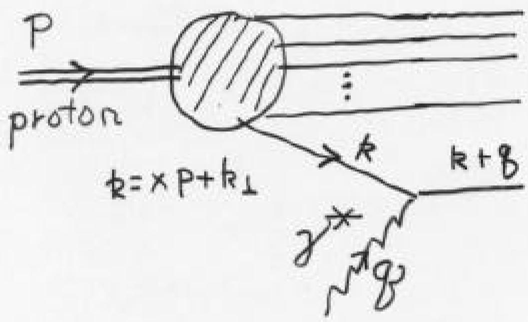
Saturation and High Field Strength QCD

1. Partons and deep inelastic scattering

Partons manifest in Bjorken frame and in Light-cone gauge in DIS



$$P = (\overset{t}{p + \tfrac{M^2}{2p}}, \overset{x}{0}, \overset{y}{0}, \overset{z}{p})$$

$$q = (q_0, q_\perp, 0)$$

$x = \dfrac{Q^2}{2p\cdot q}$ and Q^2 are invariant

$$q_0 = \frac{p\cdot q}{p} \xrightarrow[p\to\infty]{} 0 \qquad Q^2 = -q_\mu q_\mu \xrightarrow[p\to\infty]{} q_\perp^2$$

Virtual photon is absorbed by charged parton (quark)

(i) in transverse spatial region $\Delta x_\perp \sim 1/$

(ii) over time $\Delta t = \dfrac{1}{E_k + E_q - E_{k+q}} \sim 1/Q$

So, virtual photon makes localized and instantaneous measurement of quarks

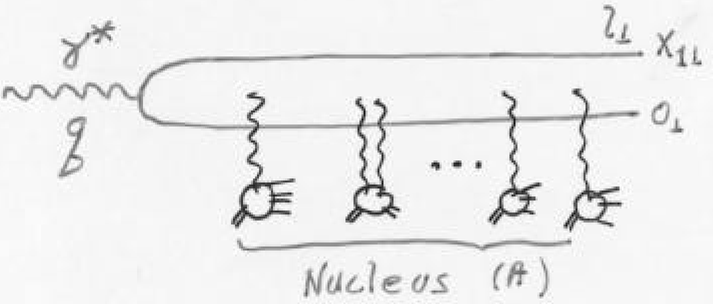
$$F_2(x,Q^2) = \nu W_2(x,Q^2) = \sum_F e_F^2 \left[x\, q_F(x,Q^2) + x\, \bar{q}_F(x,Q^2)\right]$$

9. Parton saturation

A. Quark saturation in large nucleus

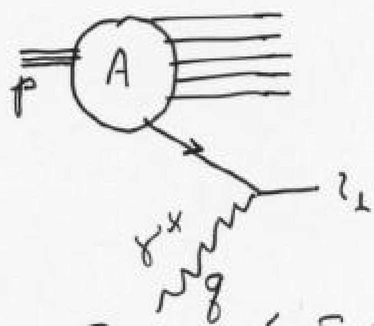
We wish to calculate $\frac{dx(q+\bar{q})}{d^2b\, d^2l_\perp}$, the phase space distribution of light sea quarks in the nuclear wavefunction at small x.

Use equality of



Dipole frame = Nuclear rest frame

and



Bjorken Frame
$q = (0,0,0,-q)$

in limit where there is no evolution in the nucleons in the nucleus.

$$\underset{\text{impact parameter}}{\frac{dx(q_F+\bar{q}_F)}{d^2b\, d^2l_\perp}} = \int d^2x_1\, d^2x_2\, e^{-il_\perp\cdot(x_{1\perp}-x_{2\perp})} \int_0^1 dz\, \overbrace{\frac{Q^2N_c}{32\pi^6}[z^2+(1-z)^2]\, \nabla_{x_1}K_0\left(\sqrt{Q^2x_{1\perp}^2 z(1-z)}\right)\cdot\nabla_{x_2}K_0\left(\sqrt{Q^2x_{2\perp}^2 \cdots}\right)}^{\frac{1}{8\pi^2}\psi_\gamma^*(x_{2\perp},z,Q)\,\psi_\gamma(x_{1\perp},z,Q)}$$

$$\cdot\left[1+e^{-(x_{1\perp}-x_{2\perp})^2\bar{Q}_s^2/4} - e^{-x_{1\perp}^2\bar{Q}_s^2/4} - e^{-x_{2\perp}^2\bar{Q}_s^2/4}\right]$$

quark sat. mom.

$$\bar{Q}_s^2 = \frac{C_F}{C_A}Q_s^2$$

gluon saturation momentum

$$Q_s^2 = \frac{4\pi^2\alpha N_c}{N_c^2-1}\, 2\sqrt{R^2-b^2}\, \rho \times G(x,\bar{Q}_s^2)$$

nuclear density; nuclear radius

$$\frac{dx(q_F+\bar{q}_F)}{d^2b\, d^2l_\perp} = \int \frac{d^2x_1 d^2x_2}{8\pi^2} \int_0^1 dz\, e^{-i(x_{1\perp}-x_{2\perp})\cdot l_\perp} \Psi_\gamma^*(x_2)\Psi_\gamma(x_1) \Big[\underset{1}{1} + \underset{2}{e^{-(x_{1\perp}-x_{2\perp})^2 Q_s^2/4}} \underset{3}{- e^{-x_{1\perp}^2 Q_s^2/4}} \underset{4}{- e^{-x_{2\perp}^2 Q}}$$

$$S(x_\perp, b, x) = e^{-x_\perp^2 Q_s^2(b,x)/4}$$

When $l_\perp^2/Q_s^2 \ll 1$ $\quad$ $3 = 4 \approx 0$

$$\frac{dx(q_F+\bar{q}_F)}{d^2b\, d^2l_\perp} = \frac{N_c}{2\pi^4}$$

while the occupation number is

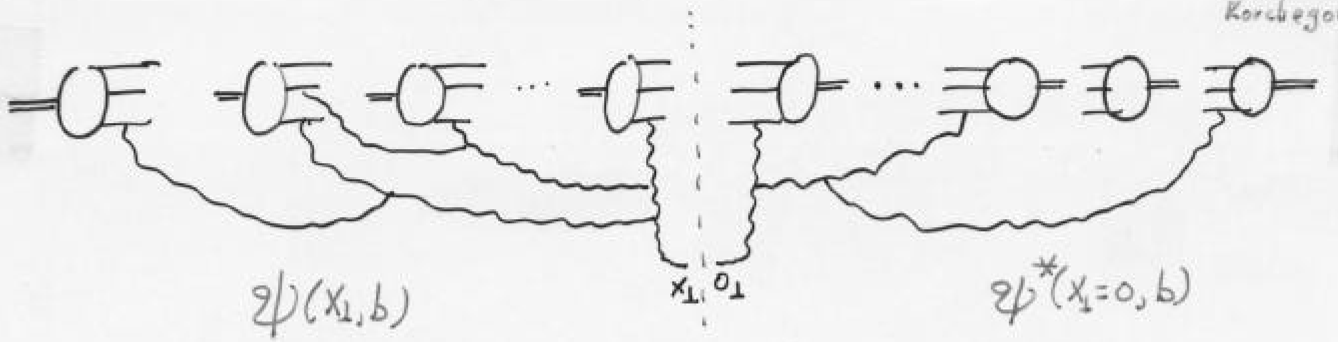
$$f_q \simeq \frac{(2\pi)^3}{\underbrace{2}_{\text{particle-antiparticle}} \cdot \underbrace{2}_{\text{spin}} \cdot N_c} \underbrace{\frac{dx(q_F+\bar{q}_F)}{d^2b\, d^2l_\perp}}_{\frac{dN_{q+\bar{q}}}{dy\, d^2b\, d^2l_\perp}} = \frac{1}{\pi} \qquad dy \approx db_3\, dl_3$$

occupation number $= O(1)$ and unitarity limit are the same phenomenon!

B. Gluon saturation (McLerran-Venugopalan model)

Quark saturation occurs at the one Fermion loop level and thus is purely quantum. Gluon saturation occurs already at the classical level.

Korchegov



Bjorken frame

One Finds

$$\frac{d^2 xG}{d^2b\, d^2l_\perp} = \frac{N_c^2-1}{4\pi^4 x N_c}\int d^2x_\perp\, e^{-il_\perp\cdot x_\perp}\, \frac{\left(1-e^{-l_\perp^2 Q_s^2/4}\right)}{x_\perp^2}$$

McLerran et al
Kovchegov, A.M.

$$f_g = \frac{(2\pi)^3}{2(N_c^2-1)}\frac{dxG}{d^2b\, d^2l_\perp} = \frac{1}{\alpha N_c}\underbrace{\int \frac{d^2x_\perp}{\pi x_\perp^2}\left(1-e^{-l_\perp^2 Q_s^2/4}\right)}$$

$$= O(1) \qquad l_\perp/Q_s \approx 1$$

$$= \ln Q_s^2/l_\perp^2 \qquad l_\perp^2/Q_s^2 \ll 1$$

Higher order corrections can change the constants but $f_g = \frac{c}{\alpha N_c}$, $l_\perp/Q_s \sim 1$ and $f_g \sim \frac{c'}{\alpha N_c}\ln Q_s^2/l_\perp^2$, $l_\perp^2/Q_s^2 \ll 1$ should be general.

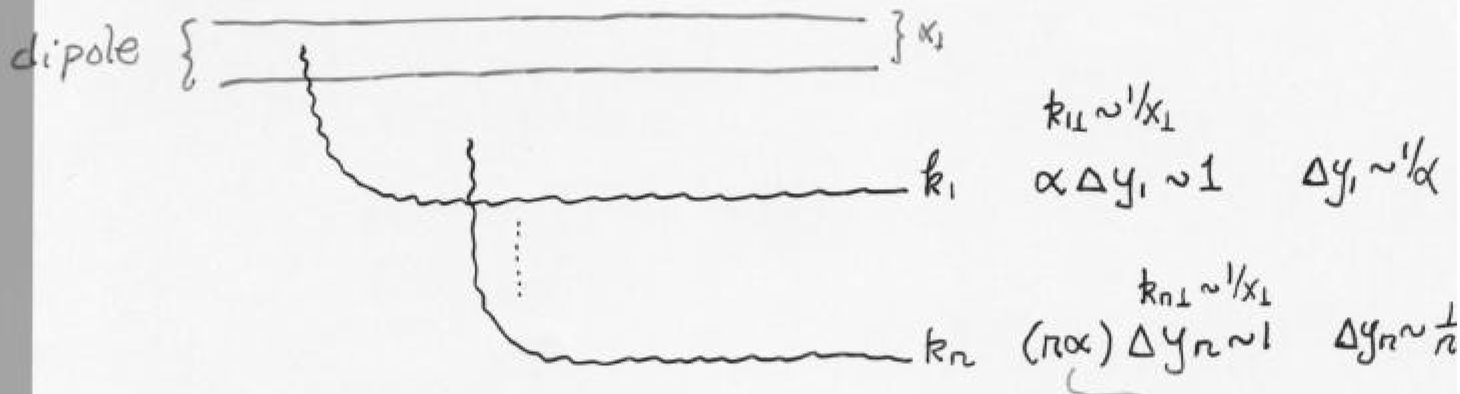
3. How to get large gluon occupation numbers

For large nucleus, in additive approximation

$$\frac{dxG^{additive}}{d^2b\, d^2l_\perp} = 2\sqrt{R^2-b^2}\,\rho\,\frac{dxG}{d^2l_\perp}$$ becomes large because

$R\rho \sim \frac{A}{R^2}$ grows like $A^{1/3}$.

For hadron small-x (BFKL) evolution is what gives large gluon densities. The idea is very simple. Take high energy "valence" dipole as example.



charge random on scale $x_\perp$

After N emissions (in wavefunction)

$$Y_N = \Delta y_1 + \Delta y_2 \cdots \Delta y_N = \frac{c}{\alpha} \ln N + c'$$

or

$$N(Y) \sim e^{\frac{\alpha}{c} Y}$$

More precise calculation gives BFKL (Balitsky, Fadin, Kuraev, Lipatov)

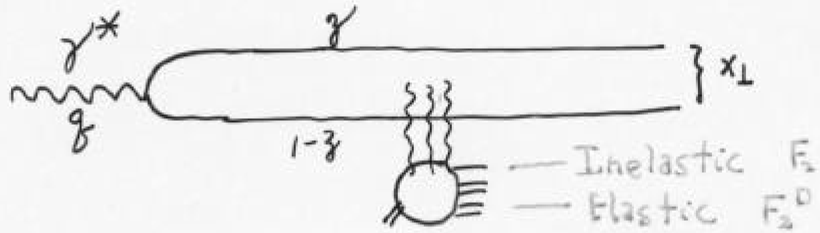
$$N(Y) = \text{const} \frac{e^{\frac{4 \ln 2}{\pi} \alpha N_c Y}}{\sqrt{\alpha Y}}$$

Non linear evolution becomes important when occupation numbers $\sim 1/\alpha N_c$ are reached

4. Phenomenology of saturation at HERA

A. Golec-Biernat Wüsthoff model

$$F_2 = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2x_\perp \int_0^1 dz \sum_f e_f^2 |\psi_{Tf}(x_\perp, z, Q)|^2 \; \sigma_0 \left(1 - e^{-x_\perp^2 \bar{Q}_s^2/4}\right)$$



$$F_2^D = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2x_\perp \int_0^1 dz \sum_f e_f^2 |\psi_{Tf}(x_\perp, z, Q)|^2 \; \frac{1}{2}\sigma_0 \left(1 - e^{-x_\perp^2 \bar{Q}_s^2/4}\right)^2 + q\bar{q}g$$

Impact parameter averaged picture

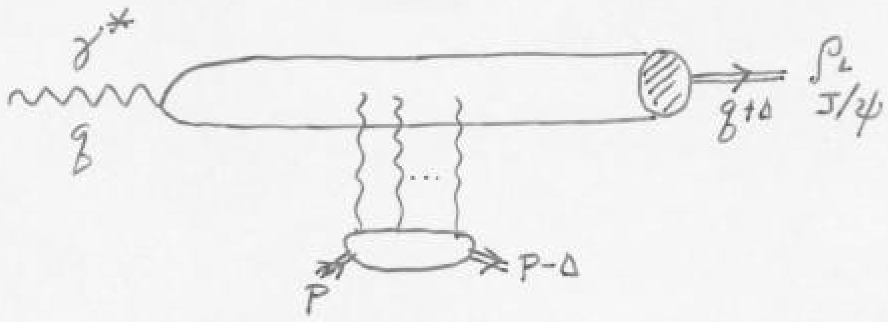
$\sigma_0 = 23$ mb $\qquad \bar{Q}_s^2 = \left(\frac{x_0}{x}\right)^\lambda$ in GeV^2 $\qquad x_0 = 3\times 10^{-4}$, $\lambda = 0.3$

3 parameters

$\bar{Q}_s^2 = 4/9\, Q_s^2 \qquad Q_s^2 = 2\text{-}4$ GeV^2 in HERA regime

G-B W model gives impressive fits to HERA data in small x and moderate Q^2 regime

B. Diffractive vector meson production



$t = -\Delta^2$

Munier, Stasto

$$S(b, x_\perp, x) = 1 - \frac{1}{2\pi^{3/2} N} \int d^2\Delta\, e^{-i\Delta\cdot b} \sqrt{\frac{d\sigma}{dt}}$$

$N = (q/\rho_L,\ q/\gamma^*)$

Write $\quad S(x_\perp, b, x) = e^{-x_\perp^2 \bar{Q}_s^2(x,b)/4}$ $\quad$ ok if gluon densities high enough

$$\bar{Q}_s^2 = \frac{2\pi^2\alpha}{N_c} \frac{dxG}{d^2b}$$

At $\quad x = 5\times 10^{-4} \quad Q^2 = 3.5\ \mathrm{GeV}^2 \quad x_\perp \approx 0.4\ \mathrm{Fm}$ $\quad$ Find

$$\bar{Q}_s^2(b=0) \approx 0.5\ \mathrm{GeV}^2 \Rightarrow Q_s^2(b=0) \approx 1.1\ \mathrm{GeV}^2$$

(Smaller than G-B W by Factor of 2)?

$$\frac{dxG}{d^2b} \approx 4.8/\mathrm{fm}^2 \qquad xG \approx 6$$

5. RHIC Phenomenology

Consider a zero impact parameter (central

ion-ion collision.

A. Just before and during the collision

Start with McLerran-Venugopalan wavefunction.

Krasnitz, Nara and Venugopalan solve discretized Yang-Mills equations as the nuclei overlap. Gluon appear to be freed on time scale $\tau \sim 3/Q_s$.

Wavefunction (just before collision)

$$\frac{dN_g}{dy\, d^2k_\perp d^2x_\perp} = \frac{N_c^2-1}{4\pi^3} \frac{1}{\alpha N_c} \int_1^\infty \frac{dt}{t}\, e^{-t k_\perp^2/Q_s^2}$$

$$f_g = \frac{1}{\alpha N_c} \int_1^\infty \frac{dt}{t}\, e^{-t k_\perp^2/Q_s^2}$$

$\langle k_\perp \rangle \simeq 0.6\, Q_s$

At time gluons are f

Krasnitz, Nara, Venugop

$$\frac{dN_g}{dy\, d^2x_\perp d^2k_\perp} = \frac{N_c^2-1}{4\pi^3} \frac{1}{\alpha N_c} \frac{0.11}{e^{\sqrt{k_\perp^2+m^2}/T_{eff}}}$$

$T_{eff} = 1.07\, Q_s$

$m = 0.08\, Q_s$

$$f_g \approx \frac{1}{\alpha N_c} \frac{0.11}{e^{k_\perp/T_{eff}}-1}$$

$\langle k_\perp \rangle \simeq 1.5\, Q_s$

C = Freeing Fraction $\approx 1/2$

a lot of transverse momentum created during the collision

Occupation numbers come out rather small, so that classical evolution is not reliable after gluons have been freed. (while being freed?)

B. Classical field theory vs kinetic theory

When $f_g \sim 1/\alpha$ kinetic theory does not apply, and classical field theory is the right approach. Thus in the initial stages of the collision when gluons are being freed, classical Yang-Mills theory is the proper dynamics.

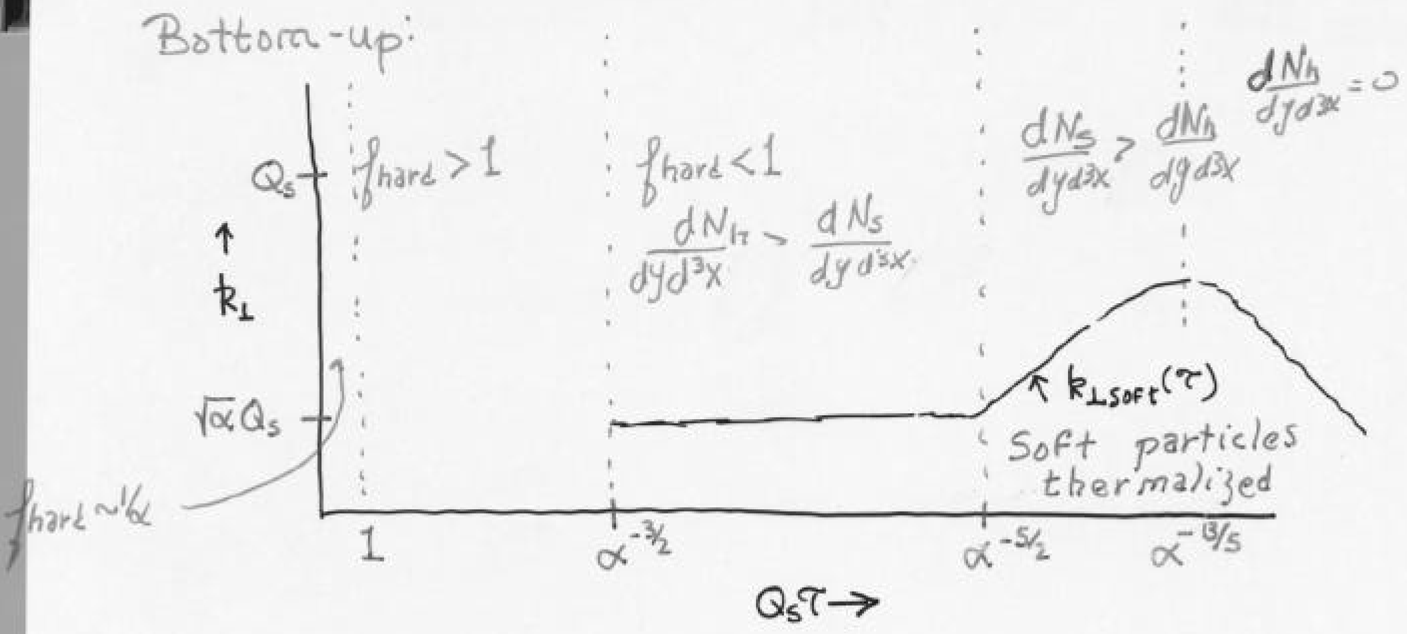
When $1 < f < 1/\alpha$ classical field theory and the Boltzmann equation are equivalent

When $f \lesssim 1$ classical field theory, which only keeps the f_g in statistical factors $(1+f_g)$ is an underestimate of the interaction strength while Boltzmann is systematic.

"Bottom-up" picture uses Boltzmann equation, including elastic and inelastic collision terms, after gluons are produced.

C. "Bottom-up" picture; Kharzeev, Levin, Nardi picture

(Baier, Mueller, Schiff, Son (BMSS)



$Q_s\tau \sim 1$ gluons having momentum $\sim Q_s$ become free.

$Q_s\tau < \alpha^{-5/2}$ dominant collision term is inelastic. BMSS include branching in the medi[um]

Soft particle density grows

$Q_s\tau > \alpha^{-5/2}$ soft particles equilibrate. Energy transferred from hard to soft by coherent branching of hard particles

$Q_s\tau \sim \alpha^{-13/5}$ all energy transferred to soft equilibrated gluons.

Probably overly optimistic to believe in these separate regimes at RHIC

Consider central collision

reasonably general. OK for all M-V based initial conditions

$$\frac{dN_g^{in}}{dy\, d^2x_\perp} = c\, \underbrace{2\sqrt{R^2-b^2}\,\rho\, xG}_{\frac{dN_g}{dy\, d^2x_\perp} \text{ in wavefunction}}$$

gluon freeing factor

Go to later times and charged hadrons

$$\left\langle \frac{2}{N_{part}} \frac{dN_{ch}}{dy} \right\rangle = R \cdot c \cdot \overset{\text{charged/total}}{\frac{2}{3}}\; xG(x, \langle Q_s^2 \rangle) \approx 3.8 \qquad \sqrt{s} = 200 \text{ RHIC}$$

inelasticity

Kharzeev-Levin-Nardi: $R = 1$ $\quad \frac{2}{3} c\, xG \approx 3.8$

Krasnitz-Nara-Venugopalan: $c \approx 1/2$ (calculated) take $R \approx 2\text{-}3$

Bottom-up: take $R \approx 3,\ c \approx 1,\ xG \approx 2$

$\Downarrow$

T at equilibration: $T \approx 230$ MeV

time at equilibration: $\tau \approx 3.6$ fm

KLN, KNV, Bottom-up based on same ideas, but details different