Is There a Peccei-Quinn Transition?

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I. Introduction.

- The lagrangian of QCD, besides ordinary terms, contains the total derivative term or θ-term:

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

  This term is a total derivative:

$$\mathcal{L}_\theta = \theta \partial_\mu K^\mu$$

$$K^\mu = \frac{1}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} (A^a_\nu \partial_\alpha A^a_\beta + \frac{1}{3} f_{abc} A^a_\nu A^b_\alpha A^c_\beta)$$

- This term does not affect equation of motion, but because of QCD instantons it contributes to the action

- It violates CP and contributes to neutron electric dipole moment. From that

$$\theta < 10^{-9}$$

- Peccei-Quinn Solution: additional $U(1)_{PQ}$ symmetry, which is broken at some scale f_{PQ}

- Massless NG boson is generated

$$\frac{a}{f_{PQ}} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

R.D. Peccei, H. R. Quinn, Phys.Rev.Lett. **38**, 1440 (1977)

- Nonperturbative instanton QCD effects give an axion a potential of the form, which is well approximated by

$$V_{QCD}(T) = f_a^2 m_a^2(T)[1 - \cos\left(\frac{aN}{f_a}\right)]$$

where N is model dependent and the potential is degenerate in general

$$m_a^2(T=0) \sim \frac{f_\pi^2 m_\pi^2}{f_a^2}$$

- In general the mass of the axion is T-dependent and and $T > T_{QCD}$ $m(T) \sim \left(\frac{1}{T}\right)^{3.7}$

M. S. Turner, Phys. Rept. **197**, 67 (1990)

II. The "Invisible" Axion Models

- Three historically consequential models
  - Weinberg-Wilczek axion (axion is made of two Higgs doublets)
  - KSVZ axion
  - ZDFS axion

- WW model is excluded by accelerator/astrophysics experiments ($f_{PQ} \sim v_{EW}$)

- KSWZ axion: $\delta\mathcal{L} = \Phi \bar{Q}_R Q_L + h.c.$
  - Φ develops large vev ($f =< \Phi >\gg v_{EW}$)
  - Axion field is $a = f_a \mathrm{Arg}\Phi$ with the low energy coupling to gluons $\frac{1}{32\pi^2}\frac{a}{f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$

J. E. Kim, Phys. Rev. Lett **43**, 103 (1979); M. Shifman, A. Vainstein and V. Zakharov, Nucl. Phys. B **166**, 493 (1980)

- ZDFS model : WW model + extra SM singlet Σ
  - The axion is made of two Higgs doublets and a SM singlet

$$a = \frac{1}{V}(v_\phi Im\phi_0 - v_\chi Im\chi_0 + v_\Sigma Im\Sigma_0)$$

  - The expectation value of Σ is big ($v_\Sigma \gg v_{EW}$)
  - The PQ breaking scale is

$$f_a = \sqrt{v_\phi^2 + v_\chi^2 + v_\Sigma^2} \approx v_\Sigma \gg v_{EW}$$

A. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 60 (1980); M. Dine and W. Fishler, and M. Srednicki, Phys. Lett. B **104**, 199 (1981)

- KSVZ and ZDFS models are minimal extensions of SM

- f_a is arbitrarily large, what constraints can we get on it?

III. Lower and upper constraints on f_a.

From astrophysical bound (dwarf cooling, emission from stars, SNa1987)

$$f_a > 10^9 \text{GeV}, \quad m_a < 10^{-2} \text{eV}$$

G. Raffelt, Phys.Lett.B 166:402 (1986); Phys.Rev.D33:897 (1986); *Tegernsee 1997, Beyond the desert 1997* 808-815

Upper limit on f_a comes from the estimate of axion abundance.

- If PQ transition occurs before/during inflation, $\theta(x)$ is a homogeneous function over enormous distances, and in our patch of the Universe it is likely, that

$$\theta_0 = \frac{a}{f_a} \sim 1$$

- No anthropics we want to consider

- Axion starts to oscillate over one of the minima, say $\theta = 0$

- While Universe cools down to $T \sim \Lambda_{QCD}$:

$$m_a \to m_a(T < \Lambda_{QCD}) \sim f_\pi m_\pi / f_a$$

- But typical momentum of the axion is much smaller then its mass at this time, since :

$$k < \frac{1}{t_{QCD}} \sim \frac{1}{10^{-4} sec}$$

  while from previous astrophysics bound we know that $m_a > \frac{1}{10^{-12} sec}$

- Think of axion as of large coherent state of cold particles

- If one assumes sudden switch on of the mass then the initial density of these cold axions turns out too large

$$\rho_a(t_{QCD}) \sim f_a^2 \partial_\mu \theta \partial^\mu \theta \sim f_\pi^2 m_\pi^2 \theta_0^2 \sim T_{QCD}^4)$$

- Not a big problem: careful estimate taking into account that $m(t) = m(T(t))$ solves the case

- After this one obtains

$$\Omega_a h^2 \approx \Delta \left(\frac{f_a}{10^{12}\text{GeV}}\right)^{\frac{7}{6}} \theta_0^2$$

- $\Omega_a h^2 < 1$ imposes

$$f_a < 10^{12}\text{GeV}, \quad m_a > 10^{-5}\text{eV}$$

Axion may be produced from decays of axionic domain walls and strings, => the estimate above may grow by on order of magnitude

If PQ transition occurred after inflation, the estimate doesn't change really: same upper bound

P. Sikivie, Gif Summer School (1982)

IV. Domain walls and Axion Isocurvature fluctuations are PROBLEMS!

There two are problems with the each of the case: if PQ occurs before or after inflation

- If PQ transition occurs after inflation $\rightarrow$

DOMAIN WALLS

- Since, in general there are N degenerate vacua (consider $N \neq 1$)

- If PQ transition happens after inflation any values of θ from 0 to 2π are allowed, => in different parts of the Universe axion can fall into different vacua

- Imagine toy potential with Z_2 symmetry

$$\frac{\lambda}{4}(\phi^2 - f^2)^2$$

- There are two degenerate vacua $\phi = \pm f$

- Imagine that there are two regions of space, one with $\phi = f$, another is $\phi = -f$, i.e. impose b.c. on the solution to the classical eqn. of motion

$$\phi(x = -\infty) = -f, \quad \phi(x = \infty) = f$$

- The solution is

$$\phi(x) \sim f \tanh(mx)$$

  with $m \sim \sqrt{\lambda} f$.

  There is an energy density in the middle region, i.e. there is a domain wall

$$V(x) \sim (\nabla\phi(x))^2 \sim f_a^2 m_a \sim f_a f_\pi m_\pi$$

- Take $f_a \sim 10^{11}$GeV the mass per unit surface $\mu \sim (10^3\text{GeV})^3$ enormous mass inside of a Hubble radius today:

$$M_{DW} \sim \mu H_0^{-2} \sim 10^{91}\text{GeV}$$

- Compare with the $M_{univ} \sim 10^{80}$GeV

- The equation of state of a collection of DW is $P = -\frac{2}{3}\rho$, =>

$$a(t) \sim t^2, \quad \rho \sim \frac{1}{a}$$

- Since $\rho_{matt} \sim 1/a^3$, and $\rho_{rad} \sim 1/a^4$, the whole cosmology is completely different

If PQ transition occurs before or during inflation, domain walls get diluted, but there will be

ISOCURVATURE FLUCTUATIONS

- Assume at inflation axion is massless, then axion charge fluctuate

  $$\ddot{a}_k + 3H\dot{a}_k + \frac{k^2}{a^2}a_k = 0,$$

  which on super horizon scales ($k \ll H$) leads to

  $$\mathcal{P}(k) = \frac{k^3}{2\pi^2}|a_k|^2 = \frac{H^2}{(2\pi)^2},$$

  which is flat on large scales.

- Implication for CMB:

  $$\frac{\delta T}{T} \sim \frac{\delta\rho_a}{\rho_a} = \Omega_a \frac{H_I}{2\pi f_a} > 1,$$

  but should be less then 10^{-5} (at least) from COBE, WMAP etc.

V. Supersymmetric Peccei-Quinn field and the effective scale of PQ symmetry breaking

In the previous models f_a is essentially free parameter. Is that a constant? What is PQ transition and when it occurs.

- Imagine PQ field $S = |S|e^{a/|S|}$ is a flat direction. The potential is exactly zero to all orders in perturbation theory.

  First simple insight on how SUSY can help.

- What kind of corrections should we expect?
  - SUSY breaking → zero curvature term
  $$\delta V_1 \sim \pm m_{3/2}^2 |S|^2$$
  - PQ field may couple to inflaton
  $$\delta V_2 \sim \pm \int d^4\theta \frac{I^+ I S^+ S}{M_p^2} \rightarrow \pm H^2 |S|^2$$
  - NR operators yield planck suppressed corrections ($\lambda_n > 0$, n is integer)
  $$\delta V_3 = \lambda_n \frac{|S|^{2n+4}}{M_p^{2n}}$$

Take $n = 1$ case, look for a combination of signs to have phase transition: first choice is:

$$V_1 = (-H^2 - m_{3/2}^2)|S|^2 + \lambda \frac{|S|^6}{M_p^2} + V_0 + ...,$$

where by ... we mean other possible terms.

Here $V_0 \sim m_{3/2}^3 M_p$ in order to cancel the cosmological constant at present.

PQ phase transition occurs before or during inflation

- At early times

$$f_a \sim (H M_p)^{\frac{1}{2}}.$$

- During inflation f_a is larger then say $f_a = 10^{12}$GeV, i.e.

$$f_a \sim (H_I M_p)^{\frac{1}{2}} \sim 10^{16}\text{GeV}.$$

- At $H \sim m_{3/2}$ f_a is freezed at a value:

$$f_a \sim (m_{3/2} M_p)^{\frac{1}{2}} \sim 10^{11}\text{GeV}$$

  So, there is no DW, and astrophysical constraint is satisfied.

- Effectively the scale of PQ symmetry breaking is higher at inflation

- There is much less to CMB anisotropy: with $H_I \sim 10^{13}$GeV, $f_a \sim 10^{16}$GeV, and $\Omega_a \sim 0.3$

  $$\Omega_a H_I / 2\pi f_a(H_I) \sim 10^{-5},$$

  which is about at barely acceptable level.

  A little better in this case with the rigid "axion" case. Remember it was

  $$\Omega_a H_I / 2\pi f_a(H_I) \sim 1,$$

  but keep thinking further:

- Higher order terms (in n) can pull $|S|$ closer to a planck scale at inflation, i.e. take $n = 2, 3$,etc./or inflation with lower scale?

  This will decrease the ratio of f_a to H_I, and constraint from CMBR will be relaxed.

- Is there a better solution?

- It is possible that during inflation axion acquires large mass, since there a plenty of operators coming from, e.g. $N = 1$ supergravity

- Consider coupling:

$$\mathcal{L} = \int d^2\theta \frac{I}{M_p^2} S^{m+1} + \text{h.c.} \tag{1}$$

- Imagine S is the axion field, $|S| \sim (H_I M^n)^{\frac{1}{n+1}}$ during inflation.

  This gives to the axion the mass $m_a^2 > H_I^2$ at some choices of n and m. But then power spectrum is suppressed on large scales:

$$\mathcal{P}(k) \approx \left(\frac{H}{2\pi}\right)^2 \left(\frac{1}{c}\right) \left(\frac{k}{aH}\right)^3 . \tag{2}$$

- No isocurvature fluctuations detectable in CMB! (No DW either.)

- But there is no PQ phase transition either! This symmetry occurs much later when couplings to inflaton fade away with time leaving no trace!

- Big idea: Nothing requires exact PQ symmetry to exist. We need it now to solve strong CP problem. But at earlier times-certainly not!

- The PQ symmetry as accidental symmetry is plausible both theoretically and phenomenologically

- The model (or kinds of models) presented above virtually remove any problems with axion cosmology leaving it a plausible candidate for a dark matter compatible with present day allowed for f_a window!

We missed one more case, namely PQ can be realized with the potential

$$V_2 = (H^2 - m_{3/2}^2)|S|^2 + \lambda \frac{|S|^6}{M_p^2} + V_0 + ...$$

This case yields phase transition after inflation

$$t \sim m_{3/2}, \quad \text{with} f_a \sim (m_{3/2} M_p)^{\frac{1}{2}}$$

Astrophysical constraint is satisfied again/no isocurvature fluctuation issue

But QCD DOMAIN WALLS are still there!

- In different patches axion still takes different values and oscillates around different minima

- Unless we, somehow make it homogeneous after PQ transition occurred but before QCD potential is formed! Let's elaborate the model =>next page

- Suppose inflaton transforms under some discrete symmetry (there are myriad of possibilities)

  The Model:

  Consider

$$W = \frac{S' S^{n+2}}{M^n} + I^2 \frac{S^{m+1}}{M^m}$$

  This model at $I = 0$, has an approximate PQ symmetry

$$S \to e^{i\alpha} S, \quad S' \to e^{-i(n+2)\alpha} S'$$

  Also, assume soft breaking terms is there:

$$V_{soft} = m_{3/2} I^2 \frac{S^{m+1}}{M^m} + ...$$

- Note that $< I^2 > \approx M_{pl}^2 e^{i\delta_I} \neq 0$ right/soon after inflation (a linear term would average to zero)

- Initially the first term in W dominates for $m < n+2$ (easy to check)

- Second term breaks PQ: initially very strongly, much later it disappears

- At the moment when both terms in the superpotential are comparable, which occurs at some time between the end of inflation and $H \leq \Gamma_I$ one can estimate that axion has a potential

$$V(a) \approx m_{3/2}^2 |S|^2 \cos((m+1)a/|S| + \delta),$$

- In different patches of the Universe axion starts damped oscillations around the same minimum but with different initial θ_0

- Eventually, $\theta_i(t) \approx \theta_0/t \to 0$, i.e. settle to a particular minimum of the potential above

There is still denger that the inflaton induced potential, and potentially a new domain wall problem.

But, there are two possible ways to avoid this.

- a. There could be two or more inflatons each of which would generate a potential to the axion with different periodicities (or a linear combination of inflatons with different Z_N charges)

- b. There could be several terms of the form (natural Taylor expansion of a coupling $I^2 f(S/M)$

  $$I^2 \frac{S^{m_1+1}}{M_1^m} + I^2 \frac{S^{m_2+1}}{M_2^m} + ...,$$

  where m_i are not very different(this is very important). Then the domain walls would collapse settling axion to unique vacuum everywhere before V_{QCD} turns on. Here one only needs to ensure that there is enough time.

  Clearly, there is plenty of room in model building, but not any serious constraints on such model.

VI. Conclusions.

- In supersymmetry there are two generic ways to arrange PQ phase transition.

- In both cases at some early point the PQ symmetry may be badly broken.

- In both cases it leads to a large mass for the axion

- This, if, PQ phase transition arranged before or during inflation suppresses isocurvature fluctuations, which would be an issue if axion would stay massless during inflation

- If PQ phase transition arranged after inflation, but PQ symmetry is badly broken again for some time. The potential for axion is generated, and for a wide class of models drives it to be homogeneous over enormous distances (without inflation!) At QCD phase transition no domain walls, thus, appear.

  The late time cosmology of axion is unaltered, and f_a is predicted to be $\sim 10^{11}$GeV.