

DECONSTRUCTION

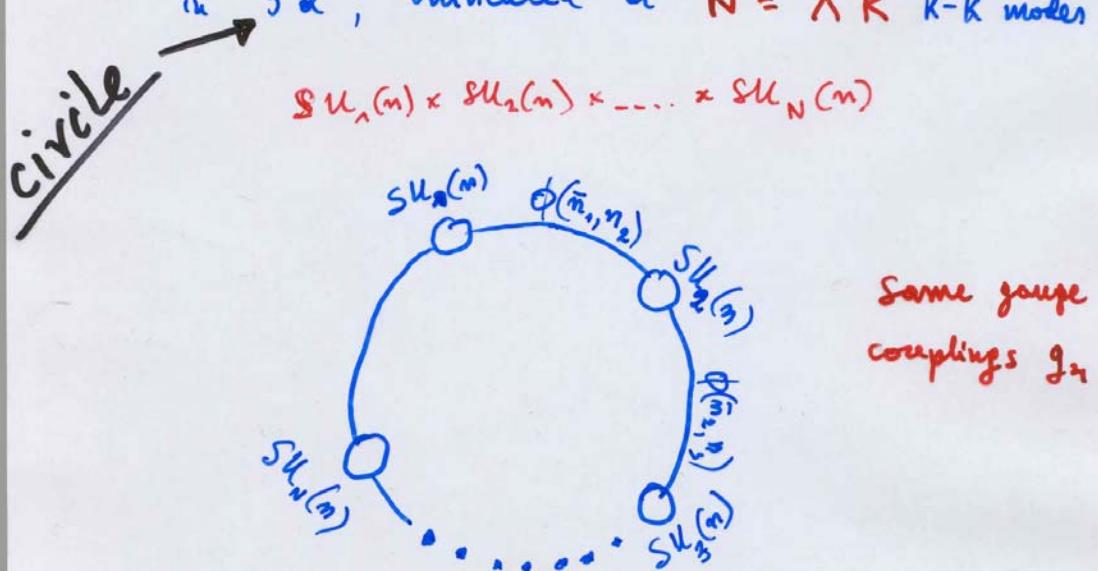
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Proposed as an UV regulator
(completion) of 5d theories

Is it useful?
What have we learned?
What perhaps we can still learn?

The observation (Arkani-Hamed, Cohen, Georgi)
Hill, SP, Wang

there exists a 4d gauge invariant and
renormalizable field theory which in its IR
mimics the physics of $SU(n)$ gauge theory
in 5d, truncated at $N = \Lambda R$ K-K modes



$$SU(n)^N \quad \phi_i: (\bar{m}_i, n_{i+1}) \quad i=1, N$$

$$\phi_N(\bar{m}_N, n_1)$$

Assume potential for ϕ_i gives VEV $\phi_i = v_1$

Then $SU(n)^N \rightarrow "SU(n)" \text{ diagonal}$

The lagrangian

$$\mathcal{L} = -\frac{1}{4} \sum_{i=0}^N F_{ij\mu\nu}^a F^{aj\mu\nu} + \\ + \sum_{i=1}^N D_\mu \phi_i^+ D_\mu^\dagger \phi_i + V(\phi_i)$$

$$D_\mu^j = \partial_\mu + ig_u A_{\mu j}^a T^a - ig_u A_{\mu j}^a T^a$$

Assumption

$V(\phi_i)$ has full $SU^N(n) \times SU^N(n)$
 $\phi \rightarrow L\phi R^+$ (chiral) symmetry and is
symmetric under interchange $\phi_j \leftrightarrow \phi_i$

Suppose the dynamics (described by
 $V(\phi_i)$) is such that the diagonal
components of ϕ_i acquire VEVs v

$$\langle \phi_i \rangle = v \mathbb{1}$$

$$\sum_i (\mathcal{D}_\mu \phi_i)^+ \mathcal{D}^\mu \phi_i \rightarrow g_4^2 v^2 \sum_i (A_i^{a\mu} - A_{i-1}^{a\mu})^2$$

$$M_n = 2g_4 v \sin \frac{\pi n}{N} \quad 0 \leq n \leq N-1$$

$$\approx 2g_4 v \frac{\pi n}{N} \quad \text{for } n \ll N$$

Identify

$$\underbrace{\frac{1}{R}}_{= \frac{1}{2g_4 v}} \rightarrow M_n = \frac{n}{R}$$

The coupling of the $SU(n)_D$:

$$g_D = \frac{g_4}{\sqrt{N}}$$

Identify

$$\underbrace{g_D}_{= \frac{g_5}{\sqrt{2\pi R}}} \rightarrow \text{same interactions as in effective } 5d \rightarrow 4d$$

$$g_5, R, \Lambda \rightarrow g_4, N, v$$

$$\begin{aligned} \Lambda &= g_4 v \\ N &= \Lambda R \end{aligned}$$

The physics below the scale v of spontaneous gauge symmetry breaking is the same as that of 5d theory with cut-off

Non-linear σ -model approximation to the full 4d theory which includes also $V(\phi_i)$

$$\bar{\Phi}_i \Rightarrow v \exp(i\phi_i^a T^a/v)$$

where ϕ_i^a , $i = 1 \dots N$ are Goldstone bosons of the spontaneously broken full (chiral) symmetry of $V(\bar{\Phi}_i)$

$$\bar{\Phi} \rightarrow L \bar{\Phi} R^\dagger \quad SU(N) \times SU(n)^N \rightarrow SU(N)(n)$$

N-1 GB eaten up by gauge bosons; one remains in the physical spectrum ($\equiv A_5^{(0)}$);

$$N \cdot \underbrace{\binom{N}{2}}_{= N \cdot 2N^2 \text{ states}} = \left\{ \begin{array}{l} \text{There remain } 2N \text{ singlets + } N \text{ adjoint} \\ \text{with } M \sim O(V) \end{array} \right. \quad \begin{array}{l} \text{with } M \sim O(V) \\ N(N^2-1) + N(N^2-1) + 2N = 2N^3 \end{array}$$

(9)

Supersymmetric extension

Wakai, Erlich, Grajeau, Kribs

Raise scalar fields in previous model
to 4D $N=1$ chiral multiplets

and introduce V_j , $j=1 \dots N$ vector
superfields associated to the gauge group
 $SU(n)_j$.

$$\bar{\Phi}_i(\psi_i, \varphi_i) \quad V_j(A_j^\alpha, \lambda_j^\alpha)$$

\downarrow
bi-fundamentals

After $SU(n)^N \rightarrow SU(n)_D$

4D $N=2$ massless vector

$$\begin{pmatrix} A^\mu \\ \lambda & \psi \\ \psi \end{pmatrix} \quad \begin{array}{l} 2 \text{ components} \\ 4 \text{ components} \\ 2 \text{ components} \end{array}$$

4D $\mathcal{N}=2$ massive vector

$$\begin{pmatrix} A^\mu \\ \lambda \\ \psi \end{pmatrix} \quad \begin{array}{l} 3 \text{ components} \\ 4 \text{ components} \\ 1 \text{ component} \end{array}$$

5D $\mathcal{N}=1$ vector multiplet

$$\begin{pmatrix} A^\mu \\ \psi \\ \phi \end{pmatrix} \quad \begin{array}{l} 3 \text{ components} \\ 4 \text{ components} \\ 1 \text{ component} \end{array}$$

Works for hypermultiplets as well.

$$4D \mathcal{N}=1 \quad SU(n)^N \rightarrow 4D \mathcal{N}=2 SU(n)_D$$

Another extension

Non-perturbative equivalence of the two theories

Csaki, Erlich, Khoze, Poppitz, Shadmi, Shirman

④ → Quantum corrections in 5d:

UV (insensitive) observables
sensitive

UV insensitive: e.g. mass of A_5 ;

but one still needs gauge invariant regularization procedure ($\Lambda \rightarrow \infty$)

Deconstruction: unambiguous calculation,
for finite N (calculable corrections
due to finite N to the 5d results)

Power-law-like running of the couplings
as a limit of the logarithmic running for
finite N

(Finite) corrections to the Goldstone boson mass
($\equiv A_5^{(0)}$)

Deconstruction - is it useful?

→ Better understanding of 5d theories
at the quantum level (regularization problems)



- Falkowski, Grojean, SP
- one loop radiative corrections to the mass of K-K gauge boson excitation
 - understanding the properties of AdS gauge theories (in RSI background)
- Falkowski, Kim, Randall, Sundrum, Weiner
- * IR brane

- Falkowski, Nilles, Olechowski, SP
- *
Dudas, Falkowski, SP
- constraints on supersymmetric U(1) gauge theories in 5d from the requirement of ~~possessing~~ having (deconstructed) UV completion
 - Chern-Simons term

- Non-perturbative problems in 5d

Loop correction to KK gauge boson masses
in 5d QED (from a single
5d fermion of electric charge e_5)

Cheng
Matchev
Schmalz
Puchwein, Kunszt

$$\delta m_n^2 = - \frac{\xi(3) e^2}{4\pi^4 R^2} + \underline{m_n^2 + \text{uncalculable}}$$

$$(n > 0)$$

$$e = \frac{e_5}{\sqrt{2\pi R}}$$

Deconstructed 5d QED

$U(1)^N$, $\phi_p (e, -e)$ under $U(1)_p \times U(1)_{p+1}$

N pairs of chiral fermions

$(\psi_p, \chi_p)_{p=1 \dots N}$ with charges (e, e) under $U(1)_p$

$$\mathcal{L} = \sum_{p=1}^N \left(i \bar{\psi}_p \gamma^\mu D_{\mu, p} \psi_p + i \bar{\chi}_p \gamma^\mu D_{\mu, p} \chi_p + \right.$$

$$\left. + f_2 e \bar{\phi}_p \bar{\chi}_p \psi_{p+1} - e v \bar{\chi}_p \psi_p + \text{l.c.} \right)$$



$$\delta m_n^2 = \frac{e^2 v^2}{N} \left[-\frac{\xi(3) e^2}{\pi^2} + \frac{11 \pi^2 e^2}{108} \frac{n^4}{N} - \dots \right]$$

or, in 5d language

$$\delta m_n^2 = -\frac{e^2}{16\pi^4 R^2} \left(4\xi(3) - \frac{11\pi^3 n^4}{54\pi R} + \frac{5\xi(5)}{(\pi R)^2} + \dots \right)$$

Interpretation in terms of loop-induced operators:

$$W(p,q) = \phi_p \dots \phi_{q-1}$$

$$\begin{aligned} D_p W(p,q) &= \partial_p W(p,q) + ig A_{p,p} W(p,q) \\ &\quad - ig W(p,q) A_{p,q} \end{aligned}$$

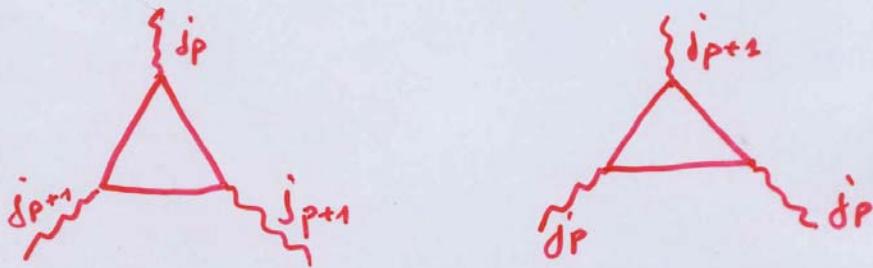
$$\mathcal{L} \sim \text{Tr} \sum_{p,q=1, p \neq q}^N D_p W(p,q) D^q W(q,p) + \dots$$

Deconstruction of supersymmetric $U(n)$, mixed anomalies and 5d limit

Take $U(1)^N$ and links (^{chiral} superfields)

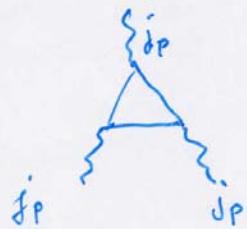
$$\phi_p(-q_p, q_p) \quad \phi_{p+2}(-q_p, q_{p+2})$$

$$\phi_{p+2}(-q_{p+2}, q_{p+2})$$

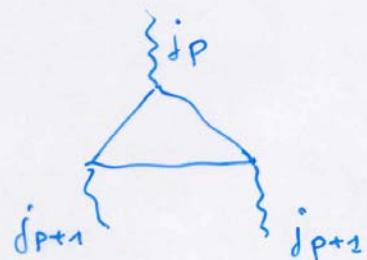


Anomalous because only ϕ_p, ϕ_{p+2}
in the loop and not ϕ_{p+1}

Gauge anomalies



Anomalous only for
 $p=1$
 $p=N$



Possible regularization schemes for triangle diagrams

- the anomaly is placed in the current appearing only once in the correlator

$$\delta d_{an} = -\frac{i}{4\pi^2} \sum_p d^2 \theta \Lambda_p (W_{p+1}^\alpha W_{a,p+1} - W_{p-1}^\alpha W_{a,p}) + h.c.$$

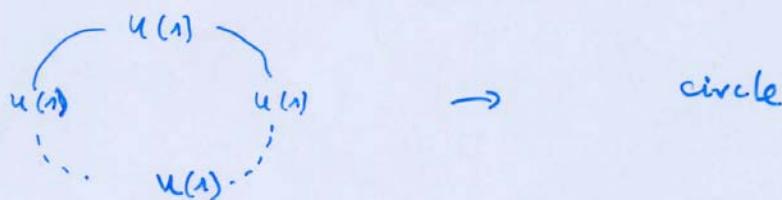
- the anomalous divergences are placed symmetrically

$$\delta d_{an} = -\frac{i}{12\pi^2} \sum_p (d^2 \theta \Lambda_p (W_{p+1} W_{p+1} - W_{p-1} W_{p-1} - 2 W_p W_{p+1} + 2 W_p W_{p-1})) + h.c. \cancel{+}$$

5 d limit

(electroweak
deconstruction)

Take



Mixed anomalies

no anomalies

- cancellation by Green-Schwarz
 - or general Wess-Zumino terms
- must correspond to Chern-Simons terms

If we insist on the 5 d limit, we must get

{	5 d Lorentz invariant theory
	N=2 supersymmetry

- 1) These are strong constraints: (only for symmetric anomaly renormalization and correlates the Kähler kinetic terms ~~for supersymmetry~~ with the form of W^2 terms)
- 2) 5 d theory with UV completion by deconstruction has Chern-Simons term

→ New model building tool

Arkani-Hamed
Cohen, Georgi,
Kaplan, Nelson

- new approach to the hierarchy problem ("little Higgs")

Dudas
Falkowski
sp

- a mechanism for supersymmetry breaking

→ Link to quiver theories

- extra-dimensional nature of the Higgs-branch of such theories
- role of superconformal (finite) field theories

realistic low energy models
as an IR limit of super-
conformal theories?

Deconstruction

(renormalizable theory)

Basic idea (totally unrealistic example)

Higgs boson as pseudo-Goldstone boson

of the "chiral" $SU(n)^N \times SU(n)^N$
symmetry in the scalar sector

$$\phi_i \rightarrow u_i \phi v_i^+ \quad \text{to } SU_N(n)^N$$

$$u_i = v_i$$

The symmetry is broken spontaneously but
also explicitly, by gauge interactions
 $SU(n)^N$ (different from $SU_N(n)^N$)

$$\phi_i \rightarrow u_i \phi_i u_{i+1}^+ \quad v_i = u_{i+1}$$

Gauge invariant operators which contribute
to the Goldstone boson mass

$$[\mathrm{Tr} \phi_1 \phi_2 \dots \phi_N]^2$$

For $N > 2$ its dim > 4 and no counterterms!

small effects

degenerate massless unrenormalizable

Mass corrections to the Goldstone boson
 $(\equiv A_5^0)$ in the linear (renormalized)
 theory

$$(\text{finite}) \quad \delta m^2 = \frac{4 g_4^4 v^2}{16 \pi^2 N^3} \sim \frac{g_D^2}{16 \pi^2} M_1^2$$

$$\text{where } M_1^2 \sim \frac{g_4^2 v^2}{N^2}$$

Linear \rightarrow non-linear S model (the details of
 the UV completion not specified) \rightarrow same
 result (because δm^2 is finite)

Compare e.g. to $m_{\pi^+}^2 - m_{\pi^0}^2$.

$$\delta m_\pi^2 = \left(\frac{e}{4\pi}\right)^2 (4\pi f_\pi)^2 \quad \underline{\text{cloud}}$$

So $\Lambda = 4\pi f_\pi$ is replaced by $\Lambda' = \frac{\Lambda}{N}$

(for the same value of δm^2 the details of
 unknown physics are irrelevant up to $\frac{1}{N\Lambda}$)

Take $N=2$ (the littlest Higgs)

Scalar sector with

$$SU_L^{(1)}(n) \times SU_R^{(1)}(n) \times SU_L^{(2)}(n) \times SU_R^{(2)}(n)$$

\swarrow \downarrow
 $\phi_1(\bar{3}, 3)$ $\phi_2(\bar{3}, 3)$

$$\phi_1 \rightarrow L_1^+ \phi_1 R_1^- \quad \oplus \quad \phi_2 \rightarrow L_2^+ \phi_2 R_2^-$$

Broken by $\phi_1 = v \mathbf{1}$ to

$$SU_V^{(1)}(n) \times SU_V^{(2)}(n)$$

Generators of $SU_V^{(i)}$:

$$T_L^1 + T_R^1 = Y_1, \quad T_L^2 + T_R^2 = Y_2$$

Broken generators

$$T_L^1 - T_R^1 = X_1, \quad T_L^2 - T_R^2 = X_2$$

or linear combinations $X_1 \pm X_2$

Add gauged group

$$g_1 \text{SU}_1(n) \times g_2 \text{SU}_2(n)$$

such that

$$\phi_1(\bar{3}_1, 3_2) \quad \phi_2(\bar{3}_2, 3_1)$$

i.e. $\phi_1 \rightarrow G_1^+ \phi_1 G_2 \quad \phi_2 \rightarrow G_2^+ \phi_2 G_1$

We see that

$$G_1 = L_1 + R_2 \quad , \quad G_2 = L_2 + R_1$$

to match (*)

For the generators

$$\begin{aligned} G_1 &= \frac{1}{2}(Y_1 + Y_2 + X_1 - X_2) & \left\{ \begin{array}{l} G_1 + G_2 = Y_1 + Y_2 \\ G_1 - G_2 = X_1 - X_2 \end{array} \right. \\ G_2 &= \frac{1}{2}(Y_1 + Y_2 + X_2 - X_1) \end{aligned}$$

" $X_1 - X_2$ " is gauged away

" $X_1 + X_2$ " remains in the physical spectrum
as a pseudo-GB

$O \subset \wp$ darf, rauswerfen, ergänzen

$$O = \{ x +_r y +_r r -_r v \mid v \in \omega \}$$

(beschränkt man) so liegen mindestens zwei
-entfernen \Rightarrow bestimmt unterschiedliche Länge
! einer mindestens eine Menge

$$O = \frac{\text{W}, \text{W}}{\text{L} \oplus \text{L}}$$

! nur zwei
! zusammenhängend

$$O \neq \frac{\text{W}, \text{W}}{\text{L} \oplus \text{L}}$$

zusammenhängendes Paar \rightarrow OI
(VdT OI) $O \subset \Lambda \Leftarrow$

Little Higgs - one of few
calculable alternatives to supersymmetry
for the electroweak breaking
but . . .

the usual price for the onset of
non-perturbative physics at low energy
(flavor, unification, etc..)