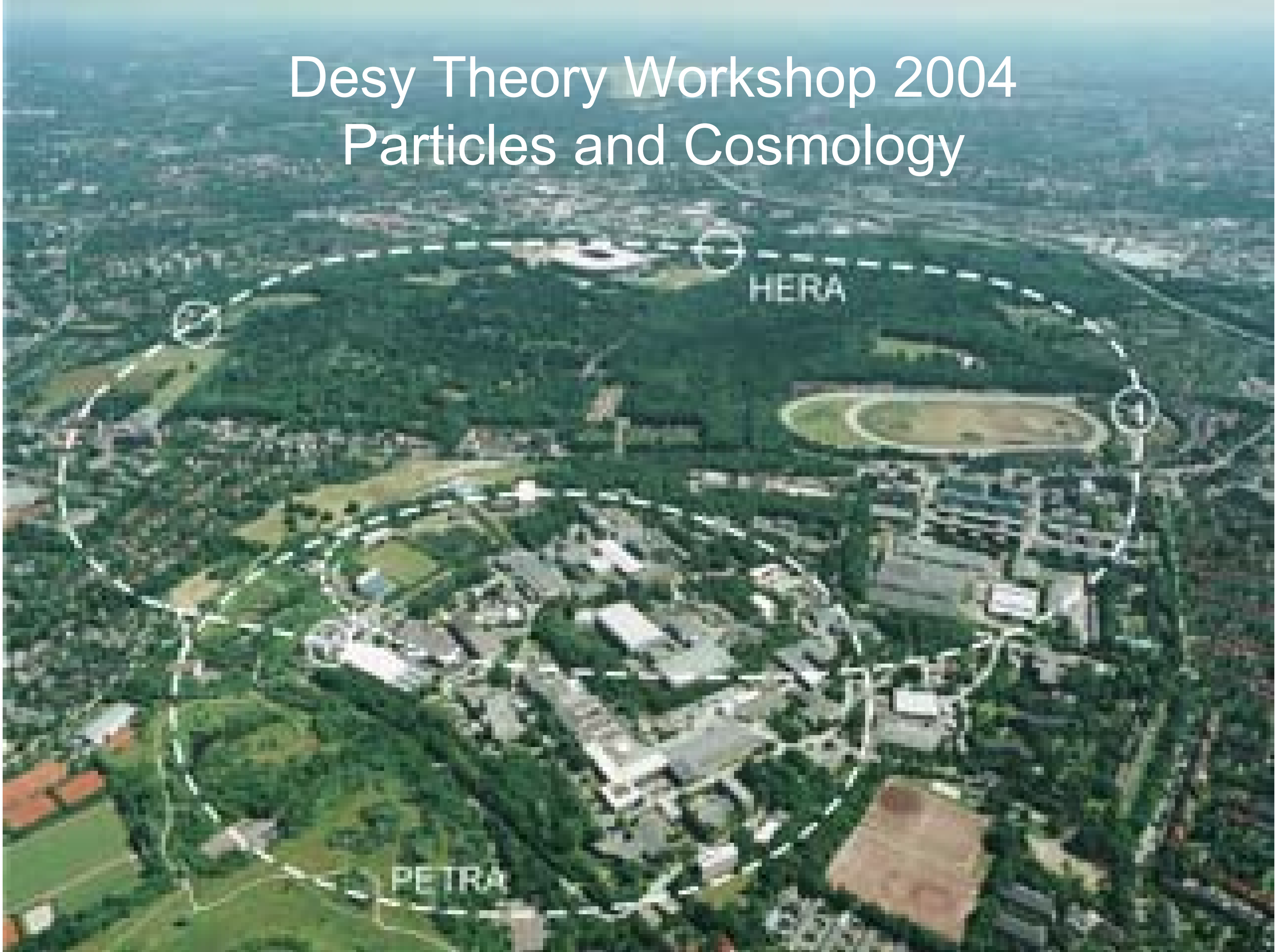
Desy Theory Workshop 2004
Particles and Cosmology



Structure Formation under the influence of a variable dark Energy

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A Scenario of Cosmological Structure Formation

- Small Fluctuations in Dark Matter
- Baryonic Matter condenses in the potential wells
- Z~15-20: First generation of Stars
- Cosmic Reionization
- Temporary Suppression of Star formation
- Growth of DM halos (accretion, mergers)
- Galaxy Formation, Star formation
- Large Scale Structure of the Universe

Cosmology of Structure Formation

Origin of Density Fluctuations

- Quantum field theory
- Inflation
- Power Spectrum of Fluctuations
- Comparison with Observations

Growth of Density Fluctuations

- Gravity as the dominant force
- Gravitational Instability, Jeans Theory
- Linear perturbation theory:

- ❑ Newtonian vs. Relativistic
- ❑ Radiation vs. Matter dominated
- ❑ Sub-Horizon vs. Super-Horizon
- ❑ Jeans-length vs. Horizon

Cosmology of Structure Formation

Growth of Density Fluctuations

- Nonlinear Structure Formation
- Spherical Collapse
- Zeldovic Approximation
- Adhesion Approximation
- Press-Schechter-Theory
- Numerical N-Body Simulations

Linear Perturbation Theory in Newotonian Limit

- Continuity Equation $$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$
- Euler Equation $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{2}\vec{\nabla} p - \vec{\nabla}\Phi$$
- Poisson Equation $$\Delta\Phi = 4\pi G \rho$$
- Equation of State (adiabatic case) $$p = p(\rho)$$

Newtonian Perturbation theory

- First order Approximation:

$$\begin{cases} \rho = \rho^0 + \varepsilon \rho^1 \\ v = v^0 + \varepsilon v^1 \\ \Phi = \Phi^0 + \varepsilon \Phi^1 \end{cases}$$

- Results in:

$$\begin{cases} \dfrac{\partial \rho^1}{\partial t} + \rho^0 \nabla \cdot v^1 = 0 \\ \dfrac{\partial v^1}{\partial t} + (v^0 \cdot \nabla) \cdot v^1 = -\dfrac{1}{\rho^0} \nabla p^1 - \nabla \Phi^1 \\ \Delta \Phi^1 = 4\pi G \rho^1 \end{cases}$$

Newtonian Perturbation theory

- In adiabatic case:

$$\nabla p^1 = \frac{\partial p^1}{\partial \rho^1} \cdot \nabla \rho^1 = v_s^{\,2} \cdot \nabla \rho^1$$

$$v_s^{\,2} = \left.\frac{\partial p^1}{\partial \rho^1}\right|_{s = const.}$$

- In Euler Equation:

$$\frac{\partial v^1}{\partial t} = -\frac{v_s^{\,2}}{\rho^0} \nabla \rho^1 - \nabla \Phi^1$$

Newotonian Perturbation theory

- After using the perturbed Poisson and Continuity equation:

$$\frac{\partial^2 \rho^1}{\partial t^2} - v_s{}^2 \Delta \rho^1 = 4\pi G \rho^0 \rho^1$$

- Ansatz:

$$\rho^1 = A e^{i(kx \pm \omega t)}$$

- Results in:

$$\omega(k)^2 = v_s k^2 - 4\pi G \rho^0$$

Newotonian Perturbation theory

- Exponential growth occurs if:

$$|k| < \sqrt{\frac{4\pi G \rho^0}{v_s^{\ 2}}} \equiv k_J$$

- The Jeans-length

$$\lambda_J = \frac{2\pi}{k_J}$$

determines which fluctuations grow.

Perturbations in the Expanding Universe

- Comoving Coordinates:

$$\underset{\text{physical length}}{x} = \underset{\text{comoving length}}{r} \cdot \underset{\text{scale factor}}{a}$$

$$\Rightarrow v \equiv \dot{x} = \underbrace{\dot{r} \cdot a}_{v_{pec}} + r \cdot \dot{a}$$

- In 0th approximation:

$$v_{pec} = 0 \Rightarrow v = r \cdot \dot{a} = \frac{x}{a} \cdot \dot{a} = x \cdot \frac{\dot{a}}{a}$$

Perturbations in the Expanding Universe

- After introducing comoving coordinates:

$$\frac{\partial \rho^1}{\partial t} + 3\rho^1 \frac{\dot{a}}{a} + \frac{\dot{a}}{a} \nabla \rho^1 + \rho^0 \, \nabla \cdot v^1 = 0$$

$$\frac{\partial v^1}{\partial t} + \frac{\dot{a}}{a} v^1 + \frac{\dot{a}}{a} (v^0 \cdot \nabla) \cdot v^1 = -\frac{1}{\rho^0} \nabla p^1 - \nabla \Phi^1$$

$$\Delta \Phi^1 = 4\pi G \rho^1$$

$$\nabla p^1 = v_s^2 \nabla \rho^1$$

- Coordinate transformation allows: $v_0 = 0$

Perturbations in the Expanding Universe

- Ansatz for separating the variables:

$$\left.\begin{aligned}\rho^1(t,x) &= \widehat{\rho^1}\cdot e^{ik\frac{x}{a}}\\ v^1(t,x) &= \widehat{v^1}\cdot e^{ik\frac{x}{a}}\\ \Phi^1(t,x) &= \widehat{\Phi^1}\cdot e^{ik\frac{x}{a}}\end{aligned}\right\} \rightarrow \begin{cases}\dot{\widehat{\rho^1}} + 3\frac{\dot{a}}{a}\widehat{\rho^1} + i\frac{\rho^0}{a}(v^1 k) = 0\\ \dot{\widehat{v^1}} + \frac{\dot{a}}{a}\widehat{v^1} + i\frac{k}{a}\left(\frac{v_s^2}{\rho^0} - \frac{4\pi G a^2}{|k|^2}\right)\widehat{\rho^1} = 0\\ \Phi^1 = -\frac{a^2}{k^2}4\pi G\widehat{\rho^1}\end{cases}$$

We obtain two differential equations

Perturbations in the Expanding Universe

- We separate the velocity into parallel and orthogonal part:

$$\widehat{v^1} = \lambda k + \widehat{v^1_{\perp}}$$

with: $\widehat{v^1_{\perp}} \cdot k = 0$ $\lambda = \dfrac{\widehat{\mathrm{v}^1} \cdot k}{|k|^2}$

- Then we obtain:

$$\dot{\widehat{v^1_{\perp}}} + \frac{\dot{a}}{a}\widehat{v^1_{\perp}} = 0$$

$$\dot{\lambda} + \frac{\dot{a}}{a}\lambda + \frac{i}{a}\left(\frac{v_s^{\,2}}{\rho^0} - \frac{4\pi G a^2}{|k|^2}\right)\widehat{\rho^1} = 0$$

Perturbations in the Expanding Universe

- Which results in: $\widehat{v^1_\perp} \sim a^{-1}$

  (the orthogonal part doesn't grow!)

- And: $$\widehat{\dot{\rho}^1} + 3\frac{\dot{a}}{a}\widehat{\rho^1} + i\,\frac{\rho^0}{a}|k|^2\lambda = 0$$

- After defining $\delta \equiv \dfrac{\widehat{\rho^1}}{\rho^0}$ and using $\rho^0 = \rho_0 \dfrac{{a_0}^3}{a^3}$

- We get: $\dot{\delta} = -i\,|k|^2\,\lambda a^{-1}$

- And finally: $$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left(\frac{{v_s}^2|k|^2}{a} - 4\pi G\rho^0\right)\delta = 0$$

Perturbations in the Expanding Universe

- Main Results (cf. Padmanabhan 1993)

Epoch	Physics	DM	Rad.	Bar.
$a < a_{enter}$ $\lambda > D_H$	Radiation dominated Relativistic theory	a^2	a^2	a^2
$a_{enter} < a < a_{eq}$ $\lambda < D_H$	Radiation dominated Nonrelativistic theory	$\ln(a)$	Osc.	Osc.
$a_{eq} < a < a_{dec}$ $\lambda < D_H$	Matter dominated Nonrelativistic theory	a	Osc.	Osc.
$a_{dec} < a$ $\lambda < D_H$	Matter dominated Nonrelativistic theory	a	free prop.	$"a"$

Variable Dark Energy

- Motivation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

(Regard Λ as some peculiar form of matter)

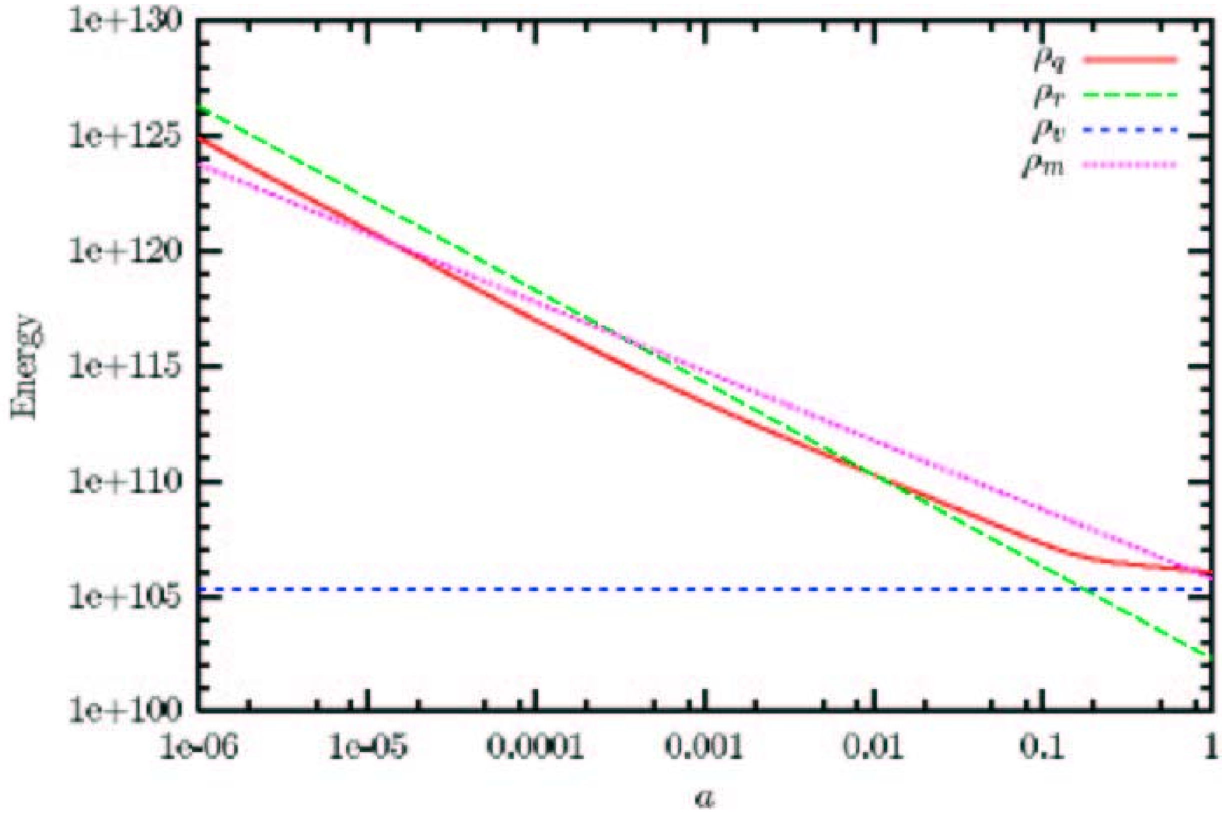
- Problems:

Cosmological Constant is very small: $\frac{\Lambda}{M_p^4} = 10^{-124}$

And why is: $\Lambda \sim \Omega_m$

Variable Dark Energy

- Possible Solution:



Variable Dark Energy

- Assume:

$$L = \frac{1}{2} g_{\mu\nu} \partial^{\mu} \partial^{\nu} \Phi - V(\Phi)$$

$$T_{\mu\nu} = \partial_{\mu}\Phi \, \partial_{\nu}\Phi - L g_{\mu\nu}$$

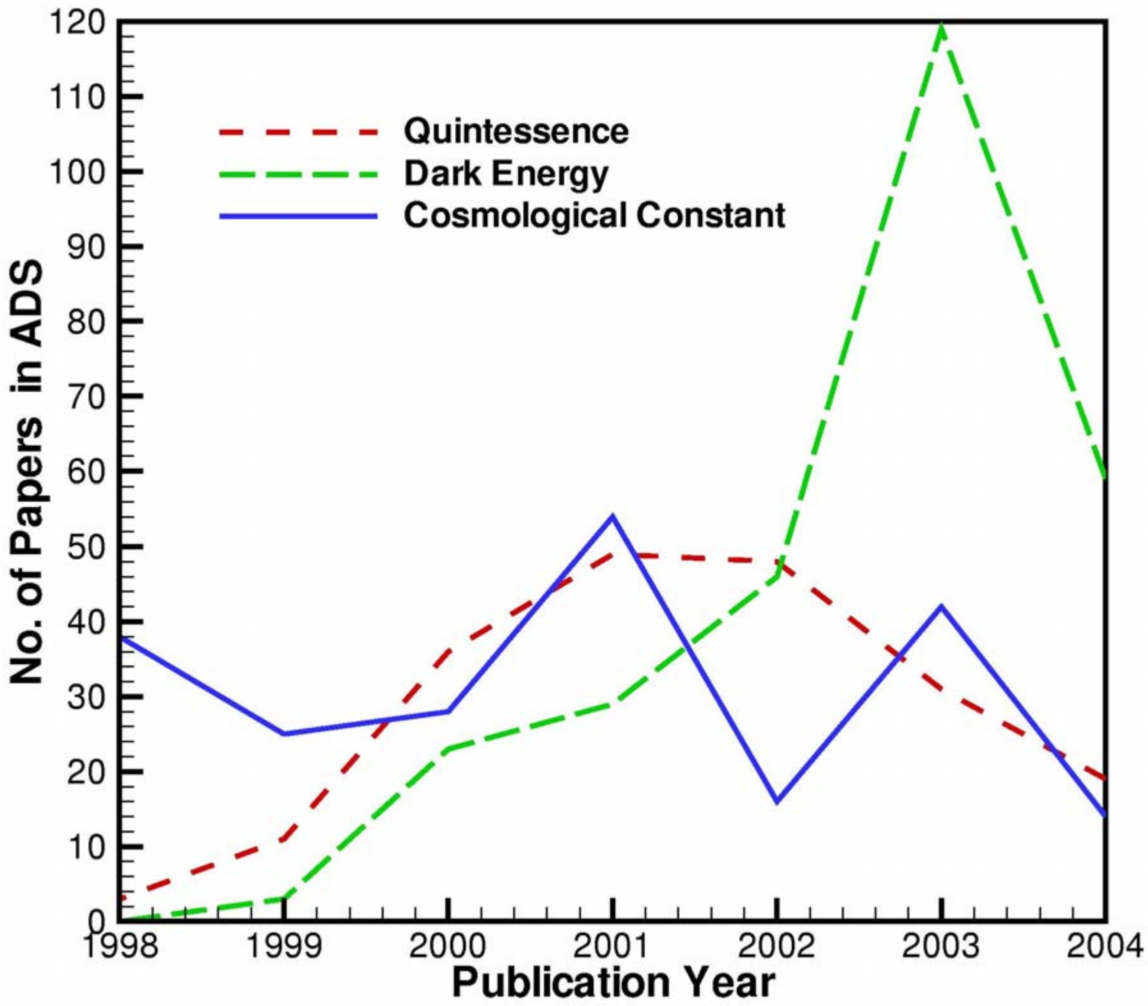
$$\Rightarrow \begin{cases} \rho_{\Phi} = \frac{1}{2}\dot{\Phi}^2 + V \\ p_{\Phi} = \frac{1}{2}\dot{\Phi}^2 - V \end{cases}$$

- If: $\dot{\Phi} \ll V(\Phi) \quad \Rightarrow \quad \rho(\Phi) = -p(\Phi)$

Variable Dark Energy

Quintessence Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998)
$m^2\phi^2, \lambda\phi^4$	Frieman et al (1995)
$V_0/\phi^\alpha, \alpha > 0$	Ratra & Peebles (1988)
$V_0 \exp(\lambda\phi^2)/\phi^\alpha$	Brax & Martin (1999,2000)
$V_0(\cosh\lambda\phi - 1)^p$	Sahni & Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$	Sahni & Starobinsky (2000), Ureña-López & Matos (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland & Nunes (2000)
$V_0(\exp M_p/\phi - 1)$	Zlatev, Wang & Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$	Albrecht & Skordis (2000)

Variable Dark Energy



Variable Dark Energy

A Simple Parametrization Scheme (Wetterich 2004)

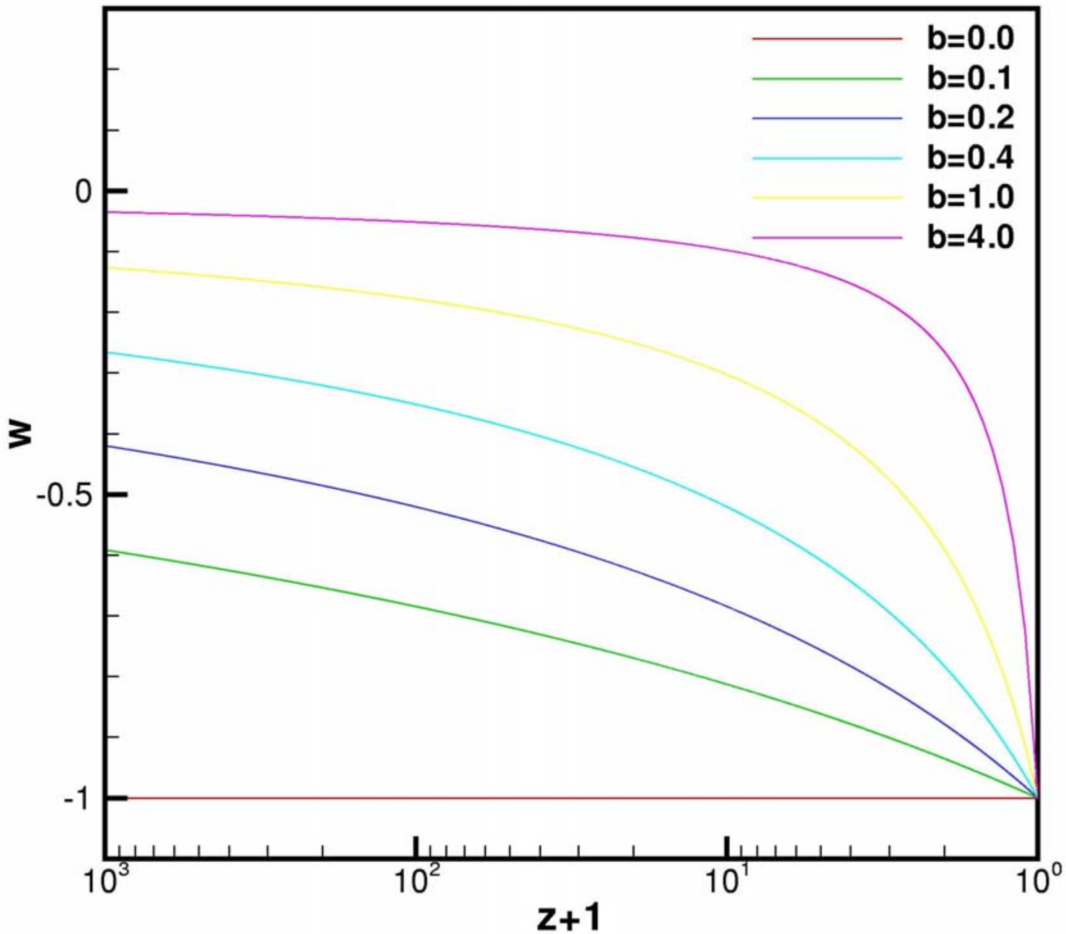
- Fraction of Dark Energy, now: $\Omega_h(0)$
- Eq. of State, now: w_0
- Bending Parameter: b

} Measure the deviation from a pure Λ-Term

$$w_h(z) = \frac{w_0}{1 + b\ln(1+z)}$$

Variable Dark Energy

- Assume $w_0=-1$ (and $\Omega_h(0)=0.7$) :



Structure Formation with variable Dark Energy

- Evolution of the Density Contrast:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\rho^0\delta = 0$$

- Rewrite in dependence of redshift :

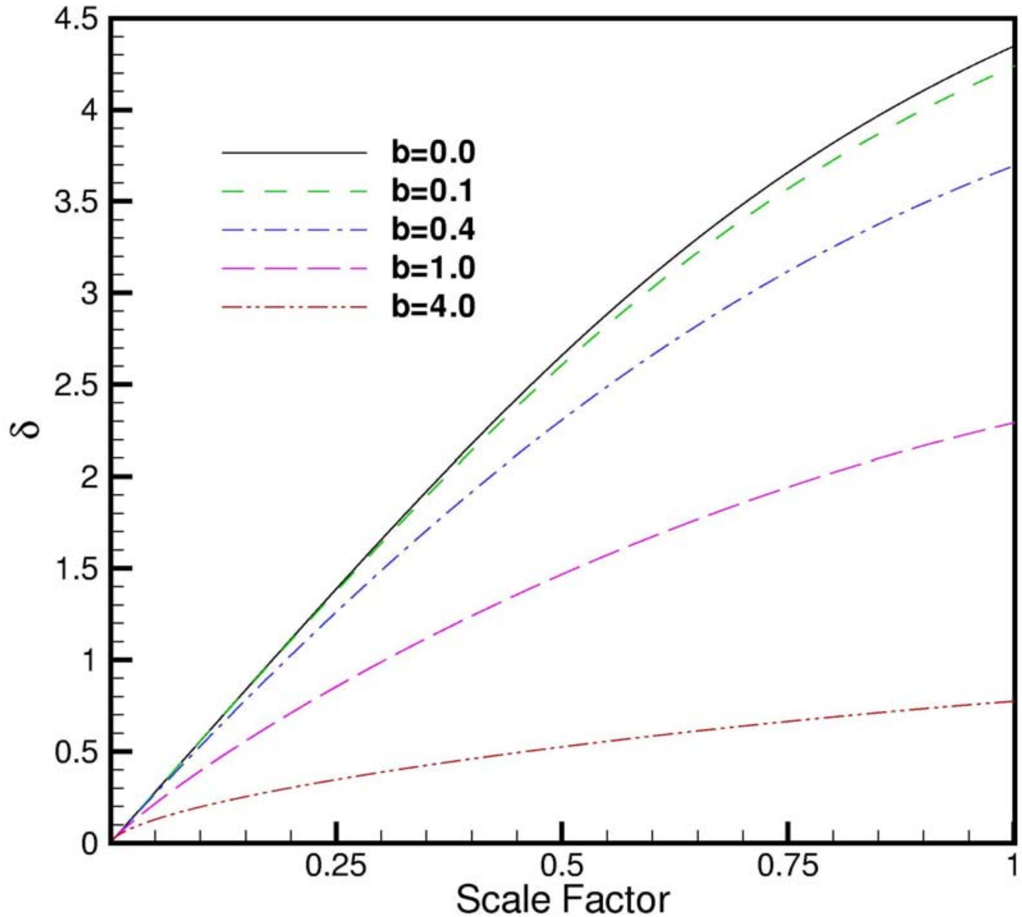
$$\frac{d^2\delta}{dz^2}\left(\frac{dz}{dt}\right)^2 + \frac{d\delta}{dz}\frac{d^2z}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta}{dz}\frac{dz}{dt} - 4\pi G\rho^0\delta = 0$$

- Use Friedmann Eq. with w(z):

$$H(z)^2 = H_0^2\left(\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{3+3w(z)}\right)$$

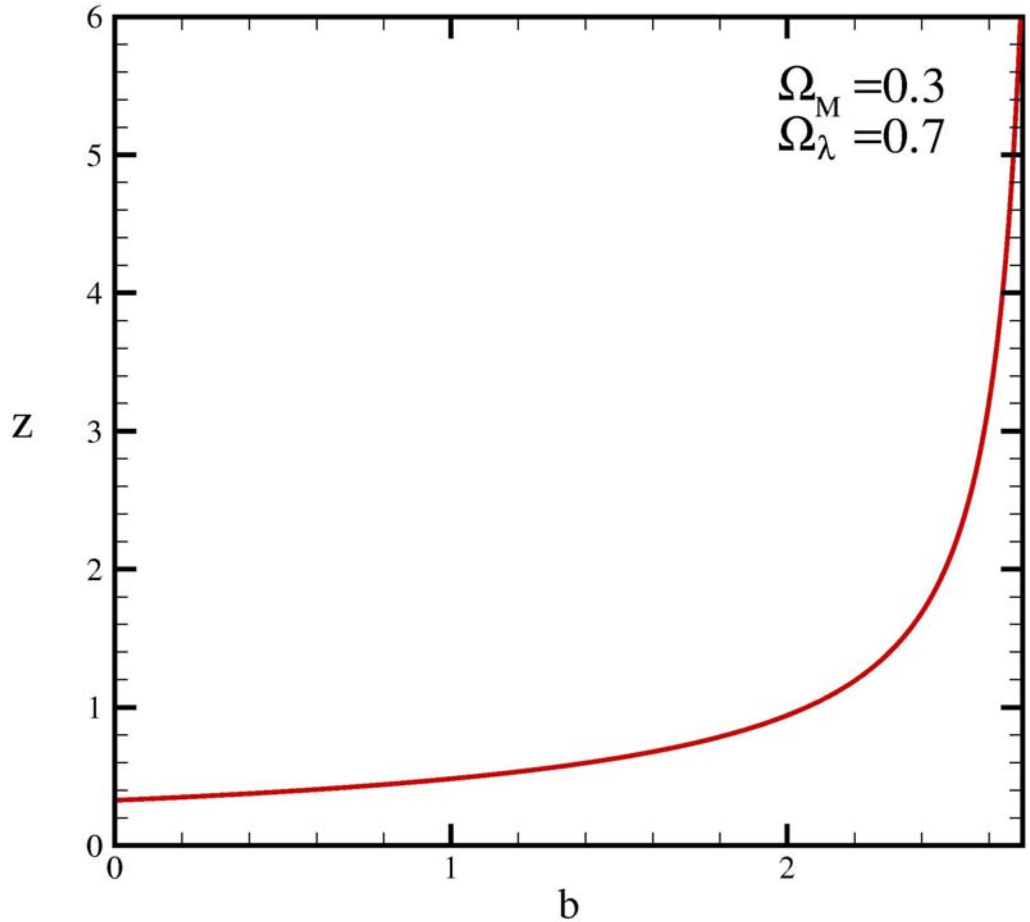
Structure Formation with variable Dark Energy

- Initial Conditions: $\delta\big|_{z_{in}=1100} = 10^{-5}$



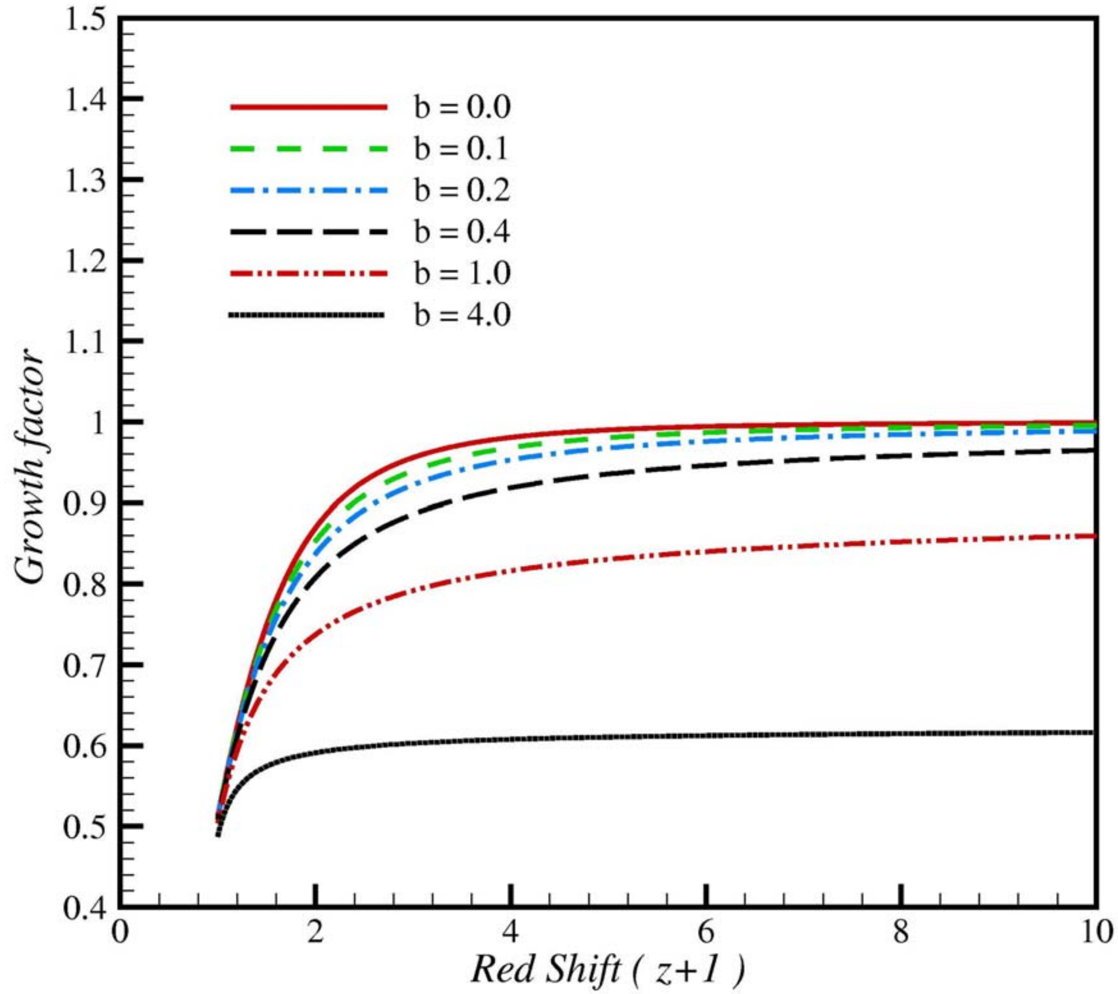
Structure Formation with variable Dark Energy

- Redshift of equivalence of dark energy and matter
  depending on parameter b:



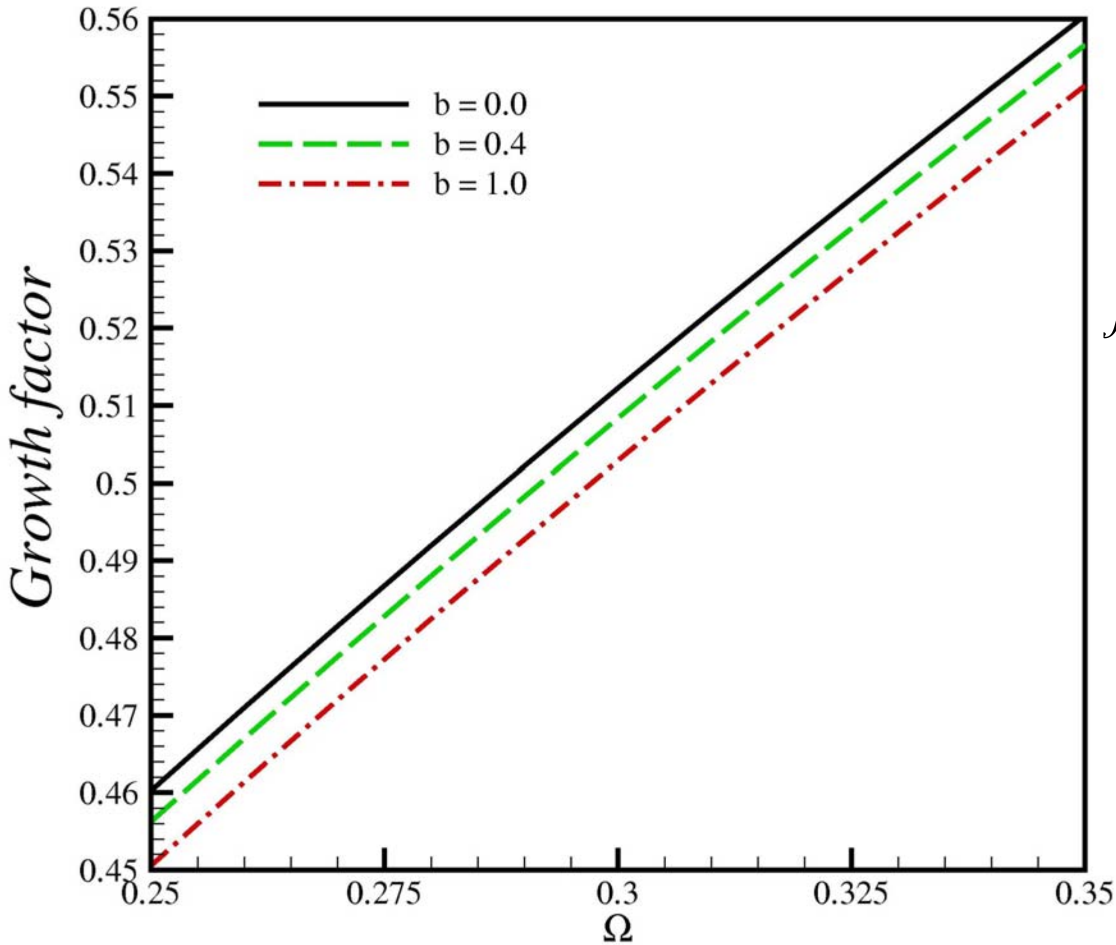
Structure Formation with variable Dark Energy

- Evolution of the growth factor with b≠0.0:



Structure Formation with variable Dark Energy

- Growth factor depends mildly on b:



$$\Omega_\Lambda = 0.7$$

$$f(\Omega_M, b) \approx \Omega_M{}^{\alpha} + \beta \cdot b$$

$$\alpha \approx 0.6$$

$$\beta \approx -0.01$$

Outlook

- Growth of structure with spherical collapse model
- Calculate the effects on CMB
  - Newtonian
  - Relativistic (Super-horizon)
- Consequences for the observed Power spectrum P(K) at z=0 ?

Spherical Model

$$H^2 = H_i^{\,2}\left[\Omega_M^{\ i}\left(\frac{R}{R_i}\right)^{-3} + \Omega_\Lambda^{\ i} - (\Omega_{tot} - 1)\left(\frac{R}{R_i}\right)^{-2}\right]$$

$$\left(\frac{\dot{R}}{R}\right)^2 = H_i^{\,2}\left[\Omega_M^{\ i}(1+\delta_i)\left(\frac{R}{R_i}\right)^{-3} + \Omega_\Lambda^{\ i} - (\Omega_M(1+\delta_i) + \Omega_\Lambda^{\ i} - 1)\left(\frac{R}{R_i}\right)^{-2}\right]$$

•

•

$$\dot{R}^2 = H_i^{\,2}\left\{\begin{aligned} &\Omega_M^{\ i}(1+z_i)^3\delta_i R_i^{\,2} \\ &+\Omega_\Lambda^{\ 0}(1+z_i)^{3+3w(z)}\left(\frac{R}{R_i}\right)^{-3-3w(z)} \\ &+\Omega_M^{\ 0}(1+z_i)^{3+3w(z)}\left(\frac{R}{R_i}\right)^{-3-3w(z)} \\ &-\left[\Omega_M^{\ 0}(1+z_i)^3 + \Omega_\Lambda^{\ 0}(1+z_i)^{3+3w(z)} - 1\right]R_i^{\,2} \end{aligned}\right\}$$

Spherical model

imation, since negative
:e and more spherical as
el, & Shu 1965). To esti-
simple peak biasing for-
lumenthal (1992) for the

s appear in a relatively
ar why there is an upper
e use it to calculate the
ıctor of voids in different

et al. 1991):

$$\dot{R}^2 = H_0^2[-\Omega_0(1+z_i)^3 R_i^2 \delta_i - (\Omega_0 + \lambda_0 - 1)(1+z_i)^2 R_i^2 + \Omega_0(1+z_i)^3 R_i^3(1+\delta_i)/R + \lambda_0 R^2] . \tag{1}$$

We combine this equation with the equation for the background's redshift:

$$\frac{1}{1+z}\frac{d(1+z)}{dt} = -H_0 P(z) , \tag{2}$$

Spherical Model

Initial Conditions:

$$\delta_i = -10^{-2}$$

$$R_i = 1.0$$

$$z_i = 20$$