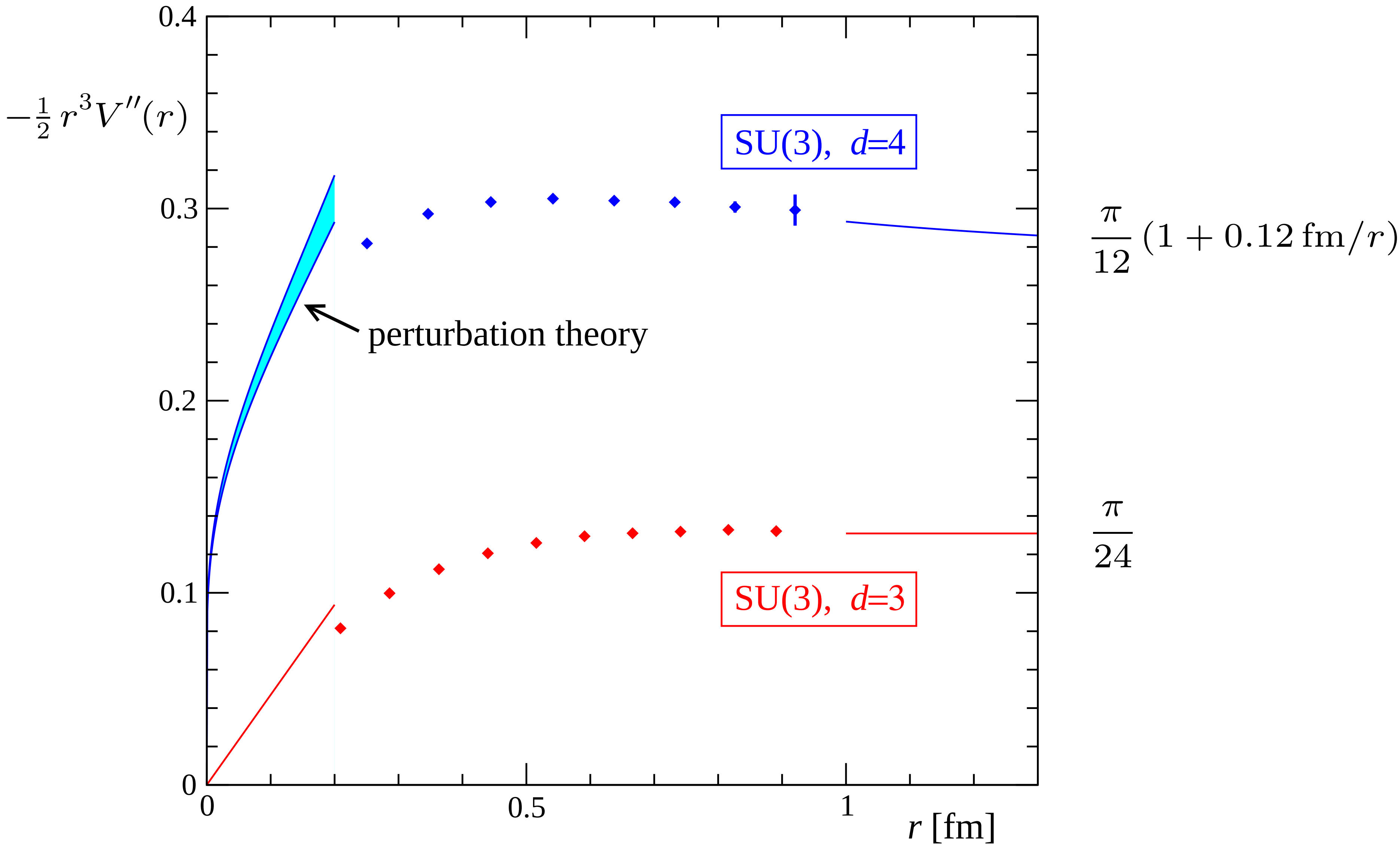
$$\langle P(r)^* P(0) \rangle = \langle \mathrm{tr}\{[U^* \otimes U]\,[U^* \otimes U] \ldots [U^* \otimes U]\} \rangle$$

$$\uparrow$$

$$\sim \mathrm{e}^{-2\sigma r a}$$

$\Rightarrow$ exponential reduction of the statistical errors! M.L. & Weisz 2001


1.0 fm
0.5 fm
$V'(r)$ [GeV/fm]
1.5
1.4
1.3
1.2
1.1
1
σ
SU(3), d=4
0
2
4
6
8
$(r\ [\mathrm{fm}])^{-2}$



M.L. & P. Weisz, JHEP 07 (2002) 049 [hep-lat/0207003]

Conclusions

Effective string theory confirmed

- Central charge $= d - 2$
  $\Rightarrow$ excludes additional fermionic string modes
- String behaviour in $V(r)$ sets in at about $0.5\,\mathrm{fm}$

Universality & higher-order terms?

- Other gauge groups
  Caselle et al. 1997, Majumdar 2002
- Low-lying energy values E_n
  Michael & Perantonis 1990, Juge, Kuti & Morningstar 1998ff

Fundamental "duality" QCD $\leftrightarrow$ string theory?

Polyakov 1981ff, Maldacena 1998, Witten 1998