SO(10) Orbifold GUTs

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Why orbifold GUTs?

- **Attractive SU(5) model in 5d** (Kawamura '00)
  (Altarelli,Feruglio '01, Kobakhideze '01, Kawamoto,Kawamura '01, Hall, Nomura '01, Hebecker, March-Russell '01, $\cdots$)

- **Extra dimension is compactified on orbifold**
  (Dixon, Harvery, Vafa, Witten '85,...)
  - **GUT symmetry breaking**
  - **Doublet-triplet splitting**
  - **Absence of fast proton decay via dim. 5 ops.**
    (Hall, Nomura '01)
  - $\cdots$

Why SO(10) GUTs?

(Gerogi '74, Fritzsch-Minkowski '75)

- **Unify one family of quarks and leptons including right-handed neutrino N into one 16-plet (q, u^c, e^c, ℓ, d^c, N)**

$\Rightarrow$ **Seesaw** (Yanagida '79, Gell-Mann, Ramond, Rlansky '79)

OBS: Neutrino oscillations

$\Rightarrow$ **Leptogenesis** (Fukugita, Yanagida '86)

OBS: Baryon asymmetry of the Universe

Models of SO(10) orbifold GUTs

In 6 dimensions,

TA, Buchmüller, Covi [hep-ph/0108021, 0204358, 0209144, 0304142]

Hall, Nomura, Okui, Smith [hep-ph/0108071]

Li [hep-ph/0108120, 0112255]

Watari, Yanagida [hep-ph/0108152]

Haba, Kondo, Shimizu [hep-ph/0112132, 0202191]

Babu, Barr, Kyae [hep-ph/0202178]

Hall, Nomura [hep-ph/0207079]

Haba, Shimizu [hep-ph/0210146, 0212384]

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In 5 dimensions,

Dermisek, Mafi [hep-ph/0108139]

Barr, Dorsner [hep-ph/0205088]

Albright, Barr [hep-ph/0209173]

Kyae, Shafi [hep-ph/0211059, 0212331]

Kim, Raby [hep-ph/0212348, 0304104]

Kitano, Li [hep-ph/0302073]

Shafi, Tavartkiladze [hep-ph/0303150]

Kyae, Lee, Shafi [hep-ph/0309205]

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Today's talk

SO(10) Orbifold GUTs

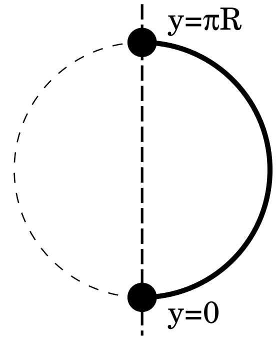
1. SO(10) gauge symmetry breaking

2. Doublet-triplet splitting

3. Fermion masses and mixings

How break SO(10) by orbifold?

Gauge Symmetry Breaking by Orbifold



Let us consider 5d theory with gauge group G compactified on an orbifold S^1/Z_2.

- Z_2 **symmetry:** $x_5 = y$: $y \to y$

- **Parity transformations of vector fields**

$$V_\mu(x,y) \to PV_\mu(x,-y)P^{-1} = +V_\mu(x,y) \quad (\mu = 0,1,2,3)$$

$$V_5(x,y) \to PV_\mu(x,-y)P^{-1} = -V_5(x,y)$$

$$V_\mu = V_\mu^A T^A,\ V_5 = V_5^A T^A\ (T^A : G)$$

- **If P acts on the internal space**

$$PT^aP^{-1} = +T^a\ (T^a : H),\ PT^\alpha P^{-1} = -T^\alpha\ \ (T^\alpha : G/H)$$

we find: [KK mass]

$$V_\mu^a(x,-y) = +V_\mu^a(x,y) \sim \sum v_{\mu,m}^a(x)\cos(my/R), \quad [\frac{m}{R}\ (m \geq 0)]$$

$$V_\mu^\alpha(x,-y) = -V_\mu^\alpha(x,y) \sim \sum v_{\mu,m}^\alpha(x)\sin(my/R), \quad [\frac{m}{R}\ (m \geq 1)]$$

$\Rightarrow$ $G \to H$ **at the fixed point.**

$\Rightarrow$ **Zero modes of V_μ form gauges fields for H.**

Multiplets in the bulk are split in mass!!

Conditions for Z_2 Orbifold Breaking ($G \to H$)

This breaking mechanism is NOT always guaranteed.

Conditions

1. H is symmetric subgroup of G.

$$[T^a, T^{a'}] \subseteq T^{a''}, \; [T^a, T^\alpha] \subseteq T^{\alpha'}, \; [T^\alpha, T^{\alpha'}] \subseteq T^a$$

$$(T^a : H, \; T^\alpha : G/H)$$

2. Rank conserving breaking.

Ex) SU(5)$\to$ G$_{\rm SM}$=SU(3)$\times$SU(2)$\times$U(1)

– $G_{\rm SM}$ is indeed symmetric subgroup of SU(5).

– SU(5) and $G_{\rm SM}$ are rank 4.

What about SO(10)?

Q. *Is it possible by orbifold breaking?*

SO(10)$\to$ G$_{\rm SM}$=SU(3)$\times$SU(2)$\times$U(1)

A. *Unfortunately, the answer is NO!!*

G_{SM}=SU(3)×SU(2)×U(1) is ...

1. NOT a symmetric subgroup of SO(10)

   SU(4)×SU(2)×SU(2) = G_{PS}

   SU(5)×U(1) = G_{GG}

   SO(8)×U(1)

2. rank 4, but SO(10) is rank 5

We found:

- We cannot break SO(10) directly into G_{SM} only by orbifold compactification.
- We have to invoke the conventional Higgs mechanism.
  - This is a big difference between SU(5) and SO(10) orbifold GUTs!!

Models for SO(10) $\to G_{SM}$

are constructed in 5 or 6 dimensions.

SO(10) in Six Dimensions

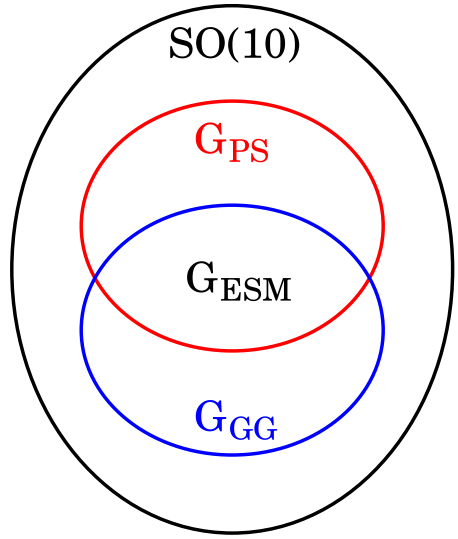
(TA, Buchmüller, Covi '01, Hall, Nomura, Okui, Smith '01)

$\mathbf{G}_{\mathrm{ESM}}$ = SU(3)×SU(2)×U(1)$_Y$×U(1)$_X$

= $\mathbf{G}_{\mathrm{PS}} \cap \mathbf{G}_{\mathrm{GG}}$

$\mathbf{G}_{\mathrm{PS}}$ = SU(4)×SU(2)×U(1)

$\mathbf{G}_{\mathrm{GG}}$ = SU(5)×U(1)



- SO(10) → $\mathbf{G}_{\mathrm{ESM}}$ is realized

  by orbifold breaking
  into TWO different subgroups G_{PS} and G_{GG}
  in TWO orthogonal compact dimensions.

  Z_2^{PS}: SO(10) → $\mathbf{G}_{\mathrm{PS}}$, Z_2^{GG}: SO(10) → $\mathbf{G}_{\mathrm{GG}}$.

- N=1 SUSY in 6d ⇔ N=2 SUSY in 4d (8 real supercharges)

  – This extended SUSY is also broken by orbifold.

  Z_2^I: N=2 SUSY in 4d → N=1 SUSY in 4d

N=1 SUSY SO(10) GUT in 6d
on orbifold $T^2/Z_2^{\mathrm{I}} \times Z_2^{\mathrm{PS}} \times Z_2^{\mathrm{GG}}$

Orbifold $T^2/Z_2^I \times Z_2^{PS} \times Z_2^{GG}$

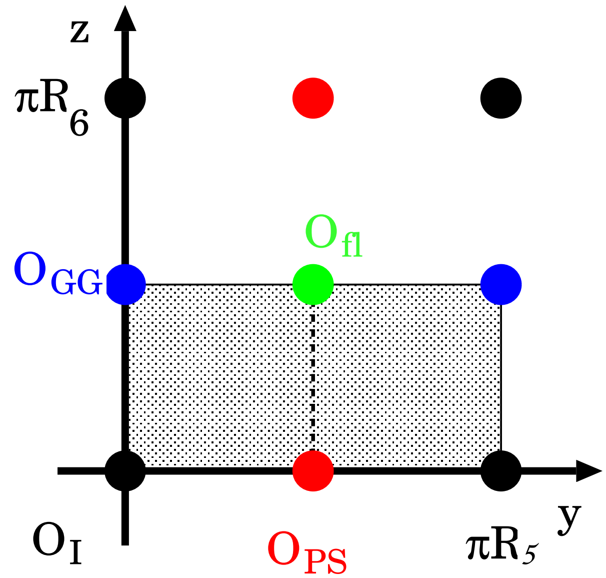
- Z_2 **symmetries**

  $Z_2^{\rm I}$: $(y, z) \to (-y, -z)$

  $Z_2^{\rm PS}$: $(y', z) \to (-y', -z)$

  $Z_2^{\rm GG}$: $(y, z') \to (-y, -z')$

  $y' = y + \pi R_5/2$
  $z' = z + \pi R_6/2$



- **Parity transformations**

  $$PVP^{-1} = +V \text{ and } P\Sigma P^{-1} = -\Sigma$$

  Gauge multiplet in the 6d bulk

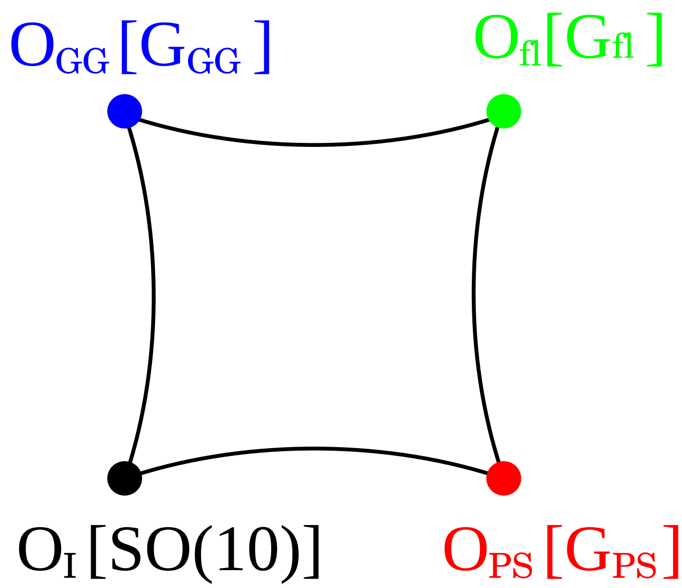
  $V = (V_\mu, \lambda_1)$, $\Sigma = (V_{5,6}, \lambda_2)$ in N=1 4d SUSY

$$P_{\rm I} = \text{diag}(+\mathbf{1}, +\mathbf{1}, +\mathbf{1}, +\mathbf{1}, +\mathbf{1})$$
$$P_{\rm PS} = \text{diag}(-\mathbf{1}, -\mathbf{1}, -\mathbf{1}, +\mathbf{1}, +\mathbf{1})$$
$$P_{\rm GG} = \text{diag}(\sigma_2, \sigma_2, \sigma_2, \sigma_2, \sigma_2)$$

- $P_{\rm I}$, $P_{\rm PS}$, $P_{\rm GG}$ **break extended SUSY in 4d.**
- $P_{\rm PS}$ **and** $P_{\rm GG}$ **break SO(10) into** G_{PS} **and** G_{GG} **at fixpoints** O_{PS} **and** O_{GG}.

The result

- A *"pillow"* with the four fixed points as corners.



- In 4d,

  we obtain $N = 1$ SUSY model
  with $G_{\rm ESM}$ = SU(3)×SU(2)×U(1)×U(1)$_X$.

  – We have to invoke the conventional Higgs mechanism to break an additional U(1)$_X$.

  – This Higgs VEV can give heavy Majorana masses for right-handed neutrinos!

SO(10) in Five Dimensions (Dermisek, Mafi '01)

SO(10) $\to G_{\rm PS}$ or $G_{\rm GG}$ is possible similar to SU(5) orbifold GUTs in 5d.

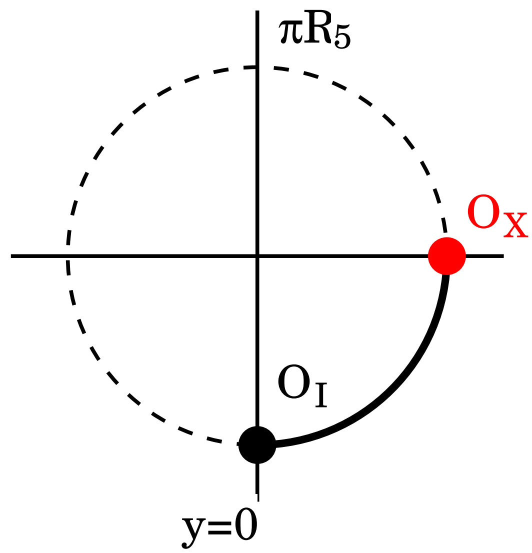
Orbifold $S^1/Z_2^I \times Z_2^X$

- **Z_2 symmetries**

  $Z_2^{\rm I}$: $y \to -y$

  $Z_2^{\rm X}$: $y' \to -y'$ (X=PS or GG)

  $y' = y + \pi R_5/2$



The result

A *"stick"* with the two fixed points as ends.

EX) SO(10) $\to$ G$_{\rm PS}$

- **G$_{\rm SM}$ is obtained by**
  - G$_{\rm PS}$ $\to$ G$_{\rm SM}$ by Higgs $(4,1,2) + (4^*,1,2)$ on $O_{\rm PS}$.
  - SO(10) $\to$ SU(5) by Higgs $16 + 16^*$.

How realize doublet-triplet splitting?

Doublet-Triplet Splitting

Multiplets in the bulk are split in mass!!

A *"pillow"* model

Introduce two SO(10) 10-plets in the bulk

$H^{6\text{d}} = (H, H')$ in N=1 4d SUSY

- **Parity** Z_2^{I}

$H_1^{6\text{d}} = (H_1, \not{H}'_1)$ and $H_2^{6\text{d}} = (H_2, \not{H}'_2)$

$+\ ,\ -\qquad\qquad +\ ,\ -$

- **Parities** Z_2^{PS} **and** Z_2^{GG}

$SO(10)$		10		
G_{PS}	$(1,2,2)$	$(1,2,2)$	$(6,1,1)$	$(6,1,1)$
G_{GG}	$(5^*,-2)$	$(5,+2)$	$(5^*,-2)$	$(5,2)$
		$(P_2^{\text{PS}},\ P_2^{GG})$		
H_1	$(+,+)$	$(+,-)$	$(-,+)$	$(-,-)$
H_2	$(+,-)$	$(+,+)$	$(-,-)$	$(-,+)$

$\Rightarrow$ Z_2^{PS} **yields doublet-triplet splitting.**

$\Rightarrow$ **Need TWO 10-plets to have two Higgs doublets** H_d **and** H_u **in MSSM.**

Anomaly Cancellation

N=1 SUSY SO(10) GUT in 6d (*"pillow"* model)

Irreducible bulk and brane anomalies should be canceled.

(e.g. TA, Buchmüller, Covi, '02)

Bulk SO(10) Anomalies

- SO(10) is not safe in 6d.
- Cancellation of irreducible anomaly constrains the bulk fields.

$$\mathcal{A}(45) = -2\mathcal{A}(10), \ \mathcal{A}(16) = \mathcal{A}(16^*) = -\mathcal{A}(10)$$

(e.g. Hebecker and March-Russel '01)

- Remarkably,
  irreducible bulk gauge anomalies DO CANCEL between 45 gauginos and two Higgs 10-plets!!

Cf.

- Irreducible brane anomalies are indeed canceled in the pillow model.
- Reducible (and mixed) anomalies can be canceled by the Green-Schwartz mechanism.
- Pure gravitational anomaly can be canceled by adding gauge singlets.

How obtain realistic fermion masses and mixings?

Major Issues

Break GUT mass relations!!

(e.g. $m_s \neq m_\mu$, $m_d \neq m_e$)

- **In the conventional 4d GUTs**
  - **Introduce a complicated Higgs sector.**

- **In the orbifold GUTs**
  - **Put matters on GUT breaking fixed points.**
    (Hebecker, March-Rusel '01,$\cdots$)
  - **Use mixings with bulk split multiplets.**
    (Hall, Nomura '01,$\cdots$)

Realize large neutrino-mixings!!

- **Recent neutrino experiments provide**
  - Atmospheric ν oscillation
    $\delta m_{\rm atm}{}^2 \simeq 2.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{\rm atm} \simeq 1.0$.
  - Solar ν oscillation
    $\delta m_{\rm sol}{}^2 \sim 7 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{\rm sol} \sim 0.9$.

- **Small quark-mixings v.s. Large neutrino-mixings**

Many interesting proposals:

Hall, Nomura, Okui, Smith '01, Watari, Yanagida '01,

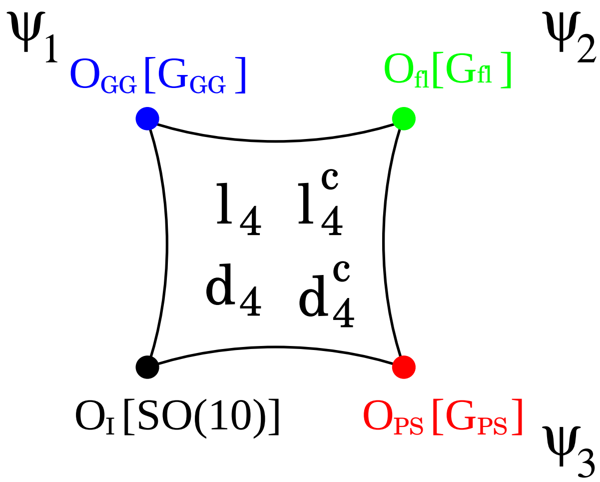
Haba, Kondo, Shimizu '01, Barr, Dorsner '02, Hall, Nomura '02,

TA, Buchmüller, Covi, '02 '03, Albright, Barr '02, Haba, Shimizu '02 '02,

Kyae, Shafi '02, Kim, Raby '02, Kitano, Li '03, Shafi, Tavartkiladze '03,

$\cdots$.

(T.A, Buchmüller, Covi '03)



- **Brane Matter Fields**

$$\psi_1 = (q_1, u^c, e^c) + (\ell_1, d^c) + N_1^c \qquad (1, a) = (R, \widetilde{X})$$
$$\psi_2 = (q_2, s^c, N_2^c) + (\ell_2, c^c) + \mu^c \qquad (1, a)$$
$$\psi_3 = (q_3, \ell_3) + (t^c, b^c, \tau^c, N_3^c) \qquad (1, a)$$

- **Bulk Matter Fields**

$$\phi(16) = (Q_\phi, U_\phi^c, E_\phi^c, \ell_4, D_\phi^c, N_\phi^c) \qquad (1, a)$$
$$\phi^c(16^*) = (Q_\phi^c, U_\phi, E_\phi, \ell_4^c, D_\phi, N_\phi) \qquad (1, -a)$$
$$H_5(10) = (H_5^c, d_4^c, H_5, G_5) \qquad (1, 2a)$$
$$H_6(10) = (H_6^c, G_6^c, H_6, d_4) \qquad (1, -2a)$$

- **Bulk Higgs Fields**

$$H_1(10) = (H_1, G_1^c, H_1^c, G_1) \qquad (0, -2a) \qquad \langle H_1 \rangle = v_1$$
$$H_2(10) = (H_2^c, G_1^c, H_2, G_1) \qquad (0, -2a) \qquad \langle H_2 \rangle = v_2$$
$$\Phi(16) = (Q_\Phi, U_\Phi, E_\Phi, L_\phi, D^c, N^c) \qquad (0, a) \qquad \langle N^c \rangle = v_N$$
$$\Phi^c(16^*) = (Q_\Phi^c, U_\Phi^c, E_\Phi^c, L_\phi^c, D, N) \qquad (0, -a) \qquad \langle N \rangle = v_N$$

- **Additional Bulk Fields**

$$H_3(10) = (H_3^c, G^c, H_3, G_3) \qquad (2, 2a)$$
$$H_4(10) = (H_4^c, G_4^c, H_4, G) \qquad (2, -2a)$$

Brane Superpotential

$$
\begin{aligned}
W &= M^d H_5 H_6 + M^l_\alpha \psi_\alpha \phi^c + M_{12} H_1 H_3 + M_{23} H_2 H_3 \\
&+ \frac{1}{2} h^{(1)}_{\alpha\beta} \psi_\alpha \psi_\beta H_1 + \frac{1}{2} h^{(2)}_{\alpha\beta} \psi_\alpha \psi_\beta H_2 + f_\alpha \Phi \psi_\alpha H_6 + f_5 \Phi^c \phi^c H_5 \\
&+ f^D \Phi^c \Phi^c H_3 + f^G \Phi \Phi H_4 + \frac{1}{2} \frac{h^N_{\alpha\beta}}{M_*} \psi_\alpha \psi_\beta \Phi^c \Phi^c \\
&+ \frac{k_1}{M_*} H_1^2 H_5^2 + \frac{k_2}{M_*} H_1 H_2 H_5^2 + \frac{k_3}{M_*} H_2^2 H_5^2 + \frac{k_4}{M_*} \Phi \Phi^c H_1 H_3 \\
&+ \frac{k_5}{M_*} \Phi \Phi^c H_2 H_3 + \frac{g^d_\alpha}{M_*} \Phi^c \psi_\alpha H_5 H_1 + \frac{g^u_\alpha}{M_*} \Phi^c \psi_\alpha H_5 H_2 \\
&+ \frac{g^d}{M_*} \Phi \phi^c H_5 H_1 + \frac{g^u}{M_*} \Phi \phi^c H_5 H_2 \\
&+ \frac{k^d_\alpha}{M_*} \Phi \Phi^c \psi_\alpha \phi^c + \frac{k^l_\alpha}{M_*} \Phi \Phi^c \psi_\alpha \phi^c + \frac{k^l}{M_*} \Phi \Phi \phi^c \phi^c \; .
\end{aligned}
$$

$\psi_\alpha = (\psi_i, \phi)$, $\alpha = 1, 2, 3, 4$

Cf.

- **Bulk fields are properly normalized.**
- **All volume factors due to the bulk fields are absorbed into the unknown couplings.**

Mass Matrices

$$W = d_\alpha m^d_{\alpha\beta} d^c_\beta + e^c_\alpha m^e_{\alpha\beta} e_\beta + n^c_\alpha m^D_{\alpha\beta} \nu_\beta + u^c_i m^u_{ij} u_j + \frac{1}{2} n^c_i M_{ij} n^c_j \ .$$

$$m^u = \begin{pmatrix} h^u_{11} v_2 & 0 & 0 \\ 0 & h^u_{22} v_2 & 0 \\ 0 & 0 & h^u_{33} v_2 \end{pmatrix}, m^N = \begin{pmatrix} h^N_{11} \frac{v_N^2}{M_*} & 0 & 0 \\ 0 & h^N_{22} \frac{v_N^2}{M_*} & 0 \\ 0 & 0 & h^N_{33} \frac{v_N^2}{M_*} \end{pmatrix}.$$

$\Rightarrow$ **3×3 diagonal matrices**

$\Rightarrow$ **No mixing with bulk matters**

$$m^d \sim m^e \sim m^D \sim m = \begin{pmatrix} \mu_1 & 0 & 0 & \widetilde{\mu}_1 \\ 0 & \mu_2 & 0 & \widetilde{\mu}_2 \\ 0 & 0 & \mu_3 & \widetilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix}.$$

$\Rightarrow$ **4×4 matrices**

$\Rightarrow$ **Mixings with bulk matters** d_4, d^c_4, ℓ_4, ℓ^c_4

$\Rightarrow$ $\mu_i, \widetilde{\mu}_i = \mathcal{O}(v_{1,2})$ and $\widetilde{M}_\alpha = \mathcal{O}(\Lambda_{\rm GUT})$

$\Rightarrow$ **Suppose** $\widetilde{M}_1 \sim \widetilde{M}_2 \sim \widetilde{M}_3 \sim \widetilde{M}_4 \sim \Lambda_{\rm GUT}$

then, m **are determined by** μ_i **and** $\widetilde{\mu}_i$**.**

Assumptions

- $\mu_1 : \mu_2 : \mu_3 \sim m_u : m_c : m_t$
- $\widetilde{\mu}_1 < \widetilde{\mu}_2 < \widetilde{\mu}_3$ but $\dfrac{\mu_2}{\mu_3} \ll \dfrac{\widetilde{\mu}_2}{\widetilde{\mu}_3}$ and $\dfrac{\mu_1}{\mu_3} \ll \dfrac{\widetilde{\mu}_1}{\widetilde{\mu}_3}$
- $\mu_3 \sim \widetilde{\mu}_3$

$\Rightarrow$ **Matrix** m **are controlled by**

$$\widetilde{\mu}_3, \quad \widetilde{\mu}_2, \quad \widetilde{\mu}_1.$$

Cf. $\mu_3 \sim \widetilde{\mu}_3, \quad \mu_2 \sim \mu_3 \dfrac{m_c}{m_t} \sim \widetilde{\mu}_3 \dfrac{m_c}{m_t}, \quad [\mu_1].$

None of

the GUT

mass relations

hold exactly!!!

Mass Matrices

$$W = d_\alpha m^d_{\alpha\beta} d^c_\beta + e^c_\alpha m^e_{\alpha\beta} e_\beta + n^c_\alpha m^D_{\alpha\beta} \nu_\beta + u^c_i m^u_{ij} u_j + \tfrac{1}{2} n^c_i M_{ij} n^c_j \ .$$

Up quarks / Right-handed neutrinos

$$m^u = \begin{pmatrix} h^u_{11} v_2 & 0 & 0 \\ 0 & h^u_{22} v_2 & 0 \\ 0 & 0 & h^u_{33} v_2 \end{pmatrix} .$$

$$m^N = \begin{pmatrix} h^N_{11} \frac{v_N^2}{M_*} & 0 & 0 \\ 0 & h^N_{22} \frac{v_N^2}{M_*} & 0 \\ 0 & 0 & h^N_{33} \frac{v_N^2}{M_*} \end{pmatrix} .$$

Down quarks / Charged leptons / Dirac neutrinos

$$m^d = \begin{pmatrix} h^d_{11} v_1 & 0 & 0 & g^d_1 \frac{v_N}{M_*} v_1 \\ 0 & h^d_{22} v_1 & 0 & g^d_2 \frac{v_N}{M_*} v_1 \\ 0 & 0 & h^d_{33} v_1 & g^d_3 \frac{v_N}{M_*} v_1 \\ f_1 v_N & f_2 v_N & f_3 v_N & M^d \end{pmatrix} .$$

$$m^e = \begin{pmatrix} h^d_{11} v_1 & 0 & 0 & h^e_{14} v_1 \\ 0 & h^e_{22} v_1 & 0 & h^e_{24} v_1 \\ 0 & 0 & h^d_{33} v_1 & h^e_{34} v_1 \\ M^l_1 & M^l_2 & M^l_3 & M^l_4 \end{pmatrix} .$$

$$m^D = \begin{pmatrix} h^D_{11} v_2 & 0 & 0 & h^D_{14} v_2 \\ 0 & h^u_{22} v_2 & 0 & h^D_{24} v_2 \\ 0 & 0 & h^u_{33} v_2 & h^D_{34} v_2 \\ M^l_1 & M^l_2 & M^l_3 & M^l_4 \end{pmatrix} .$$

Lopsided Mass Matrices

$$W = d_\alpha m^d_{\alpha\beta} d^c_\beta + e^c_\alpha m^e_{\alpha\beta} e_\beta + n^c_\alpha m^D_{\alpha\beta} \nu_\beta$$

$$m^d \sim m^e \sim m^D \sim m = \begin{pmatrix} \mu_1 & 0 & 0 & \widetilde{\mu}_1 \\ 0 & \mu_2 & 0 & \widetilde{\mu}_2 \\ 0 & 0 & \mu_3 & \widetilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix} .$$

$$m' = U^T m V = \begin{pmatrix} \hat{m} & 0 \\ 0 & \widetilde{M} \end{pmatrix}$$

$$\widehat{m} = \begin{pmatrix} e_{11}\,\widetilde{\mu}_1, & e_{12}\,\widetilde{\mu}_1, & e_{13}\,\widetilde{\mu}_1 \\ e_{21}\,\widetilde{\mu}_2, & e_{22}\,\widetilde{\mu}_2, & e_{23}\,\widetilde{\mu}_2 \\ e_{31}\,\widetilde{\mu}_3, & e_{32}\,\widetilde{\mu}_3, & e_{33}\,\widetilde{\mu}_3 \end{pmatrix}$$

with $e_{ij} \sim 1$ (but $e_{i1} \sim 0.1$)

- **Small mixings in d_L (and e_R)!!**
  - **Mixings in $V_{\rm KM}$ are small**

- **Large mixings in e_L, ν_L (and d_R)!!**

$\Rightarrow$ **Lopsided fermion mass matrices**

(Sato,Yangida '98, Irges,Lavignac, Ramond '98, Albright, Babu, Barr '98)

The results

Quark Masses and Mixings

$$[m_t : m_c : m_u \sim \mu_3 : \mu_2 : \mu_1]$$

$$m_b \sim \widetilde{\mu}_3 (\sim \mu_3), \quad m_s \sim \widetilde{\mu}_2, \quad m_d \sim \widetilde{\mu}_1 \frac{\mu_2}{\widetilde{\mu}_2} \sim \widetilde{\mu}_1 \frac{\widetilde{\mu}_3}{\widetilde{\mu}_2} \frac{m_c}{m_t}$$

$$V_{us} = \theta_c \sim \frac{\widetilde{\mu}_1}{\widetilde{\mu}_2}, \quad V_{cb} \sim \frac{\widetilde{\mu}_2}{\widetilde{\mu}_3}, \quad V_{ub} \sim \frac{\widetilde{\mu}_1}{\widetilde{\mu}_3}$$

Using m_b, m_s and θ_c, we find

$$V_{cb} \sim \frac{m_s}{m_b} \simeq 2 \times 10^{-2}, \; V_{ub} \sim \theta_c \frac{m_s}{m_b} \simeq 4 \times 10^{-3}$$

$$\frac{m_d}{m_s} \sim \theta_c \frac{m_c m_b}{m_t m_s} \simeq 0.03$$

Neutrino Masses and Mixings

$$[M_3 : M_2 : M_1 \sim m_t : m_c : m_u] \qquad \gamma \sim \frac{\mu_2}{\widetilde{\mu}_2} \sim 0.1$$

$$m_\nu = -(m^D)^T \frac{1}{M^N} m^D \sim \frac{\mu_3^2}{M_3} \begin{pmatrix} \gamma^2 & \gamma & \gamma \\ \gamma & 1 & 1 \\ \gamma & 1 & 1 \end{pmatrix}$$

This ν mass matrix can account for all neutrino data.
(Sato,Yangida '98, Irges,Lavignac, Ramond '98, Albright, Babu, Barr '98)

We find $\theta_{13} \sim \gamma \sim 0.1$

Using $m_3 \sim \sqrt{\delta m^2_{\rm atm}} \sim 0.05$ eV, **we find**

$$M_3 \sim 10^{15} \text{ GeV}, \; M_2 \sim 3 \times 10^{12} \text{ GeV}, \; M_1 \sim 10^{10} \text{ GeV}$$

Summary

SO(10) Orbifold GUTs

- SO(10) Breaking
  - Orbifold breaking + Higgs mechanism
    - Models in 6d (*"pillow"*) or 5d (*"stick"*)
    - Higgs VEV may give heavy Majorana masses

- Doublet-Triplet Splitting
  - Multiplets in the bulk are split in mass

- Fermion Masses and Mixings
  - Breaking GUT mass relations
    - Matters on the SO(10) breaking fixpoints
    - Mixing with the bulk split matter

$\Rightarrow$ Characteristic and rich phenomenology!!

Issues not discussed here are

- Proton Decay
- Gauge Coupling Unification
- $\cdots$