Elements of Grand Unification

- Problems with the Standard Model
- The Two Paths
- Grand Unification
- The Georgi-Glashow $SU(5)$ Model
- Beyond $SU(5)$
  - Larger Groups
  - Supersymmetry
  - Extra Dimensions
- Additional Implications
- To GUT or not to GUT

Reviews

- **PL,** *Grand Unified Theories and Proton Decay*, **Phys. Rep. 72, 185 (1981)**
- **G. Ross,** *Grand Unified Theories* **(Benjamin, 1985)**

Problems with the Standard Model

Standard model: $SU(3) \times SU(2) \times U(1)$ (extended to include ν masses) + general relativity

Mathematically consistent, renormalizable theory

Correct to 10^{-16} cm

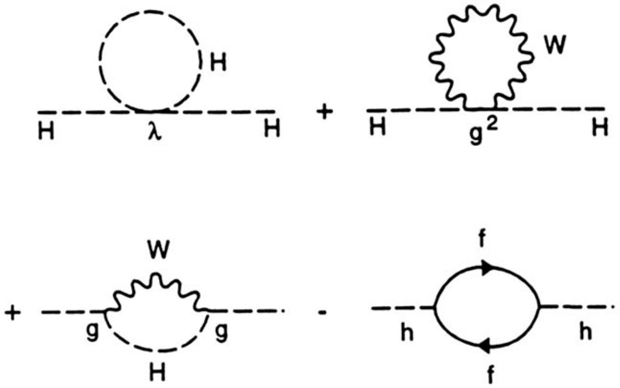
However, too much arbitrariness and fine-tuning ($O(20)$ parameters, not including ν masses/mixings, which add at least 7 more, and electric charges)

Gauge Problem

- complicated gauge group with 3 couplings
- charge quantization ($|q_e| = |q_p|$) unexplained
- Possible solutions: strings; grand unification; magnetic monopoles (partial); anomaly constraints (partial)

Fermion problem

- Fermion masses, mixings, families unexplained
- Neutrino masses, nature?
- CP violation inadequate to explain baryon asymmetry
- Possible solutions: strings; brane worlds; family symmetries; compositeness; radiative hierarchies. New sources of CP violation.



Higgs/hierarchy problem

- Expect $M_H^2 = O(M_W^2)$
- higher order corrections: $\delta M_H^2/M_W^2 \sim 10^{34}$

Possible solutions: supersymmetry; dynamical symmetry breaking; large extra dimensions; Little Higgs

Strong CP problem

- Can add $\frac{\theta}{32\pi^2} g_s^2 F\tilde{F}$ to QCD (breaks, P, T, CP)
- $d_N \Rightarrow \theta < 10^{-9}$
- but $\delta\theta|_{\text{weak}} \sim 10^{-3}$
- Possible solutions: spontaneously broken global $U(1)$ (Peccei-Quinn) $\Rightarrow$ axion; unbroken global $U(1)$ (massless u quark); spontaneously broken CP + other symmetries

Graviton problem

- gravity not unified
- quantum gravity not renormalizable
- cosmological constant: $\Lambda_{\rm SSB} = 8\pi G_N \langle V \rangle > 10^{50} \Lambda_{\rm obs}$ (10^{124} for GUTs, strings)
- Possible solutions:
  - supergravity and Kaluza Klein unify
  - strings yield finite gravity.
  - Λ?

The Two Paths: Unification or Compositeness

The Bang

- unification of interactions
- grand desert to unification (GUT) or Planck scale
- elementary Higgs, supersymmetry (SUSY), GUTs, strings
- possibility of probing to M_P and very early universe
- hint from coupling constant unification
- tests
  - light ($< 130 - 150$ GeV) Higgs (LEP 2, TeV, LHC)
  - *absence* of deviations in precision tests (usually)
  - supersymmetry (LHC)
  - possible: m_b, proton decay, ν mass, rare decays
  - SUSY-safe: Z'; seq/mirror/exotic fermions; singlets
- variant versions: large dimensions, low fundamental scale, brane worlds

The Whimper

- onion-like layers
- composite fermions, scalars (dynamical sym. breaking)
- *not* like to atom → nucleus $+e^- \rightarrow p + n \rightarrow$ quark
- at most one more layer accessible (LHC)
- rare decays (e.g., $K \rightarrow \mu e$)
  - severe problem
  - no realistic models
- effects (typically, few %) expected at LEP & other precision observables (4-f ops; $Zb\bar{b}$; ρ_0; S, T, U)
- anomalous VVV, new particles, future $WW \rightarrow WW$
- recent variant: Little Higgs

Grand Unification

Pati-Salam, 73; Georgi-Glashow, 74

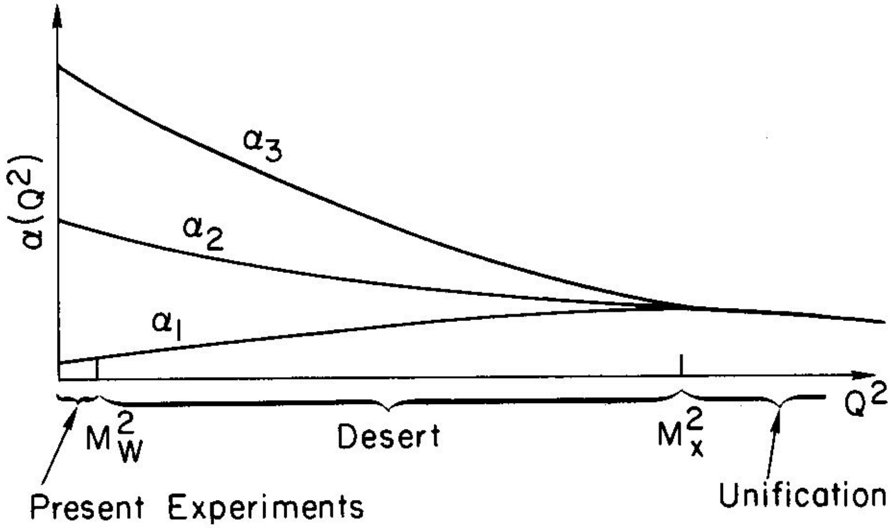
Strong, weak, electromagnetic unified at $Q \gtrsim M_X \gg M_Z$

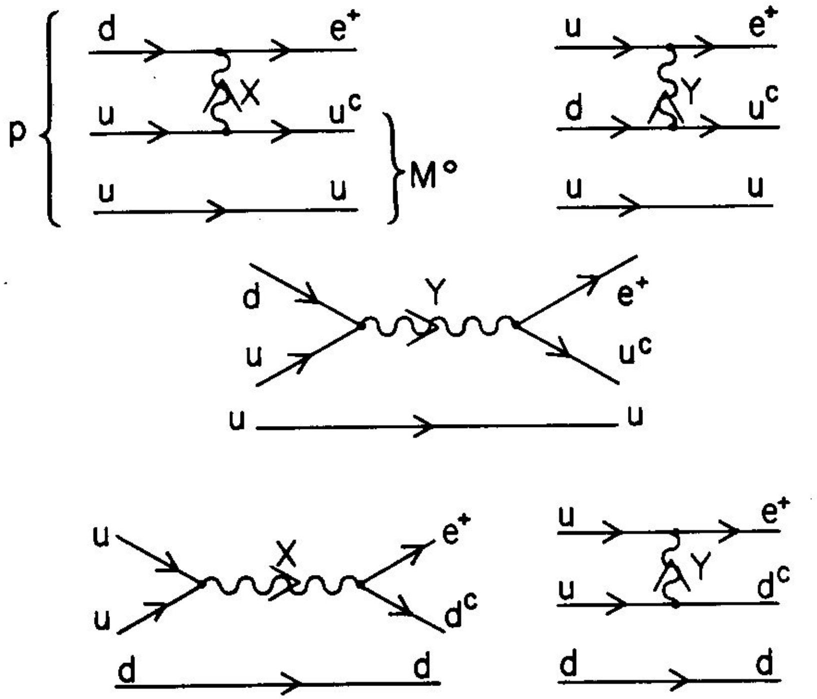
- **Simple group**

$$G \underset{M_X}{\longrightarrow} SU(3)\times SU(2)\times U(1)$$

- **Gravity *not* included**
  (perhaps not ambitious enough)

- **Couplings meet at $M_X \sim 10^{14}$ GeV (w/o SUSY)**
  (works much better with SUSY $\rightarrow M_X \sim 10^{16}$ GeV)





$$q_X = \frac{4}{3}$$

$$q_Y = \frac{1}{3}$$

- $q, \bar{q}, l, \bar{l}$ unified (in same multiplets)⇒
  - Charge quantization (no $U(1)$ factors)
  - Proton decay mediated by new gauge bosons, e.g. $p \rightarrow e^+ \pi^0$

    (other modes in SUSY GUTS)
  - $\tau_p \sim \frac{M_X^4}{\alpha^2 m_p^5} \sim 10^{30}$ yr for $M_X \sim 10^{14}$ GeV

    (10^{38} yr in SUSY, but faster $p \rightarrow \bar{\nu} K^+$)

The Georgi-Glashow $SU(5)$ Model

$$SU(5) \underset{M_X}{\longrightarrow} SU(3)\times SU(2)\times U(1) \underset{M_Z}{\longrightarrow} SU(3)\times U(1)_{\mathrm{ELM}}$$

In GUTs, SUSY, convenient to work with left-chiral fields for particles and antiparticles

Right-chiral related by CP (or CPT)

$$d^c_L \underset{CPT}{\longleftrightarrow} d_R, \quad u^c_L \underset{CPT}{\longleftrightarrow} u_R, \quad e^+_L \underset{CPT}{\longleftrightarrow} e^-_R$$

Each family in $5^* + 10$ of $SU(5)$

$$\begin{array}{ccc} SU(2) \updownarrow \left(\begin{matrix} \nu_L^0 \\ e_L^{-0} \end{matrix} \quad \overset{X,Y}{\longleftrightarrow} \quad d_L^{0c} \right) & \qquad & \left(e_L^{+0} \quad \longleftrightarrow \overset{X,Y}{\begin{matrix} u_L^0 \\ d_L^0 \end{matrix}} \longleftrightarrow \quad u_L^{0c} \right) \updownarrow SU(2) \\ \mathbf{5}^* & & \mathbf{10} \end{array}$$

$\psi^0 \Rightarrow$ weak eigenstate (mixture of mass eigenstates)

Color ($SU(3)$) indices suppressed

The $SU(n)$ Group

(Type A Lie algebra; $n^2 - 1$ generators; rank (# diagonal generators) $= n - 1$)

Group element: $U(\vec{\beta}) = e^{i\vec{\beta}\cdot\vec{T}}$

- $\vec{\beta} = n^2 - 1$ real numbers
- $\vec{T} = n^2 - 1$ generators (operators)
- $\vec{L} = n^2 - 1$ representation matrices of dimension $m\times m$ satisfying same Lie Algebra (commutation rules) as the $\vec{T} \Rightarrow U^{(m)}(\vec{\beta}) \equiv e^{i\vec{\beta}\cdot\vec{L}}$ form $m\times m$ dimensional representation of group (will suppress (m))

Fundamental representation (n):

- $L^i \equiv \frac{\lambda^i}{2}$ are $n\times n$ Hermitian matrices with Tr $L^i = 0 \Rightarrow e^{i\vec{\beta}\cdot\vec{L}}$ are $n\times n$ unitary matrices with $\det = 1$
- Normalize to Tr $(L^i L^j) = \frac{\delta_{ij}}{2}$
- For $n = 2, \lambda^i \rightarrow \tau^i$ (Pauli matrices)
- For $n = 3, \lambda^i \rightarrow$ Gell-Mann matrices

Arbitrary n: construct L^i using non-Hermitian basis

- Define matrices L^a_b, where (a, b) label the matrix (they are not indices) by

$$\left(L^a_b\right)_{cd} \equiv \left(L^a_b\right)^c{}_d = \delta^a_d \delta^c_b - \frac{1}{n} \delta^a_b \delta^c_d$$

- $\left(L^a_b\right)^\dagger = L^b_a \Rightarrow$ non-Hermitian for $a \neq b$
- $\frac{L^a_b + L^b_a}{2}$ and $i\frac{L^a_b - L^b_a}{2}$ are Hermitian

- For $a \neq b$, $\left(L^a_b\right)_{ba} = 1$, others 0, e.g., $SU(2)$ raising and lowering operators

$$L^2_1 = \frac{\tau^1 + i\tau^2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad L^1_2 = \frac{\tau^1 - i\tau^2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

- L^a_a is diagonal with

$$\left(L^a_a\right)^a{}_a = \frac{n-1}{n} \qquad \left(L^a_a\right)^b{}_b = -\frac{1}{n} \text{ for } b \neq a$$

- The n matrices L^a_a are not independent $(\sum_a L^a_a = 0) \Rightarrow$ use alternate diagonal basis

$$L_k \equiv \frac{1}{\sqrt{2k(k+1)}} \text{diag}\,(\underbrace{1\ 1 \cdots 1}_{k}\ -k\ \underbrace{0 \cdots 0}_{n-k-1}),\ k = 1, \ldots, n-1$$

Lie algebra:

- $\left[L^a_b, L^c_d\right] = \delta^a_d L^c_b - \delta^c_b L^a_d$
- Same algebra for generators T^a_b and other representations

Field transformations:

- Let $\psi^c, c = 1, \ldots, n$ transform as fundamental n, i.e.

$$\left[T^a_b, \psi^c\right] = -\left(L^a_b\right)^c{}_d\,\psi^d$$

implies

$$\begin{aligned} \psi^c &\to e^{-i\vec{\beta}\cdot\vec{T}}\psi^c e^{i\vec{\beta}\cdot\vec{T}} \\ &= U(\vec{\beta})^c{}_d\psi^d = \left(e^{i\vec{\beta}\cdot\vec{L}}\right)^c{}_d\psi^d = \left(e^{i\vec{\beta}\cdot\vec{L}}\psi\right)^c \end{aligned}$$

- **Antifundamental χ_a transforms as n^*,**

$$\left[T^a_b, \chi_c\right] = -\left(L^a_b(n^*)\right)_c{}^d \chi_d$$

where

$$\begin{aligned} L^a_b(n^*) &= -\left(L^a_b\right)^T = -L^b_a \\ \left(L^a_b(n^*)\right)_c{}^d &\equiv \left(L^a_b(n^*)\right)_{cd} \end{aligned}$$

($\chi_c \sim \epsilon_{cd_1\cdots d_{n-1}}\phi^{d_1\cdots d_{n-1}}$, totally antisymmetric products of $n-1$ fundamentals)

$$\begin{aligned} \chi_c &\rightarrow e^{-i\vec{\beta}\cdot\vec{T}}\chi_c e^{i\vec{\beta}\cdot\vec{T}} \\ &= \chi_d U^\dagger(\vec{\beta})^d{}_c = \left(\chi e^{-i\vec{\beta}\cdot\vec{L}}\right)_c \end{aligned}$$

- $\chi_c\psi^c$ **is** $SU(n)$ **invariant**
- **Adjoint field** ($n\times n^*$): $\phi^a_b \rightarrow \left(U(\vec{\beta})\phi U^\dagger(\vec{\beta})\right)^a_b$

$SU(n)$ Gauge Theory

- n^2-1 Hermitian generators T^i, $i=1,\cdots,n^2-1$ and corresponding gauge fields A^i

- Define $n\times n$ gauge matrix $\frac{A}{\sqrt{2}} = \sum_{i=1}^{n^2-1} L^i A^i = \sum \frac{\lambda^i A^i}{2}$
  ($A^a_b \equiv (A)_{ab} = A^{b\dagger}_a$ are non-Hermitian for $a \neq b$)

- $n=2$:

$$A = \begin{pmatrix} \frac{A^3}{\sqrt{2}} & A^1_2 \\ A^2_1 & -\frac{A^3}{\sqrt{2}} \end{pmatrix}, \qquad A^1_2 = \frac{A^1 - iA^2}{\sqrt{2}}$$

- $n=3$:

$$A = \begin{pmatrix} \frac{A^3}{\sqrt{2}} + \frac{A^8}{\sqrt{6}} & A^1_2 & A^1_3 \\ A^2_1 & -\frac{A^3}{\sqrt{2}} + \frac{A^8}{\sqrt{6}} & A^2_3 \\ A^3_1 & A^3_2 & \frac{-2A^8}{\sqrt{6}} \end{pmatrix}, \qquad \left\{ \begin{array}{l} A^1_2 = \frac{A^1 - iA^2}{\sqrt{2}} \\ A^1_3 = \frac{A^4 - iA^5}{\sqrt{2}} \\ A^2_3 = \frac{A^6 - iA^7}{\sqrt{2}} \end{array} \right.$$

Covariant Derivatives

Fundamental:

$$[D_\mu \psi]^a = \left[\partial_\mu \delta^a_b + ig(\vec{A}_\mu \cdot \vec{L})^a{}_b\right] \psi^b$$
$$= \left[\partial_\mu \delta^a_b + i\frac{g}{\sqrt{2}}(A_\mu)^a_b\right] \psi^b$$

Antifundamental:

$$[D_\mu \chi]_a = \left[\partial_\mu \delta^a_b + ig(\vec{A}_\mu \cdot \vec{L}(n^*))_a{}^b\right] \chi_b$$
$$= \left[\partial_\mu \delta^a_b - i\frac{g}{\sqrt{2}}(A_\mu)^b_a\right] \chi_b$$

Antisymmetric $n\times n$ representation:

Let $\psi^{ab}=-\psi^{ba}$ be $\frac{n(n-1)}{2}$ fields with

$$\begin{aligned}\left[T^a_b,\psi^{cd}\right] &= -(L^a_b)^c{}_e\psi^{ed}-(L^a_b)^d{}_e\psi^{ce}\\ [D_\mu\psi]^{ab} &= \partial_\mu\psi^{ab}+i\frac{g}{\sqrt{2}}(A_\mu)^a_c\psi^{cb}+i\frac{g}{\sqrt{2}}(A_\mu)^b_d\psi^{ad}\end{aligned}$$

(singlet for $n=2$; 3^* for $n=3$; 10 for $n=5$)

The Georgi-Glashow $SU(5)$ Model

$5^2-1=24$ generators $T^a_b-\frac{1}{5}\delta^a_b T^c_c,\ \ a,b,c=1,\ldots,5$
($\sum_{a=1}^{5}[T^a_a-\frac{1}{5}\delta^a_a T^c_c]=0$)

$SU(5)$contains $SU(3)\times SU(2)\times U(1)$

$SU(3)$: $T^\alpha_\beta-\frac{1}{3}\delta^\alpha_\beta T^\gamma_\gamma,\ \ \alpha,\beta,\gamma=1,2,3$

$SU(2)$: $T^r_s-\frac{1}{2}\delta^r_s T^t_t,\ \ \ r,s,t=4,5$

$U(1)$: $-\frac{1}{3}T^\alpha_\alpha+\frac{1}{2}T^r_r=-\frac{1}{3}(T^1_1+T^2_2+T^3_3)+\frac{1}{2}(T^4_4+T^5_5)$

$Q=T^3+Y=\frac{1}{2}(T^4_4-T^5_5)+Y$
(Q and Y are part of simple group ⇒cannot pick arbitrarily (charge quantization))

24 gauge bosons (adjoint representation) decompose as

$$24 \longrightarrow \underbrace{(8,1,0)}_{G^\alpha_\beta} + \underbrace{(1,3,0)}_{W^\pm, W^0} + \underbrace{(1,1,0)}_{B} + \underbrace{(3,2^*,-\tfrac{5}{6})}_{A^\alpha_r} + \underbrace{(3^*,2,+\tfrac{5}{6})}_{A^r_\alpha}$$

under $SU(3) \times SU(2) \times U(1)$

12 new gauge bosons, A^α_r, A^r_α, $\alpha = 1,2,3$, $r = 4,5$, carry flavor and color

$$A^4_\alpha \equiv X_\alpha \; [3^*, Q_X = \tfrac{4}{3}] \qquad A^\alpha_4 \equiv \bar{X}^\alpha \; [3, Q_{\bar{X}} = -\tfrac{4}{3}]$$

$$A^5_\alpha \equiv Y_\alpha \; [3^*, Q_Y = \tfrac{1}{3}] \qquad A^\alpha_5 \equiv \bar{Y}^\alpha \; [3, Q_{\bar{Y}} = -\tfrac{1}{3}]$$

$(X, Y) \sim 2$; $(\bar{X}, \bar{Y}) \sim 2^*$ under $SU(2)$

$$
\begin{aligned}
A &= \sum_{i=1}^{24} A^i \frac{\lambda^i}{\sqrt{2}} \\
&= \left(\begin{array}{ccc|cc}
G^1_1 - \frac{2B}{\sqrt{30}} & G^1_2 & G^1_3 & \bar{X}^1 & \bar{Y}^1 \\
G^2_1 & G^2_2 - \frac{2B}{\sqrt{30}} & G^2_3 & \bar{X}^2 & \bar{Y}^2 \\
G^3_1 & G^3_2 & G^3_3 - \frac{2B}{\sqrt{30}} & \bar{X}^3 & \bar{Y}^3 \\
\hline
X_1 & X_2 & X_3 & \frac{W^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\
Y_1 & Y_2 & Y_3 & W^- & -\frac{W^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}}
\end{array}\right)
\end{aligned}
$$

with $W^\pm = \frac{W^1 \mp i W^2}{\sqrt{2}}$

Fermions still in highly reducible representation: each family of L-fields in $5^* + 10$ (antifundamental and antisymmetric)

$$\underbrace{5^*}_{(\chi_L)_a} \quad \rightarrow \quad \underbrace{(3^*, 1, \tfrac{1}{3})}_{(\chi_L)_\alpha} + \underbrace{(1, 2^*, -\tfrac{1}{2})}_{(\chi_L)_r}$$

$$\underbrace{10}_{\psi_L^{ab} = -\psi_L^{ba}} \quad \rightarrow \quad \underbrace{(3^*, 1, -\tfrac{2}{3})}_{\psi_L^{\alpha\beta}} + \underbrace{(3, 2, \tfrac{1}{6})}_{\psi_L^{\alpha r}} + \underbrace{(1, 1, 1)}_{\psi_L^{45}}$$

$$(a, b = 1, \ldots, 5; \quad \alpha, \beta = 1, 2, 3; \quad r = 4, 5)$$

$$\begin{array}{cc} \begin{array}{c} X, Y \\ \longleftrightarrow \\ SU(2) \updownarrow \begin{pmatrix} \begin{matrix} \nu_L^0 \\ e_L^{-0} \end{matrix} & d_L^{0c} \end{pmatrix} \\ 5^* \end{array} & \begin{array}{c} X, Y \\ \longleftrightarrow \quad \longleftrightarrow \\ \begin{pmatrix} e_L^{+0} & \begin{matrix} u_L^0 \\ d_L^0 \end{matrix} & u_L^{0c} \end{pmatrix} \updownarrow SU(2) \\ 10 \end{array} \end{array}$$

$$\mathbf{5}^* : \quad \chi_{La} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L$$

$$\mathbf{10} : \quad \psi_L^{ab} = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ \hline u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{array} \right)_L$$

(family indices and weak-eigenstate superscript 0 suppressed)

R-fields (CP conjugates): $\psi_R^c \equiv C\overline{\psi_L}^T$

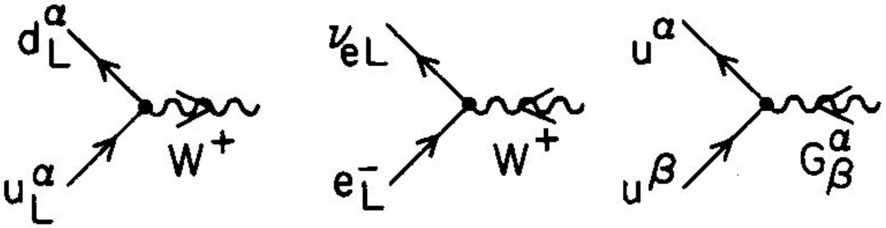
C = (representation dependent) charge conjugation matrix, e.g., $i\gamma^2\gamma^0$

$$5: \qquad \chi_R^{ca} = C\overline{\chi_{La}}^T = \begin{pmatrix} d^1 \\ d^2 \\ d^3 \\ e^+ \\ -\nu_e^c \end{pmatrix}_R$$

$$10^*: \quad \psi_{Rab}^c = C\overline{\psi_L^{ab}}^T = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u^3 & -u^2 & -u_1^c & -d_1^c \\ -u^3 & 0 & u^1 & -u_2^c & -d_2^c \\ u^2 & -u^1 & 0 & -u_3^c & -d_3^c \\ \hline u_1^c & u_2^c & u_3^c & 0 & -e^- \\ d_1^c & d_2^c & d_3^c & e^- & 0 \end{array}\right)_R$$

Fermion Gauge Interactions

$$
\begin{aligned}
-\mathcal{L}_f \;&=\; \underbrace{g_5 \sum_{i=1}^{8} \left[\bar{u}\; \not{G}^i \frac{\lambda^i}{2} u + \bar{d}\; \not{G}^i \frac{\lambda^i}{2} d \right]}_{\text{(QCD with } g_s = g_5\text{)}} \\
&+\; \underbrace{g_5 \sum_{i=1}^{3} \left[(\bar{u} \;\; \bar{d})_L \; \not{W}^i \frac{\tau^i}{2} \begin{pmatrix} u \\ d \end{pmatrix}_L + (\bar{\nu}_e \;\; \bar{e})_L \; \not{W}^i \frac{\tau^i}{2} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \right]}_{\text{(Weak } SU(2) \text{ with } g = g_5\text{)}} \\
&-\; \mathcal{L}_{U(1)_Y} - \mathcal{L}_{X,Y} + \text{ additional families}
\end{aligned}
$$

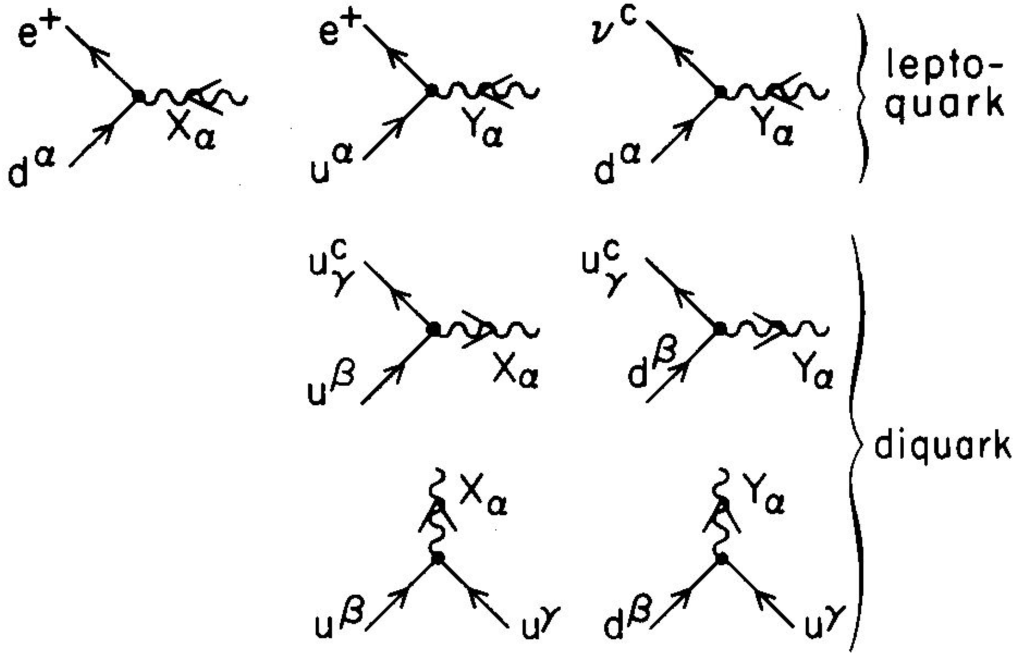


$$-\mathcal{L}_{U(1)_Y} = \sqrt{\frac{3}{5}} g_5 \underbrace{\left[-\frac{1}{2} (\bar{\nu}_L \not{B} \nu_L + \bar{e}_L \not{B} e_L) + \frac{1}{6} (\bar{u}_L \not{B} u_L + \bar{d}_L \not{B} d_L) + \frac{2}{3} \bar{u}_R \not{B} u_R - \frac{1}{3} \bar{d}_R \not{B} d_R - \bar{e}_R \not{B} e_R \right]}_{\text{(Weak } U(1)_Y \text{ with } g' = \sqrt{\frac{3}{5}} g_5 \text{)}}$$

$\sqrt{\frac{3}{5}} Y$ is properly normalized generator, $\mathrm{Tr}(L^i L^j) = \frac{\delta^{ij}}{2}$

$$-\mathcal{L}_{X,Y} = \underbrace{\frac{g_5}{\sqrt{2}}\left[\bar{d}_{R\alpha}\,\not{\bar{X}}^{\alpha} e_R^{+} + \bar{d}_{L\alpha}\,\not{\bar{X}}^{\alpha} e_L^{+} - \bar{d}_{R\alpha}\,\not{\bar{Y}}^{\alpha} \nu_R^{c} - \bar{u}_{L\alpha}\,\not{\bar{Y}}^{\alpha} e_L^{+}\right]}_{\text{(Leptoquark vertices)}}$$

$$+ \underbrace{\frac{g_5}{\sqrt{2}}\left[\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\,\not{\bar{X}}^{\alpha} u_L^{\beta} + \epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\,\not{\bar{Y}}^{\alpha} d_L^{\beta}\right]}_{\text{(Diquark vertices)}} + \textbf{H.C.}$$



$$q_X = \frac{4}{3}$$

$$q_Y = \frac{1}{3}$$

Proton Decay

Combination of leptoquark and diquark vertices leads to baryon (B) and lepton (L) number violation ($B-L$ conserved)

$$p \to e^+\bar{u}u, e^+\bar{d}d \Rightarrow p \to e^+\pi^0, e^+\rho^0, e^+\omega, e^+\eta, e^+\pi^+\pi^-, \cdots$$

$$p \to \bar{\nu}\bar{d}u \Rightarrow p \to \bar{\nu}\pi^+, \bar{\nu}\rho^+, \bar{\nu}\pi^+\pi^0, \cdots$$

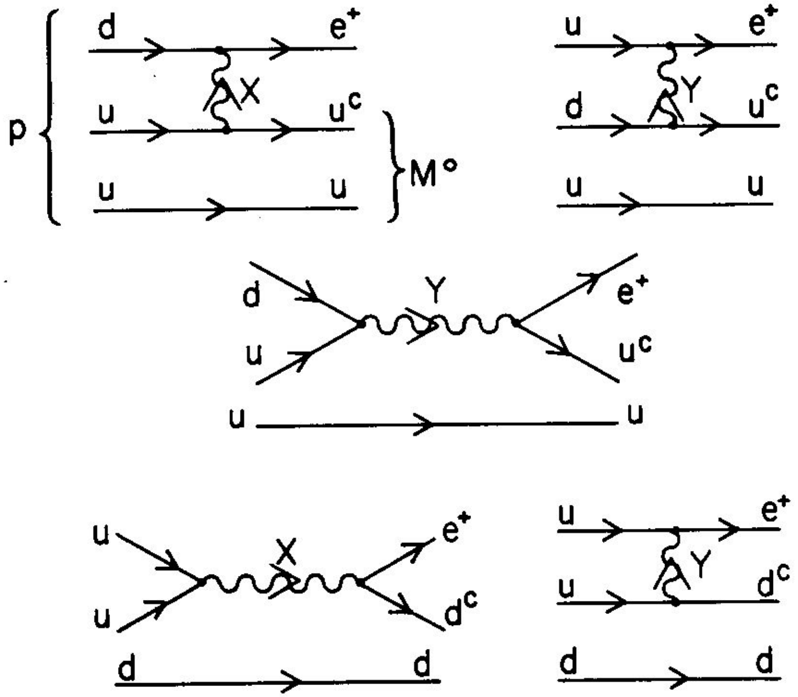
Expect $\tau_p \sim \frac{M^4_{X,Y}}{\alpha_5^2 m_p^5}$ where $\alpha_5 \equiv \frac{g_5^2}{4\pi}$

(cf $\tau_\mu \sim m_W^4/g^4 m_\mu^5$)

$\tau_p \gtrsim 10^{33}$ yr and $\alpha_5 \sim \alpha \Rightarrow M_{X,Y} \gtrsim 10^{15}$ GeV (GUT scale)

Also bound neutron decay

Additional mechanisms/modes in SUSY GUT



Spontaneous Symmetry Breaking

Introduce adjoint Higgs, $\Phi = \sum_{i=1}^{24} \phi^i \frac{\lambda^i}{\sqrt{2}}$

$$V(\Phi) = \frac{\mu^2}{2} \text{Tr}(\Phi^2) + \frac{a}{4} \left[\text{Tr}(\Phi^2) \right]^2 + \frac{b}{2} \text{Tr}\ (\Phi^4)$$

(have assumed $\Phi \longrightarrow -\Phi$ symmetry)

Can take $\langle 0|\Phi|0\rangle =$ diagonal by $SU(5)$transformation

$\langle 0|\Phi|0\rangle \neq 0$ for $\mu^2 < 0$

For $b > 0$ minimum is at

$$\langle \Phi \rangle = \begin{pmatrix} \nu & 0 & 0 & 0 & 0 \\ 0 & \nu & 0 & 0 & 0 \\ 0 & 0 & \nu & 0 & 0 \\ 0 & 0 & 0 & -\frac{3\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3\nu}{2} \end{pmatrix}$$

with $\nu = \frac{-2\mu^2}{15a+7b}$

$$\Rightarrow SU(5) \underset{M_X}{\longrightarrow} SU(3) \times SU(2) \times U(1)$$

with

$$M_X^2 = M_Y^2 = \frac{25}{8} g_5^2 \nu^2$$

(Need $a > -\frac{7}{15} b$ for vacuum stability. $SU(5) \underset{M_X}{\longrightarrow} SU(4) \times U(1)$ for $b < 0$)

To break $SU(2) \times U(1)$, introduce (fundamental) Higgs 5

$$H^a = \begin{pmatrix} H^\alpha \\ \phi^+ \\ \phi^0 \end{pmatrix}$$

$H^\alpha = (3, 1, -\frac{1}{3})$, color triplet with $q_H = -1/3$
$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = (1, 2, \frac{1}{2})$, SM Higgs doublet

Problem 1: Need to give $\langle \phi^0 \rangle = \frac{\nu_0}{\sqrt{2}}$ with $\nu^0 \sim 246$ GeV (weak scale) $\sim 10^{-13} \nu$

Problem 2: Need $M_H \gtrsim 10^{14}$ GeV $\gtrsim 10^{12} M_\phi$ to avoid too fast proton decay mediated by H^α (doublet-triplet splitting problem)

$$\begin{aligned} V(\Phi, H) &= \frac{\mu^2}{2}\,\mathsf{Tr}(\Phi^2) + \frac{a}{4}\left[\,\mathsf{Tr}(\Phi^2)\right]^2 + \frac{b}{2}\,\mathsf{Tr}\,(\Phi^4) \\ &+ \frac{\mu_5^2}{2} H^\dagger H + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \alpha H^\dagger H\ \mathsf{Tr}\left(\Phi^2\right) + \beta H^\dagger \Phi^2 H \end{aligned}$$

(μ_5^2, λ, α terms don't split doublet from triplet, but β does)

Can satisfy $\nu_0 \ll \nu$ and $M_H \gg M_\phi$, but requires fine-tuned cancellations

Yukawa Couplings

No Φ couplings to χ_{La}, ψ_L^{ab} allowed by $SU(5)$, but can have

$$\begin{aligned} L_{\text{Yuk}} &= \gamma_{mn}\, \chi^T_{mLa} C\, \psi^{ab}_{nL}\ H^\dagger_b \\ &+ \Gamma_{mn}\, \epsilon_{abcde}\, \psi^{Tab}_{mL}\ C\, \psi^{cd}_{nL}\ H^e + \text{HC} \end{aligned}$$

(γ, Γ are family matrices (Γ symmetric); m, n = family indices; ϵ = antisymmetric with $\epsilon_{12345} = 1$; C = charge conjugation matrix, with $\psi^T_L C \eta_L = \overline{\psi^c_R} \eta_L$)

Fermion masses from $\langle 0 | H^a | 0 \rangle = \frac{\nu_0}{\sqrt{2}} \delta^5_a$

First term:

$$-\bar{d}_L M^d d_R - \underbrace{\bar{e}_L^+ M^{eT} e_R^+}_{\bar{e}_L M^e e_R} + \text{HC}$$

$$M^d = M^{eT} = \frac{1}{2}\nu_0 \gamma^\dagger$$

$\Rightarrow d$ and e have same mass matrices up to transpose ($M^d = M^{eT} \neq M^e$ used in "lopsided" models, but harder to implement in $SO(10)$)

$\Rightarrow m_d = m_e, \; m_s = m_\mu, \; m_b = m_\tau$ at M_X

Appears to be a disaster, *but*

These run, mainly from gauge loops. Gluonic loops make quark masses larger at low energies

$$\ln\left[\frac{m_d(Q^2)}{m_e(Q^2)}\right] = \underbrace{\ln\left[\frac{m_d(M_X^2)}{m_e(M_X^2)}\right]}_{=0} + \frac{4}{11-2n_q/3}\ln\left[\frac{\alpha_s(Q^2)}{\alpha_5(M_X^2)}\right] + \frac{3}{2n_q}\ln\left[\frac{\alpha_1(Q^2)}{\alpha_5(M_X^2)}\right]$$

$\Rightarrow m_b/m_\tau \sim 5/1.7$; works reasonably well, ordinary and SUSY GUT

But $\frac{m_e}{m_\mu} \sim \frac{1}{200} \neq \frac{m_d}{m_s} \sim \frac{1}{20}$ is failure of model

(need more complicated Higgs sector)

Γ_{mn} term gives independent M^u

($M^u = M^\nu_{\rm Dirac}$ in $SO(10)$ plus other (model dependent) relations))

Gauge Unification

Gauge couplings unified at M_X (simple group)

$$\begin{aligned} g_3 &\equiv g_s = g_5 \\ g_2 &= g = g_5 \\ g_1 &= \sqrt{\frac{5}{3}} g' = g_5 \end{aligned}$$

Generators must have same normalization

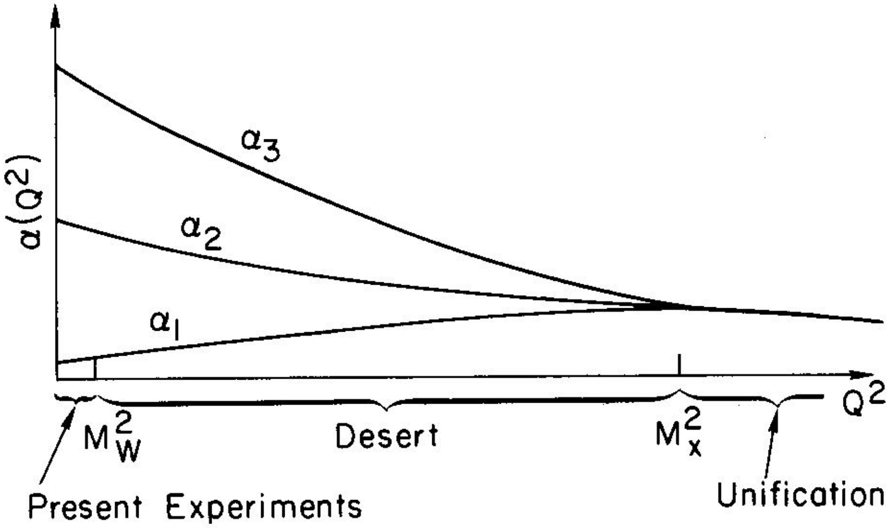
$$\mathrm{Tr}(L^i L^j) = \frac{\delta^{ij}}{2} \Rightarrow \sqrt{\frac{3}{5}} Y$$

is $SU(5)$ generator, with $g'Y = \underbrace{\sqrt{\frac{5}{3}} g'}_{g_5} \; \sqrt{\tfrac{3}{5}} Y$

Weak mixing angle at M_X

$$\sin^2\theta_W\,(M_X) = \frac{g'^2}{g^2 + g'^2} = \frac{3/5}{1 + 3/5} = \frac{3}{8}$$

$SU(5)$ broken below M_X (and some particles decouple) $\Rightarrow SU(3) \times SU(2) \times U(1)$ couplings run at different rates



Running $\alpha_i = g_i^2/4\pi$

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_X)} - b_i \ln \frac{M_Z^2}{M_X^2}$$

$$b_1 = \frac{F}{3\pi}, \quad b_2 = -\frac{1}{4\pi}\left[\frac{22}{3} - \frac{F}{3}\right], \quad b_3 = -\frac{1}{4\pi}\left[11 - \frac{F}{3}\right]$$

(F = number of families)

$$\Rightarrow \sin^2\theta_W\,(M_Z) \;=\; \frac{1}{6} + \frac{5}{9}\frac{\alpha(M_Z)}{\alpha_s(M_Z)} \sim 0.20$$

$$M_X \;\sim\; 10^{14}\ \text{GeV}$$

Reasonable zeroth order prediction

However, proton decay requires $M_X \gtrsim 10^{15}$ GeV

Precise $\sin^2\theta_W\,(M_Z) \sim 0.23$ and $\alpha_s(M_Z) \sim 0.12$ does not quite work, even with two-loop corrections ⇒SUSY

Beyond $SU(5)$

Larger gauge groups: $SU(5) \subset SO(10) \subset E_6$

- $SO(10) \rightarrow SU(5) \times U(1)_\chi$
- Extra Z' from $U(1)_\chi$ may survive to TeV scale
- Family in one IRREP:

$$\underbrace{\mathbf{16}}_{SO(10)} \rightarrow \underbrace{\mathbf{5}^* + \mathbf{10}}_{SU(5)\ \text{family}} + \underbrace{\mathbf{1}}_{\nu_R}$$

  - $\nu_R \leftrightarrow \nu_L^c$ = "right-handed" (singlet, sterile) neutrino
  - $M^u = M^\nu_{\text{Dirac}}$ in simplest Higgs scheme
  - Seesaw for large Majorana mass (need Higgs 126)
  - Right mass scales, but small mixings in simplest schemes
  - Leptogenesis

- $E_6 \rightarrow SO(10) \times U(1)_\psi$ (second possible Z')
  - Family:

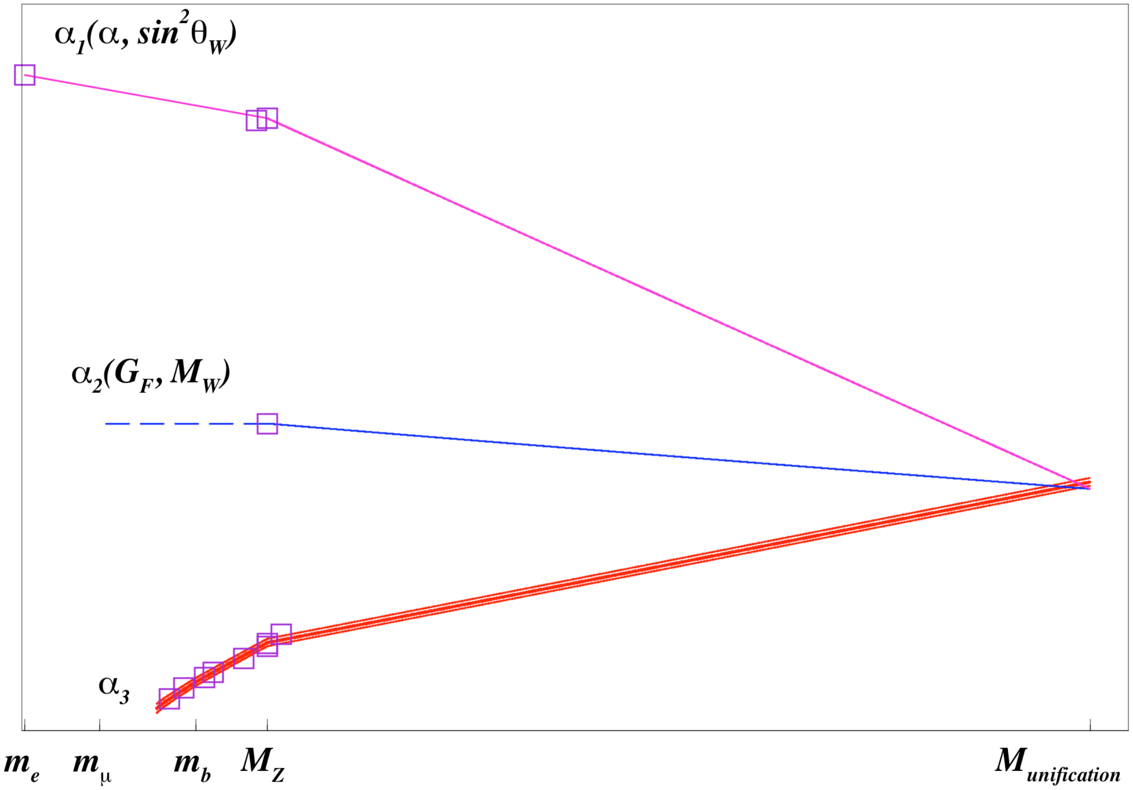
$$\underbrace{\mathbf{27}}_{E_6} \rightarrow \underbrace{\mathbf{16}}_{SO(10)} + \underbrace{\mathbf{10} + \mathbf{1}}_{\text{exotics}}$$

$$\mathbf{10} = \underbrace{\begin{pmatrix} E^0 \\ E^- \end{pmatrix}_L + \begin{pmatrix} E^0 \\ E^- \end{pmatrix}_R}_{\text{both doublets}} + \underbrace{D_L + D_R}_{\text{both singlets}}$$

$$\mathbf{1} = S_L \text{ (no charge, similar to } \nu_L^c\text{)}$$

Supersymmetric extensions: different b_i factors ⇒

- Better gauge unification agreement



- $M_X \sim 10^{16}$ GeV $\Rightarrow \tau(p \rightarrow e^+\pi^0) \sim 10^{38}$ yr
- However, new “dimension-5” operators involving superpartners may yield too rapid $p \rightarrow \bar{\nu} K^+$, etc.

Extra dimensions: new GUT-breaking mechanisms from boundary conditions

String compactifications: direct compactifications have some ingredients of GUTs

- String $\longrightarrow$ MSSM ($+$ extended?) in 4D without intermediate GUT phase avoids doublet-triplet problem, and need for large representations for GUT breaking and fermion/neutrino masses/mixings

Additional Implications

Fermion mass textures: often done in $SO(10)$ context, but need larger Higgs representations and family symmetries

Baryogenesis:

- Baryon excess could be generated by out of equilibrium decays of H^{α} (insufficient in minimal model)
- However, $B-L$ conserved and asymmetry with $B-L=0$ wiped out by electroweak sphalerons ($\Rightarrow$leptogenesis, EW baryogenesis, Affleck-Dine, $\cdots$)

Magnetic monopoles:

- Topologically stable gauge/Higgs configurations with $M_M \sim M_X/\alpha$
- Greatly overclose Universe unless subsequent inflation

To GUT or not to GUT

String → GUT → MSSM (+ extended?) *or* String → MSSM (+ extended?)

- gauge unification
- quantum numbers for family (15-plet)
- seesaw ν mass scale/leptogenesis
- m_b/m_τ

- large lepton mixings
- other fermion mass relations (need large Higgs representatons)
- additional GUT scale; no adjoints in simple heterotic
- hierarchies, e.g. doublet-triplet
- proton decay