DESY Theory Workshop 2002 "Quantum Chromodynamics", Hamburg, 25 September 2002

Non-relativistic effective theory and top quark production

Thomas Teubner (CERN–TH)

I. Why effective field theories for $Q\bar{Q}$?

II. A short introduction to Non-Relativistic QCD

III. NRQCD and $t\bar{t}$ production near threshold

- $\sigma(e^+e^- \to t\bar{t}\,)$ at NNLO
- Short Distance mass schemes
- Renormalization Group improvement: NNLL in vNRQCD
- First steps towards N^{3}LO
- Determination of m_t, α_s, Γ_t and y_t at TESLA

IV. Summary/Outlook

I. Why effective field theories for $Q\bar{Q}$?

- $Q\bar{Q}$ very suitable to measure quark masses and also the strong coupling.
- Many very different systems: from non-relativistic $t\bar{t}$, over $c\bar{c}$ to relativistic $q\bar{q}$ bound states; from perturbative to non-perturbative, hence

▶ ideal 'laboratories' to quantitatively study QCD dynamics from the *perturbative* to the *confinement* regime.

- But $Q\bar{Q}$ dynamics is a multi-scale problem:

  $m_Q,\ \boldsymbol{p} \sim m_Q v,\ E \sim m_Q v^2$

  and (quasi) bound states are described by potentials in QM.

▶ Application of fixed order Perturbation Theory not feasible! (Phenomenological) Potential models unable to *systematically* describe relativistic corrections.

- Effective Field Theories provide the framework to
  - ⋆ separate dynamical from non-dynamical scales, reducing the difficulty of single Feynman-graphs
  - ⋆ sum classes of graphs in an efficient and transparent way
  - ⋆ determine the scales of the couplings involved
  - ⋆ provide a systematic description of $Q\bar{Q}$ systems in QCD.

II. A short introduction to NRQCD

- For a Coulombic system $v \sim \alpha_s \ll 1$ (α_s/v terms leading to the LeadingOrder Coulomb Green function) and we have to expand in both, v and α_s.
- There are (at least) the three important scales m_Q (hard), $\boldsymbol{p} \sim m_Q v$ (soft), $E \sim m_Q v^2$ (ultrasoft), but only the last one has to be treated as dynamical in a non-rel. system.

How to build the Effective Field Theory?

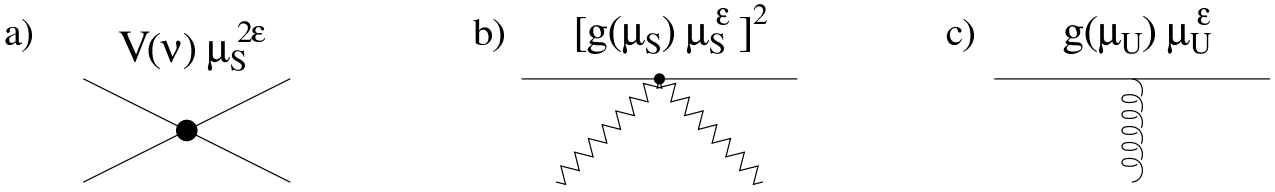
Caswell, Bodwin, Braaten, Lepage, Labelle, Grinstein, Rothstein, Luke, Manohar, Savage, Pineda, Soto, Brambilla, Vairo, ...

1. Identification of momentum regions — Beneke, Smirnov

region	$(k^0, \boldsymbol{k})$	on-shell d.o.f.
hard	(m, m)	
soft	(mv, mv)	Gluons A_p^μ
potential	(mv^2, mv)	Quarks ψ_p, χ_p
ultrasoft	(mv^2, mv^2)	Gluons A^μ

2. Write down the most general $\mathcal{L}$ for the on-shell d.o.f.

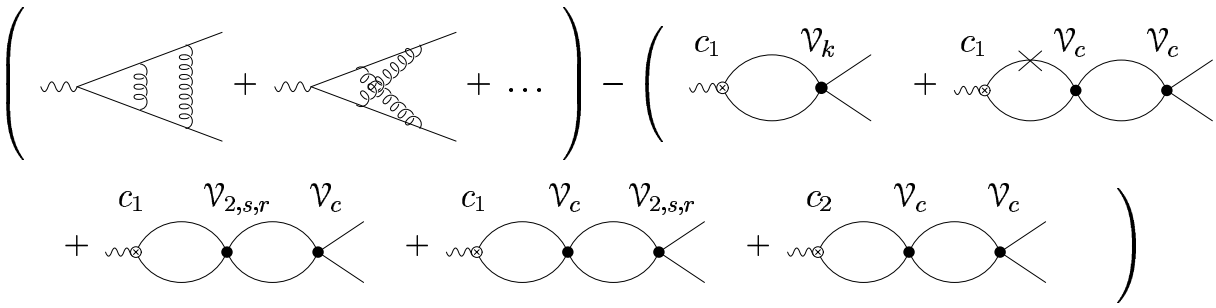
Examples of a) potential, b) soft and c) ultrasoft interactions in vNRQCD:



3. Match the EFT to full QCD at a high scale $\mu = m$

$\rightarrow$ 'integrate out' hard effects, soft quarks

Example for a matching calculation:



Difference of full QCD and order $\alpha_s^2 v^0$ EFT graphs which gives the two loop matching for $c_1(1)$. The $\times$ denotes an insertion of the $\boldsymbol{p}^4/(8m^3)$ operator, and graphs with this insertion on a different propagator are understood.

• Soft gluons are integrated out (cannot be on-shell in a non-rel. $Q\bar{Q}$ system)

$\rightarrow$ matrix elements can be calculated by solving a Schrödinger Equation.

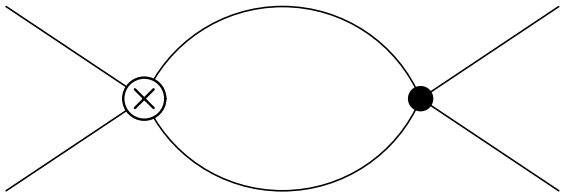
4. Calculate RenormalizationGroupEquations of all EFT operators (needed for the process up to a given order in the power-counting).

5. Evolve from the high matching down to the low dynamical scale

$\rightarrow$ large logarithms are absorbed ('*summed*') in the potentials and matching coefficients.

pNRQCD and vNRQCD are not equivalent:

- vNRQCD for *dynamical* quarks, keeps the correlation of scales $\mu_u = \mu_s^2/m$.
- pNRQCD for *static* quarks, breaks this by $1/m$ expansion (energy and three-momentum are treated independently). Consistently applicable for dynamical quarks?
- Consider the time ordered product of $1/(m|\boldsymbol{k}|)$ and $1/|\boldsymbol{k}|^2$ potentials:



Static: $$\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{m|\boldsymbol{k}|}\,\frac{1}{(k_0+i\delta)}\,\frac{1}{(k_0-i\delta)}\,\frac{1}{|\boldsymbol{k}|^2} \sim \frac{1}{m}$$

Dynamical:

$$\int \cdots \frac{1}{(k_0-\boldsymbol{k}^2/(2m)+i\delta)}\,\frac{1}{(k_0+\boldsymbol{k}^2/(2m)-i\delta)} \cdots \sim$$

$$\int \frac{d^3k}{(2\pi)^3}\,\frac{1}{m|\boldsymbol{k}|}\,\frac{m}{\boldsymbol{k}^2-i\delta}\,\frac{1}{|\boldsymbol{k}|^2} \sim 1$$

- Potentials disagree in higher orders!

III. NRQCD and $t\bar{t}$ near threshold

- Near threshold: $E \equiv m_t v^2 = \sqrt{s} - 2m_t$ small, hence

  $$v \sim \alpha_s \simeq 0.15 \ll 1 \text{ is non-relativistic}$$

  (the only truely non-rel. $Q\bar{Q}$ system we have..) and the Coulombic Singularities $\sim (\alpha_s/v)^n$ have to be summed.

- Power Counting: $R = \sigma_{t\bar{t}}/\sigma_{\mu^+\mu^-} = v \cdot$
  $\sum_n \left(\frac{\alpha_s}{v}\right)^n \left[LO\,\{1\}, NLO\,\{v, \alpha_s\}, NNLO\,\{v^2, \alpha_s v, \alpha_s^2\}\right]$

- Large hierarchy of scales in the $t\bar{t}$ system:
  $m_t \sim 175\ \mathrm{GeV} \gg m_t v \sim 25\ \mathrm{GeV} \gg m_t v^2 \sim 4\ \mathrm{GeV} \gg \Lambda_{QCD}$,
  and the (quasi-) bound state dynamics can be treated perturbatively.

- And: the large top decay width $\Gamma_{t\to Wb} \sim 1.5$ GeV provides an additional cut-off scale for non-perturbative effects, no hadronization effects!
  The price: Resonances get smeared out, no real *toponium spectroscopy*, no 'non-abelian positronium' :–(

- Treatment of unstable top quarks up to now:

  $$E \to E + i\Gamma_t$$

$\sigma(e^+e^- \to t\bar{t}\,)$ at NNLO

- The cross section is actually calculated by solving a Schrödinger Equation (in position or momentum space).
  Many people contributed: Fadin+Khoze ('87, LO), Strassler+Peskin ('91, NLO), many detailed studies: Jeżabek+Kühn+T, Sumino, Fujii, Hagiwara, Matsui, Murayama, Ng, ...

- At NNLO (for names see below..):

$$\left[-\frac{\boldsymbol{\nabla}^2}{m_t} - \frac{\boldsymbol{\nabla}^4}{4m_t^3} + V_c(\boldsymbol{r}) + V_{BF}(\boldsymbol{r}) + V_{NA}(\boldsymbol{r}) - (E + i\Gamma_t) \right] \cdot$$

$$G(\boldsymbol{r}, \boldsymbol{r}', E + i\Gamma_t) \;=\; \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}')$$

- The Coulomb-, Breit-Fermi- and Non-Abelian Potentials are derived in (p)NRQCD, where the matching (to full QCD) and regularization has to be carried out, too:

- $R = \textit{matching-coeffs.}(\Lambda) \cdot \mathrm{Im}G(\boldsymbol{r} = 0, \boldsymbol{r}' = 0, E + i\Gamma_t, \Lambda)$

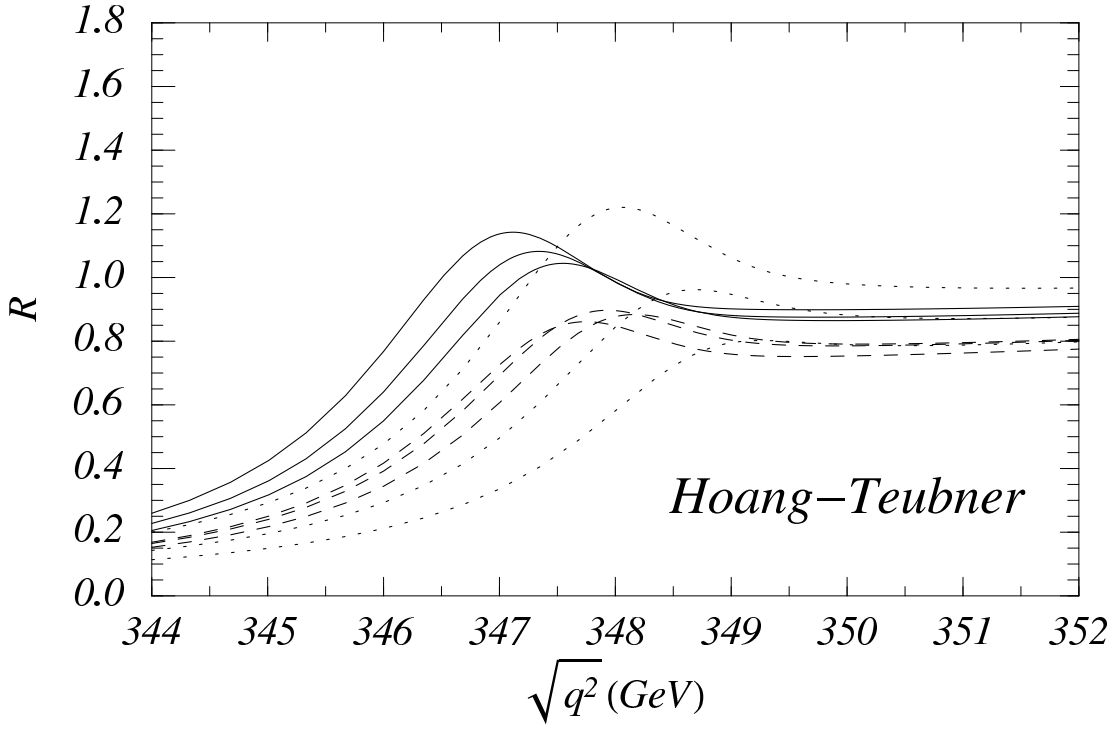
  For the total cross section:

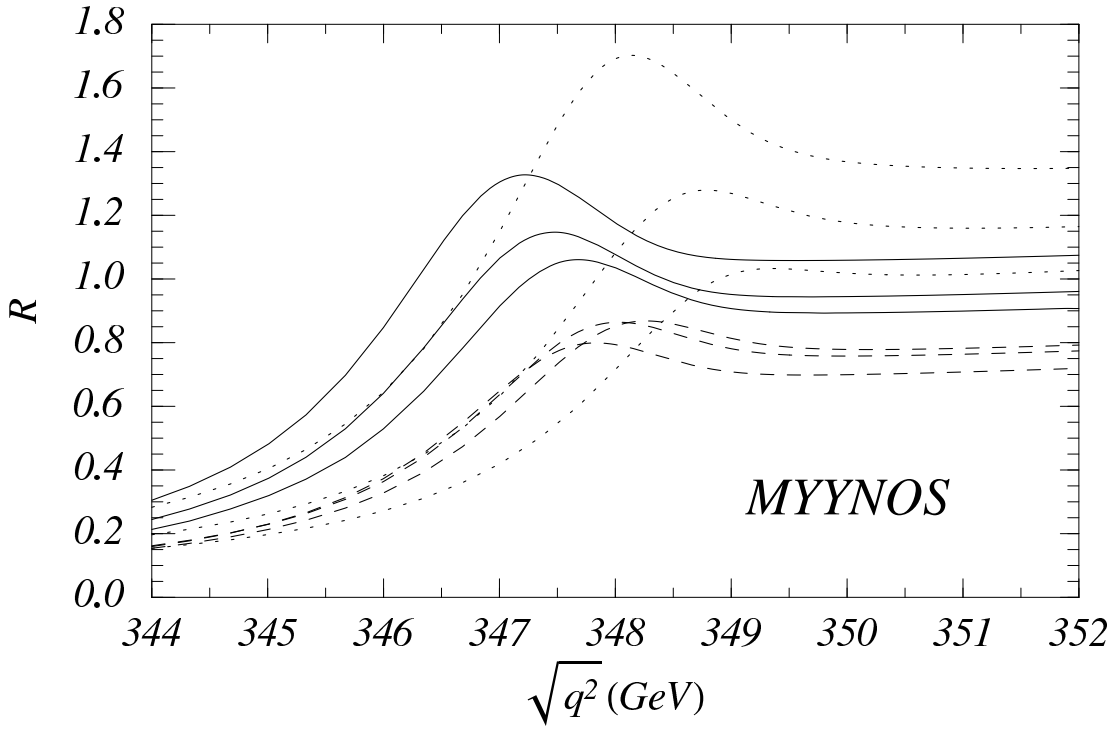
  $\sigma_{tot}^{\gamma,Z}(s) = \sigma_{pt}\left[F^v(s)R^v(s) + F^a(s)R^a(s)\right]$

- One observes large corrections to the normalization of the cross section and to the position of the $1S$ peak.

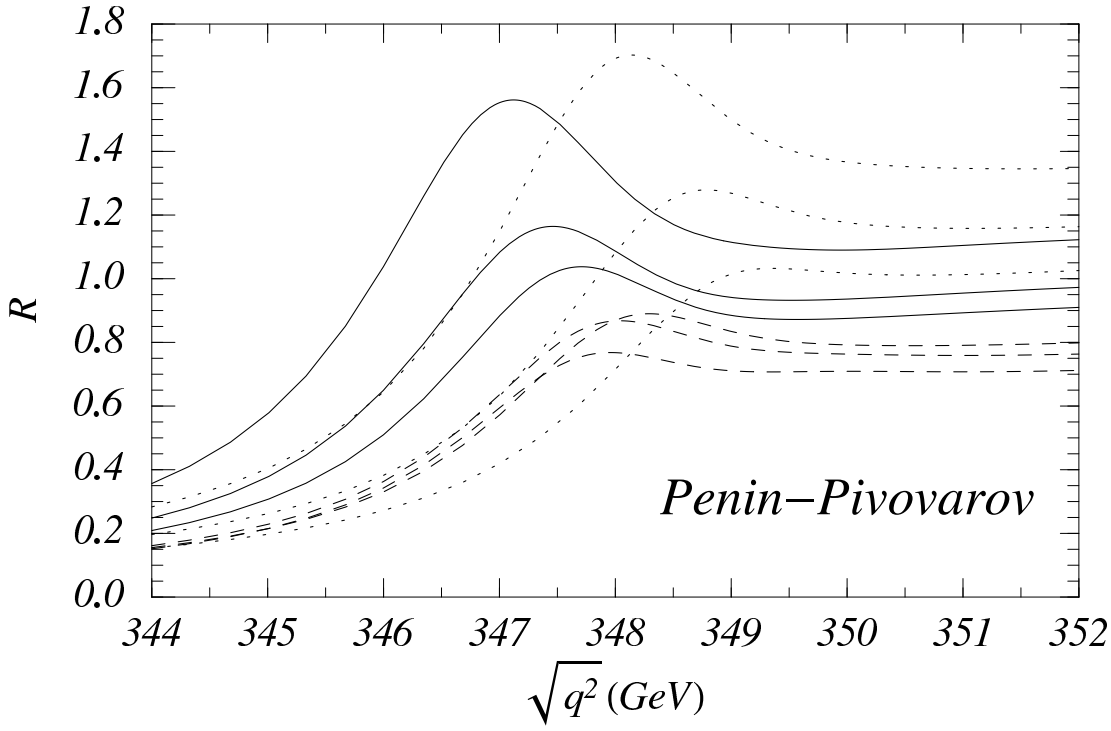
- There is also a large scheme dependence and a large uncertainty from varying the renormalization scale(s). $\longrightarrow$

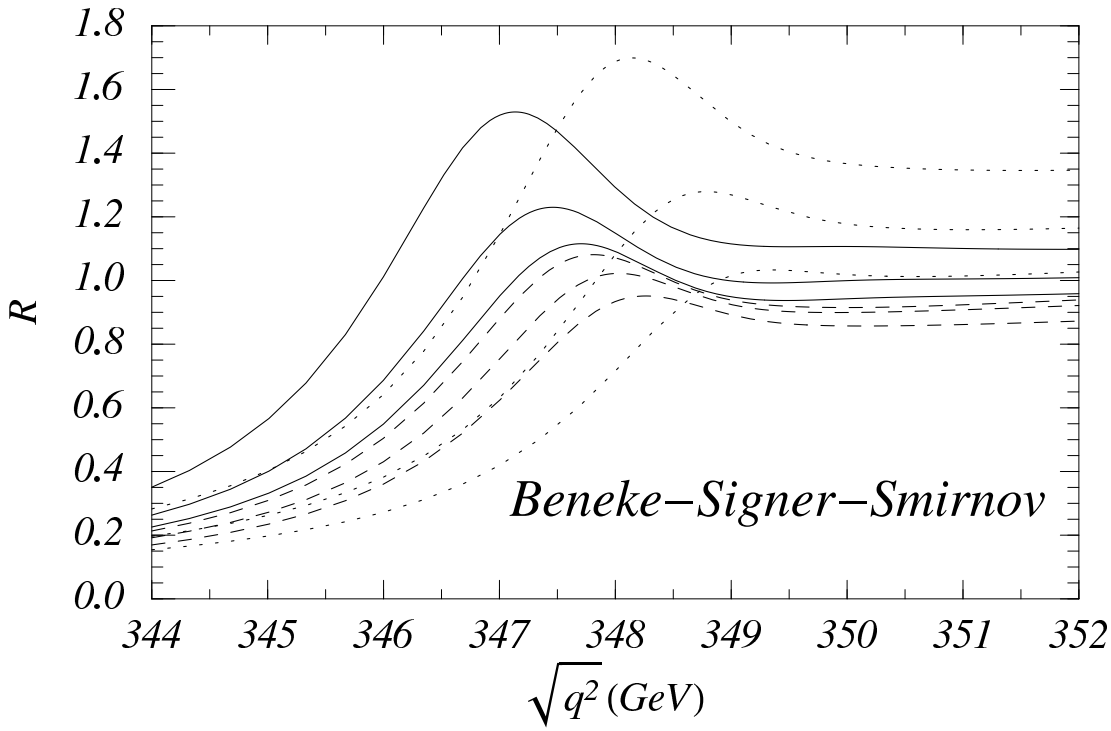
Four groups did independent calculations for $R_{t\bar{t}}$ near threshold:
Hoang-T., Melnikov-Yelkhovsky-Yakovlev-Nagano-Ota-Sumino,
Penin-Pivovarov, Beneke-Signer-Smirnov





Pole mass scheme; dotted: LO, dashed: NLO, solid: NNLO





Pole mass scheme; dotted: LO, dashed: NLO, solid: NNLO

($M_t^{\rm pole} = 175.05$ GeV, $\alpha_s(M_Z) = 0.119$, $\Gamma_t = 1.43$ GeV $\mu_{\rm soft} = 15, 30, 60$ GeV)

▶ Are peak-shifts threatening a precise mass determination from a threshold scan?

Short Distance mass schemes

- The source of the problem of the large peak shifts:
  IR sensitivity and strong correlations $m_t \leftrightarrow \alpha_s$
- On one hand:
  $\star$ threshold regime, non-rel. system, large mass, formulation of EFT: m^{pole} is very useful, and
  $\star$ m^{pole} is IR finite and gauge-independent to all orders in pQCD. Kronfeld
- On the other hand:
  $\star$ m^{pole} is NOT an observable!
  $\star$ It is defined only up to IR ambiguities of $\mathcal{O}(\Lambda_{QCD})$ ($\rightarrow$ confinement), and the large width Γ_t does not protect m_t^{pole}. Willenbrock et al.
- But: Energy of the $1S$ resonance is (in principle) an observable and unambiguous:

$$E_{static}(r) = 2m_t^{pole} + V(r)$$

  Cancellation of the (leading) IR Renormalon contributions between mass and Potential! Beneke; Hoang et al.

▶ Use a scheme where also the single contributions m, V are NOT IR-sensitive (Short-Distance).

There are various schemes available:

- 'Kinetic mass' ('Low-scale running mass') Bigi et al.

$$m^{kin}_{t,\mu_f^{kin}} = m_t^{pole} - \frac{16}{9}\frac{\alpha_s}{\pi}\mu_f^{kin} + \dots$$

(derived originally to improve PT for semileptonic B-decays)

- 'Potential Subtracted' mass $m^{PS}(\mu_f)$: Beneke

subtracts universal IR-contributions from the Coulomb-Pot.:

$$m^{PS}(\mu_f) = m^{pole} - \delta m(\mu_f)\,,$$

$$\delta m(\mu_f) = -\frac{1}{2}\int_{|q|<\mu_f}\frac{d^3q}{(2\pi)^3}\tilde{V}(q)$$

- $1S$ mass Hoang et al.

m_t^{1S} := 'half of thr perturbative mass of the fictitious ($\Gamma_t \to 0$) $t\bar{t}$ $1S$ ground state'

$\star$ quasi-observable, defined perturbatively, no extra scale

$\star$ subtraction of the same universal IR contributions

- Connection with the usual $\overline{MS}$ mass:

Steinhauser+Chetyrkin, Melnikov+Ritbergen

start e.g. from $m_t^{\overline{MS}}(m_t^{\overline{MS}}) = 165$ GeV, then

$m_t^{pole} = [165.0 + 7.6 + 1.6 + 0.51]$ GeV

$m_t^{PS}(\mu_f = 20\ \text{GeV}) = [165.0 + 6.7 + 1.2 + 0.28]$ GeV

for the LO, NLO, NNLO corrections.

- Or, from $m_t^{1S,exp.} = 175 \pm 0.1$ GeV:

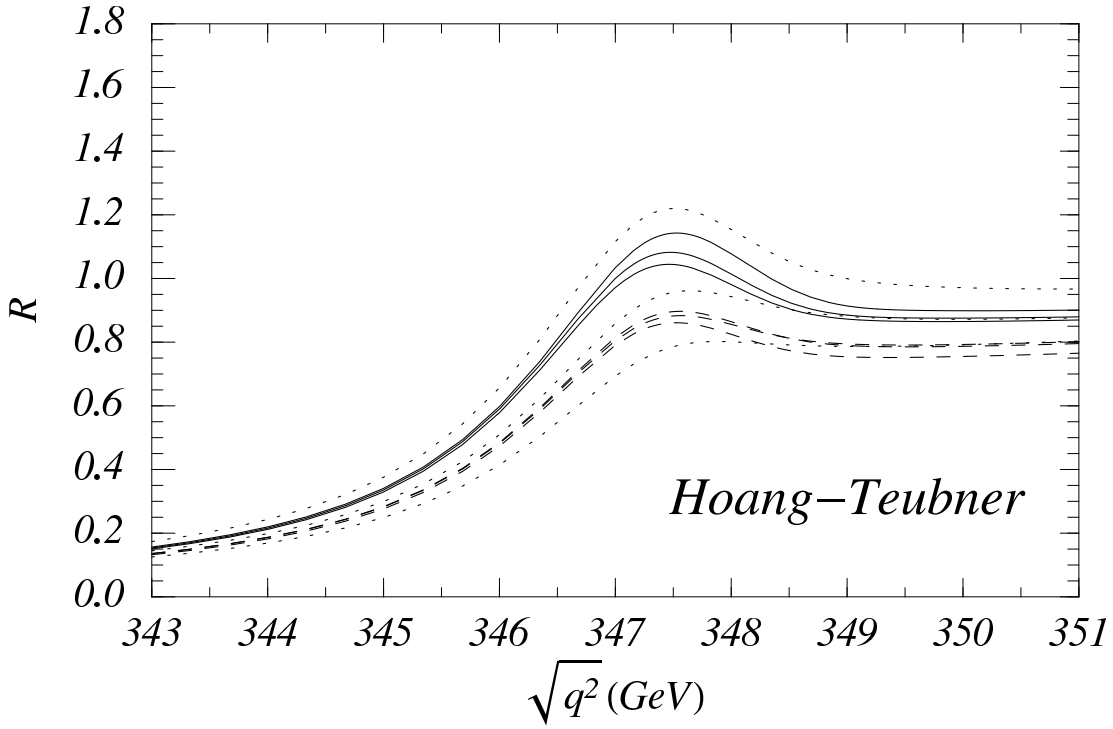
$m_t^{\overline{MS}} = [175 - 7.58 - 0.96 - 0.14 \pm 0.1 \pm x \cdot 0.007]$ GeV

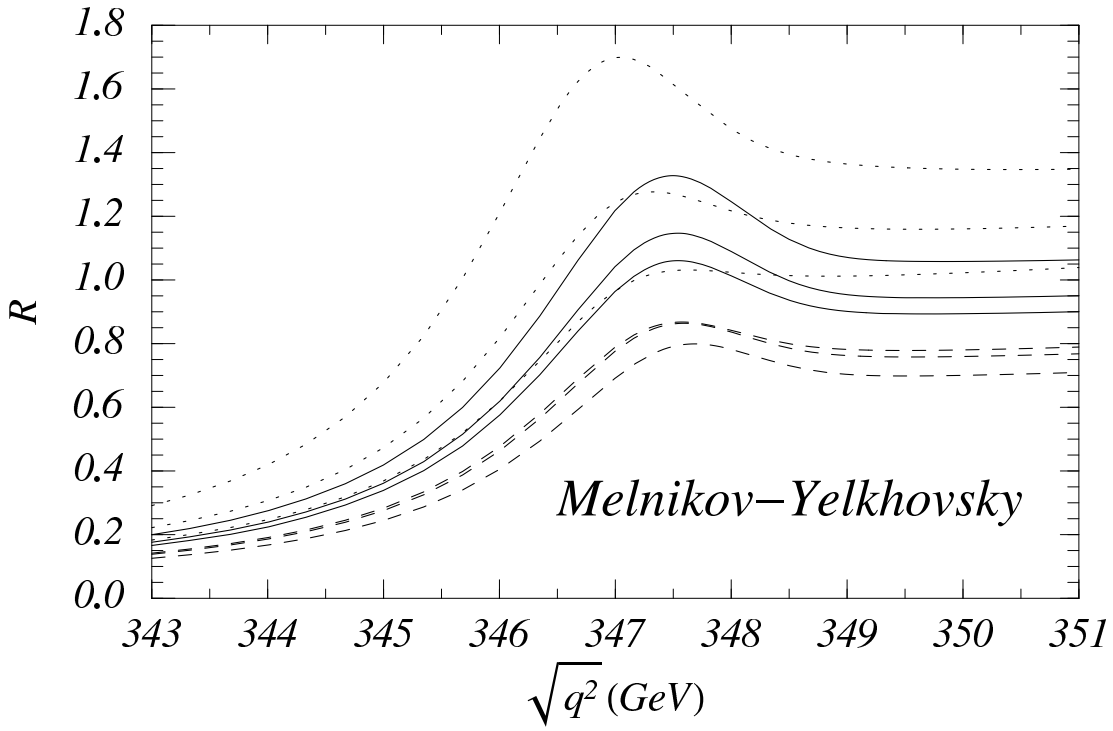
(for $\alpha_s(M_Z) = 0.118 \pm x \cdot 0.001$)

Results within Short Distance mass schemes:

Hoang-Teubner: 1S mass,

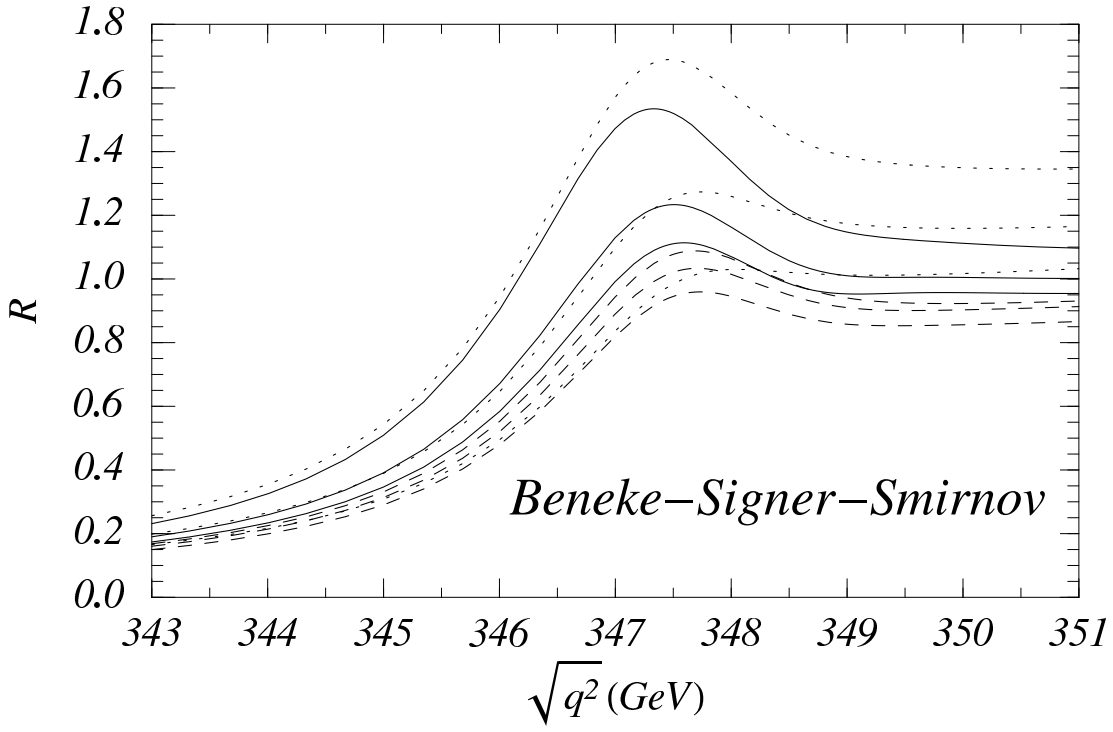
Melnikov-Yelkhovsky: kinetic mass (at scale 15 GeV)

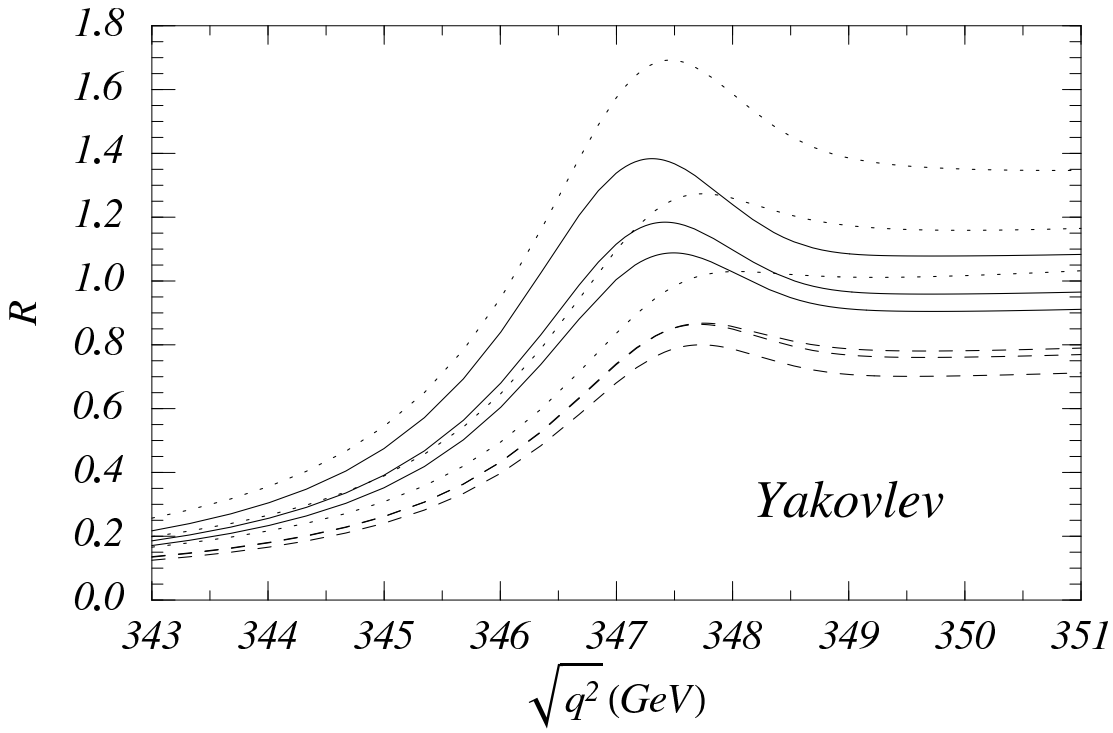




SD mass schemes; dotted: LO, dashed: NLO, solid: NNLO

Beneke-Signer-Smirnov, Yakovlev: PS mass (at 20 GeV)





SD mass schemes; dotted: LO, dashed: NLO, solid: NNLO

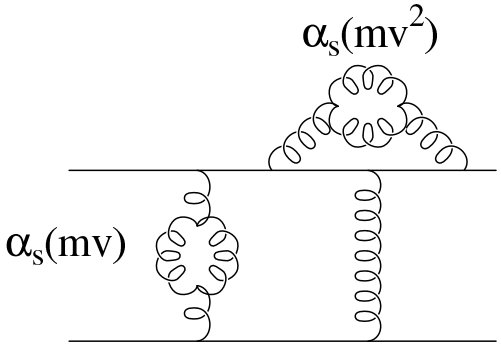
RG improvement: $t\bar{t}$ at NNLL in vNRQCD

- What is wrong here with PT? The NNLO corrections and the scale dependence of the cross section are quite large.
- BUT: Large scale hierarchy leads to sizeable

$$\log\frac{m_t}{p},\ \log\frac{p}{E},\ \log\frac{m_t}{E}$$

[remember the Lambshift: $\delta E \sim \alpha^5 \log\alpha \sim$ NNLL]

- AND: The correct choice of the (correlated) scales in α_s is important! One graph can involve more than one scale. e.g.



- Need EFT to consistently resum these log's by running from the high matching scale m_t to the low '*dynamical scales*' $m_t v$, $m_t v^2$.
- This is achieved in velocity NRQCD

Luke, Manohar, Rothstein, Stewart

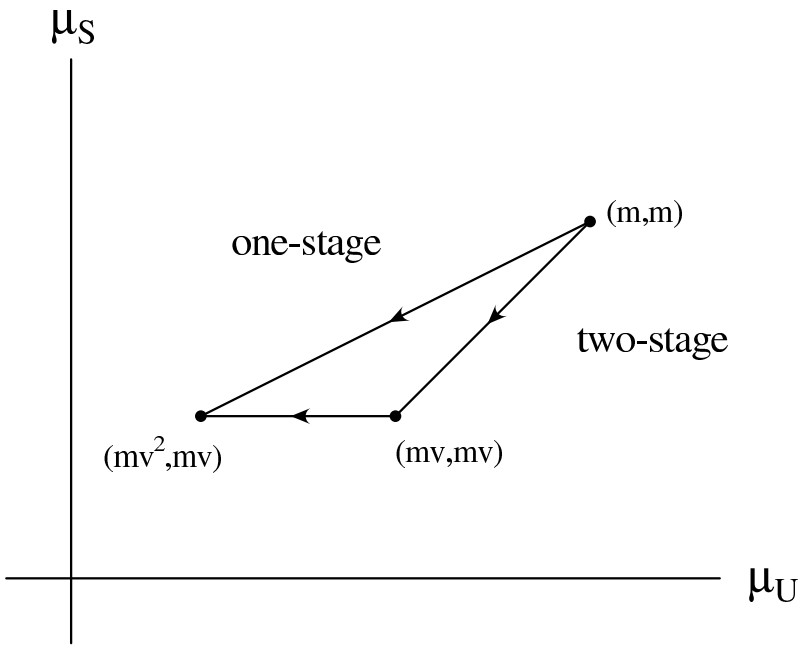
through running from $\nu = 1 \quad \rightarrow \quad \nu \approx \langle v \rangle_{t\bar{t}} \simeq \alpha_s$

$\star$ Correct running of *correlated* scales $mv^2 = (mv)^2/m$ is automatic in vNRQCD (nontrivial!)

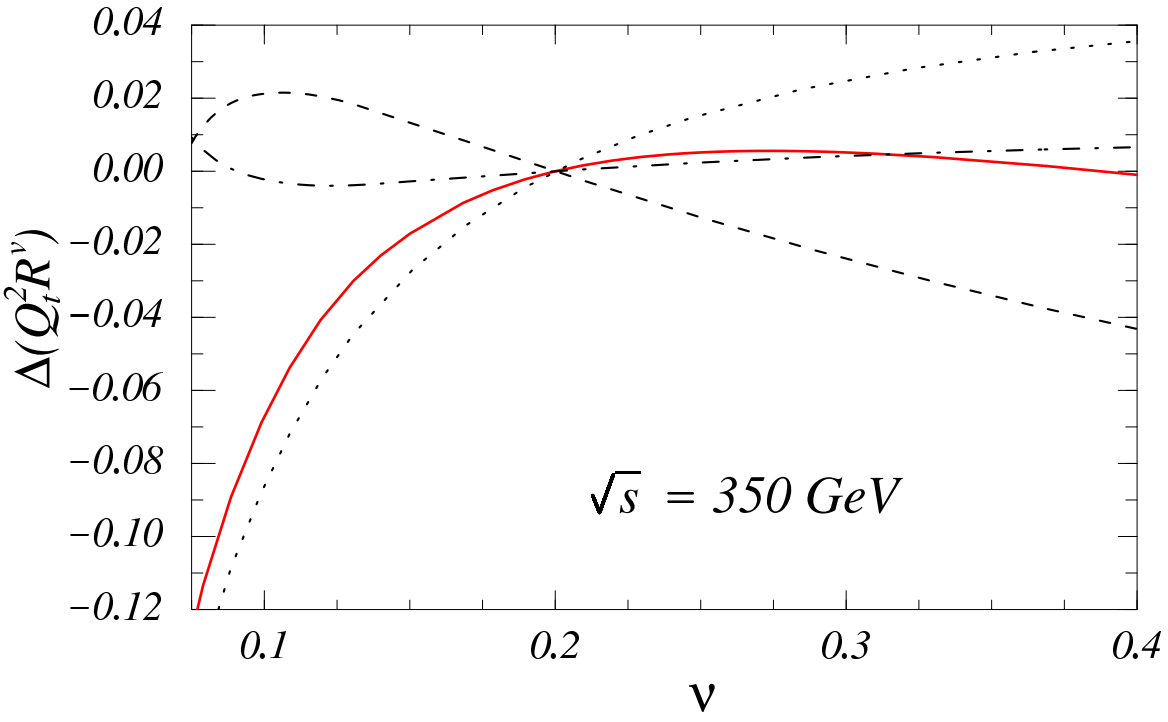
Renormalization Group Evolution:

- pNRQCD: start at $\mu = m$, scale μ down to mv, integrate out the soft d.o.f. at mv, scale down to mv^2.
- Velocity RG in vNRQCD: two scales (coupled by E.o.M.) $\mu_{soft} = m\nu$, $\mu_{ultrasoft} = m\nu^2$. Scale down *together* from $\nu = 1$ to $\nu = v$.
- Leads in general to different results (if anomalous dimensions are not constant but themselves dependent on running couplings).

  One- versus two stage running in the μ_u – μ_s plane:

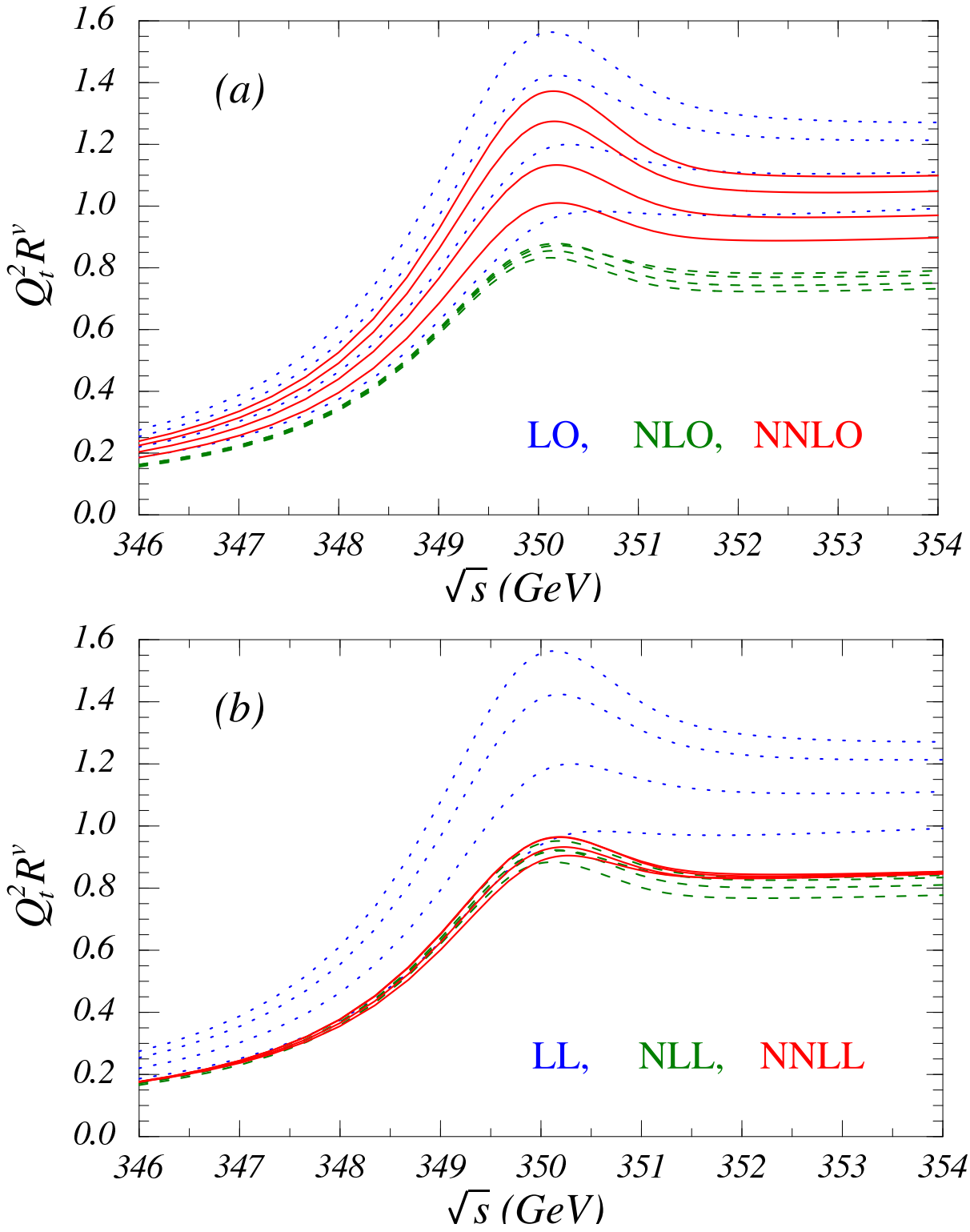


- The RGE running results in sizeable effects which partially cancel, so that σ_{tot} is much more stable at NNLL order.



Relative scale dependence of various NNLL contributions to $Q_t^2 R^v$. The contributions were divided into those from $G^c_{v,m,\nu}$ (dashed lines), δG^k (dotted line) and the sum of $\delta G^{\delta,r,kin,1S}$ (dot-dashed line), while the solid red line denotes the sum of these terms.

Vector current induced cross section $Q_t^2 R^v$ with fixed M_t^{1S} mass in a) 'fixed order', b) RG improved in vNRQCD (ν is varied between 0.1, 0.125, 0.2 and 0.4): HMST



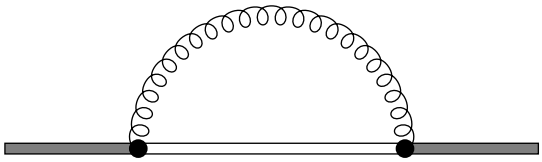
[For NNLL the Wilson coefficient of one current is only known at NLL but the missing effects are estimated to be within quoted error bands.]

First steps towards N^{3}LO

Kniehl, Penin, Smirnov, Steinhauser

- N^{3}LO Hamiltonian of potential NRQCD derived

  $\rightarrow$ relativistic corrections from emission and absorption of ultrasoft gluons at LO, via the chromoelectric dipole interaction $g_s \boldsymbol{r} \cdot \boldsymbol{E}$



$$\delta G^s(\boldsymbol{r},\boldsymbol{r}',E) = -\int \mathrm{d}^3\boldsymbol{r}'' G_C^s(\boldsymbol{r},\boldsymbol{r}'',E)\mathcal{H}^{\mathrm{us}}(r'')G_C^s(\boldsymbol{r}'',\boldsymbol{r}',E)$$
$$+ \text{ contact terms}$$

  $\rightarrow$ these *'retardation'* effects cannot be interpreted as instantaneous potentials.

- Matching performed by help of the threshold expansion.
- As a first application: $Q\bar{Q}$ energy spectrum for vanishing QCD beta function (non-RG corrections).

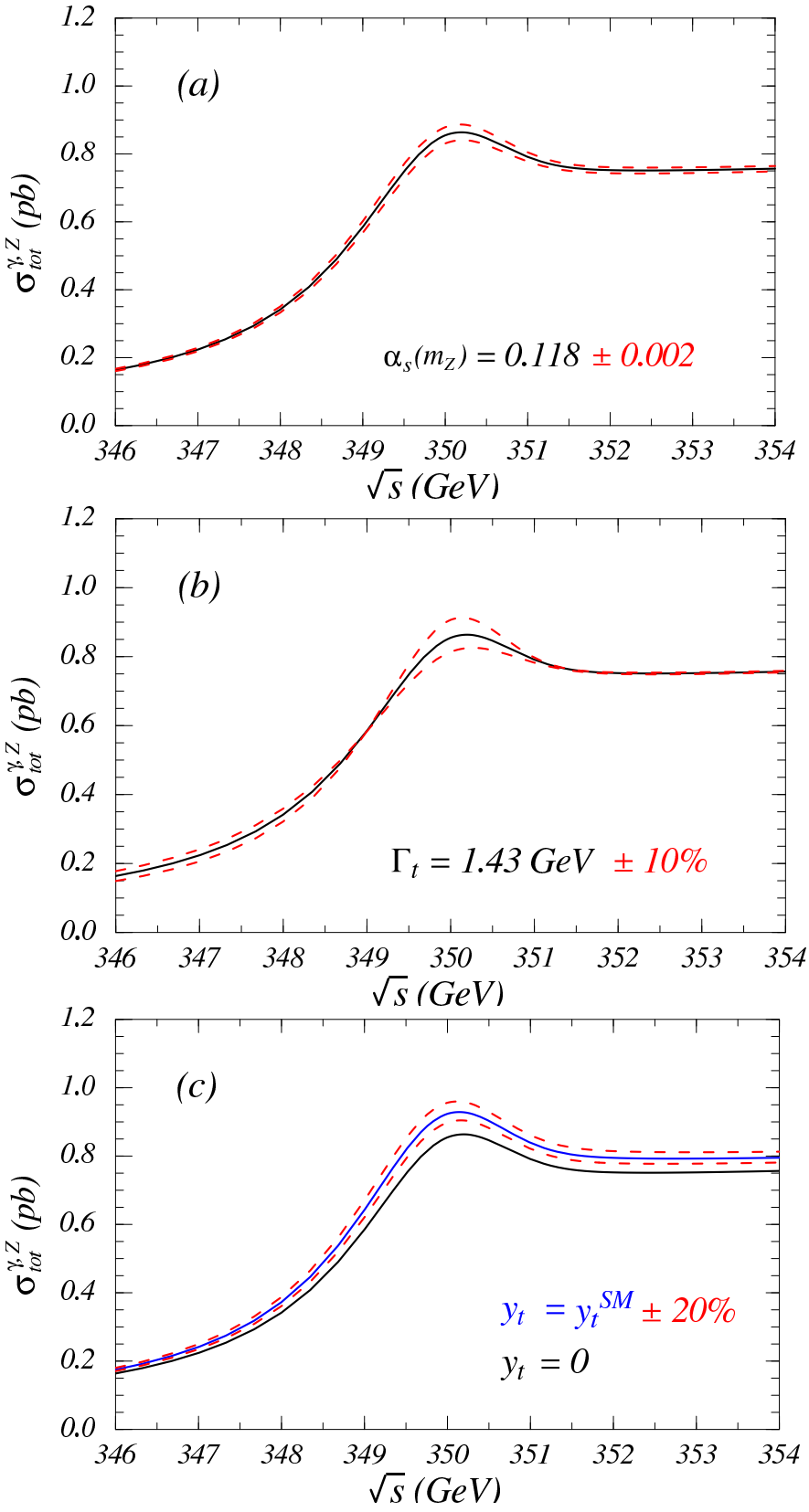
▶ The numerical result for the $1S$ energy in N^{3}LO reads (a_3 term Padé estimated):

$$E_1 = -1.53 \text{ GeV} \times \Big[1 + 0.448 + 0.322$$
$$+ (0.006 + 0.011|_{a_3} + 0.073|_{\beta_0^3}) + \ldots\Big],$$

a shift of m_t^{pole} from NNLO to N^{3}LO by -140 MeV.

Determination of m_t, α_s, Γ_t and y_t at TESLA

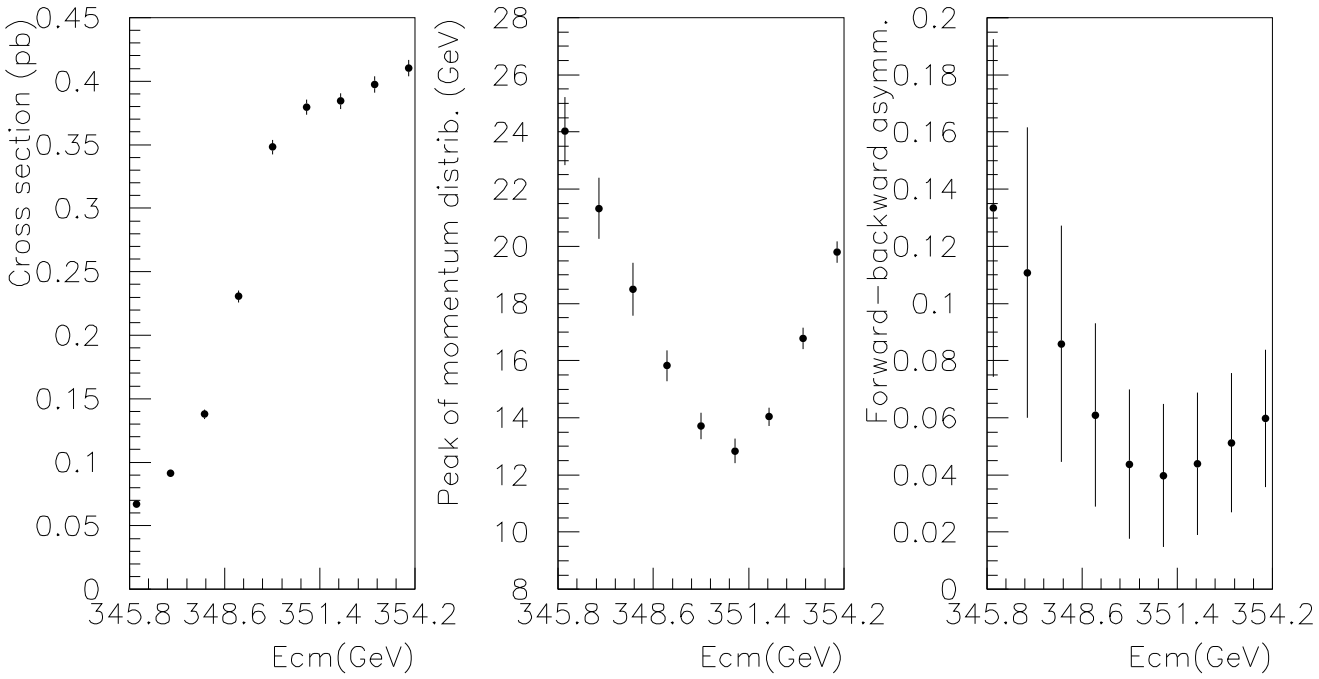
'Th'-dependences of σ_{tot} on (a) α_s, (b) Γ_t and (c) y_t: (HMST)



Scenario for a 9 point threshold scan (total $\mathcal{L} = 300$ fb^{-1}):

Martinez + Miquel

σ_{tot}, top quark momentum distribution and Forward-Backward Asymmetry as observables, with initial state radiation and beam smearing effects taken into account:



• Experimental accuracy possible at TESLA (multi-parameter fit, 3% TH-error on the total cross section assumed):

$\Delta m_t \sim 20$ MeV, $\Delta\Gamma_t \sim 30$ MeV, $\Delta\alpha_s \sim 0.0012$,
$\Delta y_t/y_t \sim 35\%$ (if $\Delta\alpha_s = 0.001$ from elsewhere).

▶ Further studies for the TH-predictions welcome to scrutinize the theoretical uncertainties.

IV. Summary/Outlook

- Effective Field Theories *which are fully equivalent to full QCD* have been derived to describe systematically the dynamics of heavy $Q\bar{Q}$ systems.
- They simplify the calculation of higher order corrections by separating scales, integrating out non-dynamical d.o.f. and allowing for the determination of the scales of α_s.
- Top quark pair production at threshold has been calculated within EFTs called potential NRQCD and velocity NRQCD up to including next-to-next-to-leading order.
- With additional help of Short Distance mass schemes the top quark mass can be measured with sub-per-mille accuracy at a future lepton collider like TESLA.

⋆ Within vNRQCD a Renormalization Group improvement up to next-to-next-to-leading logarithmic order has been achieved. This stabilizes the large normalization corrections present in fixed order, making a combined determination of m_t, Γ_t, α_s and y_t look possible.

⋆ The N^{3}LO Hamiltonian of pNRQCD has been derived together with corrections to the energy spectrum.

▶ **For the future:**

⋆ The stage is set for a N^{3}LO calculation of the full $t\bar{t}$ line-shape.

• The Renormalization Group improvement should be worked out completely in pNRQCD and compared to vNRQCD.

♢ Distributions should be computed with the same (parametric) accuracy using EFTs.

♣ Conceptually the treatment of unstable particles has to be worked out in the framework of EFTs. This might be of importance numerically for the case of $t\bar{t}$ production.