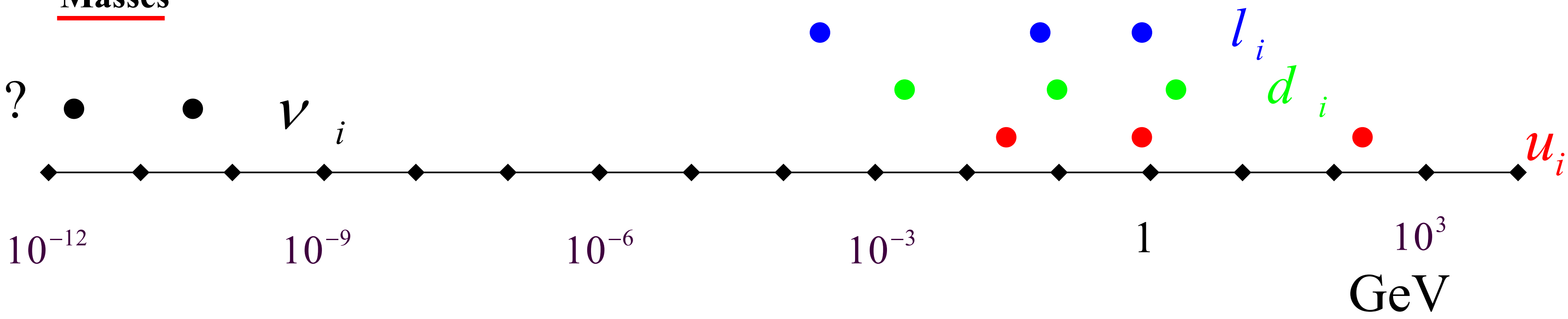
Quark and lepton masses

G.G.Ross, DESY, Sept 03

DATA :

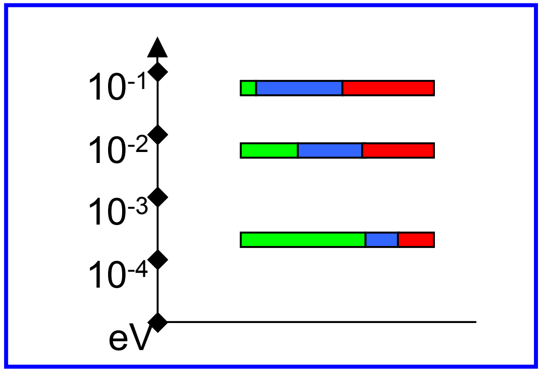
Masses



Mixing

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix}$$

$$V_{MNS} = \begin{pmatrix} 0.72-0.89 & 0.45-0.69 & <0.2 \\ 0.24-0.58 & 0.39-0.76 & 0.52-0.84 \\ 0.24-0.58 & 0.39-0.76 & 0.53-0.84 \end{pmatrix}$$



$$\approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \square\ 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal Mixing …
Non Abelian Structure?

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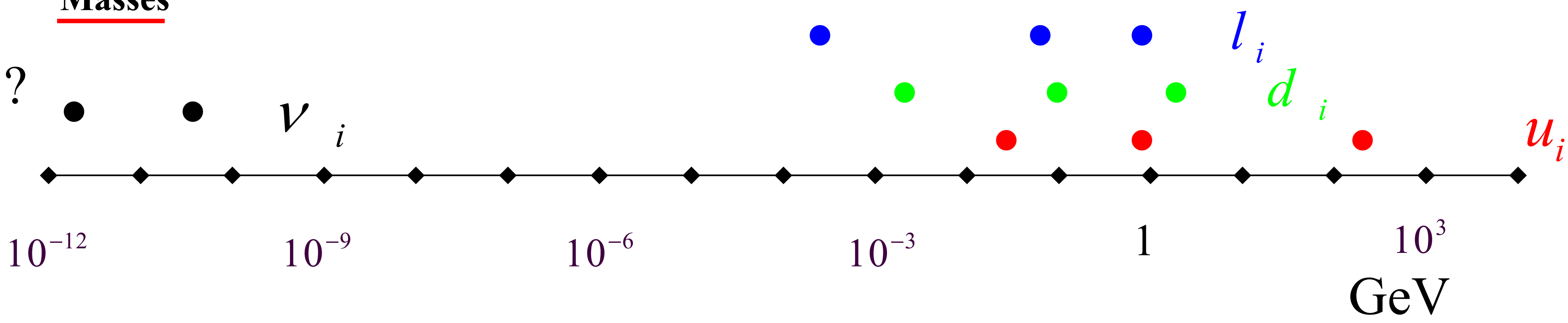
Masses †



Mixing † †

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix} \qquad V_{MNS} = \begin{pmatrix} 0.72-0.89 & 0.45-0.69 & <0.2 \\ 0.24-0.58 & 0.39-0.76 & 0.52-0.84 \\ 0.24-0.58 & 0.39-0.76 & 0.53-0.84 \end{pmatrix}$$

$$GUT^{\dagger} \otimes SU(3)^{\dagger\dagger}{}_{family} \otimes G$$

Simple(?) flavour
Group restricting
Yukawa couplings

Non-Abelian family symmetry

To relate different matrix elements need a non-Abelian structure, e.g.

$$\frac{M^{\nu}}{m_{3,3}} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3 z & \varepsilon^3 z \\ -\varepsilon^3 z & \varepsilon^2 w & \varepsilon^2 w \\ -\varepsilon^3 z & \varepsilon^2 w & 1 \end{pmatrix}$$

e.g.

- $SO(10) \otimes SU(3)_{family}$
- $\psi_i^c, \psi_i \subset (16,3)$ $\Rightarrow$ No mass while SU(3) unbroken
- Spontaneous symmetry breaking $\phi_3^{\rm i}, \overline{\phi}_{3i}$ $\phi_{23}^{\rm i}, \overline{\phi}_{23i}$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \varepsilon$$

- Fermion masses $\mathrm{P}_{\mathrm{Yukawa}} = \frac{1}{M^2} \psi_i^c \phi_3^i \psi_j \phi_3^j H + \frac{1}{M^3} \psi_i^c \phi_{23}^i \psi_j \phi_{23}^j H \Sigma$

$$+ \frac{1}{M^5} \varepsilon^{ijk} \psi_i^c \overline{\phi}_{23,j} \overline{\phi}_{23,k} \psi_l \phi_{23}^l (\phi_{23} \overline{\phi}_{23}) H$$

Form determined by additional symmetries
e.g. $Z_5 X Z_3 X Z_2$

Quarks

Symmetric fit

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & 1.5\varepsilon^3 & 0.4e^{i20}\varepsilon^3 \\ 1.5\varepsilon^3 & \varepsilon^2 & 1.3\varepsilon^2 \\ 0.4e^{i20}\varepsilon^3 & 1.3\varepsilon^2 & 1 \end{pmatrix} \qquad \varepsilon = 0.15$$

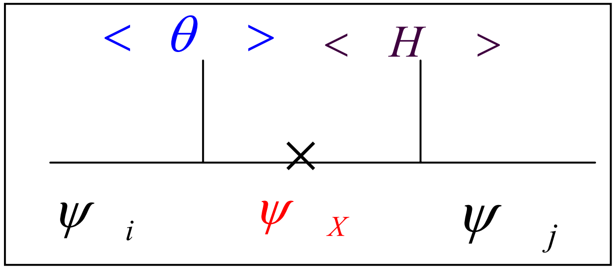
$$\frac{M^u}{m_t} = \begin{pmatrix} 0 & \varepsilon'^3 & ?\ \varepsilon'^3 \\ \varepsilon'^3 & \varepsilon'^2 & ?\ \varepsilon'^2 \\ ?\ \varepsilon'^3 & ?\ \varepsilon'^2 & 1 \end{pmatrix} \qquad \varepsilon' = 0.05$$

Roberts
Romanino
GGR
Velasco Sevilla

$$\frac{M^f}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & a\varepsilon^2 \\ \varepsilon^3 & a\varepsilon^2 & 1 \end{pmatrix} \qquad \varepsilon = \frac{\phi_{23}}{M} \qquad a = \frac{\Sigma}{M} = O(1)_{quarks}$$

$$\varepsilon^3(1 \pm \delta / M) \leftarrow \delta\left(\varepsilon^{ijk}\psi_i^c \overline{\phi}_{23,j}\psi_k\right) H \left(\phi_{23}^l \overline{\phi}_{3,l}\right)^2 / M^6, \quad \delta / M^d = O(1)$$

Messenger mass



Froggatt, Nielsen

$$M_R^u < M_R^d < M_L^q$$

(Universal masses natural for Wilson line breaking)

$$\varepsilon = \frac{\phi_{23}}{M_R^d}, \quad \varepsilon' = \frac{\phi_{23}}{M_R^u}$$

(Higgs mixing $\varepsilon = \frac{\phi_{23}}{M_{H_d}}, \quad \varepsilon' = \frac{\phi_{23}}{M_{H_u}}$)

Extension to charged leptons – GUT?

$$Det(M^l) = Det(M^d)|_{M_X}$$

$$\frac{M^l}{m_\tau} = \begin{pmatrix} 0 & 1\varepsilon^3 & 1\varepsilon^3 \\ -1\varepsilon^3 & 3\varepsilon^2 & 3\varepsilon^2 \\ -1\varepsilon^2 & 3\varepsilon^2 & 1 \end{pmatrix}$$

$$a = \frac{\Sigma}{M} = 3_{muon}$$

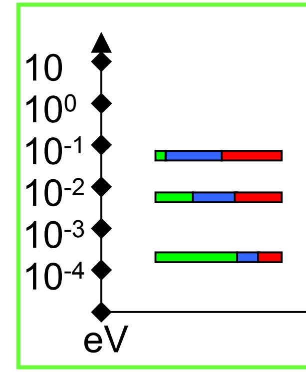
$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

$$\frac{m_d}{m_e}(M_X) = 3$$

Georgi Jarlskog

$$\frac{m_b}{m_\tau}(M_X) = 1$$

Bi-Tri maximal mixing?



Possible with "see-saw" $M_\nu = M_D^\nu \, M_M^{-1} \, M_D^{\nu T}$

Dirac Mass

$$M_D^\nu \propto \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & b\varepsilon^3 & -b\varepsilon^3 \\ \varepsilon^3 & -b\varepsilon^3 & 1 \end{pmatrix}$$

$\psi_i \phi_{23}^i \psi_j^c \phi_3^j \; H\Sigma\left(\phi_{23}^l \overline{\phi}_{23,l}\right)^2 / M^6$

subleading (b=Σ/M_L small)

(leading term just contributes to ν_1)

Magnitude ordered by $Z_5 X Z_2$

Majorana Mass

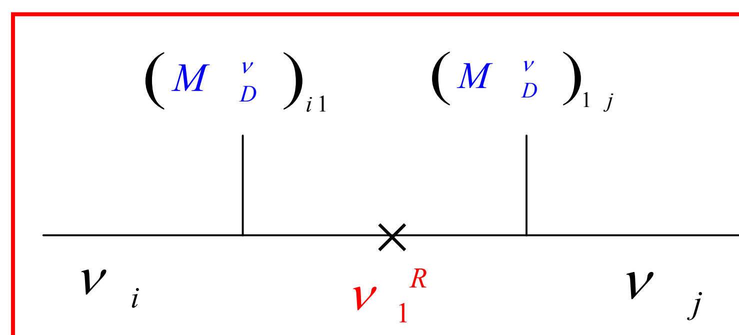
$$M_M \propto \begin{pmatrix} \varepsilon^{7.5} & & \\ & \varepsilon^7 & \\ & & 1 \end{pmatrix} = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$M_1 << M_2 << M_3$

$\psi_i^c \nu^i \psi_j^c \nu^j / M$

$$\nu_a \approx (\nu_\nu + \nu_\tau)/\sqrt{2}$$
$$\nu_b \propto b(\nu_\mu - \nu_\tau)/\sqrt{2} + \nu_e$$

$b = \sqrt{2}$ possible



Vacuum alignment

$$P_1 = U\left(\phi_3 \overline{\phi}_3 + X\right)$$
$$P_2 = W\left(\phi_{23} \overline{\phi}_{23} + X^4\right)$$
$$P_3 = V \phi_3 \overline{\phi}_2 + Z.X^2.\phi_3 \overline{\phi}_3$$
$$P_4 = Y\left(\phi_{23} \overline{\phi}_2 \phi_{23} \overline{\phi}_3 + X^5\right)$$

Restricted by $Z_5 \otimes Z_{2R}$

$$\phi_3 = \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix}$$

$$V_{P_1,P_2} = \left|\phi_3 \overline{\phi}_3 + <X>\right|^2 + \left|\phi_{23} \overline{\phi}_{23} + <X^4>\right|^2$$

$$\overline{\phi}_2 = \begin{pmatrix} 0 \\ a_2 \\ 0 \end{pmatrix}$$

$$V_{P_3,P_4} = \left|\phi_3{}^i \overline{\phi}_{2i}\right|^{2\dagger} + \left|\phi_{23}{}^2 \overline{\phi}_{2,2} \phi_{23}{}^3 \overline{\phi}_{3,3} + <X^5>\right|^2 + \left|<X^2> \phi_3 \overline{\phi}_3 / M^2\right|$$

$$\phi_{23}^2 \phi_{23}^3 = <X^5> / a_2 a_3 \equiv b^2$$

$$\frac{\phi^u{}_3 \overline{\phi}^u{}_3}{M^{u\,2}} \approx \frac{\phi^d{}_3 \overline{\phi}^d{}_3}{M^{d\,2}}$$

$$V_{soft} = +m_{23}^2 (\left|\phi_{23,1}\right|^2 + \left|\phi_{23,2}\right|^2 + \left|\phi_{23,3}\right|^2) + \ldots$$

$$\phi_{23} = \begin{pmatrix} 0 \\ b \\ b \end{pmatrix}$$ "Soft mass alignment"

Summary

- Neutrino masses, gauge coupling unification, proton decay point to GUT with high unification scale.

- Extraction of Yukawa couplings crucial to understanding fermion structure

Bounded off diagonal terms ✓ (anti)symmetric, hermitian ✓ ✓ SUSY CP ✓ ✓

$(m_d, \Delta S \neq 0 \ \Delta Q = 0)$

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \varepsilon^4 & \varepsilon^3 & \leq \varepsilon^3 \\ \leq \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \leq \varepsilon^2 & \leq \varepsilon^2 & 1 \end{pmatrix}$$

$\Delta S \neq 0 \ \Delta Q = 0$

$(A_{\alpha\beta\gamma})$

CP

$(A_{\alpha\beta\gamma})$

SUGRA

Gauge mediation?

Vives, GGR

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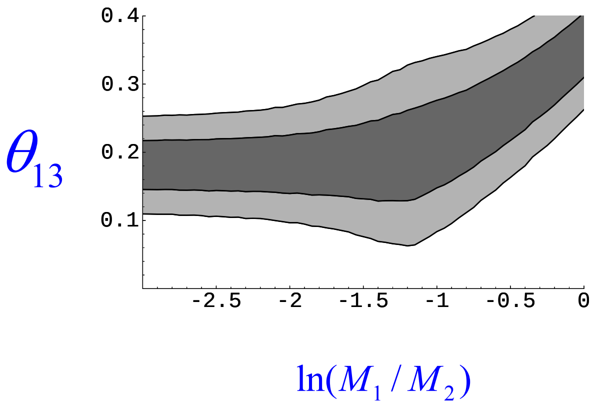
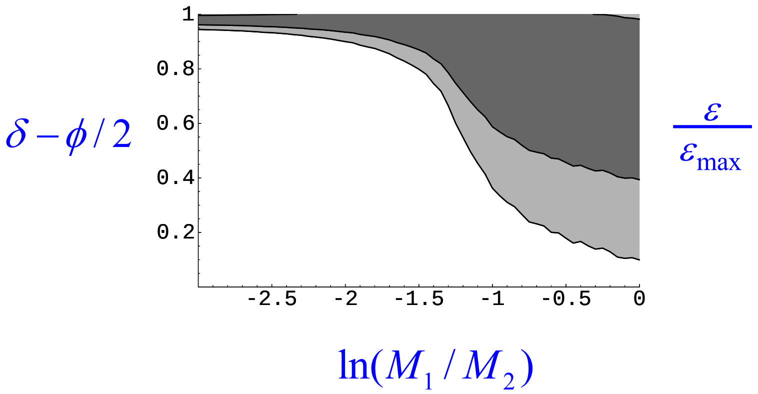
  Family symmetry ? GUT symmetry?

(1,1) neutrino texture zero :





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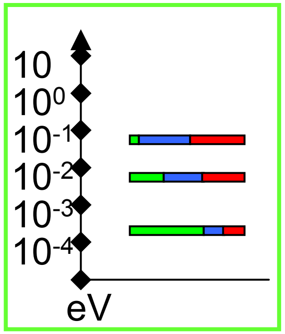
- Due to the see-saw mechanism, the quark, charged lepton and neutrino masses and mixing angles can be consistent with a similar structure for their Dirac mass matrices.

  Hints at an underlying (spontaneously broken) family symmetry? SO(10)XSU(3)?

Near Bi- tri- maximal mixing $\theta_{atmos} \square 45^0, \quad \theta_{\square} \approx 33^0$



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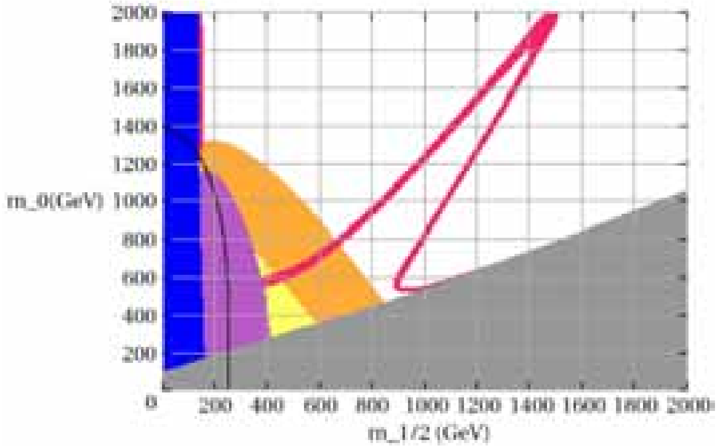
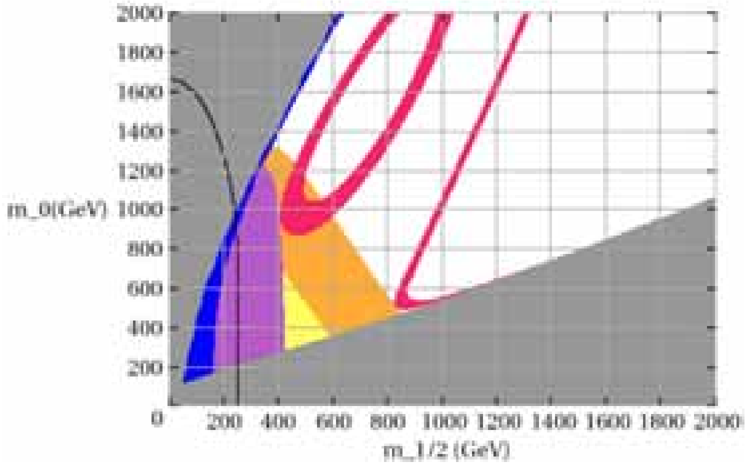
- $SU(3)_{family}$ gives a new solution to the SUSY FCNC problem

$$m_{Q_i} = m_Q, \quad m_{u_i^c} = m_{u^c}, \quad m_{d_i^c} = m_{d^c}, \ldots$$





$$m_{Q_i} = m_{u_i^c} = m_{d_i^c} = m_H = m_0$$

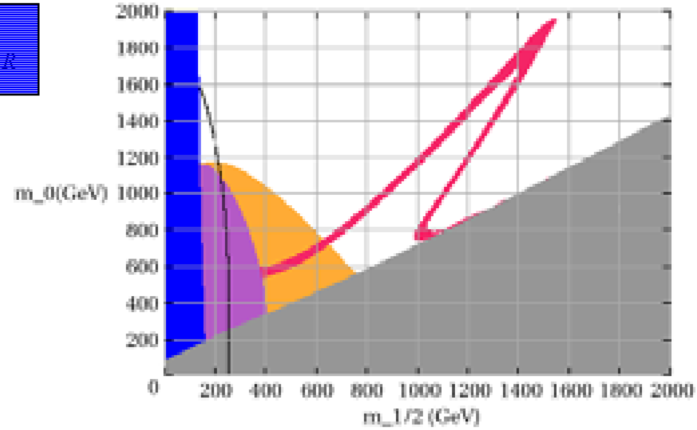
$$m_{Q_{i\neq 3}} = m_{u_i^c} = m_{d_i^c} = m_H = m_0$$

$$m_{Q_3} = m_{Q i\neq 3}(1 \pm 0.2)$$

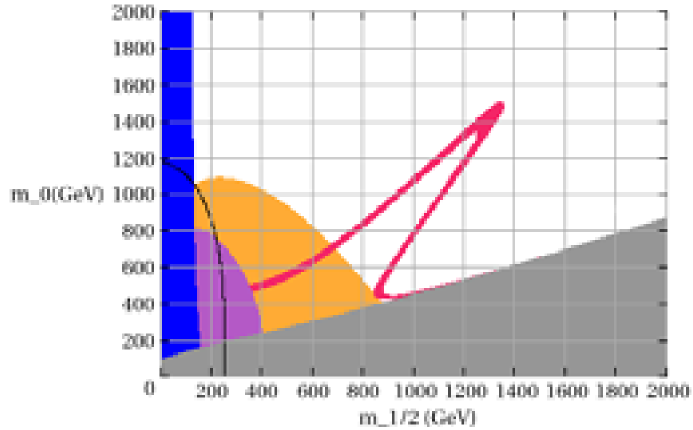
ϕ_3 family symmetry breaking

Ramage,GGR

$SO(10) \to SU(4) \otimes SU(2)_L \otimes SU(2)_R$



$$m_L^2 = 1.25 m_0^2$$
$$m_R^2 = 0.75 m_0^2$$
$$m_H^2 = m_0^2$$



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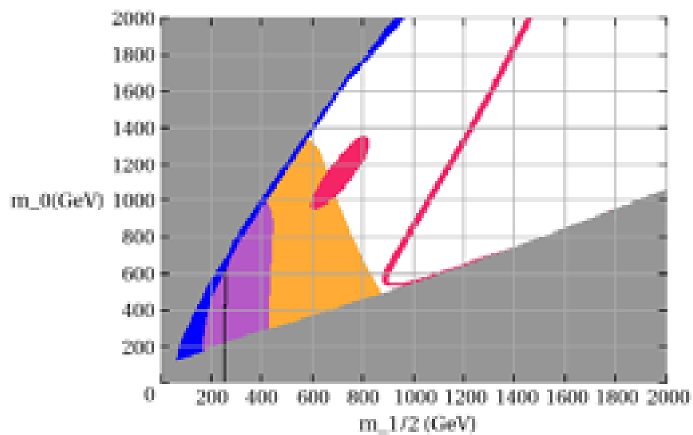

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  Vives, GGR, Velasco Sevilla

  If SUSY spontaneously broken in flavon sector SUSY CP violating angles naturally small