SUPERSYMMETRY BREAKING AND RADION STABILIZATION

M. QUIROS

DESY 2003

Outline

- SCHERK-SCHWARZ SUPERSYMMETRY BREAKING
- FIXING THE SCHERK-SCHWARZ ORDER PARAMETER
- RADION STABILIZATION
  - *Massless bulk fields*
  - *Massive (quasi-localized) fields*
- CONCLUSION

Based on works done in collaboration with:

G. Gersdorff, L. Pilo, D. Rayner, A. Riotto

SCHERK-SCHWARZ SUPERSYMMETRY BREAKING

- Scherk-Schwarz (SS) supersymmetry breaking of a five-dimensional (5D) theory compactified on $S^1/\mathbb{Z}_2$ can be interpreted as spontaneous breaking of 5D local supersymmetry [a] by the Hosotani mechanism

- To interpret SS supersymmetry breaking as a Hosotani mechanism one has to go to the off-shell version of 5D N=1 SUGRA where $SU(2)_R$ is gauged by auxiliary fields $\vec{V}_M$

- Off-shell 5D SUGRA has recently been formulated [b]. Two multiplets are necessary: the minimal supergravity multiplet $(40_B + 40_F)$ and the tensor multiplet $(8_B + 8_F)$. Their parities are given in tables 1 and 2, respectively

---

[a] G. Gersdorff and M. Quiros, hep-th/0110132

[b] M. Zucker, hep-th/9907082, hep-th/9909144, hep-th/0009083; T. Kugo and K. Ohashi, hep-ph/0006231, hep-ph/0010288, hep-th/0203276; +T. Fujita, hep-th/0106051

Table 1: Minimal supergravity multiplet

Field		$\mathbb{Z}_2 = +1$	$\mathbb{Z}_2 = -1$
e_M^A	graviton	e_μ^a, e_5^5	e_μ^5, e_5^a
ψ_M	gravitino	$\psi_{\mu L}^1, \psi_{5L}^2$	$\psi_{\mu L}^2, \psi_{5L}^1$
B_M	graviphoton	B_5	B_μ
$\vec{V}_M$	$SU(2)_R$-gauge	$V_\mu^3, V_5^{1,2}$	$V_5^3, V_\mu^{1,2}$
v^{MN}	antisymmetric	$v^{\mu 5}$	$v^{\mu\nu}$
$\vec{t}$	$SU(2)_R$-triplet	$t^{1,2}$	t^3
C	real scalar	C	
ζ	$SU(2)_R$-doublet	ζ_L^1	ζ_L^2

Table 2: Tensor multiplet

Field	$\mathbb{Z}_2 = +1$	$\mathbb{Z}_2 = -1$
$\vec{Y}$	$Y^{1,2}$	Y^3
B_{MNP}	$B_{\mu\nu\rho}$	$B_{\mu\nu 5}$
N	N	
ρ	ρ_L^1	ρ_L^2

- Fields in the upper panel of table 1 are physical fields while those in the lower panel are auxiliary fields. Fields in table 2 are all of them auxiliary fields

- The auxiliary fields that are relevant for supersymmetry breaking are: $V_5^{1,2}$, that constitute the F-term of the radion superfield [a],

$$\mathbb{T} = \left[T \equiv e_5^5 + iB_5, \psi_{5L}^2, V_5^1 + i\, V_5^2\right]$$

[a] Z. Chacko and M.A. Luty, hep-ph/ 0008103; D. Marti and A. Pomarol, hep-th/0106256; D.E. Kaplan and N. Weiner, hep-ph/0108001

- In the background of V_5^2 the Goldstino is identified with the fifth component of the gravitino: ψ_5. This is obvious from the local supersymmetry transformation,

$$\delta_\xi \psi_5 = \mathcal{D}_5 \xi + \cdots = i\sigma^2 \, V_5^2 \xi + \cdots$$

- The 5D kinetic term for the gravitino can be decomposed in four-dimensional and extra-dimensional components

- The redefinition

$$\psi_\mu = \psi'_\mu + \mathcal{D}_\mu (\mathcal{D}_5)^{-1} \psi_5$$

can be seen as a local supersymmetry transformation with parameter $(\mathcal{D}_5)^{-1}\psi_5 \equiv \xi$, gauging ψ_5 away and giving a mass to the gravitino

- This defines a "super-unitary" gauge where ψ_5 has been "eaten" by the four dimensional gravitino ψ_μ

- Using the coupling of V_5^2 to the gravitino field through the covariant derivative $\mathcal{D}_5$ in $\mathcal{L}_{grav}$ one obtains the gravitino mass eigenvalues for the Kaluza-Klein modes

$$m_{3/2}^{(0)} \propto \langle V_5^2 \rangle$$

- If we define the 5D background metric as

$$ds^2 = G_{MN} dx^M dx^N =$$
$$\phi^{-1/3} g_{\mu\nu} dx^\mu dx^\nu + \phi^{2/3} (dy)^2$$

the radion field is

$$T + \bar{T} = \phi^{1/3}$$

and the physical size of the fifth dimension is

$$R = L\phi^{1/3}$$

where L is an unphysical (arbitrary) length scale

- The gravitino zero mode acquires a mass given by

$$m_{3/2} = \frac{V_5^2}{\phi^{1/2}}$$

- In the absence of an external source of supersymmetry breaking $\langle V_5^2 \rangle$ is undetermined at tree level but can be determined in the 5D theory at one-loop (Hosotani breaking). In that case the field equations of V_5^2 will provide a functional relation of the form $V_5^2 = V_5^2(\phi)$

- In the presence of an external source of supersymmetry breaking $\langle V_5^2 \rangle$ will be determined in the 5D theory at tree-level

- In both cases we will use the Casimir energy to fix the VEV $\langle \phi \rangle$

Fixing the SS order parameter

- The relevant terms in $\mathcal{L}_{grav}$ involving V^2_M and B_{MNP} are

$$V^2_M J^M - \frac{1}{12}\epsilon^{MNPQR}\partial_M V^2_N B_{PQR} + W_A W^A$$

with

$$W^M = \frac{1}{12}\epsilon^{MNPQR}\partial_N B_{PQR} - J^M$$

- The field equations for the auxiliary fields yield

$$W_M = 0, \qquad dV^2 = 0$$

which means that the form V^2 is closed, not necessarily exact. A simple solution is

$$V^2_\mu = 0, \quad V^2_5 = \omega/L$$

- The resulting on-shell Lagrangian is

$$V^2_5 J^5$$

corresponds to the previously shown gravitino mass

- The closed form V^2 has a physical effect parametrized by the Wilson line

$$\oint dx^M V_M^2 = 2\pi\omega$$

where ω is an arbitrary (undetermined at tree-level) constant field configuration. It should be determined from the (one-loop) Casimir energy and should be provided by

$$\omega = \omega(\phi)$$

- The function $\omega(\phi)$ will be determined along with $\langle\phi\rangle$ from the Casimir energy

- Non-renormalization theorems are of course respected since:

  - $V_0(\omega) \equiv 0$
  - $V_1(\omega = 0) = 0$

- If there are independent

sources of supersymmetry breaking

attached on the branes at $y = 0, \pi L$ the VEV $\langle V_5^2 \rangle$ can be fixed at tree-level.

- One can introduce the brane Lagrangian [a]

$$W^5[2\pi\omega_0\delta(y) + 2\pi\omega_\pi\delta(y - \pi L)]$$

and consider W_M as independent variables by means of a Lagrange multiplier: $\partial_M X(W^M + J^M)$

- The field equations lead to

$$V_5^2 = \partial_5 X + [2\pi\omega_0\delta(y) + 2\pi\omega_\pi\delta(y - \pi L)]$$

- The Wilson flux amounts to fixing ω

$$\omega = \omega_0 + \omega_\pi$$

[a] J.A. Bagger, F. Feruglio and F. Zwirner, hep-th/0108010; R. Rattazzi, C. Scrucca and A. Strumia, hep-th/0305184

RADION STABILIZATION [a]

[a] G.v. Gersdorff, A. Riotto and M. Quiros, in preparation

- We will consider the Casimir energy $V_1 \oplus$ possible counterterms V_{ct} corresponding to a bulk cosmological constant and (common) brane tensions

$$\alpha \int d^5x\sqrt{G} + \frac{\beta}{2}\int d^5x\sqrt{\tilde{G}}[\delta(y)+\delta(y-\pi L)]$$

MASSLESS BULK FIELDS

- Consider N_V vector multiplets and N_h hypermultiplets [a] in the bulk. The Casimir energy is

$$V_1 \propto (2+N_V-N_h)\frac{1}{\phi^2}$$

- For $2+N_V-N_h>0$ it gives rise to a repulsive force. It can be stabilized by counterterms $\alpha>0, \beta<0$ (dS) that are not consistent with 5D supersymmetry

- For $2+N_V-N_h<0$ it gives rise to an attractive force. It can not be stabilized

[a] G.v. Gersdorff, A. Riotto and M. Quiros, hep-th/0204041

MASSIVE (QUASI-LOCALIZED) FIELDS

● In the presence of N_H hypermultiplets with a common odd-mass M the Casimir energy can be cast as [a]

$$V_1 \propto \frac{1}{x^6} f(\omega, x)$$

where

$$f = 3\left[Li_5(e^{2i\pi\omega}) - \zeta(5) + h.c.\right] + 4\,\delta F(x)$$

with

$$\delta = \frac{N_H}{2 + N_V - N_h}, \quad F \simeq e^{-2x}(3+6x+6x^2+4x^3)$$

and the variable x encodes the radion dependence

$$x = ML\pi\phi^{1/3}$$

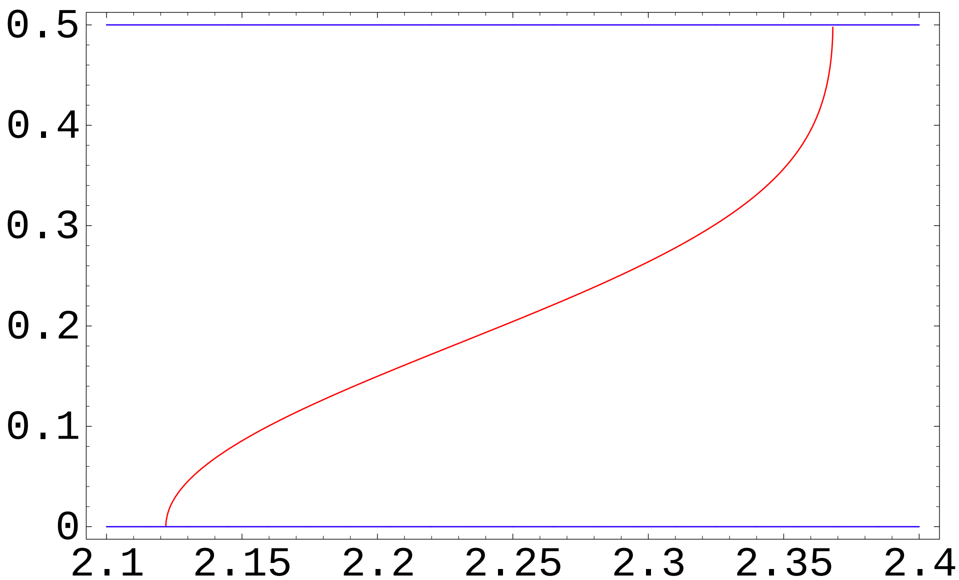
● We are assuming $2 + N_V - N_h > 0$ and the approximation where $x > 1$

[a] G.v. Gersdorff, L. Pilo, M. Quiros, D. Rayner, A. Riotto, hep-ph/0305218

- In the absence of supersymmetry breaking brane effects extremizing V_1 gives the field equation

$$\omega = \begin{cases} 1/2, \ x \geq x_1, \quad 9\,\zeta(3) = 4\delta F(x_1) \\ w(x), \ x_0 < x < x_1 \\ 0, \ x \leq x_0, \quad 3\,\zeta(3) = \delta F(x_0) \end{cases}$$

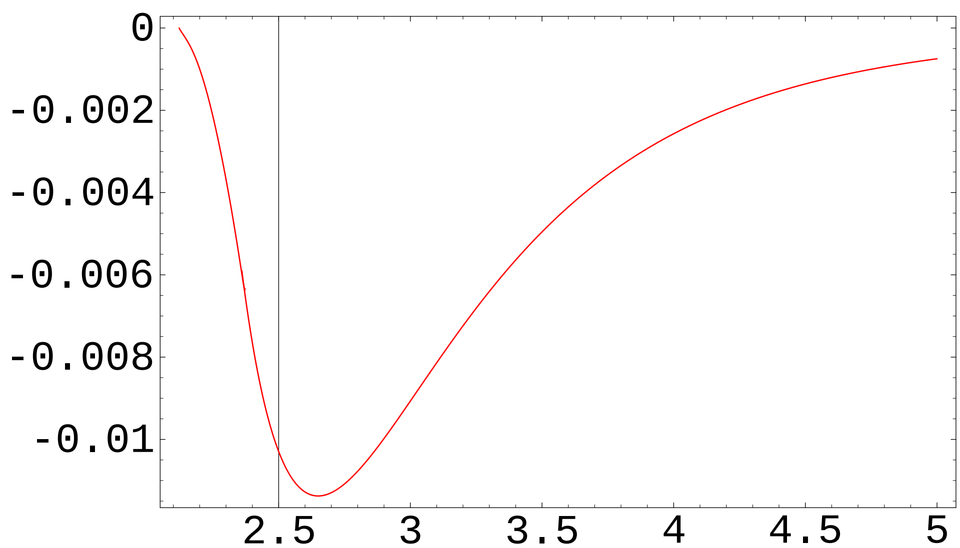
where the function $\omega(x)$ is given (for $\delta = 3$) by



- In the presence of supersymmetry breaking brane effects $\omega = \omega_0$ is fixed and the Casimir energy should be minimized only with respect to the radion field

- In the absence of supersymmetry breaking brane effects the Casimir energy has a non-trivial minimum at $x > x_1$

- For instance, for $\delta = 3$



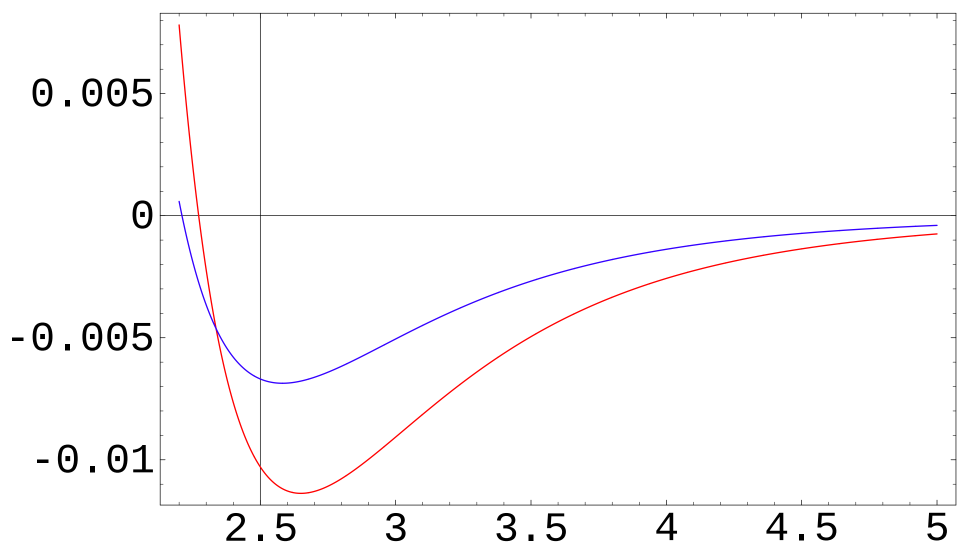
- Including counterterms consistent with 5D supersymmetry, $\alpha < 0$ and $\beta > 0$ (AdS) shifts the minimum towards larger values of x

$$\Downarrow$$

$$\boxed{\omega(\langle x \rangle) = 1/2}$$

- In the presence of supersymmetry breaking brane effects the Casimir energy has a non-trivial minimum at $x(\omega_0)$ for the fixed value of ω_0

- For instance, for $\omega_0 = 1/2, 1/4$



- Including counterterms consistent with 5D supersymmetry, $\alpha < 0$ and $\beta > 0$ (AdS) shifts the minimum towards larger values of x

- In the presence of counterterms the effective potential becomes proportional to

$$V_{eff} = V_1 + (ax+b)/x^2$$

where the dimensionless parameters a and b are related to the 5D cosmological term and branes tension by

$$a = \frac{128\pi^2}{2+N_V-N_h}\frac{\alpha}{M^5}, \quad b = \frac{64\pi^2}{2+N_V-N_h}\frac{\beta}{M^4}$$

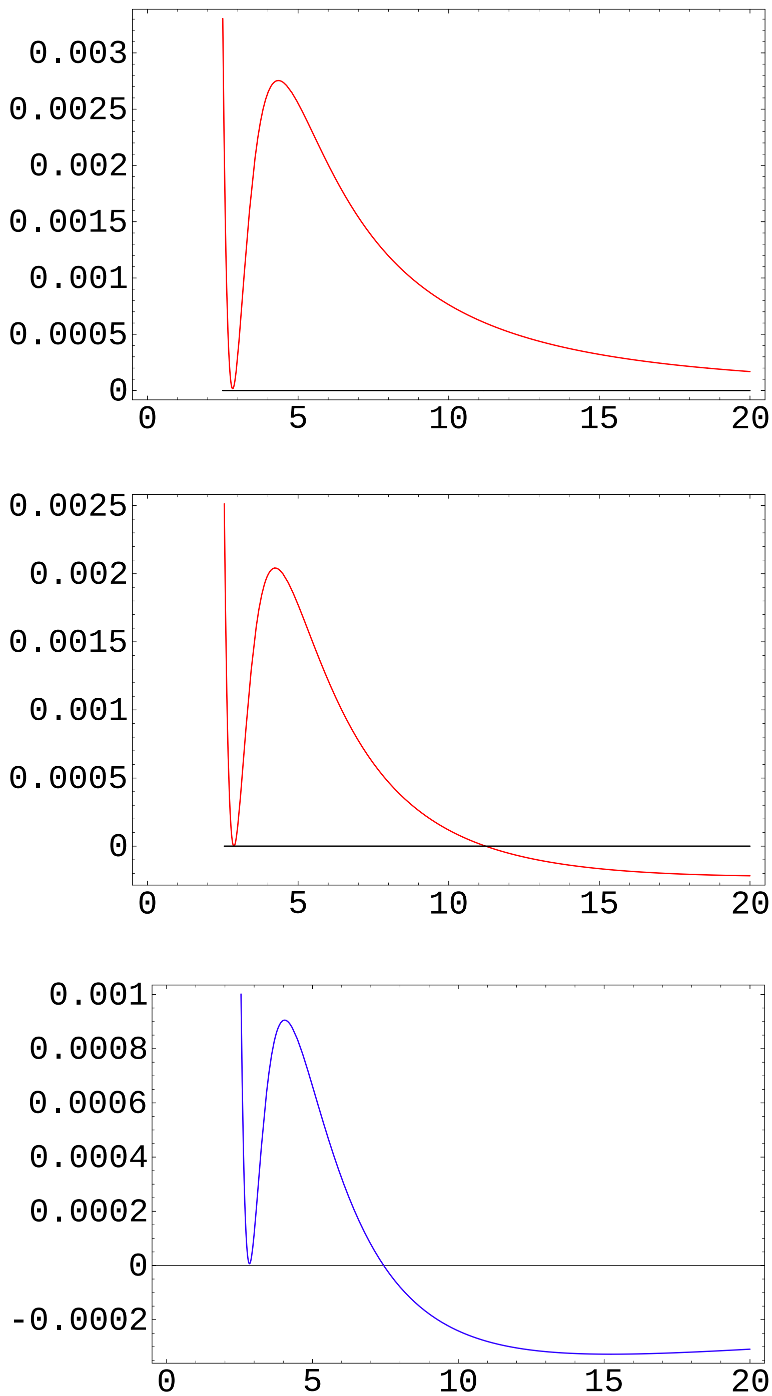
- The counterterm b is fine-tuned to make a zero 4D cosmological constant $\langle V_{eff}\rangle = 0$
- The condition for 5D supersymmetry [a] translates into

$$\frac{1}{8\pi}\frac{M}{M_5} \le \left[\frac{-3a}{2\,(2+N_V-N_h)\pi b^2}\right]^{1/3}$$

---

[a] J. Bagger and D. Belyaev, hep-th/0206024

- The effective potential for $\omega = 1/2$, $a = 0.001, 0.01$ and for $\omega = 1/4$, $a = 0.01$



CONCLUSIONS

- Scherk-Schwarz supersymmetry breaking is a spontaneous breaking of local supersymmetry when the auxiliary field $\langle V_5^2 \rangle \neq 0$ and ψ_5 is the Goldstino

- $\langle V_5^2 \rangle$ should be fixed by the 5D theory. If there is a brane source for supersymmetry breaking, $\langle V_5^2 \rangle$ is fixed at tree-level

- Otherwise matching of 5D and 4D theories is done at one-loop

- If only (massless) bulk fields are present, the Casimir energy can not stabilize the radion consistently with 5D supergravity

- In the presence of massive (quasi-localized) bulk fields the Casimir energy stabilizes the radion VEV consistently with 5D supergravity

- The scale problem

$$M_{Pl}^2 = M_5^3 R$$

with $R \sim 1/\text{TeV}$ requires a cutoff much below M_5. This can appear for instance in "little string theory" at the TeV

$$M_5^3 \simeq \frac{M_s^3}{g_s^2}$$

with M_s (the string scale) in the multi-TeV range and the string coupling $g_s \ll 1$. In a class of string theories [a] the gauge coupling has a geometrical origin and it is thus unrelated to the string coupling

- The bulk cosmological constant is then negligible in M_5 units (but consistent with 5D supergravity) and prevent the tunneling from the Minkowski vacuum to the AdS vacuum

- In that case also the backreaction on the metric is negligible

[a] I. Antoniadis, S. Dimopoulos and A. Giveon, hep-th/010333