THE CONDENSED MATTER PHYSICS OF QCD

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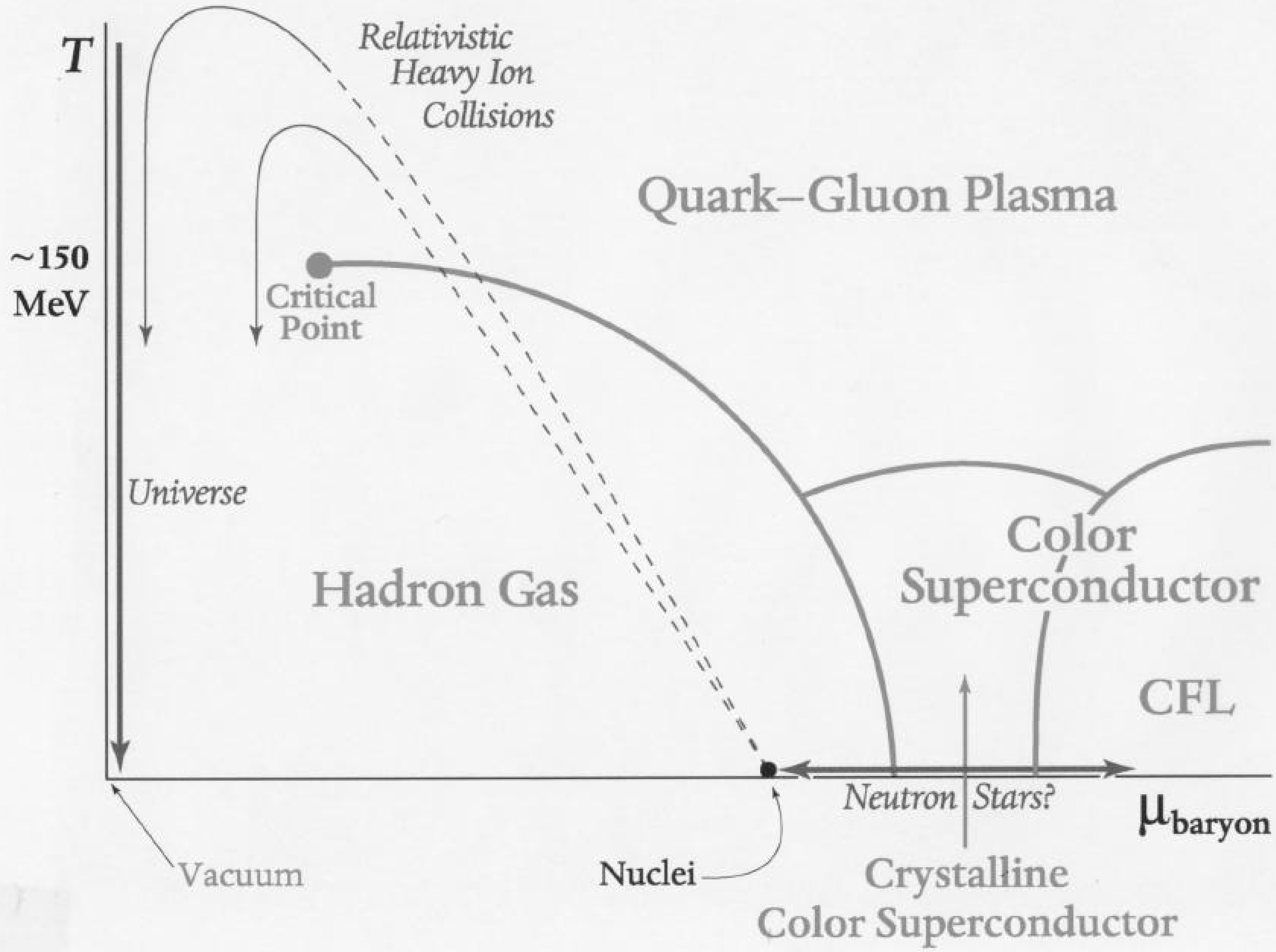
DESY THEORY WORKSHOP ON QCD

Hamburg, Sept 27, 2002

EXPLORING *the* PHASES *of* QCD



LARGE μ; SMALL T

whereas at high T entropy wins
→ quark-gluon plasma with symmetr
of the QCD Lagrangian manifest...

At large μ with small T we find quark matter with new patterns of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity

⋮

How can we use astrophysical observations of compact stars to determine the QCD phase diagram?

THE DIFFICULTY WITH DENSITY

Why are we still asking basic questions about QCD at high μ, low T, like "what is symmetry of ground state?"

NO LATTICE CALCULATIONS

$\mu \neq 0 \rightarrow$ complex Euclidean action
$\rightarrow$ sign problem that makes difficult of standard Monte Carlo $\sim e^{V}$.

Equally nasty sign problems can be solved in simpler systems. Chandrasekharan Wiese

Sign problem may also be evaded:

- at small V, small μ/T Fodor Katz; Hands Karsch et al
- calculate at Im μ; continue observabl[e] works at $\mu/T < \pi/3$. V can be large
de Forcrand Philipsen; d'Elia Lombardo
- may be used to locate critical point.
- modify the theory. (color superconduct[ivity] studied on lattice for NJL & QCD w N_c
Hands et al Kogut et al

NO EVASION POSSIBLE FOR QCD at $\mu \gg T$

- use smallness of g at $\mu \rightarrow \infty$
- use models at accessible μ.

WHY COLOR SUPERCONDUCTIVITY

Large $\mu \to$ quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom $\to$ weak interactions between quarks at Fermi surface.

BUT any attractive interaction, no matter how weak, $\to$
COOPER PAIRS ; $\langle qq \rangle$

One gluon exchange (& instanton interaction) attractive in color $\bar{3}$.
(no need to resort to phonons; $\therefore$ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)

$\langle qq \rangle$, ie Cooper pairs of quarks,
$\Rightarrow$ - electric & color currents supercond[uct]
- mass for photon & (some) gluons
- Meissner effects. (Magnetic & color magnetic fields excluded

GAP AND T_c

Much work (that I will not review)

suggests that @ $\mu \sim 500$ MeV [$\sim 10 \times$ nuclear density]

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ THIS is high T_c S.C

Two classes of methods $\sim$ agree:

i) models normalized to $\mu = 0$ physics

(Alford, K.R, Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetz, ...

ii) weak-coupling QCD calculations, valid for $\mu \rightarrow \infty$; $g \rightarrow 0$. (Quantitatively, valid for $g \lesssim 1$ which means $\mu \gtrsim 10^8$ MeV K.R, Shuster

$$\frac{\Delta}{\mu} \sim \underbrace{256 \pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2}}_{\text{Schaefer, Wilczek; Pisarski, Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz; Brown, Liu, Ren; Beane, Bedaque, Savage; K.R, Shuster; Rischke, Wang; ....}} \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right.}_{\text{Son}}$$

$\Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons:

$$\rightarrow 1 = g^2 \underbrace{\ln\frac{\Delta}{\mu}}_{\text{BCS}} \underbrace{\ln\frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

COLOR-FLAVOR LOCKED QUARK MATTER

Alford, KR, Wilczek; Schaefer, Wilczek; ....

- occurs in quark matter with $m_s = m_{u,d}$
- all 9 quarks pair, and ∴ are gapped
- superfluid
- chiral symmetry spontaneously broken
  → pseudo Goldstone mesons
- unbroken gauged $U(1)$ → massless photon
- transparent insulator (neutral without electrons)
  - index of refraction and refraction & reflection coefficients known   Litim, Manuel; Manuel, KR
- occurs in Nature's QCD phase diagram wherever $\mu > m_s^2/4\Delta$,   Alford, Berges, KR; Schaefer, Wilczek; KR, Wilczek; Alford, KR, Reddy, Wilczek
  possibly augmented by K^0 CONDENSATE   Bedaque, Schaefer; Kaplan, Reddy
- m_s here is poorly known, μ-dependent effective mass. Δ uncertain also.
- Could be single nuclear matter → CFL transition → sharp interface with charged "inversion layers"   Alford, KR, Reddy, Wilczek
- OR less symmetrically paired quark matter may intervene, between nuclear and CFL matter. To this we now turn.....

$N_f = 3$: COLOR-FLAVOR LOCKING

Alfor
KR
Wilc

Condensate pairs quarks of all colors & flavors:

$\langle u_L d_L - d_L u_L - u_L d_L + d_L u_L + d_L s_L - d_L s_L - s_L d_L + s_L d_L + s_L u_L - u_L s_L - s_L u_L + u_L$

$\neq 0$

Locks $SU(3)_{color}$ to $SU(3)_L$.

ie $SU(3)_{color+L}$ is a symmetry.

Similarly, condensate of R-quark

locks $SU(3)_{color}$ to $SU(3)_R$.

Result:

$SU(3)_{color+L+R}$ unbroken. $\leftrightarrow$ use th

Chiral symmetry broken. "EM" + "isospin".

$U(1)_{\tilde{Q}}$ unbroken. $\leftrightarrow$ classify excito

All other gauge symmetries broken

$U(1)_B$ broken. $\therefore$ superfluid.

$\tilde{Q}$ IN THE CFL PHASE

$$\langle q_a^\alpha q_b^\beta \rangle \sim \Delta_1 \delta_a^\alpha \delta_b^\beta + \Delta_2 \delta_b^\alpha \delta_a^\beta$$

$$\tilde{Q} = \underbrace{Q_{EM}}_{\begin{cases} \frac{2}{3} \text{ for } u \\ -\frac{1}{3} \text{ for } d \\ -\frac{1}{3} \text{ for } s \end{cases}} + \underbrace{\frac{1}{\sqrt{3}} T_8}_{\begin{cases} -\frac{2}{3} \text{ for } b \\ \frac{1}{3} \text{ for } r \\ \frac{1}{3} \text{ for } g \end{cases}}$$

$\tilde{Q}$ charges of quarks:

u	+1
u	+1
u	0
d	0
d	0
d	−1
s	0
s	0
s	−1

Similarly, $\tilde{Q}$ charges of gluons all integer-valued.
Also for $\tilde{Q}$ charges of Goldstone bosons.

One finds...

ordinary photon

gluon

$$A_\mu^{\tilde{Q}} = \cos\theta \, A_\mu + \sin\theta \, G_\mu^8$$

$$A_\mu^X = -\sin\theta \, A_\mu + \cos\theta \, G_\mu^8$$

where

$A_\mu^{\tilde{Q}}$ is massless. This $\tilde{Q}$-photon satisfies Maxwell's equations with dielectric const $\epsilon \neq \epsilon_0$. (medium is polarizable.) To the $\tilde{Q}$-photon, CFL matter is a transparent dielectric medium

A_μ^X is massive. Like Z-boson.

θ is analogue of Weinberg angle

$$\sin\theta = \frac{e/\sqrt{3}}{\sqrt{g^2 + e^2/3}} \simeq \frac{e}{g\sqrt{3}} \sim \frac{1}{20}$$

$\tilde{Q}$-photon is "mostly ordinary photon"

Alford KR Wilczek; Alford Berges KR

A TRANSPARENT INSULATOR

What can the $\tilde{Q}$-photon scatter off?

- the CFL condensate itself is $\tilde{Q}$-neutral.
- once you include nonzero quark masses, all excitations with $\tilde{Q} \neq 0$ are massive.
- $\therefore$ for $T \ll M_{\text{lightest excitation with } \tilde{Q} \neq 0}$

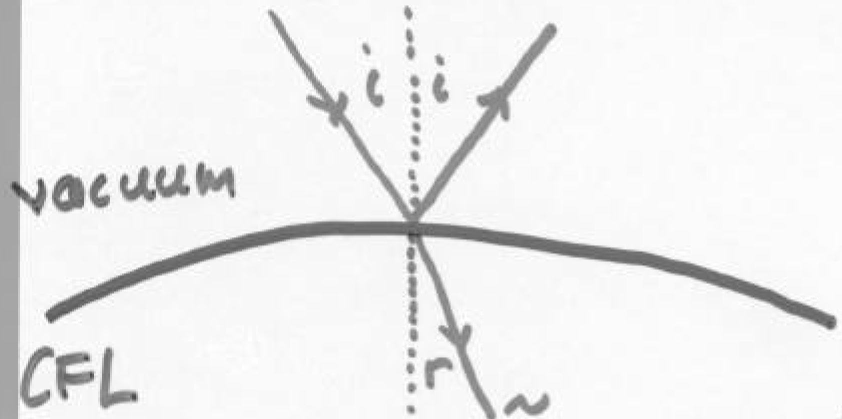
likely a kaon

the CFL phase is transparent to the $\tilde{Q}$-photon. It is a $\tilde{Q}$-insulator, with some index of refraction $n_{CFL} \neq 1$.

ILLUMINATING CFL QUARK MATTE

Manuel, KR

Suppose (just for fun) you had a quark s
in CFL phase, and shone light on it:



$\tilde{Q}$-light: $A_\mu^{\tilde{Q}} = \cos\theta \, A_\mu^{EM} + \sin\theta$

$$n_{CFL} = 1 + \frac{4\alpha}{9\pi}\frac{\mu^2}{\Delta^2}\cos^2\theta \quad \text{(Litim, Manuel)}$$

Find: $\frac{\sin i}{\sin r} = n_{CFL}$

Explicit expressions in terms of n, θ
for reflection & refraction coefficien
for light of either possible
polarization.

Its fun to think of 10 km
lenses in space, but more likely
applicable version of this is
in the static limit:

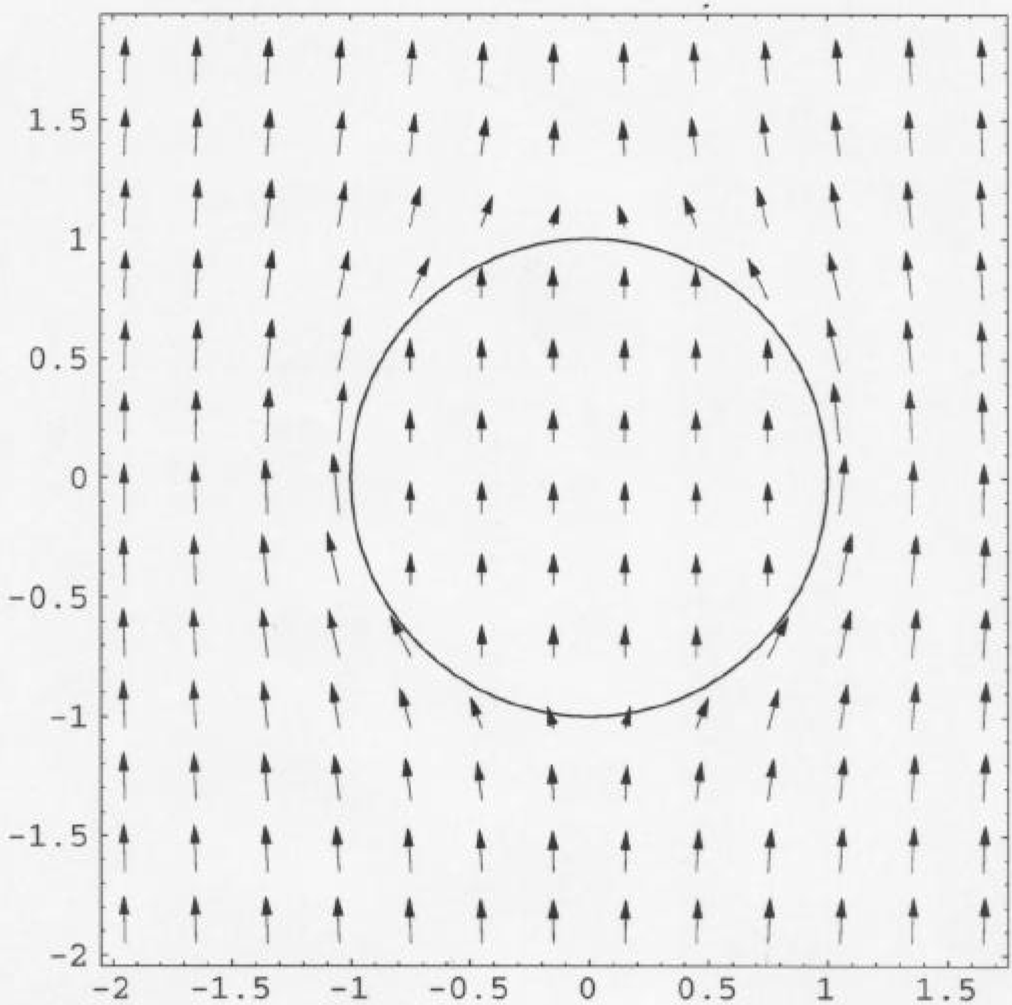
Suppose core of a neutron
star is CFL. How does it respond
to the large static $\vec{B}$ it finds
itself in?

ANSWER: (Alford, Berges, KR)
(found via magnetic b.c.'s ...)

Partial Meissner effect...

Magnetic field solution (sharp boundary)

Stitching together the inside and outside solutions, we find the solution. For $\cos\alpha_0 = 0.5$

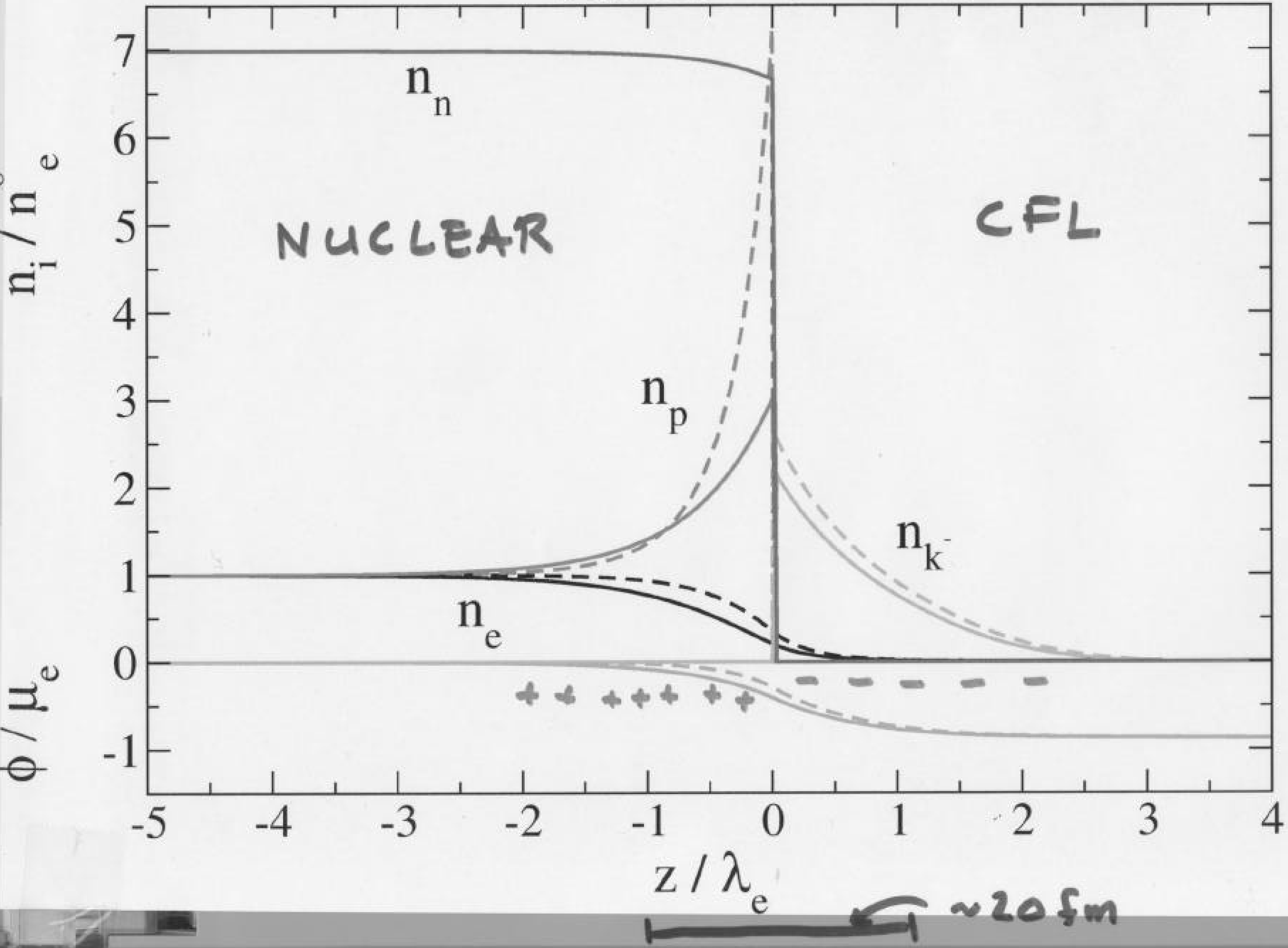


In the real world α_0 is small, so the field is mostly converted into $\tilde{Q}$ flux by the supercurrents and monopoles, and penetrates the interior. Only a weak field is excluded.

Alford, Berges, KR

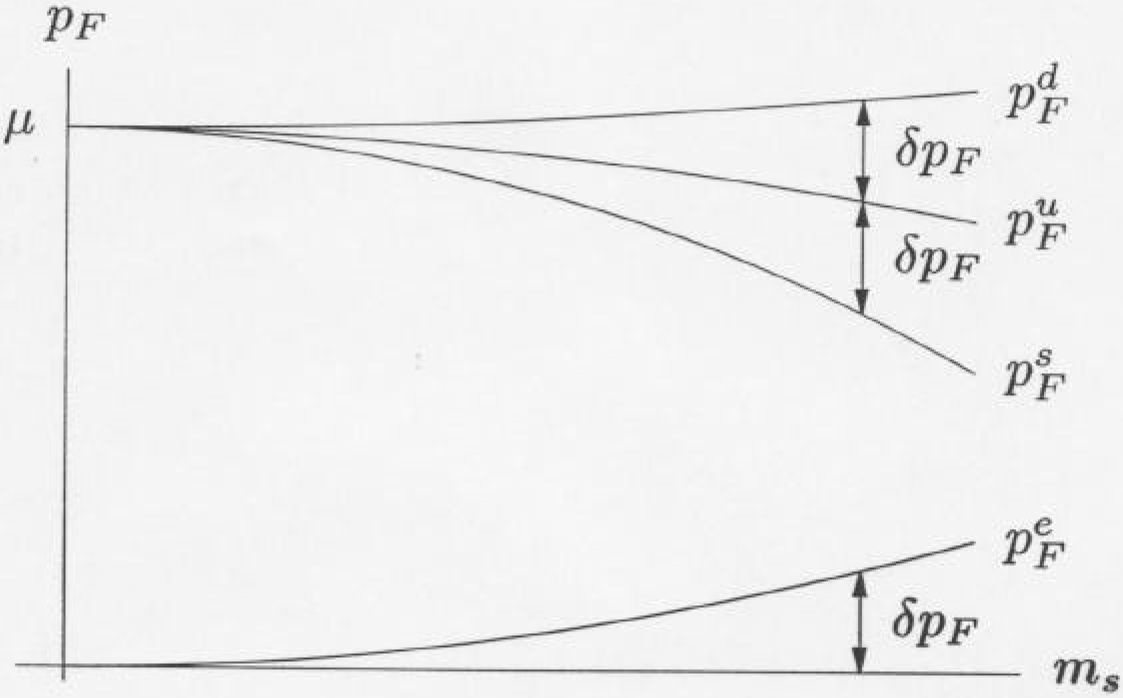
MINIMAL CFL - NUCLEAR INTERFACE

Alford, KR, Roddy, Wilczek



INTERMEDIATE DENSITY QUARK MATT

- M_s important
- For orientation, consider noninteracting quarks, $m_u = m_d = 0$ $m_s \neq 0$, impose electrical neutrality and weak eqbm



- In noninteracting quark matter, $\delta p_F \simeq \frac{m_s^2}{4\mu}$
- Motivates result that CFL pairing "breaks" when $\frac{m_s^2}{4\mu} > \Delta$
- Also, when CFL "breaks", no residual <ud> pairing either. Alford, KR

CRYSTALLINE COLOR SUPERCONDUCTIVIT

Alford Bowers K.R; Bowers Kundu K.R Shuster; Leibovich K.R Shust
Casalbuoni Gatto Mannarelli Nardulli; Giannakis Liu Ren; Bowers K

As $\mu\downarrow$, if CFL "breaks" before you get to hadronic matter, quark matter at intermediate density may have:

Pairing between quarks with <u>different p_F</u>

<u>GOAL</u>: both quarks in a pair on respective Fermi surfaces

<u>IDEA</u>: Cooper pairs with momentum!

$$(\vec{p}+\vec{q}, -\vec{p}+\vec{q}) \text{ for any } \vec{p}.$$

Each pair has total momentum $2\vec{q}$

- $|\vec{q}| \simeq 1.2\, \delta p_F$ determined energetically
- "pattern" of $\{\hat{q}_i\}$ " Bowers K

$$\langle\psi\psi\rangle \sim \Delta \sum_i e^{i\vec{q}_i\cdot\vec{x}}$$

- spontaneous breaking of rotational and translational symmetry.

<u>LOFF</u>: Larkin Ovchinnikov Fulde Ferrell (1964) considered this state for $\langle e_\uparrow e_\downarrow\rangle$ pairing with Zeeman splitting. State not seen in condensed matter. Problem is that $\vec{B}\to$ orbital effects, not just Zeeman. QCD, with its "flavor Zeeman splitting" turns out to be the natural context for LOFF's idea!

SIMPLIFICATIONS, FOR NOW

- two flavors, with Fermi surfaces split by a $\delta\mu$ introduced by hand:

$$\mu_u = \mu + \delta\mu$$
$$\mu_d = \mu - \delta\mu$$

(instead of 3 flavors with F.S. splitting from M_s, neutrality.)

See work by Kundu + KR for how to use M_s instead of $\delta\mu$.

- point-like 4-fermion interaction between quarks,  with quantum #s of g.

See Leibovich, KR, Shuster; Giannakis, Liu, Ren

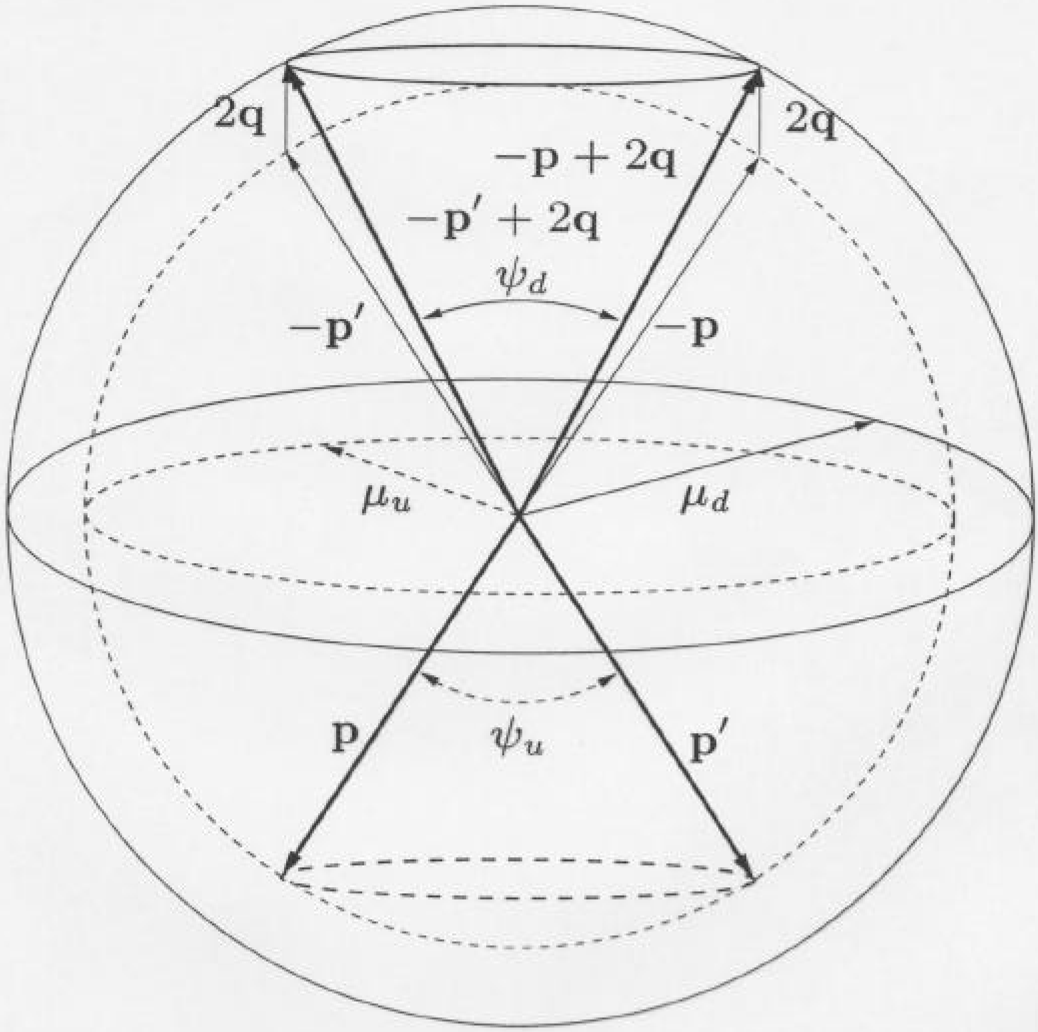
for 

- $\Delta \ll \mu$

Basic LOFF idea

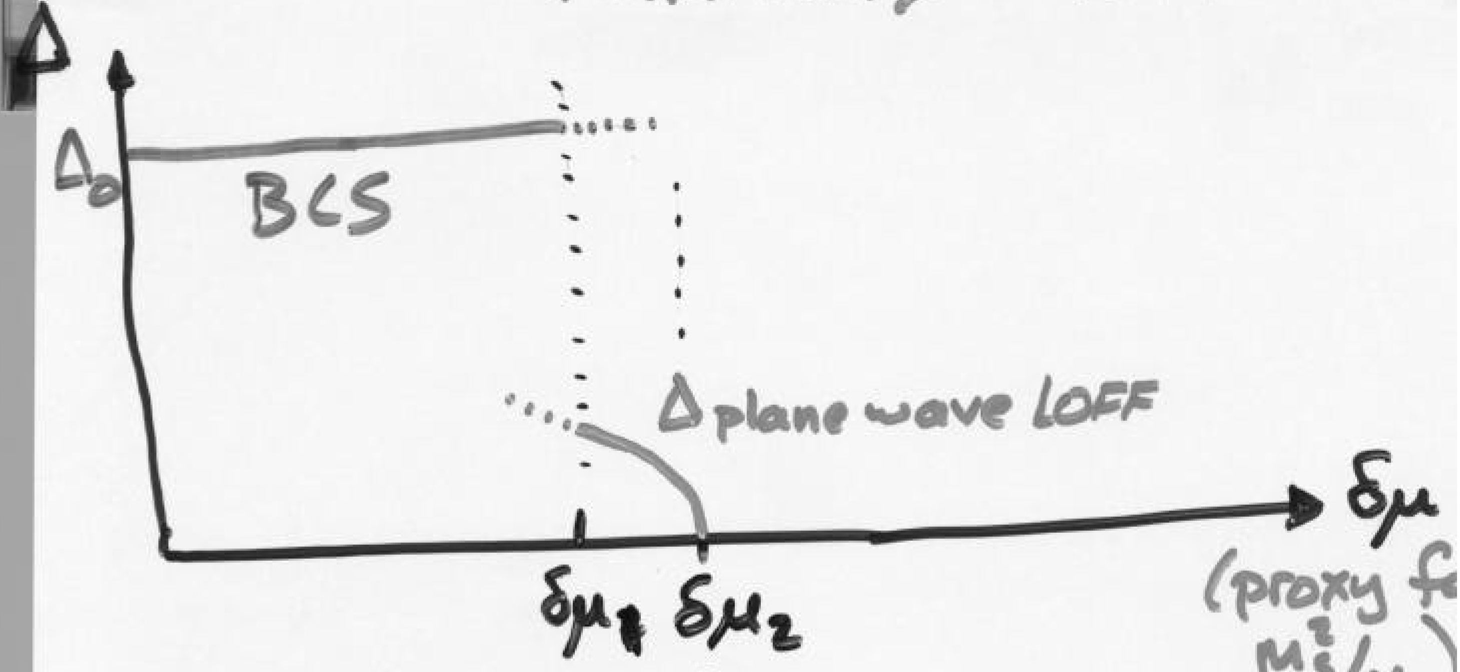
Try Cooper pairs $(\mathbf{p}, -\mathbf{p} + 2\mathbf{q})$

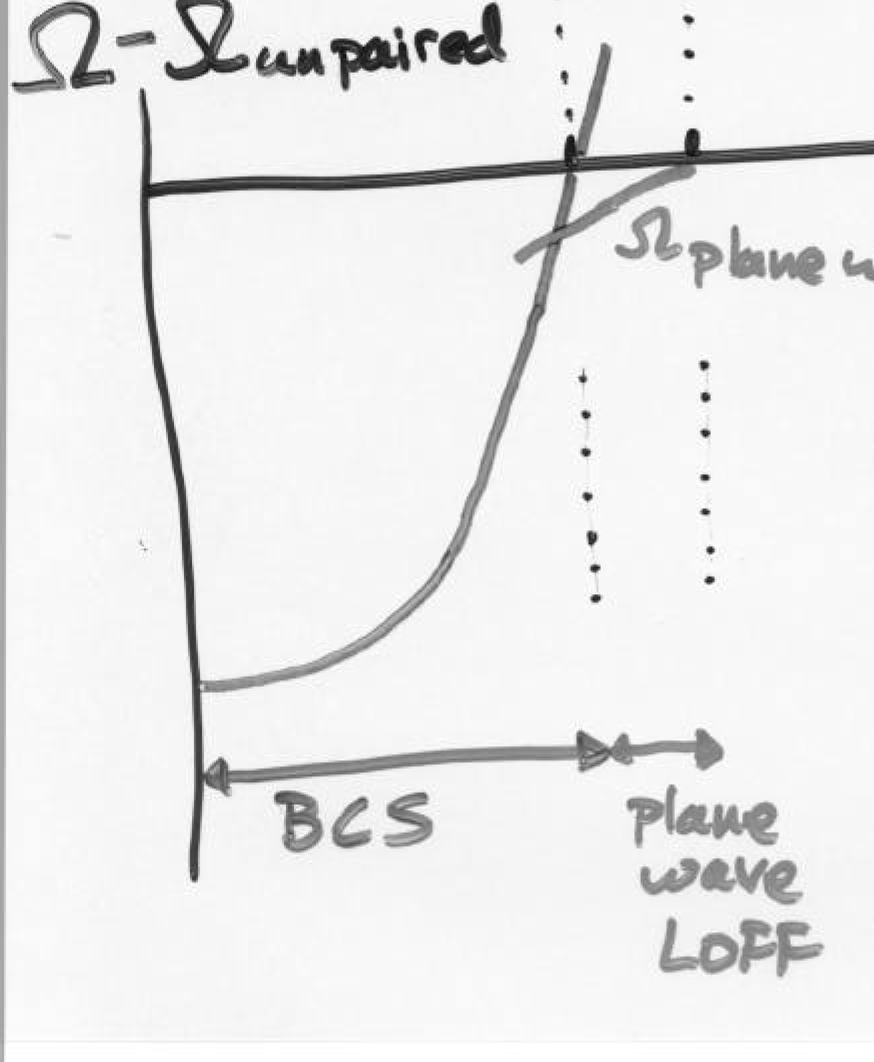
- total momentum $2\mathbf{q}$ for each and every pair
- each quark at its Fermi surface, even with $p_F^u \neq p_F^d$
- $\hat{q}$ chosen spontaneously, $|\mathbf{q}|$ determined variationally (result is $|\mathbf{q}| = q_0 \approx 1.20\delta\mu$)
- condensate forms a ring on each Fermi surface, with opening angle $\psi_u \approx \psi_d \approx 2\cos^{-1}(\delta\mu/q_0) \approx 67.1°$



SINGLE PLANE WAVE

$$\langle \psi(x)\,\psi(x) \rangle \sim \Delta\, e^{i 2 \vec{q}\cdot\vec{x}}$$





@ $\delta\mu_1$: 1st order

BCS → LOFF

@ $\delta\mu_2$: 2nd order

LOFF → unpaired

$$\frac{\delta\mu_2 - \delta\mu_1}{\delta\mu_1} = .07$$

MULTIPLE PLANE WAVES

If system unstable to formation of 1 plane wave, this allows quarks lying on one ring on each F.S. to pair. Much of F.S. remains unpaired....

Why not multiple $\vec{q}$'s? i.e. multip rings?

Want to compare many different possible $\{\vec{q}_i\}$;

$$\langle \psi(x)\, \psi(x) \rangle = \sum_{\{\vec{q}_i\}} \Delta\, e^{i 2\vec{q}_i \cdot \vec{x}}$$

and for each $\{\vec{q}_i\}$ calculate Δ and Ω

$\{q_i\}$, ie crystal structure, with lowest Ω wins.

GINZBURG - LANDAU

For $\Delta \ll \Delta_0$, ie for $\delta\mu \to \delta\mu_2$,
the free energy Ω can be evaluat
order-by-order in Δ, for <u>many</u>
crystal structures.

<u>Order Δ^2</u>: $|\vec{q}_i| = 1.2\,\delta\mu$ for all q_i's

- each q_i gives pairing on a ring with opening angle $67°$.
- the more q_i's, the better.

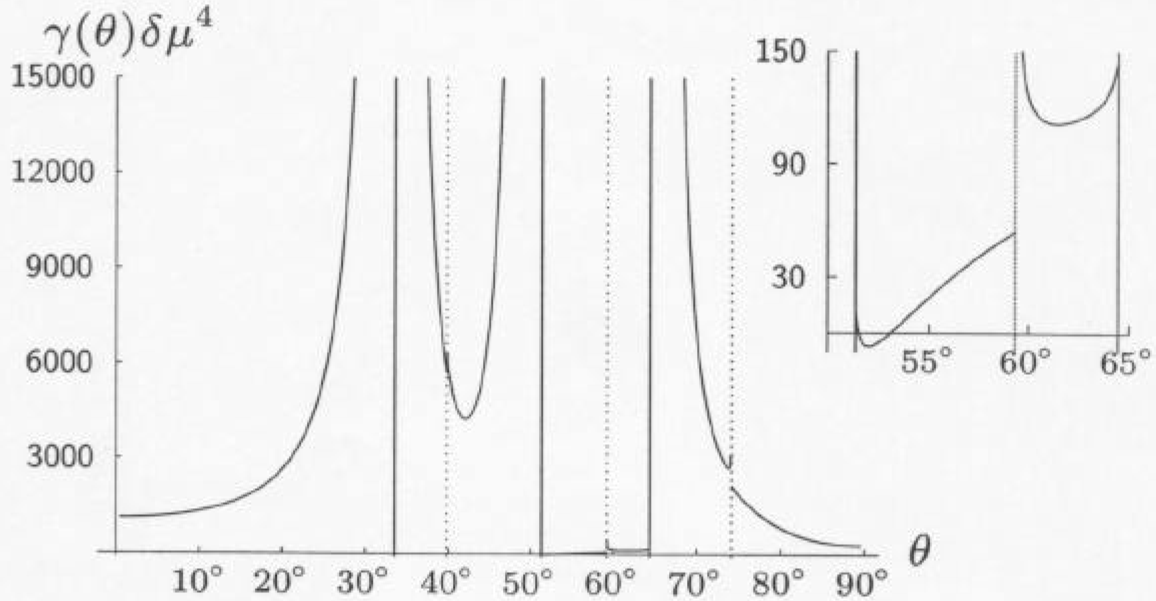
<u>Order Δ^4 and Δ^6</u>: "interaction between rin

- intersecting rings costs a lot $\Rightarrow$ at most 9 plane waves
- "regularity" (lots of different ways of making closed 4-, 6-, ... sided figures from q_i strongly favored.
- indicates that best choice is.....

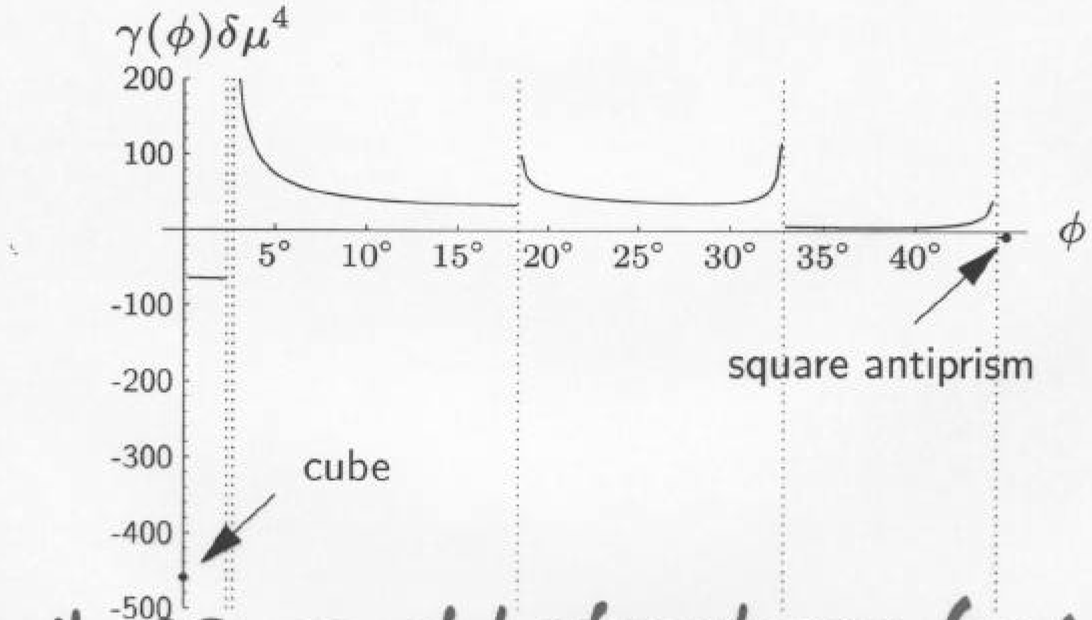
Continuous variations $\Omega = \alpha\Delta^2 + \beta\Delta^4 + \gamma\Delta^6 + \ldots$

γ FOR DIFFERENT CRYSTALS WITH 8 WAVES

- Varying the "height" of a square antiprism



- Varying the "twist" of a square prism



of all 23 crystal structures (and their continuous variations) we investigated, CUBE has most negative β and

Crystal structures

Candidate crystal structures with P plane waves, specified by their symmetry group $\mathcal{G}$ and Föppl configuration. Bars denote dimensionless equivalents: $\bar{\beta} = \beta\,\delta\mu^2$, $\bar{\gamma} = \gamma\,\delta\mu^4$, $\bar{\Omega} = \Omega/(\delta\mu_2^2 N_0)$ with $N_0 = 2\bar{\mu}^2/\pi^2$. $\bar{\Omega}_{\rm min}$ is the (dimensionless) minimum free energy at $\delta\mu = \delta\mu_2$. The phase transition (first order for $\bar{\beta} < 0$ and $\bar{\gamma} > 0$, second order for $\bar{\beta} > 0$ and $\bar{\gamma} > 0$) occurs at $\delta\mu_*$.

Structure	P	$\mathcal{G}$(Föppl)	$\bar{\beta}$	$\bar{\gamma}$	$\bar{\Omega}_{\rm min}$	$\delta\mu_*/\Delta_0$
point	1	$C_{\infty v}(1)$	0.569	1.637	0	0.754
antipodal pair	2	$D_{\infty v}(11)$	0.138	1.952	0	0.754
triangle	3	$D_{3h}(3)$	-1.976	1.687	-0.452	0.872
tetrahedron	4	$T_d(13)$	-5.727	4.350	-1.655	1.074
square	4	$D_{4h}(4)$	-10.350	-1.538	–	–
pentagon	5	$D_{5h}(5)$	-13.004	8.386	-5.211	1.607
trigonal bipyramid	5	$D_{3h}(131)$	-11.613	13.913	-1.348	1.085
square pyramid	5	$C_{4v}(14)$	-22.014	-70.442	–	–
octahedron	6	$O_h(141)$	-31.466	19.711	-13.365	3.625
trigonal prism	6	$D_{3h}(33)$	-35.018	-35.202	–	–
hexagon	6	$D_{6h}(6)$	23.669	6009.225	0	0.754
pentagonal bipyramid	7	$D_{5h}(151)$	-29.158	54.822	-1.375	1.143
capped trigonal antiprism	7	$C_{3v}(13\bar{3})$	-65.112	-195.592	–	–
cube	8	$O_h(44)$	-110.757	-459.242	–	–
square antiprism	8	$D_{4d}(4\bar{4})$	-57.363	-6.866	–	–
hexagonal bipyramid	8	$D_{6h}(161)$	-8.074	5595.528	-2.8×10^{-6}	0.755
augmented trigonal prism	9	$D_{3h}(3\bar{3}\bar{3})$	-69.857	129.259	-3.401	1.656
capped square prism	9	$C_{4v}(144)$	-95.529	7771.152	-0.0024	0.773
capped square antiprism	9	$C_{4v}(14\bar{4})$	-68.025	106.362	-4.637	1.867
bicapped square antiprism	10	$D_{4d}(14\bar{4}1)$	-14.298	7318.885	-9.1×10^{-6}	0.755
icosahedron	12	$I_h(15\bar{5}1)$	204.873	145076.754	0	0.754
cuboctahedron	12	$O_h(4\bar{4}\bar{4})$	-5.296	97086.514	-2.6×10^{-9}	0.754
dodecahedron	20	$I_h(5555)$	-527.357	114166.566	-0.0019	0.772

FCC Crystal

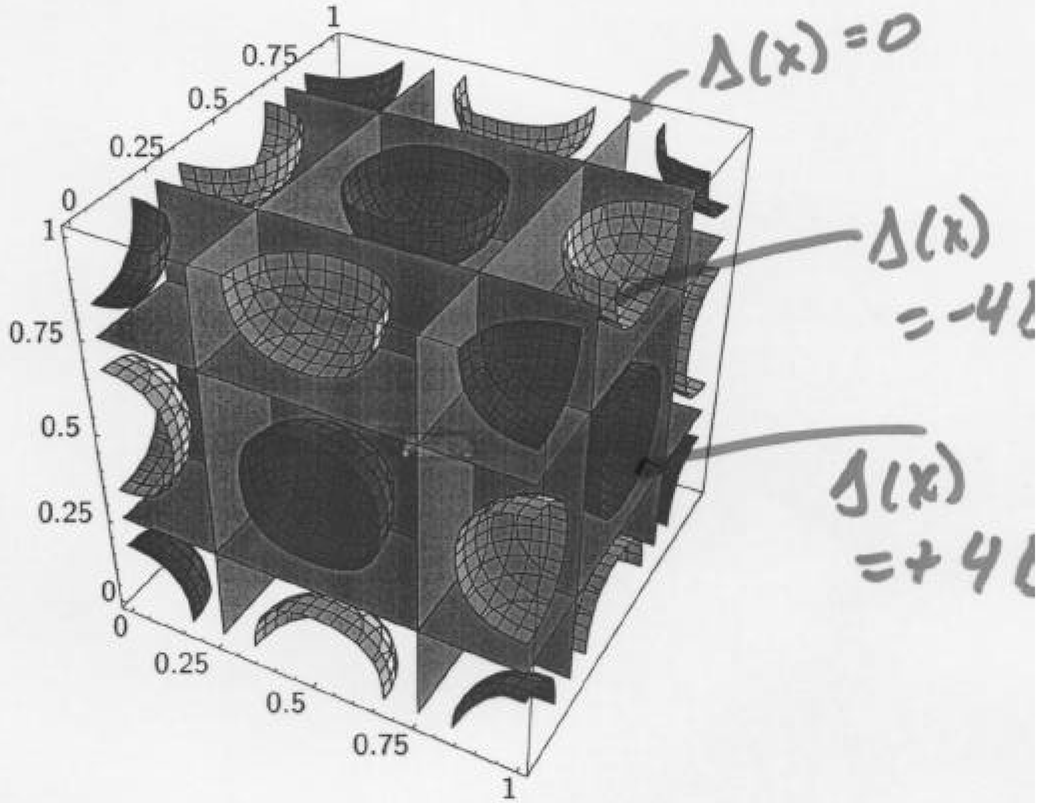
Favored according to Ginzburg-Landau analysis, that is not yet quantitatively reliable. Bowers ...

- The cube structure is the favored ground state: eight wave vectors pointing towards the corners of a cube, forming the eight shortest vectors in the reciprocal lattice of a face-centered-cubic crystal. The gap function is

$\Delta \sim \Delta_{CFL}$

$$\Delta(\mathbf{x}) = 2\Delta\left[\cos\frac{2\pi}{a}(x+y+z)+\cos\frac{2\pi}{a}(x-y+z) + \cos\frac{2\pi}{a}(x+y-z)+\cos\frac{2\pi}{a}(-x+y+z)\right]$$

A unit cell:



with contours $\Delta(\mathbf{x}) = +4\Delta$ (black), 0 (gray), -4Δ (white). Lattice constant is $a = \sqrt{3}\pi/|\mathbf{q}| \simeq 6.012/\Delta_0$.

CONCLUSIONS

- FCC cube is favored structure. Ginzburg-Landau analysis has taught us what features of a crystal structure are favored, and thus why FCC best
- BUT: $\Omega = \alpha\Delta^2 + \beta\Delta^4 + \gamma\Delta^6 + \dots$ with β, γ large and negative, and $\alpha \sim (\delta\mu - \delta$

  $\Rightarrow$ Strong 1^{st} order crystalline $\rightarrow$ unpaired transition at a $\delta\mu_* \gg \delta\mu_2$

  cf G-L analysis of liquid-solid transition

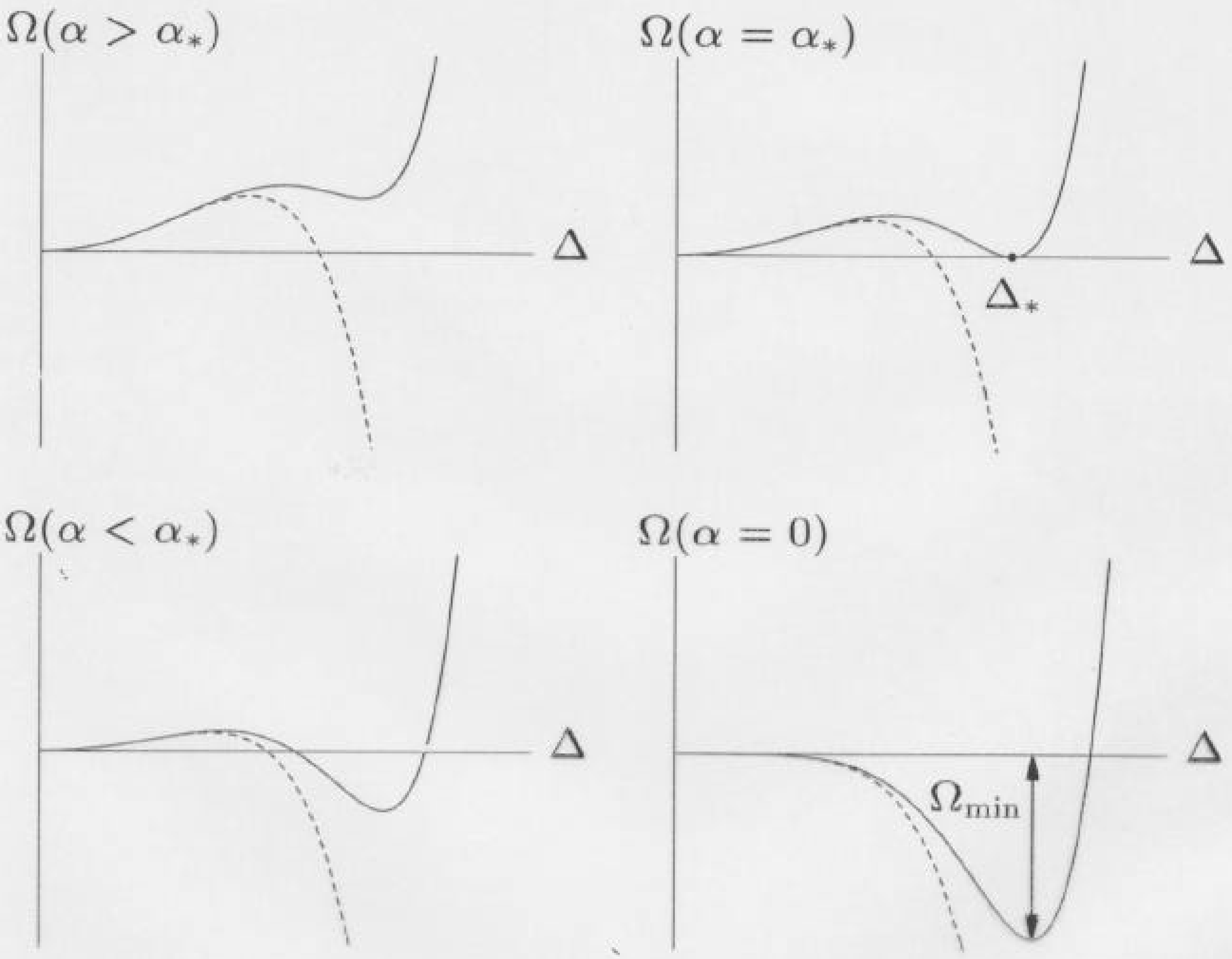
  - crystalline "window" in phase diagram <u>not</u> small
  - Δ <u>not</u> small

  $\Rightarrow$ Ginzburg-Landau cannot provi quantitative calculation of Δ,

- Make FCC ansatz, calculate Ω, Δ variationally. (In progress)

Unstable structures?

- Ginzburg-Landau instability guarantees a strong first-order transition at some $\delta\mu = \delta\mu_* \gg \delta\mu_2$
- Δ_*, $\Omega_{\rm min}$ are large, but cannot be predicted by the Ginzburg-Landau method
- Larger instability $\Rightarrow$ more robust ground state (cube has the most unstable Ginzburg-Landau free energy)



OUTLOOK AND IMPLICATIONS

- variational calculation for FCC crystal, now that we know this is the favored z
- three-flavor analysis

CRYSTALLINE SUPERFLUIDITY

- this phase may be created in gases of ultracold fermionic atoms (Combescot)
  - trap 2 hyperfine ~~states~~ states of atom;
  - arrange strong attractive interact between 2 "species"
  - arrange different number densiti for 2 "species"

VORTEX PINNING & PULSAR GLITCHES:

- rotate crystal; what happens? vortices? pinned at intersecti of crystal planes?
- if so, presence of a layer of crystalline color superconducting quark matter within neutron ~~star~~ → gl

IMPLICATIONS FOR COMPACT STARS

or, flipping that around,... How can we use observations of compact stars to determine the high density region of the phase diagr

- If core of "neutron" star is quark matter, then it IS a color supercondu

$$T_{star} \sim \text{keV} \lll T_c$$

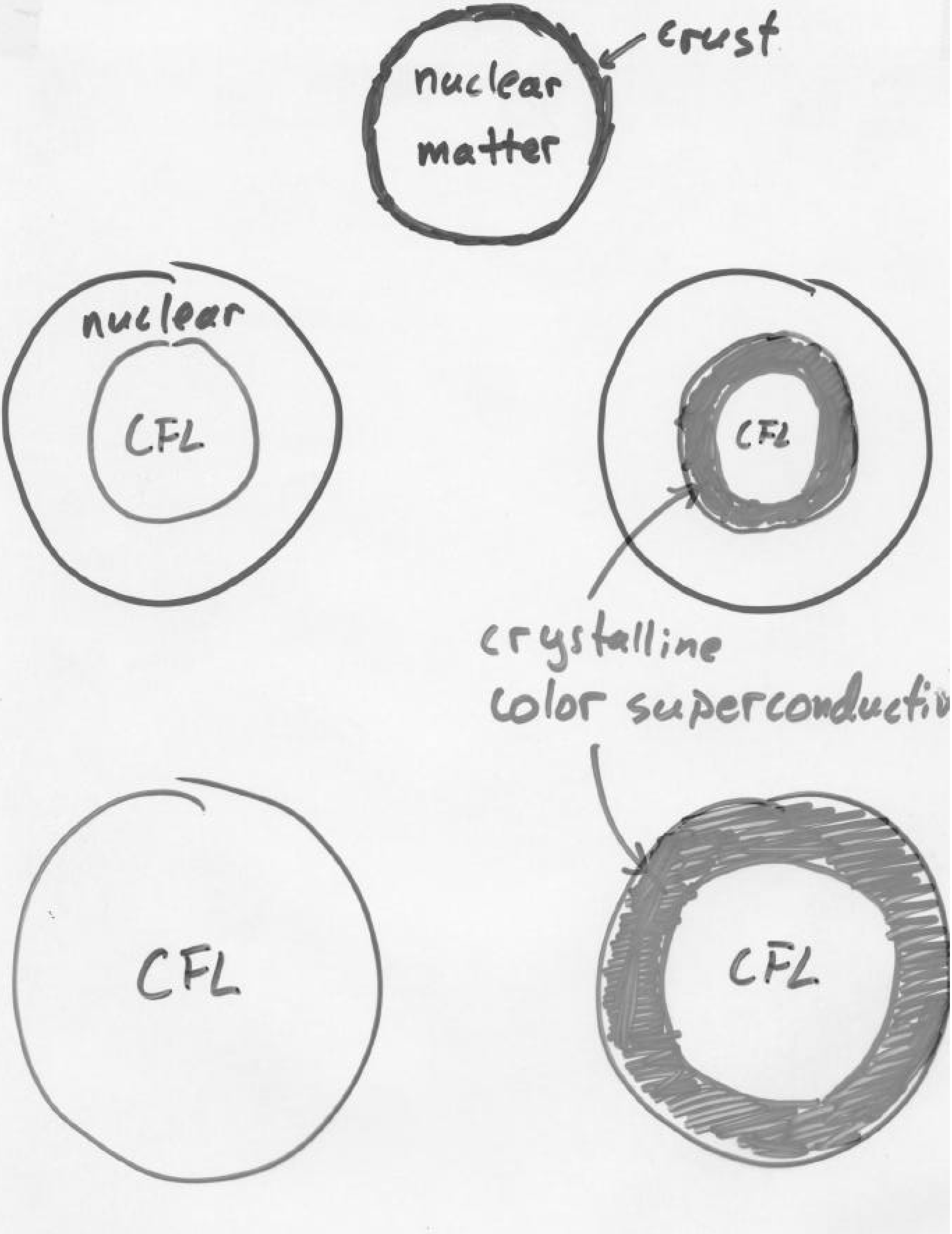
- Not known whether neutron stars have quark matter cores. Goal: understand observational consequences, so we can find ou

FIRST: Can we discover whether there is a crystalline color superconductivity windo
- As a function of increasing depth, m_s^2/μ decreases.

$\therefore$ LOFF WINDOW → LOFF SHELL

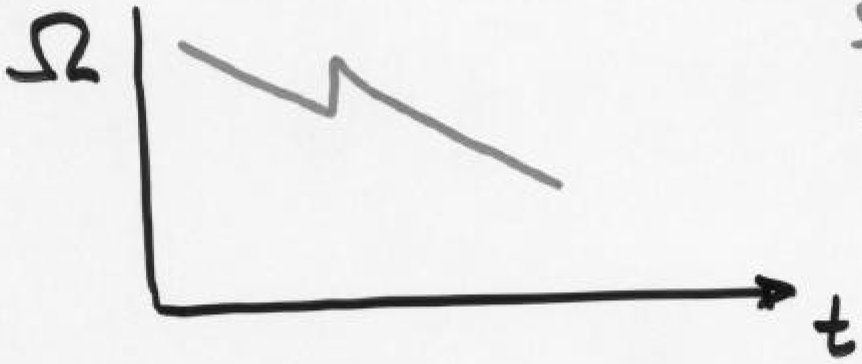
THEN: List other examples of ways to answer this question.

SEVERAL SCENARIOS



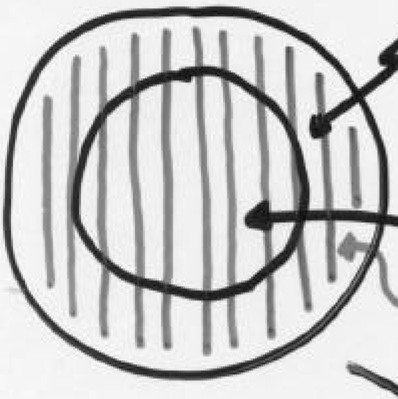
GLITCHES

Pulsars glitch:



$$\frac{\delta\Omega}{\Omega} \sim 10^{-9} \to 10^{-}$$

Conventional mechanism:



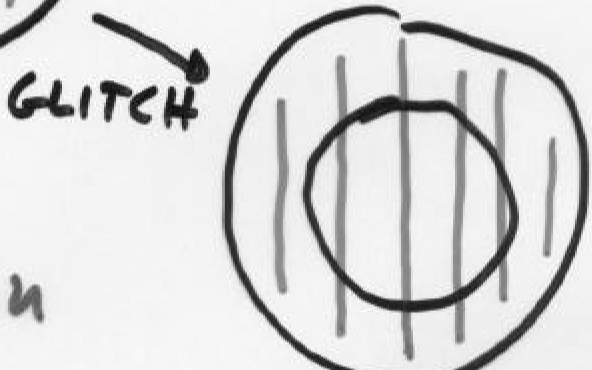
crust: nuclear crystal batha
in neutron superfluid

neutron superfluid

ROTATIONAL VORTICES

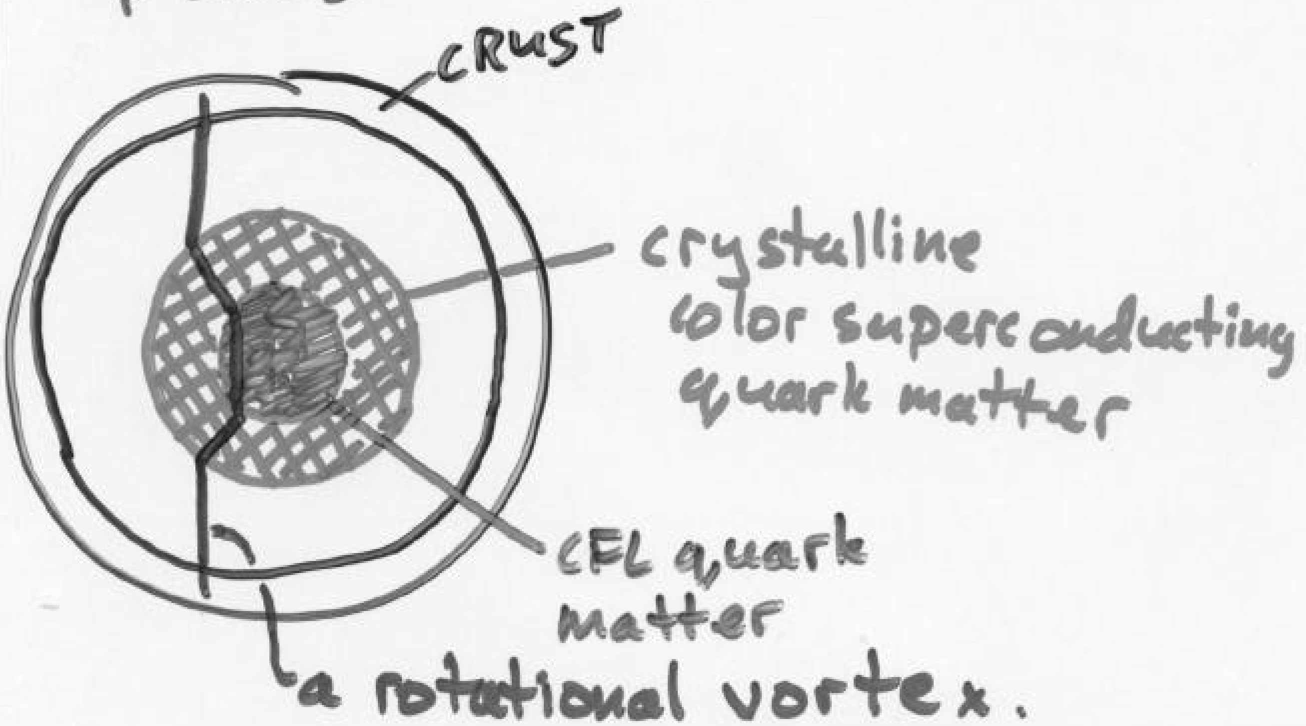
Glitches require non-uniformity (ie crystal) to impede (pin) motion of vortices.

∴ thought impossible in QM.



GLITCHES IN QUARK MATTER?

- crystalline condensate may pin vortices, since they will prefer to follow intersections of nodal planes



- could some (eg the smaller?) glitches originate in crystalline layer in core?
- could observed features of glitches rule out existence of crystalline layer?
- serious glitch phenomenology awaits calculation of pinning force. This is in progress.

If neutron stars have CFL cores, what have we learned about their properti

- Except just after supernova, $T_{star} \lll T_c$

- For given M, R a little smaller. But, uncertainty in R dominated by nuclear outer layer
- "density step" at sharp interface is <u>big</u>. $\xrightarrow{?}$ LIGO signal Alford KR Reddy Wilczek

- superfluid, with $v_s \gtrsim 1/\sqrt{3}$
- transparent insulator. (No electrons) $\vec{B}$ not in flux tubes; not frozen.
- At T < 5-10 MeV:
  - very small specific heat, neutrino emission, neutrino opacity Page Pra Lattimer Steine
  - very large thermal conductivity Shovkov et al

  ⇒ cooling of star controlled by nuclear outer layer.
- During supernova, mesons important, a ∃ phase transitions. → SN ν signals? Carter Reddy Sa Tachib
- Bare quark star would be nice. NOT seen