Cosmology with time-varying constants

Florian Bauer
Technische Universität München

DESY Theory Workshop
30.09.2004

Introduction

- Time-dependence of the cosmological constant Λ and Newton's constant G.
- General relativity with classical matter: Λ and G are true constants.
- Quantum fields on curved background
  $\rightsquigarrow$ Infinite contributions to the effective stress tensor $\langle T_{\mu\nu}\rangle$ (vev of $T_{\mu\nu}$), divergences are proportional to the metric tensor $g_{\mu\nu}$ and the Einstein tensor $G_{\mu\nu}$.
- Regularisation and renormalisation of infinite terms $\rightsquigarrow$ finite renormalised Λ and G, which now depend on a renormalisation scale μ. The dependence on the scale is described by the RGEs.
- "Usual" meaning of μ: External momentum.
  For Λ and G there is no external momentum given. Several proposals for μ in the literature: Hubble constant H, temperature of the background radiation, square root of the Ricci scalar, ...
- In this work, we take the Gibbons-Hawking temperature T of the cosmological event horizon as renormalisation scale μ. $T = (2\pi R)^{-1}$ is given by inverse of the horizon radius R.
- Why this choice?
  The universe is currently accelerating. Simplest reason: $\Lambda > 0 \rightsquigarrow$ final state of universe will be deSitter-like. deSitter space-time has a cosmological event horizon $\rightsquigarrow$ only one scale: R

Renormalisation group equations with renormalisation scale $\mu = T_{GH} = (2\pi R)^{-1}$

- RGE for Λ:

$$\Lambda(R) = \Lambda_0 \left(1 + q_1 \ln \frac{R}{R_0}\right), \quad \Lambda_0 := \Lambda(R_0).$$

  Quantity q_1 depends on the masses of the quantum fields:

$$q_1 := \frac{1}{\Lambda_0 32\pi^2} \left[m_{\mathsf{B}}^4 - m_{\mathsf{F}}^4\right].$$

- $m_{\mathsf{B,F}}$: field mass of one boson/fermion d.o.f.
- $q_1 > 0$: CC Λ increasing with the radius R of the event horizon. $q_1 < 0$: decreasing.
- RGE for G:

$$G(R) = \frac{G_0}{1 + q_2 \ln \frac{R}{R_0}}, \quad G_0 := G(R_0),$$

$$q_2 := \frac{G_0}{\pi}\left[\left(\frac{1}{6} - \xi\right) m_{\mathsf{B}}^2 + \frac{1}{12} m_{\mathsf{F}}^2\right].$$

  ξ: coupling constant in the coupling between the Ricci scalar and the boson field: $\xi R \phi^2$.

- $q_2 \sim G_0 m^2 = \frac{m^2}{M_{\mathsf{Planck}}^2} \ll 1 \leadsto$ effect of fields with high masses m small $\leadsto$ very weak running of Newton's constant G, compatible with the strong bounds on the time-variation of G.

Conservation equations

- FRW space-time with line element:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right).$$

- The radius of the event horizon depends only on the cosmological time t:

$$R(t) = a(t)\int_t^\infty \frac{dt'}{a(t')}.$$

$\rightsquigarrow$ Λ, G depend on time.

- Energy/matter content: perfect fluid with energy density ρ and pressure $p = \omega\rho$, equation of state parameter ω.

- Einstein's equations:

$$G^{\mu\nu} = 8\pi G(\Lambda g^{\mu\nu} + T^{\mu\nu}) \;\text{ with }\; \Lambda = \Lambda(R),\; G = G(R).$$

- Time-dependence $\rightsquigarrow$ modified conservation equations. Bianchi identities:

$$G^{\mu\nu}{}_{;\nu} = 0 \;\longrightarrow\; \dot{G}(\Lambda + \rho) + G(\dot{\Lambda} + \dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega)) = 0.$$

- Energy transfer between Λ and matter content $\rightsquigarrow$ "old" scaling behaviour $\rho \propto a^{-3(1+\omega)}$ no longer valid.

Cosmological evolution

- Equations for the scale factor $a(t)$:

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{8\pi}{3} G(\Lambda + \rho), \\ \frac{\ddot{a}}{a} &= \frac{8\pi}{3} G(\Lambda - Q\rho), \quad Q := \frac{1+3\omega}{2}. \end{aligned}$$

- No scaling rule for $\rho \rightsquigarrow$ eliminate it:

$$F(t) := \frac{\ddot{a}}{a} + Q\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] = \frac{8\pi}{3} G\Lambda(1+Q).$$

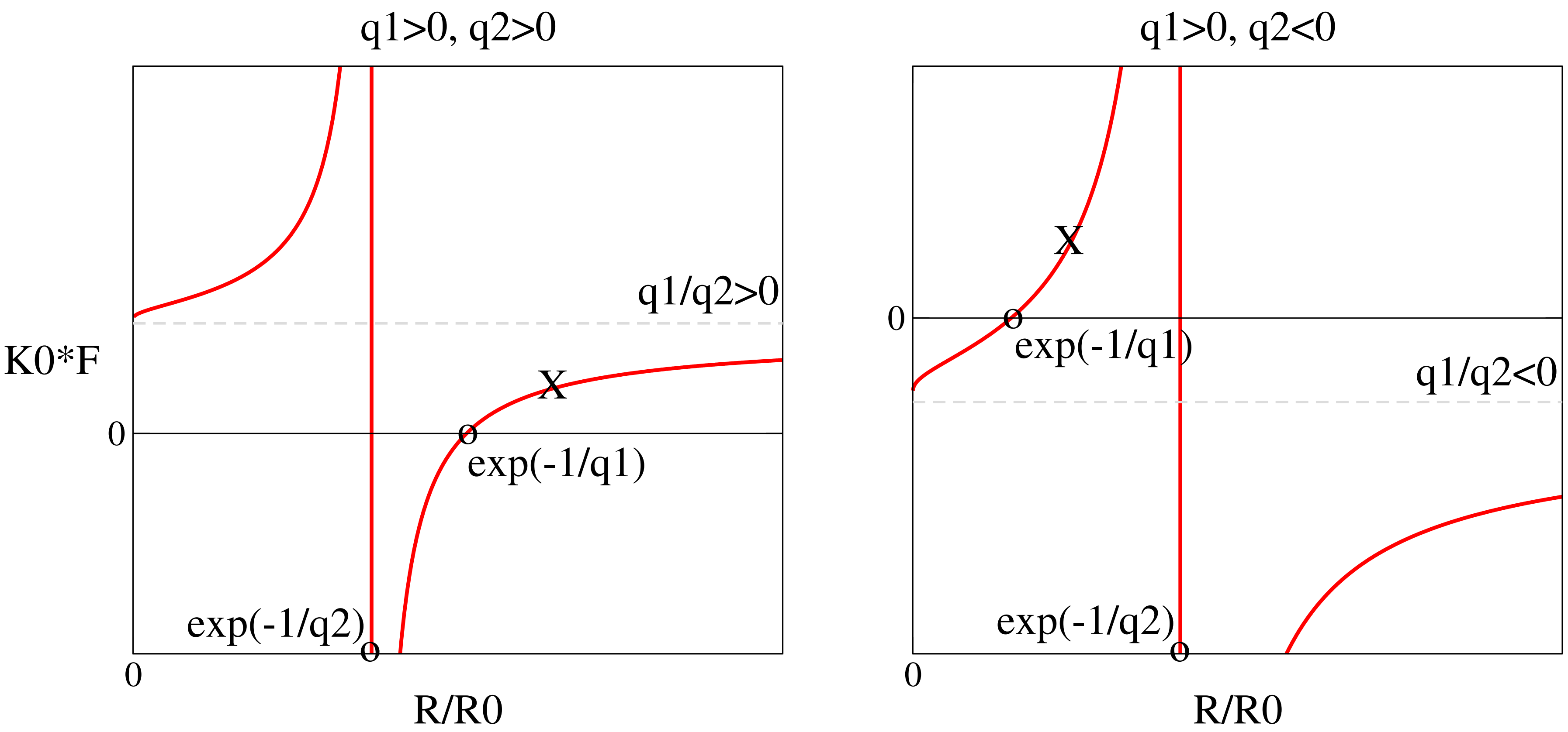
- Insert RGEs for Λ, G:

$$K_0 F = \frac{1 + q_1 \ln \frac{R}{R_0}}{1 + q_2 \ln \frac{R}{R_0}}, \quad K_0 := \frac{3}{8\pi G_0 \Lambda_0 (Q+1)} = \frac{1}{H_0^2 \Omega_{\Lambda 0}(Q+1)}.$$

- Solve for R:
$$\frac{R(t)}{R_0} = \exp\left[\frac{K_0 F(t) - 1}{q_1 - q_2 K_0 F(t)}\right].$$
- $R(t)$ involves an integral over the scale factor $a(t)$.
- For a numerical treatment we differentiate w.r.t. t
$\rightsquigarrow$ ordinary differential equation for the scale factor $a(t)$:
$$\left[\frac{\dot{a}}{a} + \frac{(q_2 - q_1)K_0 \dot{F}}{(q_1 - q_2 K_0 F)^2}\right] \cdot \exp\left[\frac{K_0 F - 1}{q_1 - q_2 K_0 F}\right] - \frac{1}{R_0} = 0.$$

$q_1 > 0$: Increasing Λ with R

$$K_0 F = \frac{1 + q_1 \ln \frac{R}{R_0}}{1 + q_2 \ln \frac{R}{R_0}}, \quad F(t) = \frac{\ddot{a}}{a} + Q \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right].$$



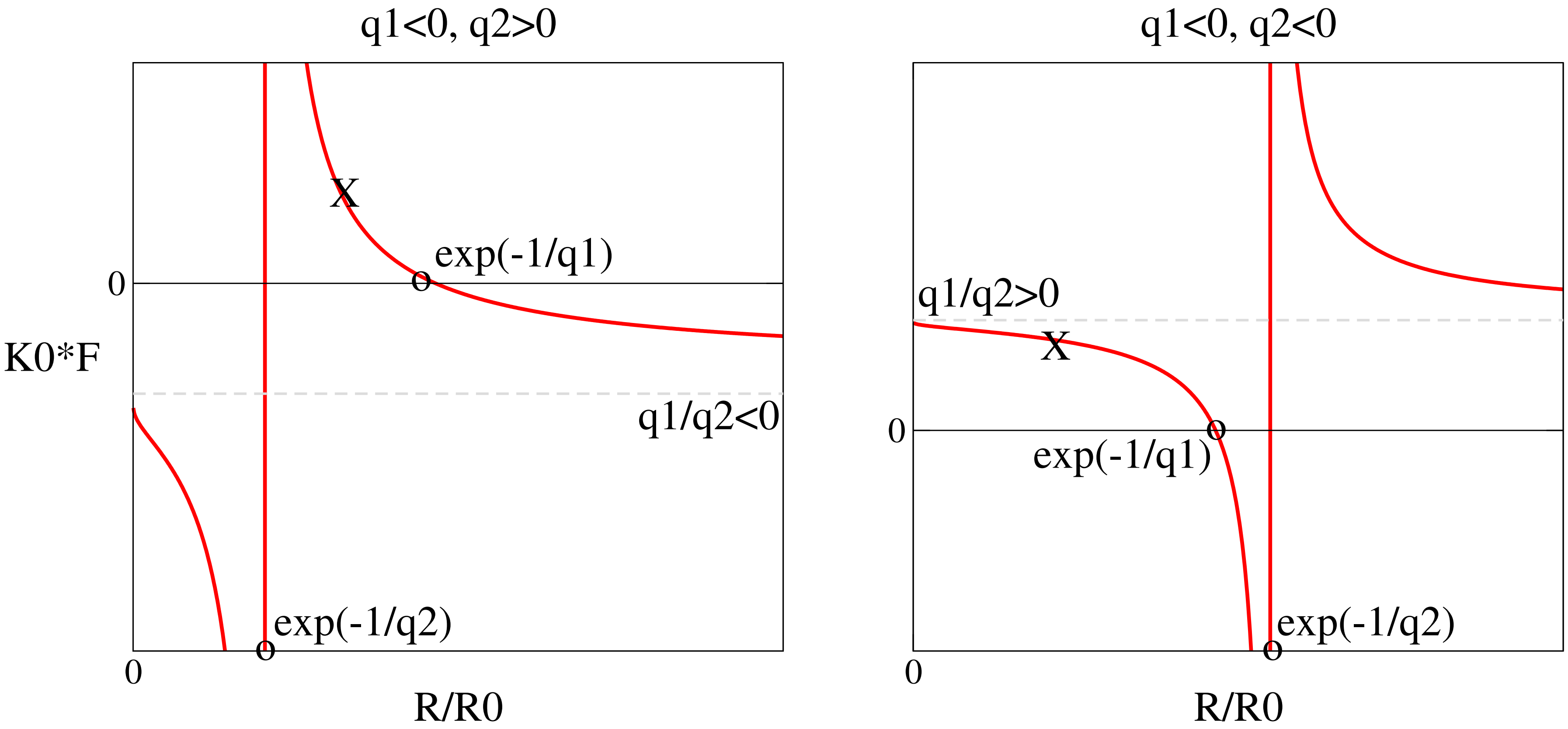
Numerical solutions for $q_1 > 0$

- $q_2 > 0$:
  For $t \longrightarrow \infty$: $R \longrightarrow \infty$, but $K_0 F \longrightarrow \frac{q_1}{q_2} =$ const.
  $\rightsquigarrow$ deSitter behaviour $a(t) \propto \exp(b \cdot t)$ with $K_0 F = b^2/H_0^2 = \frac{q_1}{q_2}$.
  deSitter space-time has a finite horizon radius $R = b^{-1}$ in contrast to $R \longrightarrow \infty$ $\rightsquigarrow$ no solution.

- $q_2 < 0$:
  For $t \longrightarrow \infty$: $K_0 F \longrightarrow \infty$, $R \longrightarrow$ const., not compatible, too.

$q_1 < 0$: Λ decreases with R

$$K_0 F = \frac{1 + q_1 \ln \frac{R}{R_0}}{1 + q_2 \ln \frac{R}{R_0}}, \quad F(t) = \frac{\ddot{a}}{a} + Q\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right].$$



Numerical solutions for $q_1 < 0$

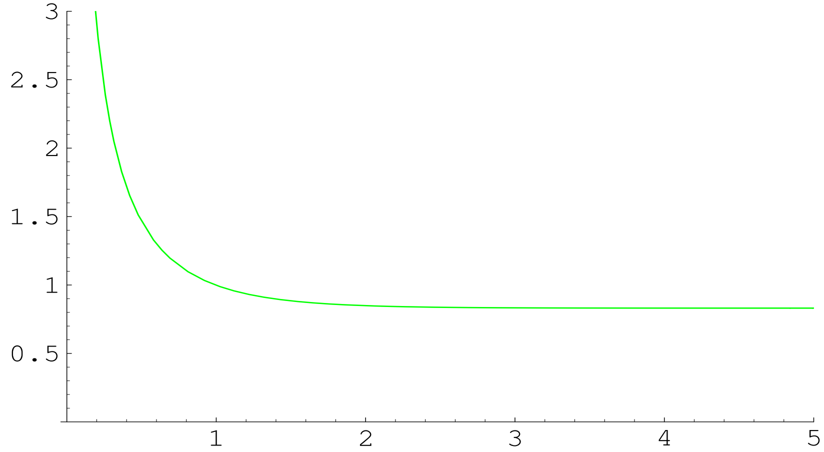
- Λ decreases with time until it reaches a stable point, where $K_0 F$ and R are finite, positive and constant. This is a final deSitter state.
- Acceptable values for q_1:
  For $q_1 < 0$ we also observed an increased age of the universe. To be compatible with the observed age $t_0 \approx 13,7\,\text{Gyr}$, we need $q_1 < 0$ and $|q_1| \lesssim \mathcal{O}(1)$, light fermions $\rightsquigarrow$ Possible candidates: neutrinos $m_\nu \sim 1\,\text{eV}$.
- Problem:

$$q_1 = \frac{1}{\Lambda_0 32\pi^2}\left[m_{\mathsf{B}}^4 - m_{\mathsf{F}}^4\right].$$

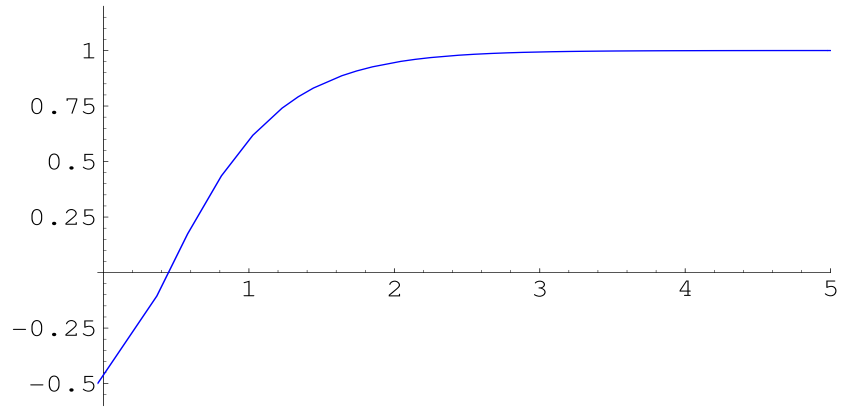
  The fields with the highest mass dominate. We have to assume some decoupling or suppression mechanism for the high-mass fields. The simple form of the RGEs in our model cannot account for this.

Example: $q_1 = -1, \; q_2 = 0, \; R_0 = 1,14$

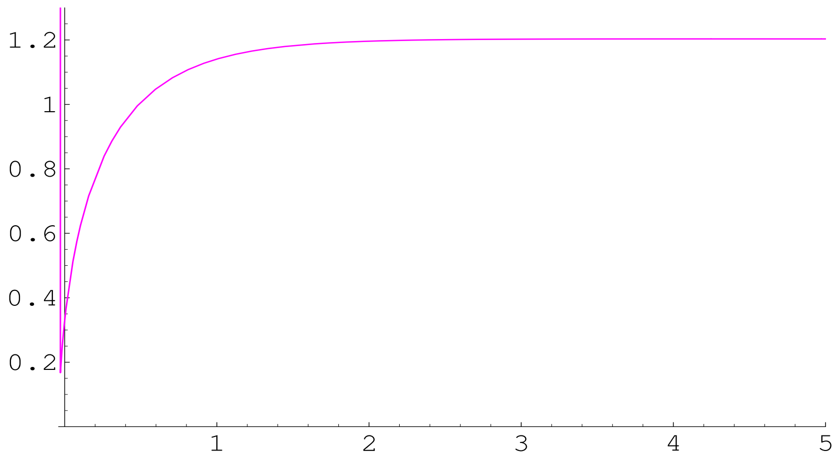
Hubble scale $H(t) = \frac{\dot{a}}{a}$



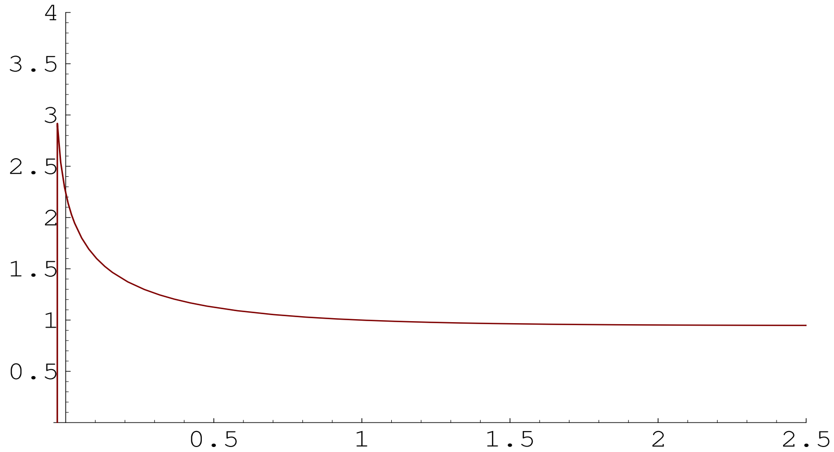
Acceleration $q(t) = \frac{\ddot{a}a}{\dot{a}^2}$



Event horizon radius $R(t)$

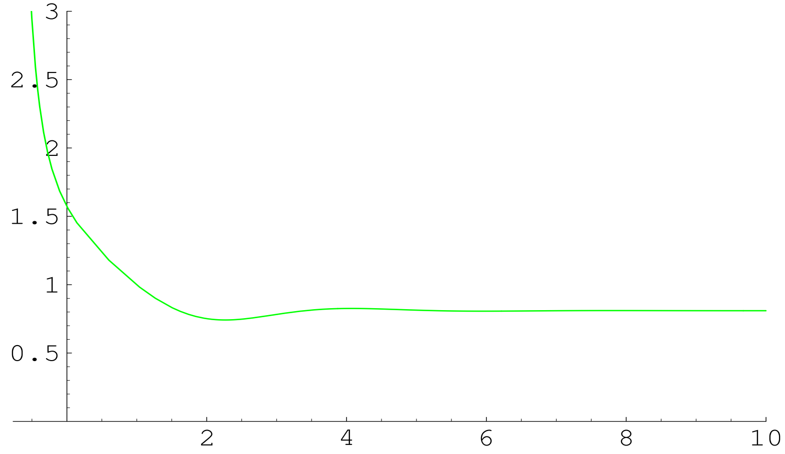


Vacuum energy density $\Lambda(t)$

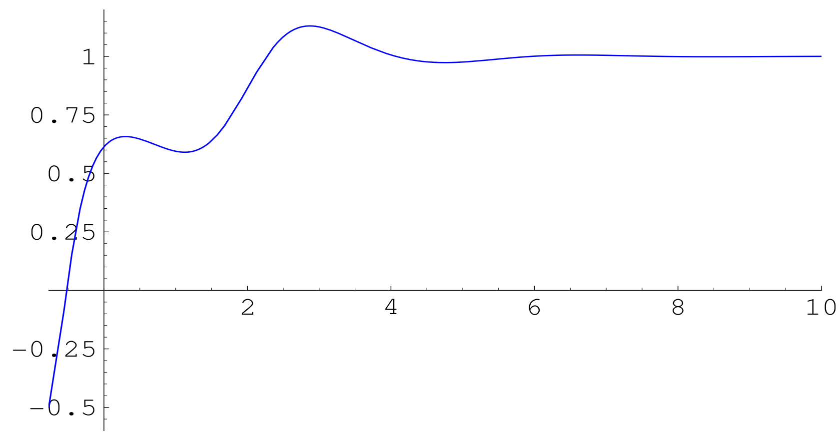


Example: $q_1 = -5,\ q_2 = 0,\ R_0 = 1,21$

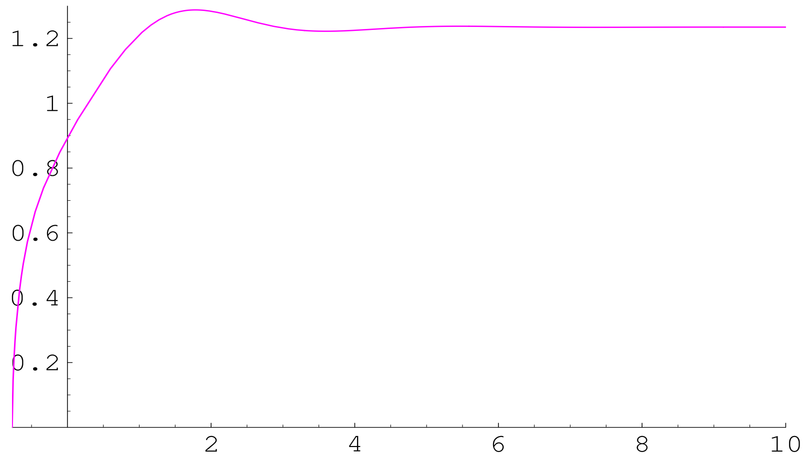
Hubble scale $H(t) = \frac{\dot{a}}{a}$



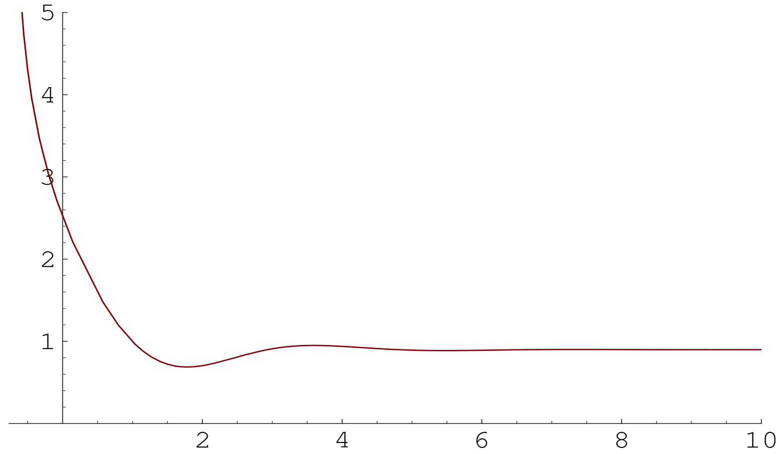
Acceleration $q(t) = \frac{\ddot{a}a}{\dot{a}^2}$



Event horizon radius $R(t)$



Vacuum energy density $\Lambda(t)$



Conclusions

- Infinite contributions of quantum fields lead to a scale dependence of Λ and G.
- We used the radius R of the cosmological event horizon as renormalisation scale.
- We found no stable solutions for $q_1 > 0$.
- To be compatible with observations, we need $q_1 < 0$ and $|q_1| \lesssim \mathcal{O}(1)$. Known candidates: neutrinos.
- A more negative q_1 increases the age of the universe.
- An unknown decoupling or suppression mechanism is active for higher mass fields.
- The bounds for the running of G are satisfied even for fields with high masses.