GUT-model Building

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<u>Outline</u>

- Motivation/"Evidence"
- * <u>Model Building Issues:</u>
- Gauge group
- Matter multiplets
- Symmetry breaking
- Doubtlet—triplet splitting
- Fermion masses and mixings
- Flavor violation
- Leptogenesis
- Proton decay
- * <u>Realistic GUTs</u>
- * <u>Experimental tests</u>
- * Conclusions

Evolution of Gauge Couplings



SUSY Spectrum

SM Particles	SUSY Partners
Q	$ ilde{Q}$
u^c	$ ilde{u}^c$
Spin = $1/2$ d^c	\tilde{d}^c Spin = 0
L	$ ilde{L}$
e^c	$ ilde{e}^c$
Spin = 0 H_u	$ ilde{H}_u$ Online 1/2
H_d	\tilde{H}_d Spin = 1/2
g	${\widetilde g}$
Spin = 1 W	\tilde{W} Spin = 1/2
B	$ ilde{B}$

 $R = (-1)^{3B+L+2S}$

Gauge Coupling Unification in [SU(3)]⁴ Quartification



K.S. Babu, Ernest Ma, S. Willenbrock, hep-ph/0307380

$SU(3)_q \propto SU(3)_L \propto SU(3)_l \propto SU(3)_R$ Quartification



Surviving symmetry: $SU(3)_C \times SU(2)_L \times U(1) \times SU(3)_I$

Structure of Matter Multiplets

$$egin{aligned} Q &= egin{pmatrix} u_1 & u_2 & u_3 \ d_1 & d_2 & d_3 \end{pmatrix} &\sim (3,2,rac{1}{6}) \ u^c &= (u_1^c & u_2^c & u_3^c) \sim (\overline{3},1,rac{-2}{3}) \ d^c &= (d_1^c & d_2^c & d_3^c) \sim (\overline{3},1,rac{1}{3}) \ L &= egin{pmatrix}
u &= \
u^c &\sim (1,2,rac{-1}{2}) \
e^c &\sim (1,1,+1) \
u^c &\sim (1,1,0) \end{aligned}$$

Matter Unification in 16 of SO(10)



u_1	:	$ \uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$ \uparrow\downarrow\uparrow\downarrow\uparrow>$
u_{3}	:	$ \uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$ \downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$ \downarrow\uparrow\uparrow\downarrow\uparrow>$
d_{3}	:	$ \downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$ \downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$ \downarrow\downarrow\downarrow\uparrow\uparrow\downarrow>$
$u^c_{\sf 3}$:	$ \downarrow\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$ \uparrow\uparrow\uparrow\downarrow\downarrow\rangle>$
d_2^c	:	$ \uparrow\uparrow\downarrow\uparrow\downarrow>$
$d^c_{\sf 3}$:	$ \uparrow\uparrow\downarrow\downarrow\downarrow\uparrow>$
u	:	$ \uparrow\downarrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$ \downarrow\uparrow\downarrow\downarrow\downarrow\downarrow>$
e^{c}	:	$ \downarrow\downarrow\uparrow\uparrow\uparrow>$
$ u^c$:	

Neutrino Masses and the Scale of New Physics

$$\mathcal{L} = \frac{LLHH}{M_R}$$

 $\langle H
angle \sim$ 246 GeV and $m_{
u_3} \sim$ 0.05 eV

from atmospheric neutrino oscillation data

$$\longrightarrow$$
 $m_R \sim 10^{14} - 10^{15} {
m GeV}$

Very Close to the GUT scale.

Leptogenesis via v_R decay explains cosmological baryon asymmetry

- * Anomaly freedom automatic in many GUTs
- ***** Electric charge quantization
- * Nonzero neutrino masses required in many GUTs
- Saryon number violation natural in GUTs needed for generating cosmological baryon asymmetry
 M_d = M^T_ρ works well for 3rd family (m_b = m_τ)

<u>GUT Gauge Groups</u>

- SU(5)
- **SO(10)**
- E₆
- E₈

. . .

- $[SU(3)]^3$
- $[SU(5)]^2$
- [SU(3)]⁴
 - ••••

<u>SU(5) GUT</u>

Matter multiplets: $\{10+\bar{5}+1\}$

$$10: \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

$$\bar{\mathbf{5}}: (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$1: \nu^c$$
Higgs: $24_H, \quad {\mathbf{5}_H, \quad \bar{\mathbf{5}}_H} \Longrightarrow \text{ Contain color triplets } {H_C, \quad \bar{H}_C}$
Yukawa Couplings $Y_u^{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H + Y_d^{ij} \mathbf{10}_i \bar{\mathbf{5}}_j \bar{\mathbf{5}}_H$

$$M_\ell = M_d^T \Rightarrow m_b = m_\tau, m_s = m_\mu, m_d = m_e$$

MSSM Higgs doublets have color triplet partners in GUTs.

 $H(1,2,1/2) \oplus H_c(3,1,-1/3) = 5$ of SU(5) $\bar{H}(1,2,-1/2) \oplus \bar{H}_c(\bar{3},1,1/3) = \bar{5}$

 H, \bar{H} must remain light H_c, \bar{H}_c must have GUT scale mass to prevent rapid proton decay

Doublet-triplet splitting

Even if color triplets have GUT scale mass, d=5 proton decay is problematic.

Symmetry Breaking

Doublet-triplet splitting in SU(5)

$$W_{D-T} = \overline{\mathbf{5}}_H (\lambda \mathbf{24}_H + M) \mathbf{5}_H$$

$$< 24_{H} >= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix} V$$
FINE-TUNED TO $O(M_{W})$
$$M_{H_{c}} = \lambda V + M \sim O(M_{GUT}) M_{H} = -\frac{3}{2}\lambda V + M$$

The GOOD

The BAD

- (1) **Predicts unification of couplings**
- (2) Uses economic Higgs sector

(1) Unnatural fine tuning

(2) Large proton decay rate

Nucleon Decay in SUSY GUTs

Gauge boson Exchange



$$p \to e^+ \pi^0$$
, $\tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{36 \pm 1} yr]^{-1}$

Higgsino Exchange

Sakai, Yanagida (1982) Weinberg (1982)



 $p \to \overline{\nu} K^+$

 $\tau_p^{-1} \approx \left[\frac{f^2}{M_{H_c}M_{SUSY}}\right]^2 \left(\frac{\alpha}{4\pi}\right)^2 m_p^5 \approx \left[10^{28} - 10^{32} yr\right]^{-1}$

SO(10) GUT

- \Leftrightarrow Quarks and leptons ~{16_i}
- \Rightarrow Contains **n**_R and Seesaw mechanism

Model with Non-renormalizable Yukawa Couplings

Higgs: $\{45_H + 10_H + 16_H + \overline{16}_H\}$

 $\mathcal{L}_{\text{Yukawa}} = f_{ij} \mathbf{16}_{i} \mathbf{16}_{j} \mathbf{10}_{H} + h_{ij} \mathbf{16}_{i} \mathbf{16}_{j} \mathbf{\overline{16}}_{H} \mathbf{\overline{16}}_{H} / M_{Pl}$ $\implies m_{\nu_{\tau}}^{D} \simeq m_{t}; \ m_{\nu_{\tau_{R}}}^{M} \simeq h_{33} \frac{M_{GUT}^{2}}{M_{Pl}}$ $m_{\nu_{\tau}} = \frac{m_{t}^{2}}{m_{\nu_{\tau_{R}}}} \simeq 0.05 \text{ eV}, \ h_{33} \sim 1$

Fits the atmospheric neutrino data well

*Small Higgs rep \implies small threshold corrections for gauge couplings *R-parity not automatic (needs a Z₂ symmetry)

Matter Unification in 16 of SO(10)



u_1	:	$ \uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$ \uparrow\downarrow\uparrow\downarrow\uparrow>$
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d_2^c	:	$ \uparrow\uparrow\downarrow\uparrow\downarrow>$
$d^c_{\sf 3}$:	$ \uparrow\uparrow\downarrow\downarrow\downarrow\uparrow>$
u	:	$ \uparrow\downarrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$ \downarrow\uparrow\downarrow\downarrow\downarrow\downarrow>$
e^{c}	:	$ \downarrow\downarrow\uparrow\uparrow\uparrow>$
$ u^c$:	

Renormalizable Yukawa Coupling Model

Higgs: $\{210_H + 126_H + 10_H\}$ Automatic R-parity $\mathcal{L}_{Yukawa} = f_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + h_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}_H$ Under $SU(2)_L \times SU(2)_R \times SU(4)_C$ $\overline{126} = (1, 3, \overline{10}) + (3, 1, 10) + (1, 1, 6) + (2, 2, 15)$ contains(1,2,1/2) of SM contains(1, 1, 0) of SM $M_u = A + B \qquad M_{\nu_D} = A - 3B$ $M_d = \alpha A + \beta B$ $M_\ell = \alpha A - 3\beta B$ $M_{\nu_M} \propto B$

Model has only 11 real parameters plus 7 phases

K.S. Babu and R. Mohapatra, Phys. Rev. Lett.70, 2845 (1993)

Quark, Lepton & Neutrino Masses & Mixings in Minimal SO(10)

Input at GUT scale

Fit

an eta = 55 $m_u = 0.85 \ MeV$ $m_d = 1.08 \ MeV$ $m_c = 222.3 \ MeV$ $m_s = 34.3 \ MeV$ $m_t = 85.5 \ GeV$ $m_b = 1.549 \ GeV$ $\delta_{CKM} = 1.508$ $V_{us} = 0.22 \ V_{ub} = 0.0027 \ V_{cb} = 0.036$

Output: Type II Seesaw

$$\begin{split} \sin^2 2\theta_{\odot} &= 0.635 \\ \sin^2 2\theta_{e3} &= 0.08 \\ \sin^2 2\theta_{atm} &= 0.892 \\ \frac{\Delta m_{atm}^2}{\Delta m_{\odot}^2} &= 15.2 \\ \epsilon_1 &= 1.0 \times 10^{-6} \Rightarrow Y_B \sim 10^{-10} \end{split} \text{KSB, C. Macesanu (2003)}$$



Minimal SO(10) GUT Prediction for Neutrino Mixings

FIG. 1. The figure shows the range of predictions for $\sin^2 2\theta_{\odot}$ and $\sin^2 2\theta_A$ for the range of quark masses in table I that fit the charged lepton spectrum and where all CP phases are set to zero. We required that the ratio $\Delta m_{\odot}^2 / \Delta m_A^2 \leq 0.05$. Note that $\sin^2 2\theta_{\odot} \geq 0.9$ and $\sin^2 2\theta_A \leq 0.9$.

H.S. Goh, R.N. Mohapatra, Siew-Phang Ng, hep-ph/0308197

See also: Fukuyama, Okada, 2002; Aulakh et. al., 2003; Goh, Mohapatra, Ng, 2003

- B-L VEV gives mass to triplets only (DIMOPOULOS-WILCZEK)
- \implies If 10_H only couples to fermions, no d=5 proton decay
- \implies Doublets from 10_H and $10'_H$ light

4 doublets, unification upset

Add mass term for $10'_{H}$

 $W_{D-T} = \lambda(\bar{10}_H 45_H 10'_H) + M 10'_H 10'_H$

<u>Realistic SO(10) Model</u>

Pati, Wilczek, KB (1998)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon + \sigma \\ 0 & -\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' + \eta' & 0 & \epsilon + \eta \\ 0 & -\epsilon + \eta & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon + \sigma \\ 0 & 3\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad L = \begin{pmatrix} 0 & -3\epsilon' + \eta' & 0 \\ 3\epsilon' + \eta' & 0 & -3\epsilon + \eta \\ 0 & 3\epsilon + \eta & 1 \end{pmatrix} m_D$$

$$M_{\nu}^{R} = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_{R}$$

"1" : $16_3 16_3 10_H$ " ϵ " : $16_2 16_3 (10_H \times 45_H)/M$ " σ " : $16_2 16_3 (10_H \times 1_H)/M$ " η " : $16_2 16_3 16_H 16_H/M$ $\langle 45_H \rangle \propto (B-L)$

Predictions

$$au(p
ightarrow ar{
u} K^+) \lesssim 10^{34} ext{ yr}$$

 $Br(p
ightarrow \mu^+ K^0) \sim 10\%$

Large Neutrino Mixing with Lopsided Mass Matrices

Quark and Lepton Mass hierarchy:

m_d : m_s : m_s	$_b~\sim$	$m_e: m_\mu: m_\tau \sim \epsilon_1: \epsilon_2: \epsilon_3$
m_u : m_c : m	$_t$ \sim	$\epsilon_1^2:\epsilon_2^2:\epsilon_3^2$
This motivates:	U =	$H^T U_{O} H$
j	D =	D_0H
	L =	$H^T L_0$
1	V =	N_0
$H = Diag(\epsilon_1,$	ϵ_2, ϵ_3	$\epsilon_1 << \epsilon_2 << \epsilon_3$

 10_i of SU(5) carry flavor charge, $\overline{5}_i$ do not. Leads to large left-handed charged lepton mixing and large right-handed down quark mixing.

KSB and S. Barr, 1995

Albright, KSB and Barr, 1998 Sato and Yanagida, 1998 Irges, Lavignac, Ramond, 1998 Altarelli, Feruglio, 1998

Example of Lopsided Mass Matrices

Gogoladze, Wang, KSB, 2003

$$U_{ij} = \begin{pmatrix} \epsilon^{6} & \epsilon^{5} & \epsilon^{3} \\ \epsilon^{5} & \epsilon^{4} & \epsilon^{2} \\ \epsilon^{3} & \epsilon^{2} & 1 \end{pmatrix} H_{u}, \quad D_{ij} = \begin{pmatrix} \epsilon^{4} & \epsilon^{3} & \epsilon^{3} \\ \epsilon^{3} & \epsilon^{2} & \epsilon^{2} \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{p} H_{d},$$
$$L_{ij} = \begin{pmatrix} \epsilon^{4} & \epsilon^{3} & \epsilon \\ \epsilon^{3} & \epsilon^{2} & 1 \\ \epsilon^{3} & \epsilon^{2} & 1 \end{pmatrix} \epsilon^{p} H_{d}, \quad \nu_{ij}^{D} = \begin{pmatrix} \epsilon^{2} & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_{1}} H_{u}$$
$$\nu_{ij}^{M} = \begin{pmatrix} \epsilon^{2} & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_{2}} \sim \mathsf{M}_{light}^{\nu}$$
$$\epsilon \sim 0.2$$

Structure enforced by Anomalous U(1) Symmetry or Discrete Z_N Gauge Symmetry via Froggatt-Nielsen Mechansim

Lopsided Mass Matrix Model in SO(10)

S.Barr and KSB,2002

$$L = \begin{pmatrix} 0 & 0 & \delta' \\ 0 & \delta & -\epsilon' \\ \rho' & \rho - \epsilon & 1 \end{pmatrix} m_D, \qquad D = \begin{pmatrix} 0 & 0 & \rho' \\ 0 & \delta & \rho - \epsilon' \\ \delta' & -\epsilon & 1 \end{pmatrix} m_D,$$
$$N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon' \\ 0 & \epsilon & 1 \end{pmatrix} m_U, \qquad U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon' \\ 0 & \epsilon & 1 \end{pmatrix} m_U$$

 $W_{\text{Yuk}} = 16_3 16_3 10_H + 16_2 16_3 10_H 45_H / M + 16_2 16_3 16_H 16'_H / M + 16_1 16_3 16_H 16'_H / M + 16_2 16_3 16_H 16'_H / M$

10 Parameters vs. 20 Observables



PREDICTIONS

Predictions

$$1, \rho, \rho' \gg \epsilon, \epsilon' \gg \delta, \delta'$$

$$m_b \simeq m_\tau = \sqrt{1 + \sigma^2} m_D$$
Buras, et al
$$\frac{m_s}{m_b} \simeq \frac{|\sigma\epsilon + \delta\rho/\sigma|}{1 + \sigma^2}$$

$$\frac{m_\mu}{m_\tau} \simeq \frac{|\sigma\epsilon' + \delta\rho/\sigma|}{1 + \sigma^2}$$

$$m_d m_s m_b \simeq m_e m_\mu m_\tau$$
Georgi-Jarlskog

$$\frac{m_c}{m_t} \simeq \epsilon \epsilon' \quad \frac{m_u}{m_t} \simeq 0 \qquad m_\mu \neq m_s$$
$$|V_{us}| \simeq \frac{\delta'}{\epsilon + \delta \rho / \sigma^2} \quad |V_{ub}| \simeq \frac{\delta'}{1 + \sigma^2}$$
$$|V_{cb}| \simeq \frac{|\epsilon(2 + \sigma^2) - \delta \rho|}{1 + \sigma^2} \qquad \eta_{CP} \simeq \frac{2\epsilon \rho Im(\delta)}{\sigma^2(1 + \sigma^2)|V_{cb}|^2}$$

$$\begin{array}{rcl} \tan \theta_{\rm atm} &\simeq & \displaystyle \frac{m_s}{m_b} \left| \frac{V_{us}}{V_{ub}} \right| \\ \tan \theta_{\odot} &\simeq & (0.4-0.6) \\ & & |U_{e3}| &\simeq & 0.06 \end{array}$$

Lepton Flavor Violation and Neutrino Mass

Seesaw mechanism naturally explains small **n**-mass.

$$\mathcal{L} = \bar{\nu}_L M_D \nu_R + \frac{1}{2} \nu_R^T M_R \nu_R + h.c.$$
$$M_\nu = -M_D M_R^{-1} M_D^T$$

Current neutrino-oscillation data suggests

$$M_R \sim (10^{12} - 10^{15}) \text{ GeV}$$

Flavor change in neutrino-sector



Flavor change in charged leptons

In standard model with Seesaw, leptonic flavor changing is very tiny.

$$Br(\mu
ightarrow e\gamma) \propto rac{1}{M_{Pl}^4} \sim 10^{-50}$$

In Supersymmetric Standard model

$$Br(\mu
ightarrow e\gamma) \propto rac{1}{M_{SUSY}^4} \sim 10^{-10}$$

For $M_R \leq \mu \leq M_{Pl}$ n_R active

> flavor violation in neutrino sector Transmitted to Sleptons

Borzumati, Masiero (1986) Hall, Kostelecky, Raby (1986) Hisano et. al., (1995)

SUSY Seesaw Mechanism

$$\mathcal{W} = f \nu^c \nu^c \Delta + Y_{\nu} \nu^c L H_u$$
$$M_D = Y_{\nu} v_u \; ; \; M_R = f v_{B-L}$$

If *B-L* is gauged, M_R must arise through Yukawa couplings.

Flavor violation may reside entirely in f or entirely in Y_n



Conclusions

- Grand Unification motivated on various grounds
- <u>Challenges in GUT-model Building:</u>

Doublet-triplet Splitting Realistic Quark and Lepton Masses Proton Decay

• Promises of GUT models:

Predictive Quark-Lepton Spectrum Naturally Small Neutrino Masses Baryon Asymmetry Generation Proton Decay in Observable Range