Electron-Positron Annihilation into Hadrons

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I. Introduction

II. $R(s)$ — experiment

$R(s)$ — theory

III. Phenomenological Applications

a_μ^{had}, $\Delta\alpha_{\text{had}}$, α_s, m_c, m_b

IV. Conclusions and Outlook

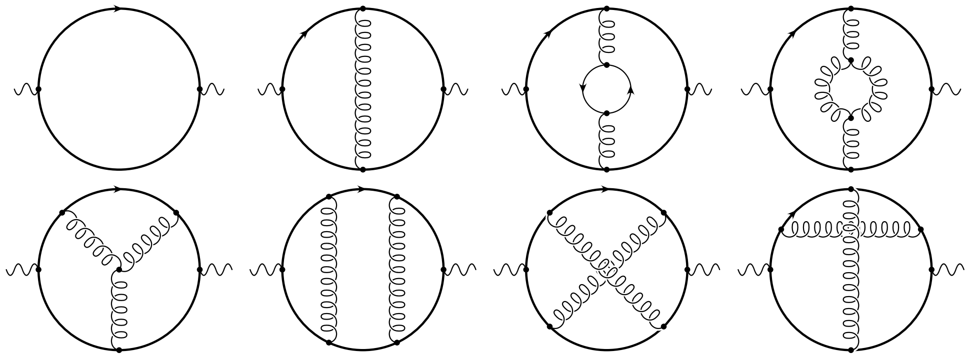
DESY Theory Workshop, September 2002

I. Introduction

Introduction

- inclusive quantity:

$$R(s) = \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} = 12\pi \text{Im}\left[\Pi(s+i\epsilon)\right]$$



$$R(s) = 3\sum_q Q_q^2\left(1+\frac{\alpha_s}{\pi}\right)+\dots$$

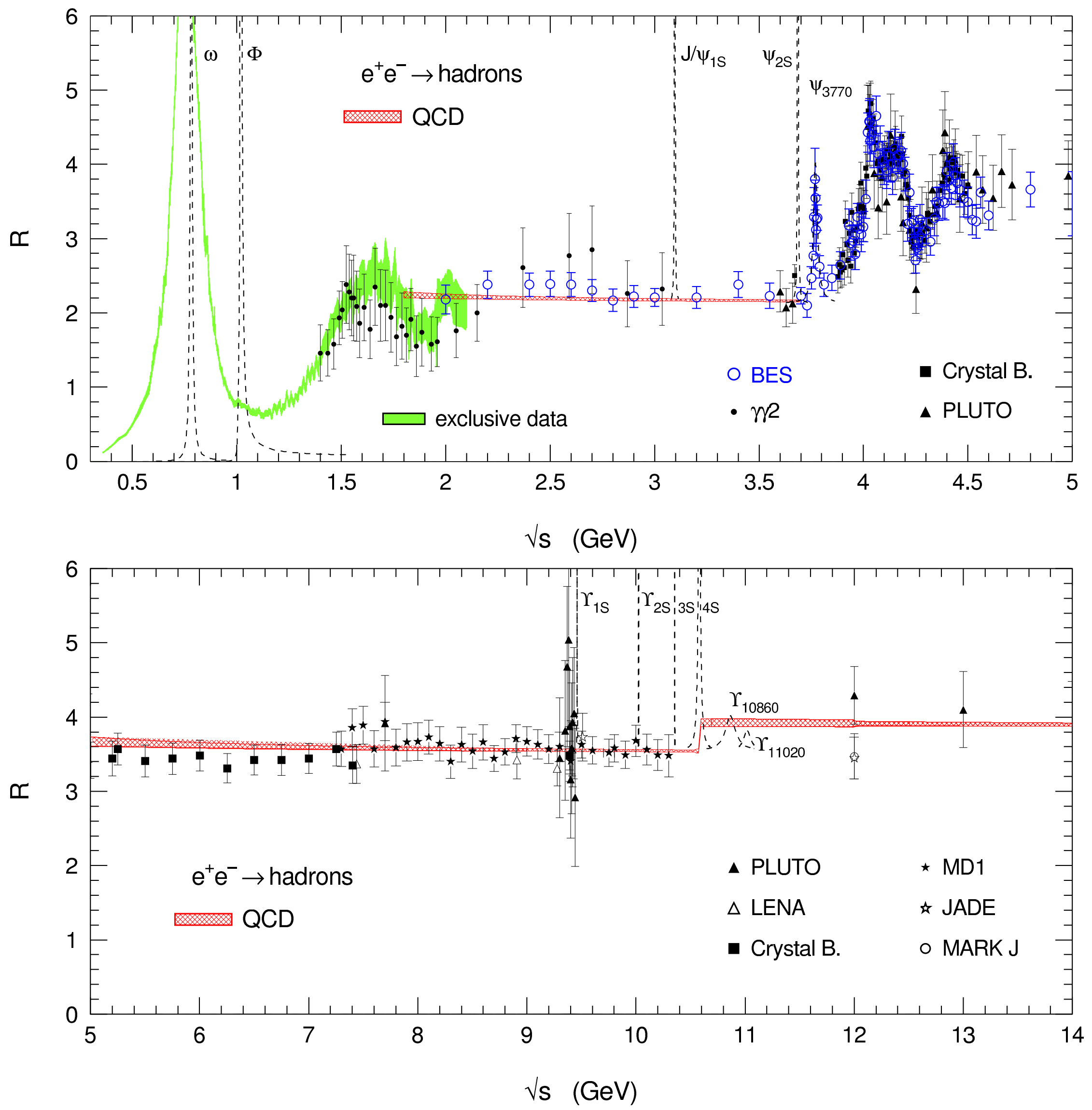
- experimentally accessible for $\sqrt{s} =$

  $2m_\pi \dots 3.6$ GeV $\dots 10.52$ GeV $\dots M_Z \dots$

  ($\sqrt{s} \lesssim 2$ GeV: exclusive; $\sqrt{s} \gtrsim 2$ GeV: inclusive)
- outside resonances; above threshold

  $\longrightarrow$ prediction from perturbative QCD possible

Introduction



[Davier, Eidelman, Höcker, Zhang'02]

Introduction

- test of perturbative QCD
- determination of α_s
- test of evolution of α_s as predicted by the RGE
- $\sqrt{s} \gg m_q \Longrightarrow m_q$ can be neglected
- $\sqrt{s} \approx m_q$

  $\longrightarrow$ effects from m_c and m_b are important

  $\longrightarrow$ determination of m_c, m_b

II. Experimental and Theoretical Status

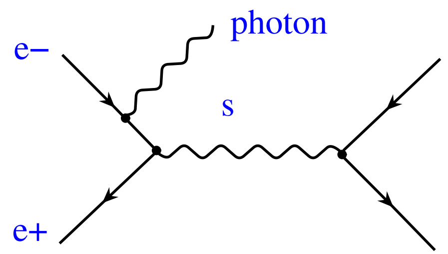
$R(s)$ — Experiment

Recent measurements of $R(s)$

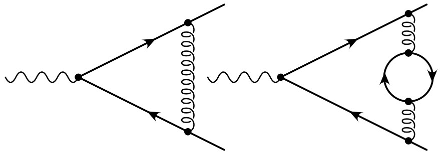
experiment	date	$\sqrt{s}$ (GeV)	sys. error	most important phen. application
CMD-2	2002	$0.61 - 0.96$	0.6%	a_μ^{had}
BES	2001	$2 - 5$	4%	$\Delta\alpha_{\text{had}}^{(5)}$, m_c
MD-1	1996	$7.2 - 10.34$	4%	α_s, $\Delta\alpha_{\text{had}}^{(5)}$
CLEO	1998	10.52	2%	pQCD-test, α_s, $\Delta\alpha_{\text{had}}^{(5)}$
. . . many more . . .				

needed: cross check of CMD-2 and BES data
precise data for $\sqrt{s} \gtrsim 5$ GeV, 10 GeV

Future: CLEO @ $3 \leq \sqrt{s} \leq 10$ GeV (?)
KLOE (?)
radiative return (KLOE, BaBar)

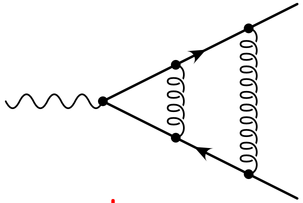


$R(s)$ — Theory



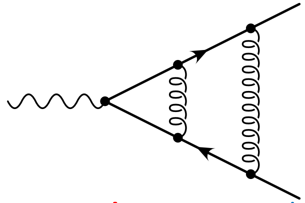
$\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2 n_l)$:
"direct" calculation

[Källen, Sabry'55; Hoang, Kühn, Teubner'95]



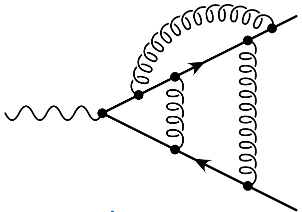
massless quarks:
recurrence relations,
3-loop propagator-type integrals

[Chetyrkin, Kataev, Tkachov'79; Dine, Sapirstein'79; Celmaster, Gonsalves'80]



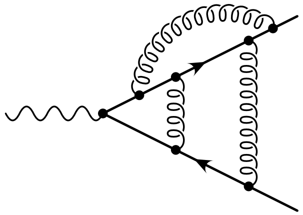
massive quarks: Padé approximation,
conformal mapping,
semi-numerical result

[Chetyrkin, Kühn, MS'95'96]



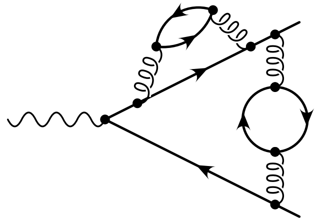
massless quarks: infra-red-rearrangement (IRR)

[Surguladze, Samuel'91; Gorishny, Kataev, Larin'91; Chetyrkin'97]



massive quarks,
$\alpha_s^3 m^2/s, \alpha_s^3 m^4/s^2$: OPE, RGE

[Chetyrkin, Kühn'90; Chetyrkin, Harlander, Kühn'00]



massless quarks, $\alpha_s^4 n_f^2$:
no recurrence relations

[Baikov, Chetyrkin, Kühn'01]

[Harlander,Kühn,MS]: `Fortran` program `rhad`, contains all corrections (in preparation)

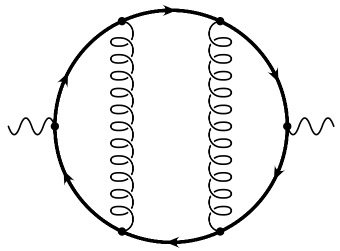
Recurrence Relations

Implemented in program packages

MINCER: massless 2-point integrals, 1,2,3 loops [Larin,Tkachov,Vermaseren'91]

MATAD: vacuum integrals, 1 mass, 1,2,3 loops [MS'96'00]

Principle structure:



$\downarrow$

```
Dg(nu1,nu2,p5)*
Dg(nu3,nu4,-p7)*
S(nu2,p4,nu4,p6,
  nu3,p4,mu2,p1,
  nu1,p2,mu1,p3)
```

$\longrightarrow$

recurrence relations
(integration-by-parts)

[Chetyrkin,Tkachov'81]

complicated integral
$=\sum$ simpler integrals

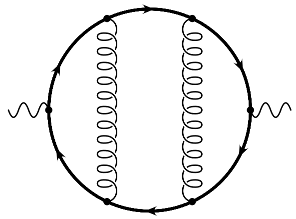
$\longrightarrow$

original integral
$= \sum_n C_n M_n$

M_n: master integral

C_n: coefficient function

Padé Method



1. evaluate diagrams for

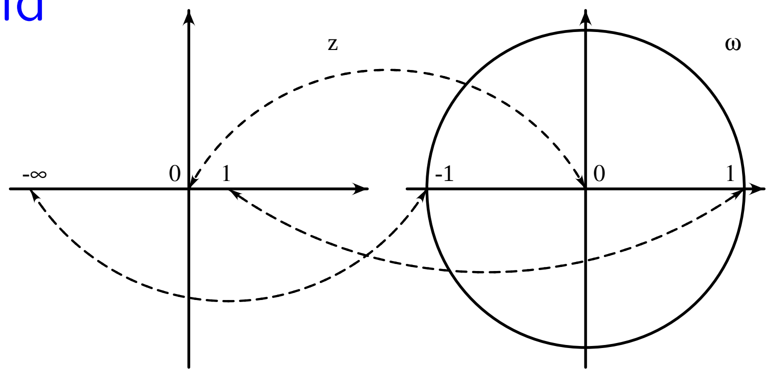
$q^2 \ll M^2 \rightarrow$ Taylor expansion $\rightarrow$ MATAD [MS'96'00]

$q^2 \gg M^2 \rightarrow$ asymptotic expansion $\rightarrow$ MATAD/MINCER [Larin et al.'91]

$q^2 \approx 4M^2 \rightarrow$ potential/threshold

2. apply conformal mapping:

z		ω
$-\infty$	$\rightarrow$	-1
0	$\rightarrow$	0
1	$\rightarrow$	1



$z = q^2/4M^2$ plane $\qquad \omega$ plane

3. combine data from different kinematical regions; perform Padé approximation:

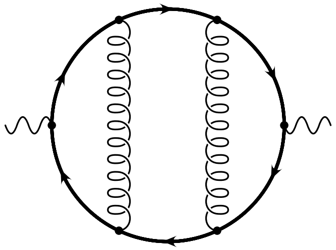
$$[n/m](\omega) = \frac{a_0 + a_1\omega + \ldots + a_n\omega^n}{b_0 + b_1\omega + \ldots + b_m\omega^m}$$

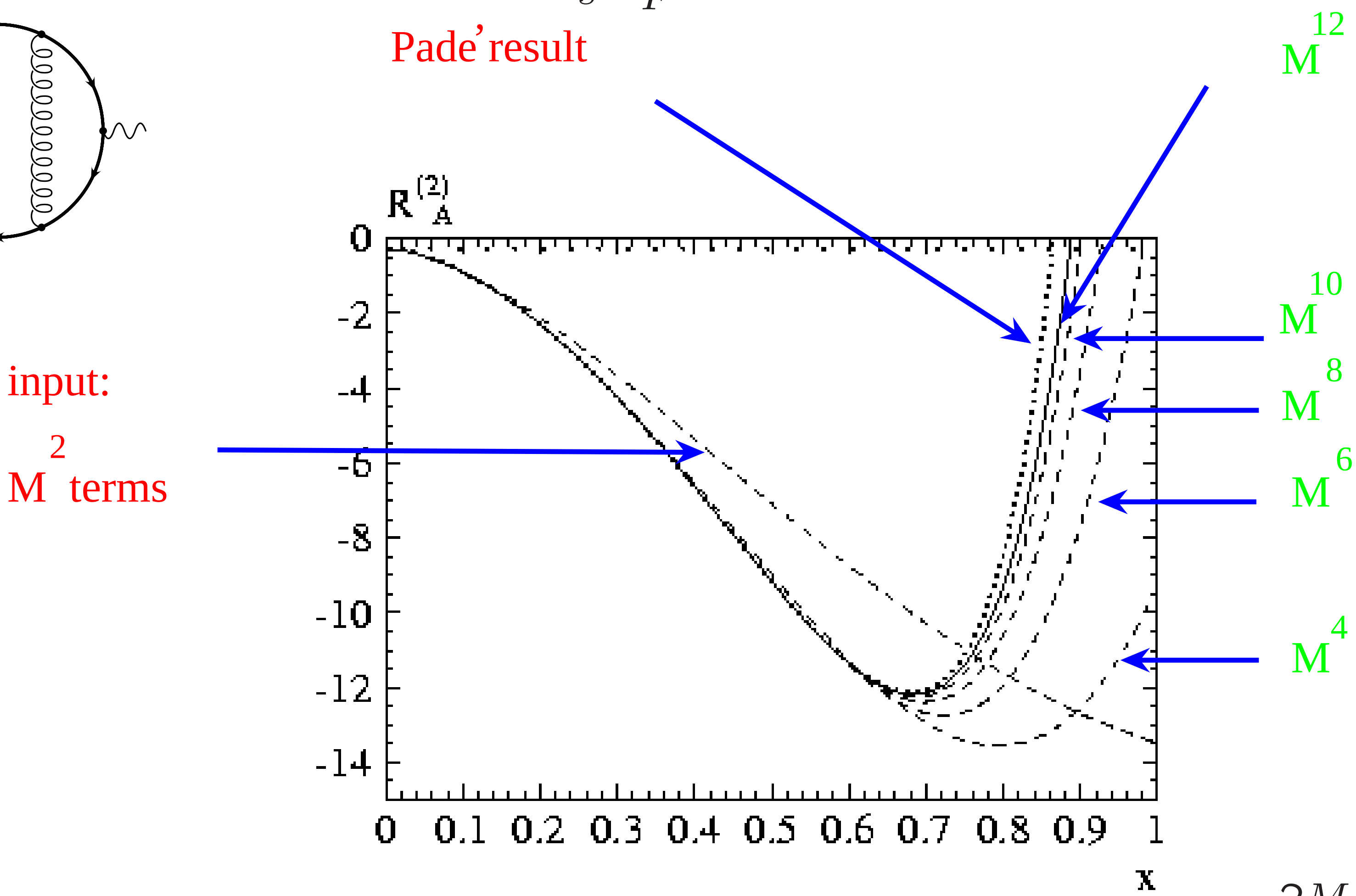
➪ semi-analytical formula for $\Pi(q^2)$ and $R(s)$

similar methods used by [Baikov, Broadhurst, Fleischer, Tarasov, Davydychev, ...]

Padé Method — Results

Abelian contribution $\propto \alpha_s^2 C_F^2$

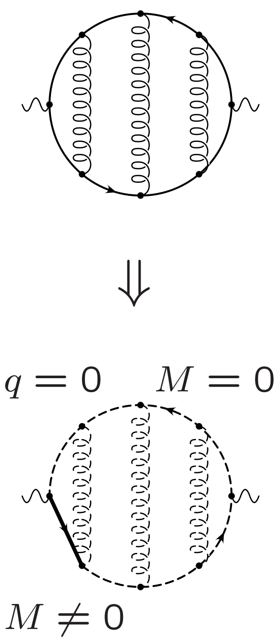




e.g.: $x \approx 0.7, M = m_b = 4.6$ GeV $\rightarrow \sqrt{s} \approx 13$ GeV

$$x = \frac{2M}{\sqrt{s}}$$

$\mathcal{O}(\alpha_s^3)$ — Computation



- Consider propagator-type diagrams
- Needed: imaginary part $\Longrightarrow$ compute: pole part

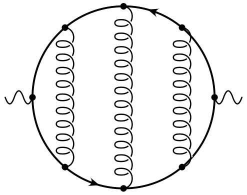
UV pole part of log-divergent diagram does not depend on masses and external momenta [Vladimirov'80]

➪ assign masses and choose external momenta in such a way, that the 4-loop integral factorizes into (3+1)-loop integrals, i.e. change IR behaviour of diagram [Chetyrkin,Smirnov'84] (InfraRed-Rearrangement, IRR)

- treat IR divergences with $R^\star$ operation [c.f.: UV div. $\leftrightarrow$ R op.]
- initial formulation: apply $R^\star$ separately to each diagram
  - ➪ $\mathcal{O}(\alpha_s^3)$ corrections [Surguladze, Samuel'91; Gorishny, Kataev, Larin'91]
- global $R^\star$ operation [Chetyrkin, '97]
  - ➪ automatic computation possible

$\mathcal{O}(\alpha_s^3 m_q^4)$ — Computation

OPE:



$$R(s)\Big|_{m_q^4} \sim \text{Im}\Pi\Big|_{m_q^4} \sim \text{Im}\sum\nolimits_n C_n \langle \mathcal{O}_n \rangle$$

relevant dimension-4 operators:

$$\mathcal{O}_1^0 = \left(G_{\mu\nu}^{0,\prime,a}\right)^2 \qquad \mathcal{O}_2^0 = m_q^{0,\prime}\bar{\psi}_q^{0,\prime}\psi_q^{0,\prime}$$

$$\mathcal{O}_6^0 = \left(m_q^{0,\prime}\right)^4$$

Mixing under renormalization $\longrightarrow$ anomalous dimension matrix $\gamma \sim (\gamma_m, \beta, \gamma_0)$

$C_1, C_2, \langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle$: #loops $\leq 3 \Longrightarrow$ `MATAD/MINCER`

C_6: massless 4-loop 2-point functions: NOT available

Way out: use RGE: $\mu^2 \frac{\mathrm{d}}{d\mu^2}\Pi\Big|_{m_q^4} = 0$

$$\Longrightarrow \frac{\partial}{\partial L} C_6 \langle \mathcal{O}_6 \rangle = \overbrace{-4\gamma_m C_6 \langle \mathcal{O}_6 \rangle - \beta\pi \frac{\partial}{\partial \alpha_s} C_6 \langle \mathcal{O}_6 \rangle - C_1 4 m_q^4 \alpha_s \frac{\partial}{\partial \alpha_s}\gamma_0 + C_2 4 m_q^4 \gamma_0}^{\#\text{loops} \leq 3 \Longrightarrow \texttt{MATAD/MINCER}}$$

$$\Longrightarrow L = \ln \frac{\mu^2}{-q^2} \text{ terms of } C_6 \Longrightarrow \text{Im}C_6$$

[Chetyrkin,Kühn'94,Chetyrkin,Harlander,Kühn'00]

$\mathcal{O}(\alpha_s^3 m_q^4)$ — Results

$$R(s) = 3\sum_q Q_q^2 \left(r_0 + \frac{m_Q^2}{s} r_{Q,2} + \left(\frac{m_Q^2}{s}\right)^2 r_{Q,4} \right)$$

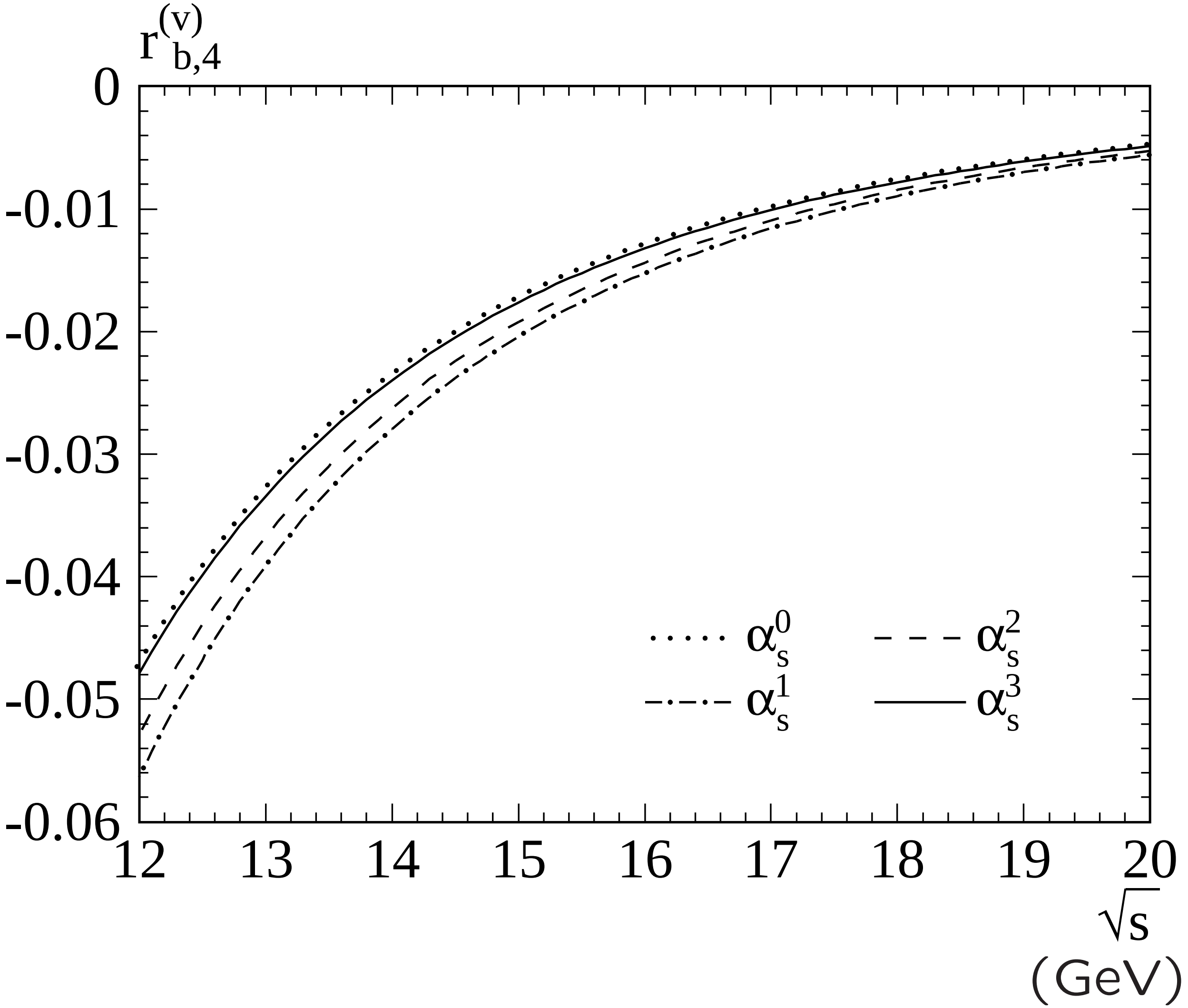
$$\begin{aligned} r_{Q,4}^{\mathrm{OS}} = & -6 + \frac{\alpha_s}{\pi}(10 - 24L_{ms}) + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{206}{3} + 218\zeta_2 + 16\zeta_2 \ln 2 + 104\zeta_3 \right. \\ & \left. - \frac{311}{2}L_{ms} - 15L_{ms}^2 + n_f\left(-\frac{35}{9} - 12\zeta_2 - \frac{8}{3}\zeta_3 + 9L_{ms} - 2L_{ms}^2 \right)\right) \\ & + \left(\frac{\alpha_s}{\pi}\right)^3 \left(\frac{91015}{108} - \frac{76}{9}\ln^4 2 + \frac{2564287}{540}\zeta_2 - \frac{4568}{9}\zeta_2 \ln 2 - \frac{128}{3}\zeta_2 \ln^2 2 \right. \\ & + \frac{56257}{18}\zeta_3 - \frac{1439}{3}\zeta_2\zeta_3 - \frac{1565}{6}\zeta_4 - \frac{3770}{3}\zeta_5 - \frac{608}{3}a_4 + L_{ms}\left(-\frac{5536}{3} + 564\zeta_2 \right. \\ & \left. - 24\zeta_2 \ln 2 + 416\zeta_3 \right) - \frac{591}{4}L_{ms}^2 + \frac{15}{2}L_{ms}^3 + n_f\left(-\frac{21011}{216} + \frac{8}{27}\ln^4 2 - \frac{3544}{9}\zeta_2 \right. \\ & - \frac{176}{9}\zeta_2 \ln 2 + \frac{32}{9}\zeta_2 \ln^2 2 - \frac{2323}{9}\zeta_3 + \frac{700}{9}\zeta_4 + \frac{440}{9}\zeta_5 + \frac{64}{9}a_4 + L_{ms}\left(\frac{2419}{12} \right. \\ & \left.\left. + \frac{44}{3}\zeta_2 + \frac{16}{3}\zeta_2 \ln 2 + \frac{28}{3}\zeta_3 \right) - \frac{157}{6}L_{ms}^2 - \frac{2}{3}L_{ms}^3 \right) + n_f^2\left(\frac{35}{18} + \frac{25}{3}\zeta_2 + \frac{112}{27}\zeta_3 \right. \\ & \left.\left. + L_{ms}\left(-\frac{94}{27} - \frac{8}{3}\zeta_2 \right) + \frac{13}{9}L_{ms}^2 - \frac{2}{9}L_{ms}^3 \right)\right) \end{aligned}$$

[Chetyrkin,Kühn'94]

[Chetyrkin,Harlander, Kühn'00]

$a_4 = \mathrm{Li}_4(1/2), L_{ms} = \ln m_Q^2/s$

$\mathcal{O}(\alpha_s^3 m_q^4)$ — Results



m_b^4 terms for bottom quarks

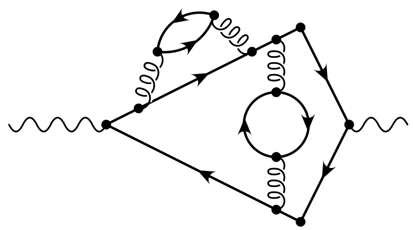
h.o. corr. significant
(bad convergence (?))

small compared to m_b^4 Born term

overall prediction for R stable

[Chetyrkin,Harlander, Kühn'00]

$\mathcal{O}(\alpha_s^4 n_f^2)$ — Computation



- compute imaginary part of 5-loop diagrams
- use IRR-trick: reduce to massless 4-loop propagator-type integrals

- recurrence relations very complicated
- use method develped by [Baikov'96'00]:

  [arbitrary 4-loop propagator-type integral] $= \sum_n C_n M_n$

M_n: master integrals

C_n: coefficient functions, rational function in $D = 4 - 2\varepsilon$

$\longrightarrow$ construct integral represenation for C_n

$\longrightarrow$ expand for $1/D \to 0$

$\longrightarrow$ reconstruct C_n

- huge disk space needed ($\approx$ 350 GB)
  500h CPU time on 8-processor alpha-SMP

$R(s)$ — Massless Result

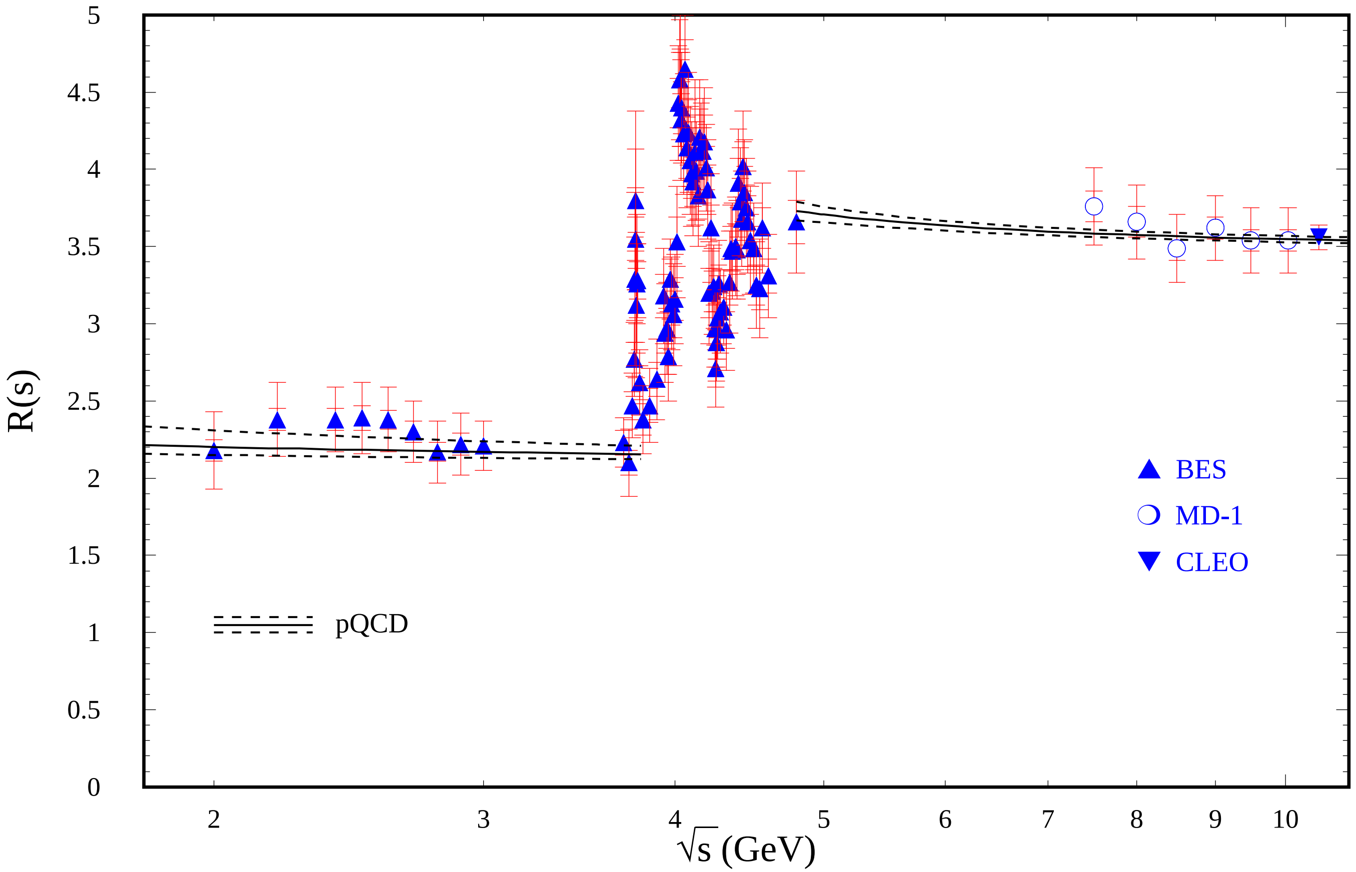
$$
\begin{aligned}
R(s) &= 3\sum_q Q_q^2 r_0 \\
r_0 &= 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{365}{24} - 11\,\zeta_3 + n_f\left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right)\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{87029}{288} - \frac{121}{8}\zeta_2 - \frac{1103}{4}\zeta_3 + \frac{275}{6}\zeta_5 + n_f\left(-\frac{7847}{216} + \frac{11}{6}\zeta_2 \right.\right. \\
&\qquad \left.\left. + \frac{262}{9}\zeta_3 - \frac{25}{9}\zeta_5\right) + n_f^2\left(\frac{151}{162} - \frac{1}{18}\zeta_2 - \frac{19}{27}\zeta_3\right)\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^4 \left[n_f^3\left(-\frac{6131}{5832} + \frac{11}{432}\pi^2 + \frac{203}{324}\zeta_3 - \frac{1}{54}\pi^2\zeta_3 + \frac{5}{18}\zeta_5\right)\right. \\
&\qquad \left. + n_f^2\left(\frac{1045381}{15552} - \frac{593}{432}\pi^2 - \frac{40655}{864}\zeta_3 + \frac{11}{12}\pi^2\zeta_3 + \frac{5}{6}\zeta_3^2 - \frac{260}{27}\zeta_5\right) + n_f B + A\right] \\
&= 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 (1.986 - 0.1153\,n_f) + \left(\frac{\alpha_s}{\pi}\right)^3 (-6.6369 - 1.2001\,n_f - 0.00518\,n_f^2) \\
&+ \left(\frac{\alpha_s}{\pi}\right)^4 (0.02152\,n_f^3 - 0.7974\,n_f^2 + n_f B + A)
\end{aligned}
$$

[Surguladze et al.'91, Gorishny et al.'91, Chetyrkin'97]

[Beneke'93]

[Baikov,Chetyrkin, Kühn'01]

$R(s)$ — Experiment vs Theory



experiment	energy (GeV)	sys.error
BES	2 – 5	4%
MD-1	7.2 – 10.34	4%
CLEO	10.52	2%

$\sqrt{s}$ (GeV)	$u,\ d,\ s$	$c,\ b$	$R^{\text{th}}(s)$	$R^{\text{exp}}(s)$
2.000	2.202	0.002	2.203	2.180± 0.070
3.730	2.154	0.002	2.156	2.100± 0.080
4.800	2.146	1.607	3.753	3.660± 0.140
8.900	2.122	1.440	3.562	3.579± 0.066
10.520	2.117	1.427	3.544	3.560± 0.010

III. Phenomenological Applications

a_μ^{had} and $\Delta\alpha_{\text{had}}^{(5)}$

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{ew}}$$

$$\Delta\alpha = \Delta\alpha_{\text{lep}} + \Delta\alpha_{\text{had}}^{(5)} + \Delta\alpha_{\text{top}}$$

a_μ^{had} and $\Delta\alpha_{\text{had}}^{(5)}$ can not be computed from first principles

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^\infty \mathrm{d}s\, \frac{K(s)}{s} R(s)$$

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = -\frac{\alpha M_Z^2}{3\pi}\text{Re}\int_{4m_\pi^2}^\infty \mathrm{d}s\, \frac{R(s)}{s(s-M_Z^2-i\epsilon)}$$

$R(s)$: experimental results or predictions from pQCD

$\approx 91\%$ from $\sqrt{s} < 1.8$ GeV

⇨ ω, ϕ, $|F_\pi^0|^2$, $e^+e^- \to \pi^+\pi^-$, $\pi^+\pi^-\pi^+\pi^-, \pi^+\pi^-\pi^0\pi^0, \ldots$;

$\tau \longleftrightarrow e^+e^-$ data

$2m_\pi - 1.8$ GeV	56.90(1.10)	data
$J/\Psi, \Psi', \Psi''$	9.24(74)	data
$\Upsilon(1S) - \Upsilon(6S)$	1.17(9)	data
$3.7 - 5.0$ GeV	15.67(34)	data
$1.8 - 3.7$ GeV	24.36(48)	pQCD
$5.0 - 11.1$ GeV	45.20(0.19)	pQCD
$11.2 - \infty$ GeV	124.77(0.12)	pQCD

[Kühn,MS'98]

$\Delta\alpha^{(5)}_{\text{had}}$

$\Delta\alpha^{(5)}_{\text{had}}(M_Z)\times 10^4$	Reference	comment
273.2(4.2)	[Martin et al. '95]	(low-order) pQCD
280(7)	[Eidelman et al. '95]	data
280(7)	[Burkhardt et al. '95]	data
275.2(4.6)	[Swartz '96]	—
281.7(6.2)	[Alemany et al. '97]	τ data
278.4(2.6)	[Davier et al. '97]	+ pQCD
277.5(1.7)	[Kühn, MS '98]	+ "charm threshold region"
277.6(4.1)	[Groote et al. '98]	sum rules (SR) (pQCD)
277.3(2.0)	[Erler '98]	τ data + UDR
277.0(1.6)	[Davier et al. '98]	τ data + pQCD + SR
277.8(2.5)	[Jegerlehner '99]	pQCDin Euclidian region (ER)
274.3(1.9)	[Martin et al. '00]	new data + pQCD (ER)
276.1(3.6)	[Burkhardt et al. '01]	new data
277.3(2.1)	[Jegerlehner '01]	pQCD (ER) + new data

Determination of α_s

- use continuum data

BES	<3.73 GeV	$\alpha_s^{(3)}(3\text{ GeV})$	$= 0.369^{+0.047\,+0.123}_{-0.046\,-0.130}$
BES	4.8 GeV	$\alpha_s^{(4)}(4.8\text{ GeV})$	$= 0.183^{+0.059\,+0.053}_{-0.064\,-0.057}$
MD-1	8.9 GeV	$\alpha_s^{(4)}(8.9\text{ GeV})$	$= 0.193^{+0.017\,+0.127}_{-0.017\,-0.107}$
CLEO	10.52 GeV	$\alpha_s^{(4)}(10.52\text{ GeV})$	$= 0.186^{+0.008\,+0.061}_{-0.008\,-0.057}$

- combine results; assume uncorrelated systematic errors

- evolve to common scale $\sqrt{s} = 5$ GeV: $\alpha_s^{(4)}(5\text{ GeV}) = 0.235^{+0.047}_{-0.047}$
- evolve to $\sqrt{s} = M_Z$: $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$ [Kühn,MS'01]

$\longrightarrow$ confirmation of running

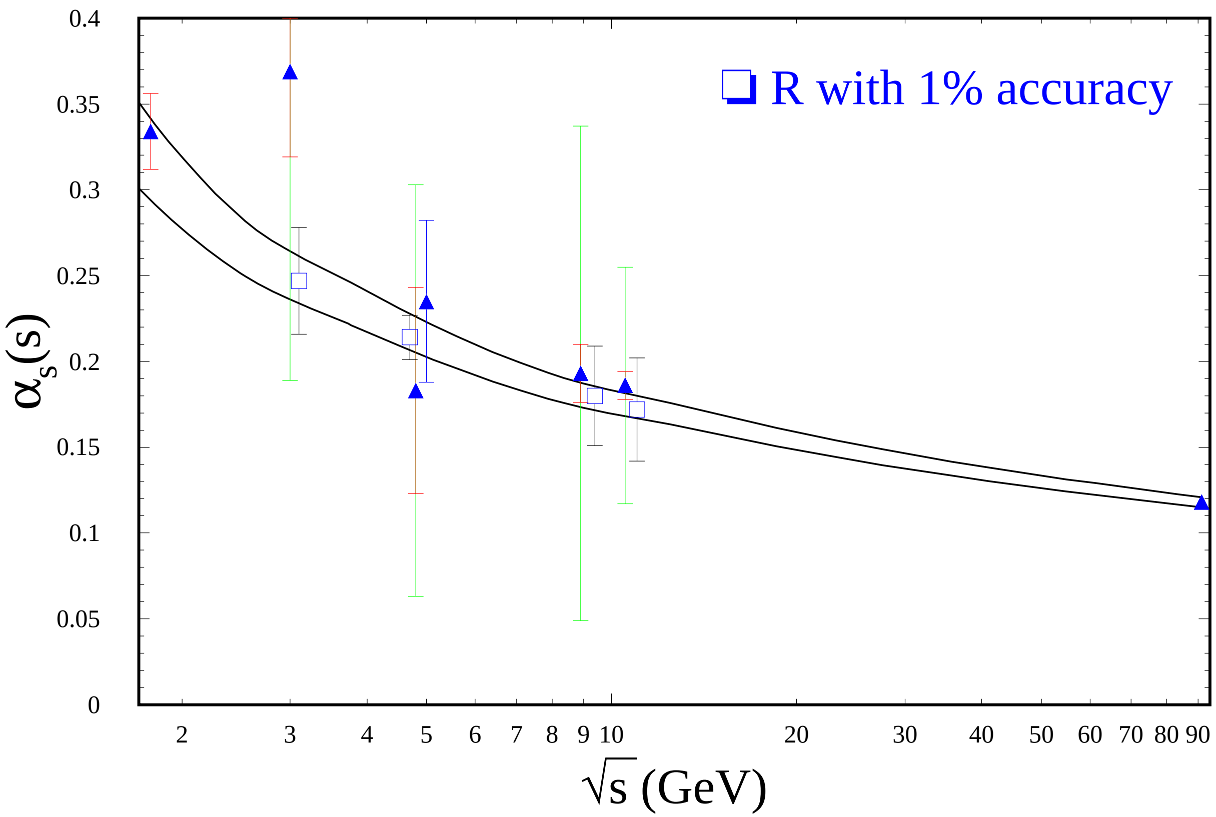
$\longrightarrow$ good agreement with other determinations

$[\alpha_s^{(5)}(M_Z) = 0.1172 \pm 0.0020]$ [PDG]

$\longrightarrow$ useful cross check

Determination of α_s

Confirmation
of running
statistical error
systematical error



For comparison:
α_s from $R(s)$ with
1% accuracy

Sum Rules — Moments

Basic idea: Compare theoretical calculation with experimental data

Consider moments of the photon polarization function:

$$\Pi^{\mu\nu}(q) = \left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)\,\Pi(q^2)$$

$$\mathcal{M}_n \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi(q^2)\Big|_{q^2=0} = \int_{s^{\mathrm{thr}}=4m_q^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} R(s) \longrightarrow \begin{cases} \mathcal{M}_n^{\mathrm{exp}} \\ \mathcal{M}_n^{\mathrm{th}} \end{cases}$$

n large $\longrightarrow$ main contribution from threshold ($s^{\mathrm{thr}} = 4m_q^2$)

$\longrightarrow$ α_s and v ($= \sqrt{1-4m_q^2/s}$, velocity of produced quarks) small

$\longrightarrow$ resummation of $(\alpha_s/v)^n$ necessary

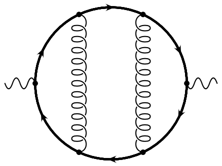
$\longrightarrow$ Υ sum rule

n small $\longrightarrow$ fixed-order calculation (in α_s) possible

$\longrightarrow$ SVZ sum rule

Determination of m_c: SVZ-SR

Low-order moments $\longrightarrow$ no resummation $\longrightarrow$ direct determination of $\overline{\text{MS}}$ mass



Theory: $\mathcal{M}_n^{\text{th}} = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$

$\bar{C}_1, \ldots, \bar{C}_8$ known analytically up to order α_s^2 [Chetyrkin,Kühn,MS'95'96]

Experiment: $\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s) = \mathcal{M}_n^{\text{exp,res}} + \mathcal{M}_n^{\text{exp,cc}} + \mathcal{M}_n^{\text{cont}}$

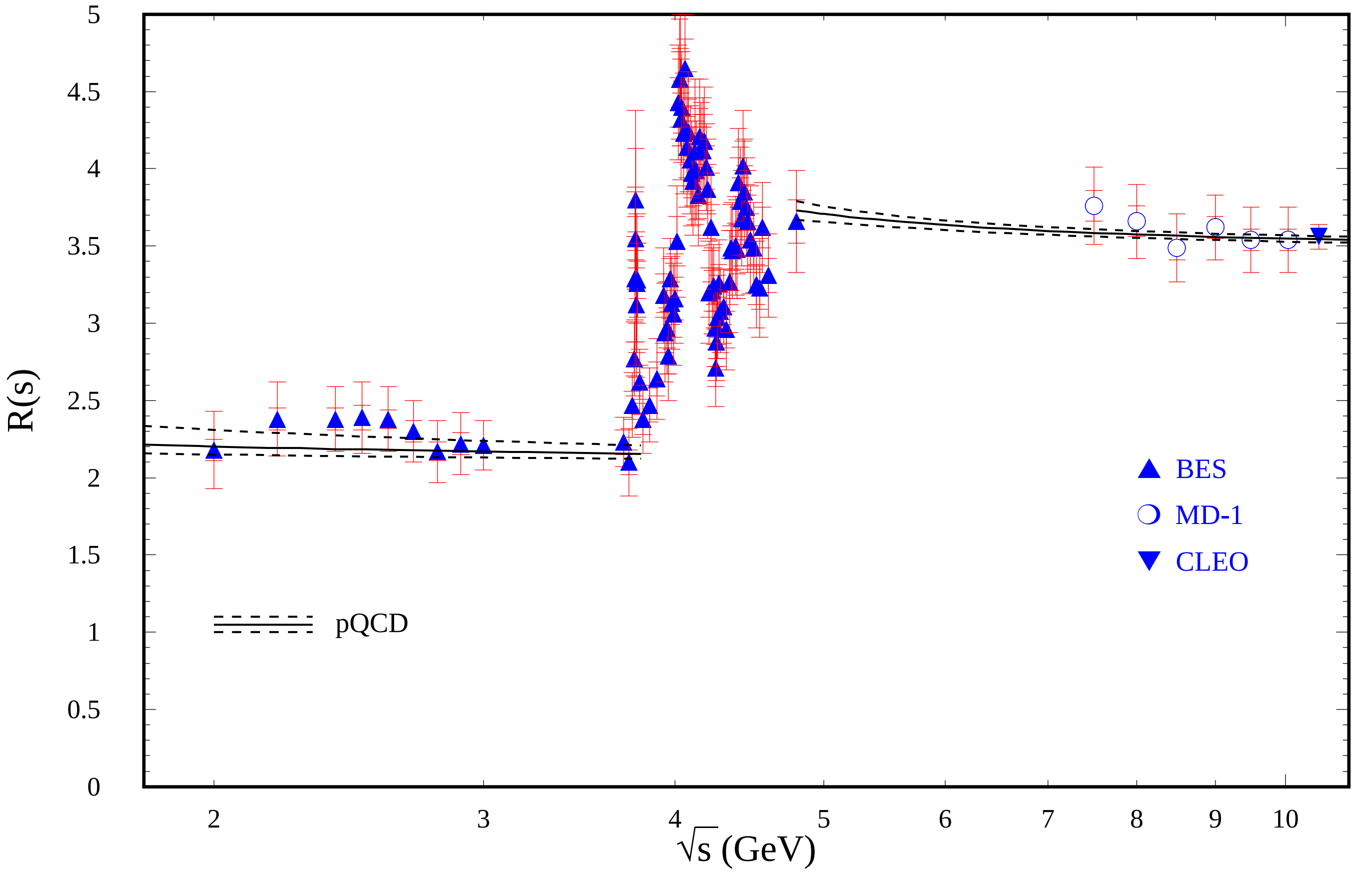
resonances (J/Ψ, Ψ')

below 4.8 GeV (BES)

above 4.8 GeV (theory)

n	$\mathcal{M}_n^{\text{exp,res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp,cc}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$
1	0.1114(82)	0.0313(15)	0.0638(10)
2	0.1096(79)	0.0174(8)	0.0142(3)
3	0.1094(79)	0.0099(5)	0.0042(1)
4	0.1105(79)	0.0057(3)	0.0014(0)

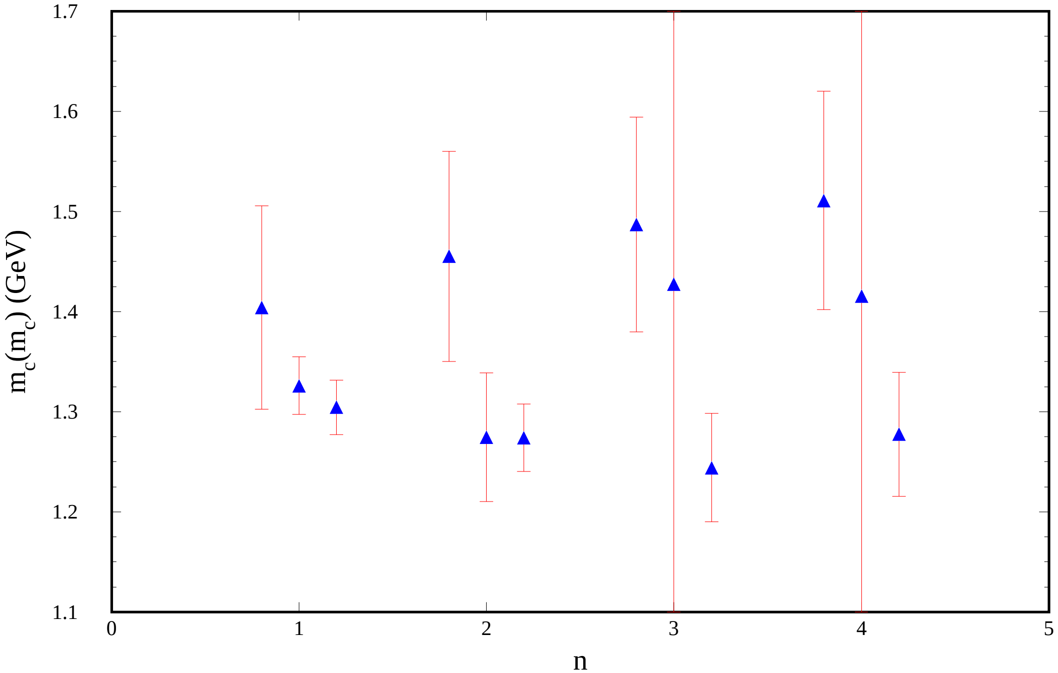
$R(s)$ — Experiment vs Theory



experiment	energy (GeV)	date	sys.error
BES	$2-5$	2001	4%
MD-1	$7.2-10.34$	1996	4%
CLEO	10.52	1998	2%

Determination of m_c: SVZ-SR

$m_c(m_c)$ for n=1,2,3 and 4 in LO, NLO and NNLO



use result for $n = 1$ $\boxed{m_c(m_c) = 1.304(27) \text{ GeV}}$ [Kühn,MS'01]

reason:

- condensates unimportant
- fixed-order result in α_s sufficient
- dominant error from experiment ($\Gamma_{ee}(J/\Psi)$, ...)

conversion to pole mass:

$$M_c^{(2\text{-loop})} = 1.514(34) \text{ GeV}$$
$$M_c^{(3\text{-loop})} = 1.691(35) \text{ GeV}$$

➪ $\overline{\text{MS}}$ mass more stable

Determination of m_c: Chauchy-SR

QCD duality:

$$\frac{1}{12\pi^2}\int^{s_{\rm max}} \mathrm{d}s\, R(s)\, p(s) = -\frac{1}{2\pi i}\oint_{|s|=s_{\rm max}} \mathrm{d}s\, \Pi(s)\, p(s)$$

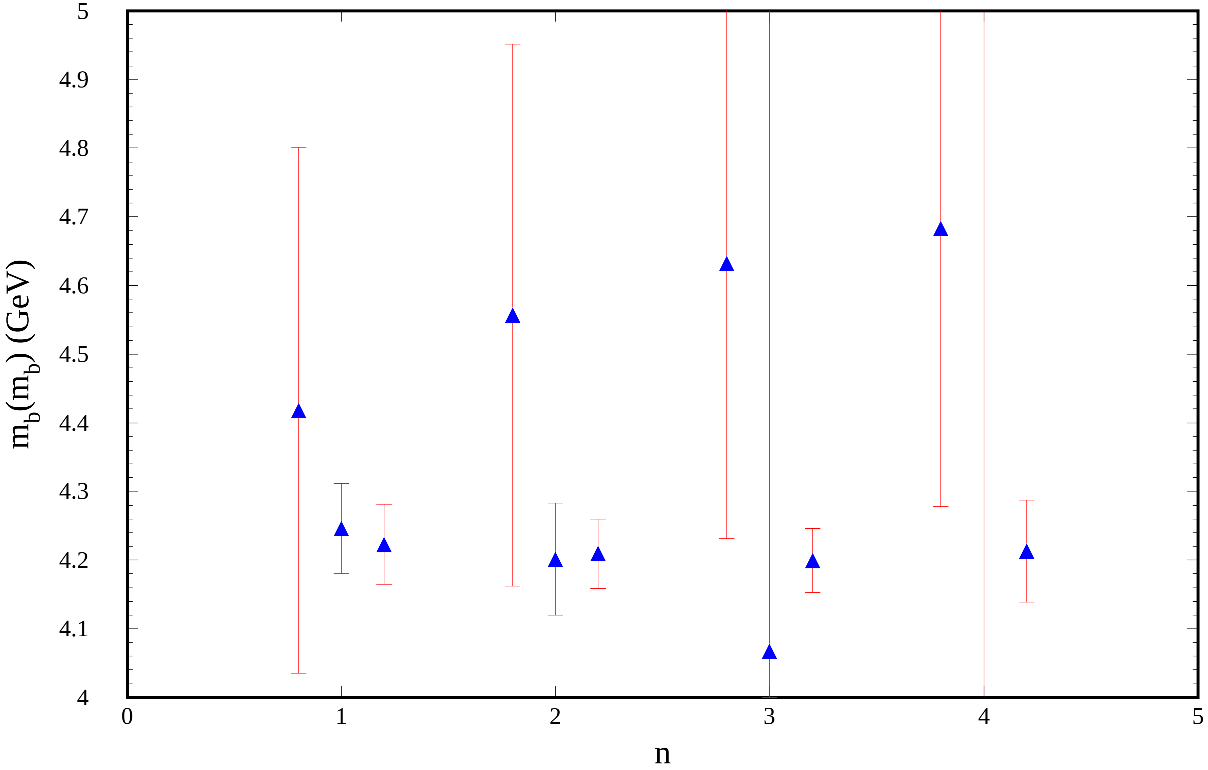
- $\Pi(s)$: pQCD; analytical result for 3-loop corrections up to $(m_c^2/s)^6$ (Large Momentum Procedure) [Chetyrkin,Harlander,Kühn,MS'97]
- $R(s)$: J/Ψ, Ψ' and BES data; $s_{\rm max} = (5\ \mathrm{GeV})^2$
- $p(s)$: arbitrary polynomial; enhance resonance contribution

- direct determination of $\overline{\rm MS}$ mass
- condensates unimportant

$\boxed{m_c(m_c) = 1.37(9)\ \mathrm{GeV}}$ [Peñarrocha,Schilcher'01]

Determination of m_b: SVZ-SR

$m_b(m_b)$ for n=1,2,3 and 4 in LO, NLO and NNLO



$$\boxed{m_b(m_b) = 4.191(51)\ \text{GeV}}$$

[Kühn,MS'01]

$$M_b^{(2-\text{loop})} = 4.651(57)\ \text{GeV}$$
$$M_b^{(3-\text{loop})} = 4.819(57)\ \text{GeV}$$

Determination of m_b: Υ sum rules

$\mathcal{M}_n^{\text{th}}$: non-relativistic expansion of $\Pi(q^2)$;
corrections to Coulomb Green function up to NNLO

$$\mathcal{M}_n^{\text{th}} = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi(q^2)\Big|_{q^2=0} \sim \frac{1}{m_b^{(2n)}}$$

$$\Pi(q^2) = \frac{N_c}{2m_b^2}\left[C_h(\alpha_s)G(0,0,k) + \ldots\right]$$

$$k = \sqrt{m_b^2 - q^2/4}$$

C_h: hard (pert.) coefficient; matching
$G = G_C + \Delta G$: non-rel. Green function
G_C: Coulomb Green function
ΔG: from non-rel. pert. theory

$\mathcal{M}_n^{\text{exp}}$: resonances: $\Upsilon(1S),\ldots,\Upsilon(6S)$
continuum contribution: small

$$\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_b(s) = \mathcal{M}_n^{\text{exp,res}} + \mathcal{M}_n^{\text{exp,cont}}$$

Determination of m_b: Υ sum rules

n large
- higher sensitivity to m_b
- suppress continuum
- α_s/v terms need to be resummed (NRQCD)

n small
- suppress non-perturbative effects
- ultra-soft scale ($m_b v^2$; $v \sim 1/\sqrt{n}$) needs to be perturbative

➪ $n = 5, \ldots, 10, \ldots, 20$

Analysis performed by several groups; differences:
- choice of n
- regularization scheme
- treatment of renormalization scale μ
- implementation of resummation
- intermediate mass definition (pole mass or related one)

	$m_b(m_b)$ (GeV)	
[Penin,Pivovarov'99]	4.21(11)	$n = 8, \ldots, 12$
[Melnikov,Yelkhovsky'99]	4.20(10)	$n = 14, 16, 18$
[Beneke,Signer'99]	4.25(8)	$n = 10$
[Hoang'00]	4.17(5)	$n = 4, \ldots, 10$

[Application to charm system: [Eidemüller,Jamin'01]]

Determination of m_b: Υ-SR with $q^2 \neq 0$

- slight modification: evaluate moments for $q^2 = -4m_b^2\xi$

$$\mathcal{M}_n(\xi) = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi(q^2)\Big|_{q^2=-4m_b^2\xi} = \int \frac{\mathrm{d}s}{(s+4m_b^2\xi)^{n+1}} R_b(s)$$

- $\xi \nearrow$: enhance higher bound states and continuum
- claim: $\xi \nearrow$: improve theoretical expansion

$\Longrightarrow$ $\boxed{m_b(m_b) = 4.24(10)\ \text{GeV}}$ $(n = 5, \ldots, 10)$ [Eidemüller'02]

Comparison: m_c

Method	$m_c(m_c)$ (GeV)	
Chauchy SR	1.37 ± 0.09	[Penarrocha,Schilcher'01]
SVZ-SR, NNLO	1.304 ± 0.027	[Kühn,MS'01]
Lattice, quenched	$1.26 \pm 0.04 \pm 0.12$	[Becirevic et al.'01]
Lattice, quenched	1.301 ± 0.034	[Rolf,Sint'02]
SR, NRQCD, NNLO	1.23 ± 0.09	[Eidemüller,Jamin'01]
SR, NRQCD, NNLO, $q^2 \neq 0$	1.19 ± 0.11	[Eidemüller'02]
SR, NNLO, $q^2 \neq 0$, high moments, NO resummation	1.275 ± 0.015	[Ioffe,Zyablyuk'02]

Comparison: m_b

Method	$m_b(m_b)$ (GeV)	
Υ-SR, NNLO	4.21 ± 0.11	[Penin,Pivovarov'99]
Υ-SR, NNLO	4.20 ± 0.10	[Melnikov,Yelkhovsky'99]
Υ-SR, NNLO	4.25 ± 0.08	[Beneke,Signer'99]
Υ-SR, NNLO	4.17 ± 0.05	[Hoang'00]
Υ-SR, NNLO, $q^2 \neq 0$	4.24 ± 0.10	[Eidemüller'02]
SVZ-SR, NNLO	4.191 ± 0.051	[Kühn,MS'01]
$\Upsilon(1S)$, NNLO	$4.210 \pm 0.090 \pm 0.025$	[Pineda'01]
$\Upsilon(1S)$, NNNLO	4.346 ± 0.070	[Penin,MS'02]

IV. Conclusions and Outlook

Conclusions/Outlook

$R(s)$ — experiment

"precision" data:

"precision" data:	$\lesssim 1$ GeV	0.6%	CMD-2
	$3.7 - 5$ GeV	4%	BES
	10.52 GeV	2%	CLEO

$R(s)$ — theory

"precision" calculations:

$\mathcal{O}(\alpha_s^2)$: full quark mass dependence

$\mathcal{O}(\alpha_s^3)$: $\alpha_s^3 \times \left\{1, \frac{m_q^2}{s}, \left(\frac{m_q^2}{s}\right)^2\right\}$

$\mathcal{O}(\alpha_s^4)$: $\alpha_s^4 \times \{n_f^3, n_f^2\}$

Applications

"precise" physics:

$$
\begin{aligned}
\delta a_\mu^{\text{had}} &\approx 7 \times 10^{-10} \\
\delta \alpha_{\text{had}}^{(5)} &\approx 2 \times 10^{-4} \\
\delta \alpha_s(M_Z) &\approx 0.01 \\
\delta m_c(m_c) &\approx 30 \text{ MeV} \\
\delta m_b(m_b) &\approx 50 \text{ MeV}
\end{aligned}
$$

Conclusions/Outlook

needed/desirable in future:

check of CMD-2 and BES data
measurement of $R(s)$ around charm/bottom
threshold region with 1-2% accuracy (CLEO (?))
measurement of $\Gamma_{ee}(J/\Psi)$ and $\Gamma_{ee}(\Psi')$
with 2% accuracy (CLEO (?))
completion of massless order α_s^4 result
full quark mass dependence at order α_s^3

- ➪ Check of pQCD
- ➪ $\Delta\alpha_{\rm had}^{(5)}$: check/replace pQCD-input
- ➪ $\delta\alpha_s(M_Z)$: comparable to current world average
- ➪ δm_c and δm_b can significantly be reduced