(Pre-)Thermalization after Inflation

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J. B., J. Serreau, Phys.Rev.Lett. 91 (2003) 111601

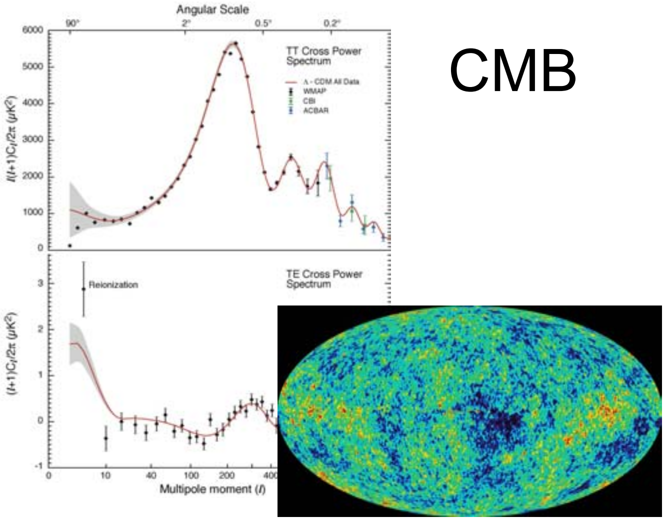
J. B., Sz. Borsanyi, C. Wetterich, Phys.Rev.Lett. (2004) hep-ph/0403234

J. B., hep-ph/0409233

Early Universe

End of Inflation

(P)reheating



CMB

far-from-equilibrium
`initial´ state

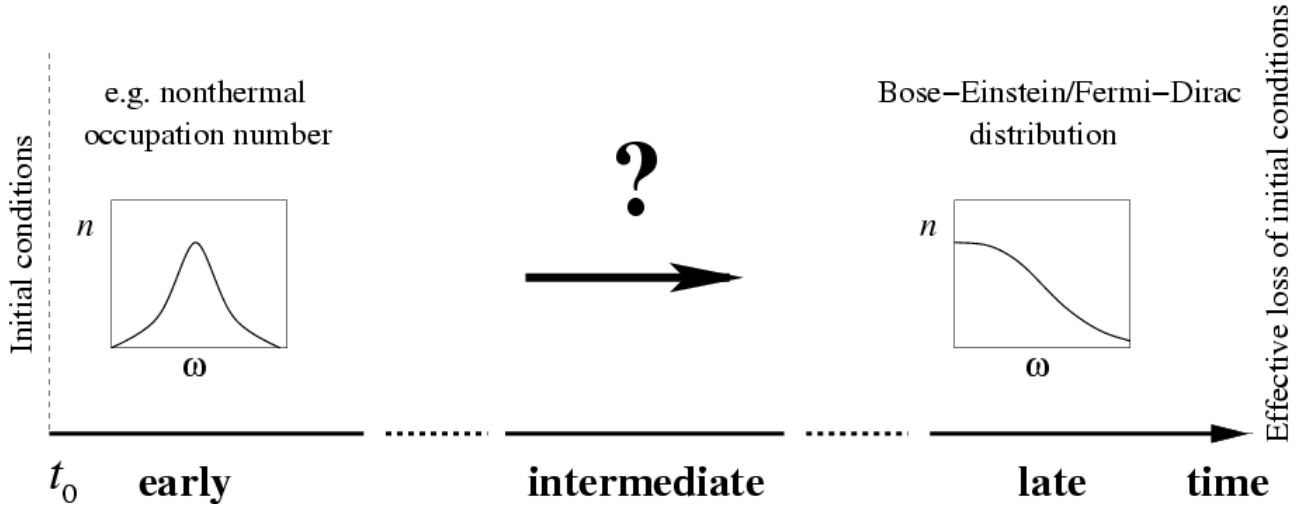
`entropy´
production

thermal spectrum
with fluctuations

time

Thermalization

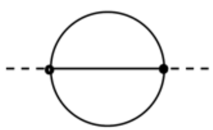
- Process of thermalization leads to loss of details about the initial conditions: late-time *`universality´*
  - ➨ precise understanding of *out of equilibrium* phenomena crucial for knowledge about the primordial universe
- Approach to thermal equilibrium requires *quantum evolution*
  - ➨ classical equilibration times are functions of Rayleigh-Jeans cutoff



Standard approximations fail out of equilibrium

`Secularity´

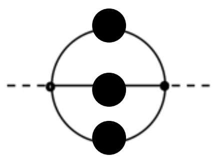
- uniform approximations in time require infinite pert. orders



$\sim \gamma t$

`Universality´

- nonlinear dynamics necessary for effective loss of information



$\rightarrow e^{-\gamma t}$

Two-particle irreducible effective actions

- systematic 2PI loop, coupling or 1/N expansions available
- *far-from-equilibrium* dynamics as well as late-time *thermalization* in QFT

J.B., Cox ´01; Aarts, J.B. ´01; J.B. ´02; Cooper, Dawson, Mihaila ´03;
J.B., Borsanyi, Serreau ´03; Cassing, Greiner, Juchem ´03; Bedingham ´03; ...

Two-particle irreducible 1/N expansion to NLO

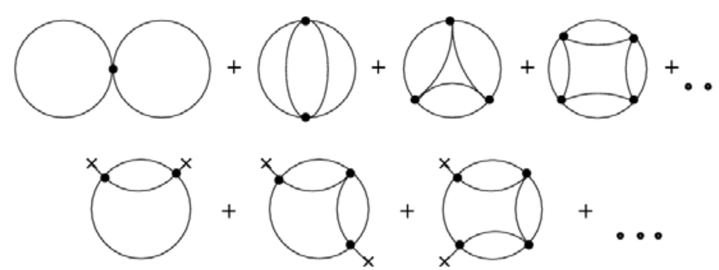
2PI effective action: Cornwall, Jackiw, Tomboulis ´74

$$\Gamma[\phi, G] = S[\phi] + \frac{i}{2}\mathrm{Tr}\ln G^{-1} + \frac{i}{2}\mathrm{Tr}\, G_0^{-1}(\phi)\, G + \Gamma_2[\phi, G]$$

- Parametrized by macroscopic field: $\phi(x) = \langle \Phi(x) \rangle$ and
- exact connected propagator: $G(x,y) = \langle T\Phi(x)\Phi(y)\rangle - \phi(x)\phi(y)$
- $\Gamma_2[\phi, G]$ contains only *two-particle irreducible* (2PI) diagrams

E.g. scalar *N*-component to NLO:

$$\Gamma_2[\phi, G] = -\frac{\lambda}{4!N}\int_x G_{aa}(x,x)G_{bb}(x,x) + \frac{i}{2}\mathrm{Tr}\ln \mathbf{B}(G)$$
$$+ \frac{i\lambda^2}{(6N)^2}\int_{xyz} \mathbf{B}^{-1}(x,z;G)G^2(z,y)\phi_a(x)G_{ab}(x,y)\phi_b(y)$$



$$\mathbf{B}(x,y;G) \equiv \delta(x-y) + \tfrac{i\lambda}{6N}G^2(x,y)$$

J.B. ´02; Aarts, Ahrensmeier, Baier, J.B., Serreau ´02

Time evolution equations

Equations of motion: (1) $\frac{\delta\Gamma[\phi,G]}{\delta\phi(x)} = 0$, (2) $\frac{\delta\Gamma[\phi,G]}{\delta G(x,y)} = 0$

spectral function » h[Φ,Φ]i

$$G(x,y) = F(x,y) - \frac{i}{2}\rho(x,y)\,\mathrm{sign}_{\mathscr{C}}(x^0 - y^0)$$

statistical propagator » h{Φ,Φ}i

$$\left[\Box_x \delta_{ac} + M^2_{ac}(x)\right]\rho_{cb}(x,y) = -\int_{y^0}^{x^0} \mathrm{d}z\, \Sigma^{\rho}_{ac}(x,z)\rho_{cb}(z,y)$$

$$\left[\Box_x \delta_{ac} + M^2_{ac}(x)\right]F_{cb}(x,y) = -\int_{0}^{x^0} \mathrm{d}z\, \Sigma^{\rho}_{ac}(x,z)F_{cb}(z,y) + \int_{0}^{y^0} \mathrm{d}z\, \Sigma^{F}_{ac}(x,z)\rho_{cb}(z,y)$$

$$\left(\left[\Box_x + \frac{\lambda}{6N}\phi^2(x)\right]\delta_{ab} + M^2_{ab}(x;\phi=0,F)\right)\phi_b(x) = -\int_0^{x^0} \mathrm{d}y\, \Sigma^{\rho}_{ab}(x,y;\phi=0,F,\rho)\,\phi_b(y)$$

Nonequilibrium:

$$F \not\sim \rho$$

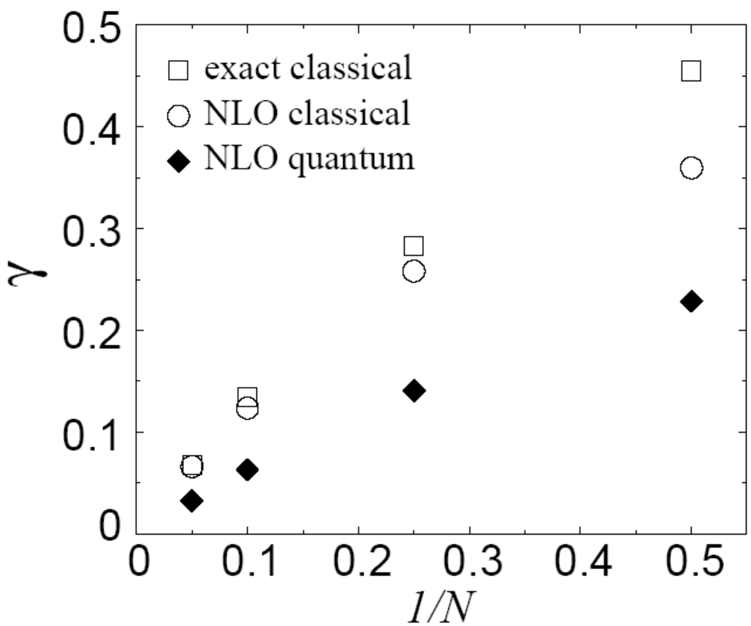
Equilibrium/Vacuum:

$$F \sim \rho$$

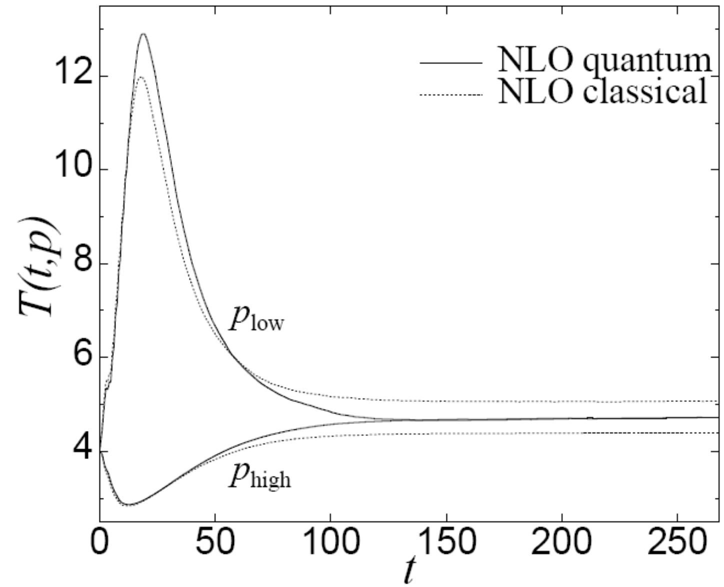
Precision tests

Aarts, J.B. ´02

Damping rate:



Mode temperature: $n(t,p) = \frac{1}{e^{\omega(t,p)/T(t,p)} - 1}$



- Convergence of classical NLO and exact (MC) results for small *N* !
- Damping reduced with quantum corrections for *n* < ½, while very good quantum-classical agreement for *n* >> ½ and not too late times
- Classical equilibration time >> quantum equilibration time (classical evolution does of course not reach Bose-Einstein distribution)

Application: Parametric resonance preheating

<u>*N*-component scalar $\lambda\phi^4$-theory</u>

macroscopic field $\phi = \langle \Phi \rangle$

fluctuations $F \sim \langle \Phi\Phi \rangle - \langle \Phi \rangle^2$

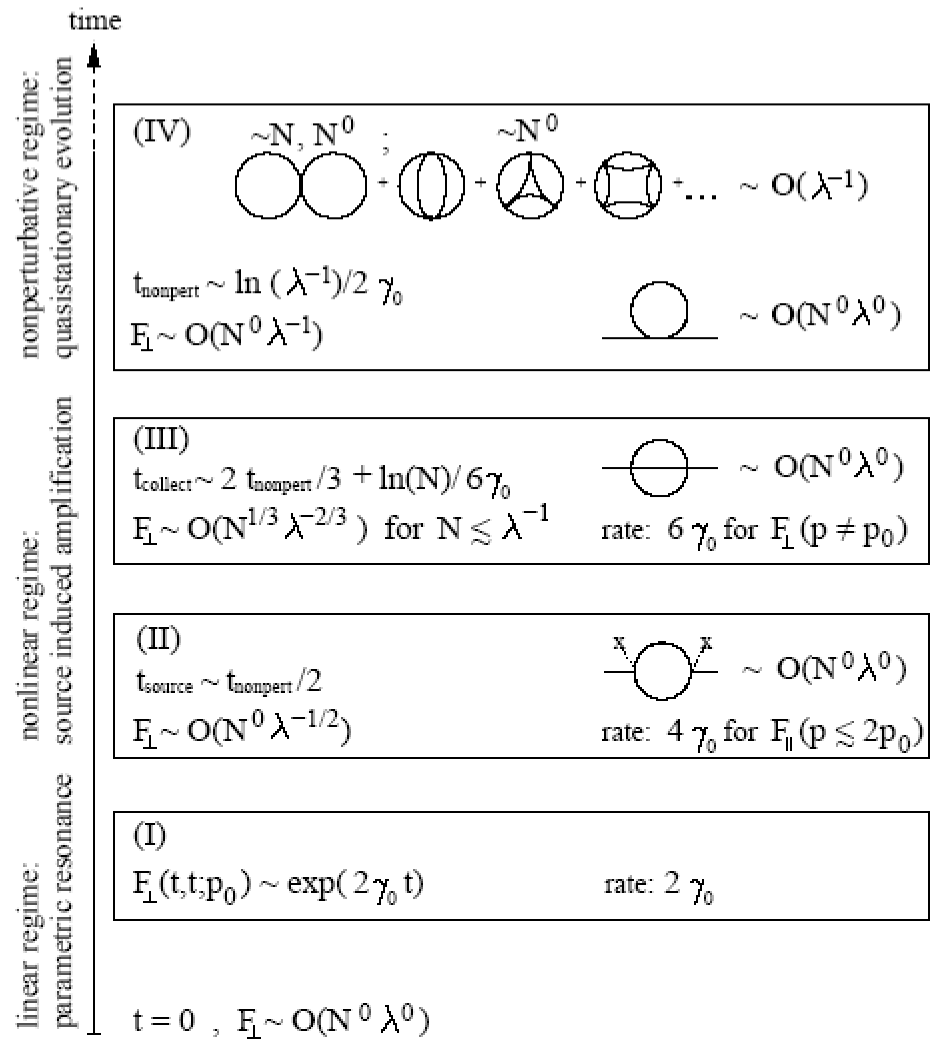
<u>Initial conditions</u>:

Large field: $\phi \sim \mathcal{O}(1/\sqrt{\lambda})\,,\ \lambda \ll 1$

Small fluctuations: $F \sim \mathcal{O}(1)$

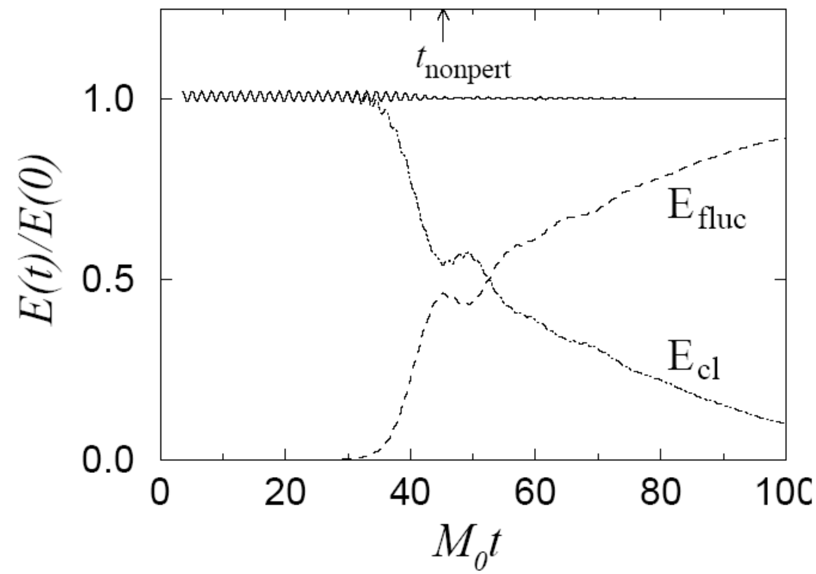
<u>Parametric resonance</u>:

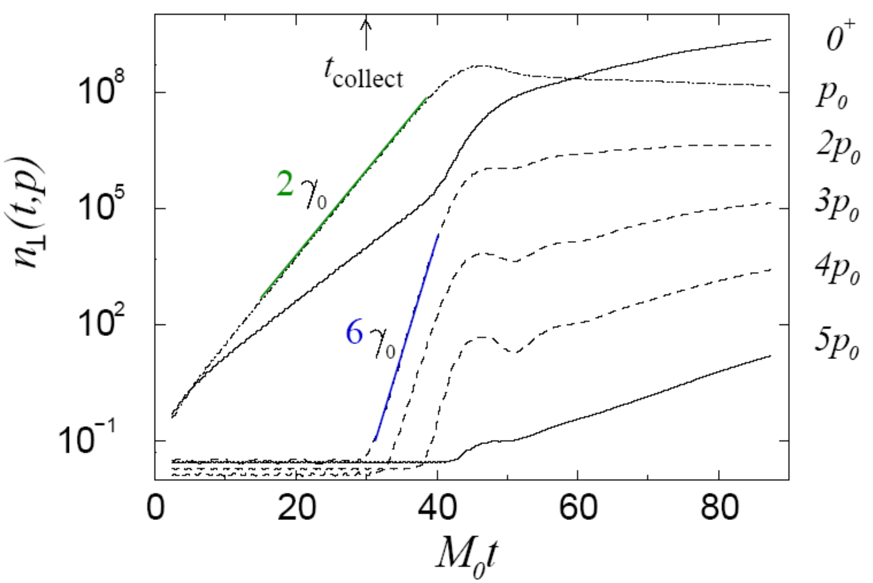
$F \sim e^{\gamma t}$ until $F \sim \mathcal{O}(1/\lambda)$

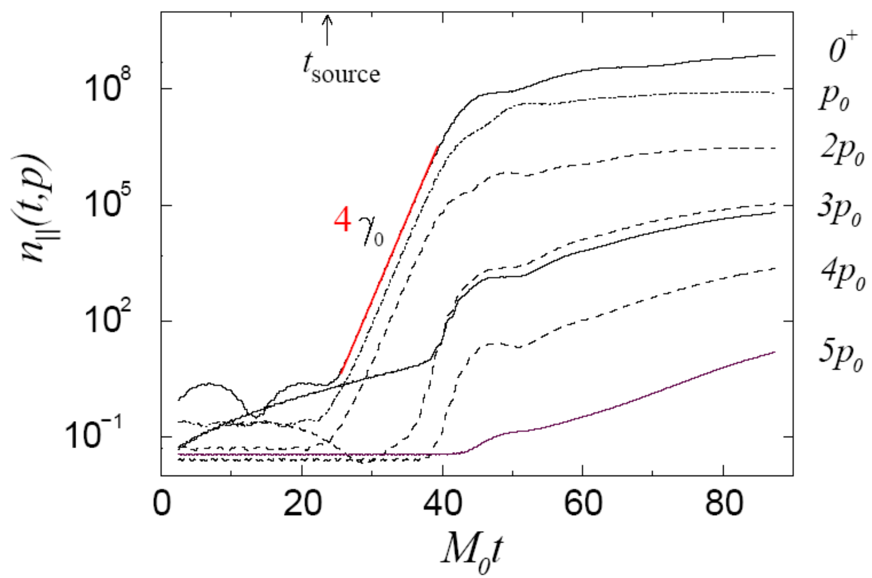


- Fluctuation dominated regime after E_{cl} ' E_{fluc} at $t = t_{nonpert}$
- *Critical slowing down for $t > t_{nonpert}$*

$N = 4$, $\lambda = 10^{-6}$:





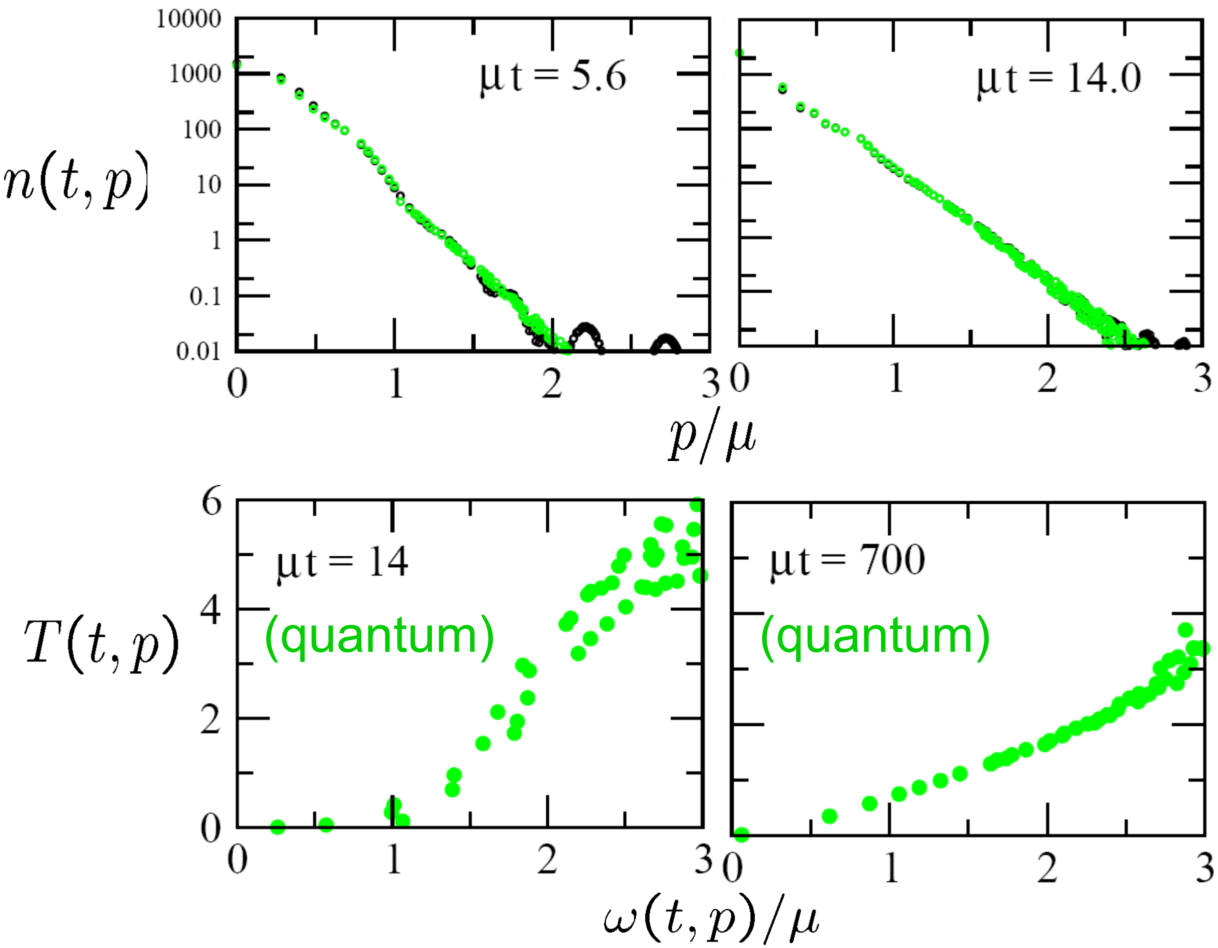


J.B., Serreau ´03

Similarly: Tachyonic preheating

Same 2PI *1/N* approximation employed by Arrizabalaga, Smit, Tranberg
hep-ph/0409177

$\mu^2_{\rm eff}(t<0) = \mu^2, \qquad \mu^2_{\rm eff}(t>0) = -\mu^2$ *(N = 4, λ = 1)*



← quantum vs. classical

Critical slowing down of thermalization

Prethermalization

- Different quantities effectively thermalize on different time scales
  - complete thermalization of all quantities may not be necessary
- *Prethermalized* quantities approximately take on their final thermal values on time scales dramatically shorter than the thermal equilibration time
  - an approximately time-independent equation of state $p = p(\varepsilon)$ with almost fixed relation between p and ε may form far from equilibrium
  - similarly, a suitably defined effective temperature may prethermalize

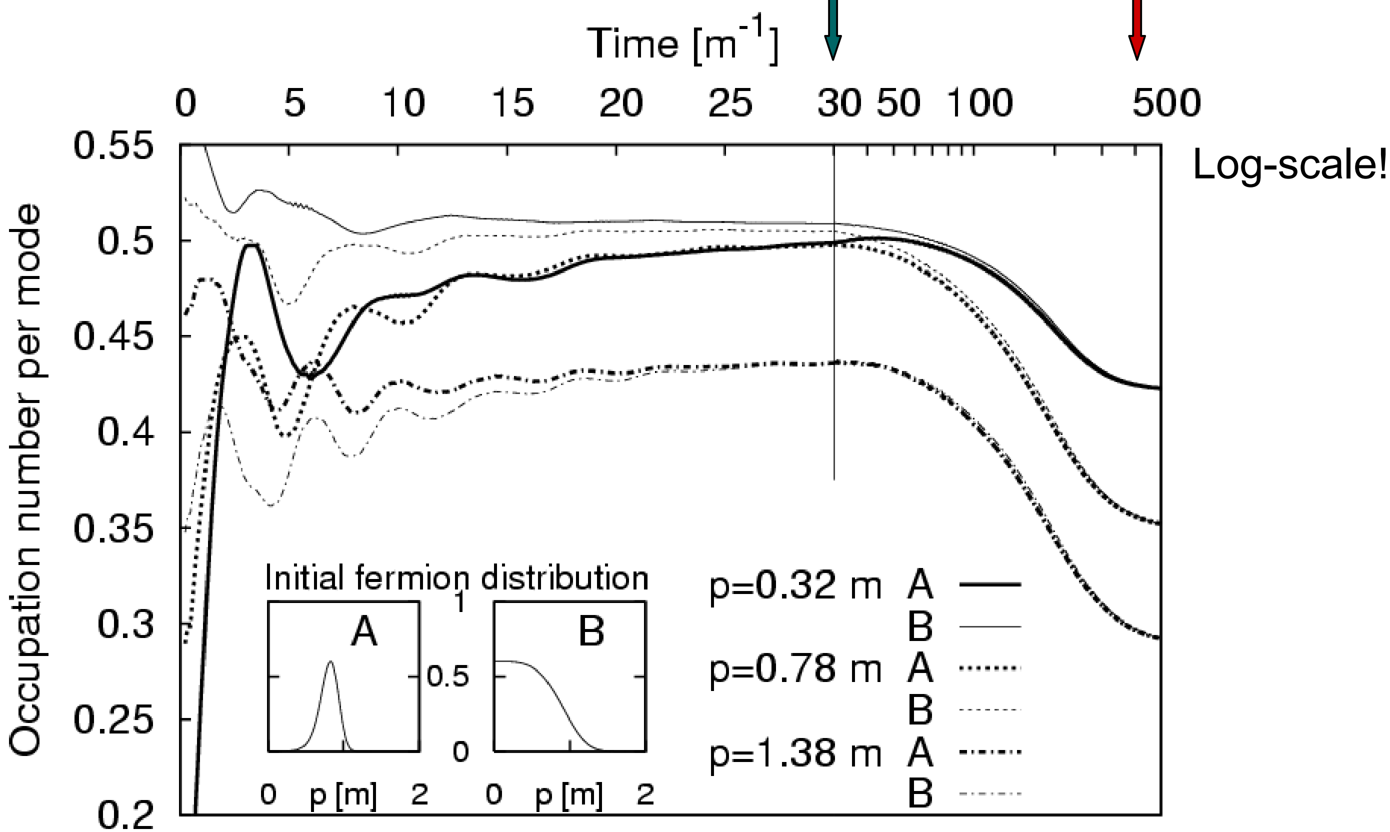
`quasi-thermal´ description in a far-from-equilibrium situation

Characteristic time scales

Yukawa model with coupling *h* » *1* (*O(4)*,2PI *1/(N_F=2)* to NLO)

Loss of initial details for mode quantities t_{damp}

Thermalization t_{eq}

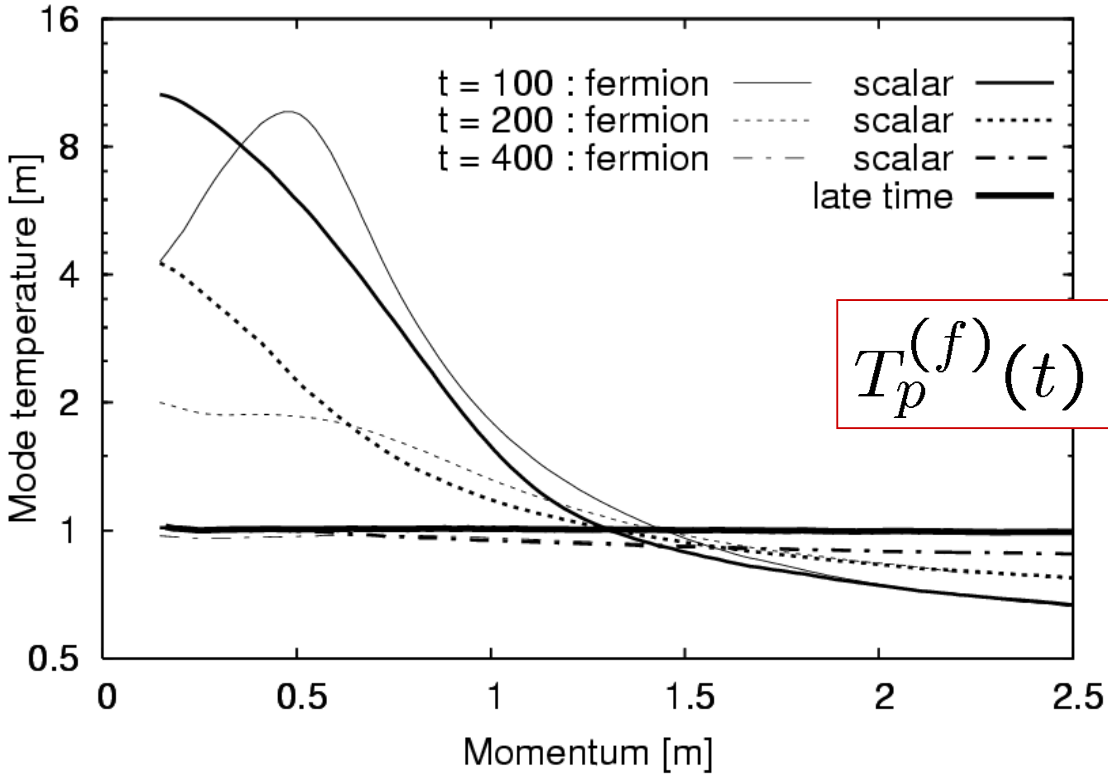


J.B., Borsányi, Wetterich ´04

Thermalization

- Mode temperature $T_p(t)$: $\quad n_p(t) \stackrel{!}{=} \dfrac{1}{\exp\left[\omega_p(t)/T_p(t)\right] \pm 1}$

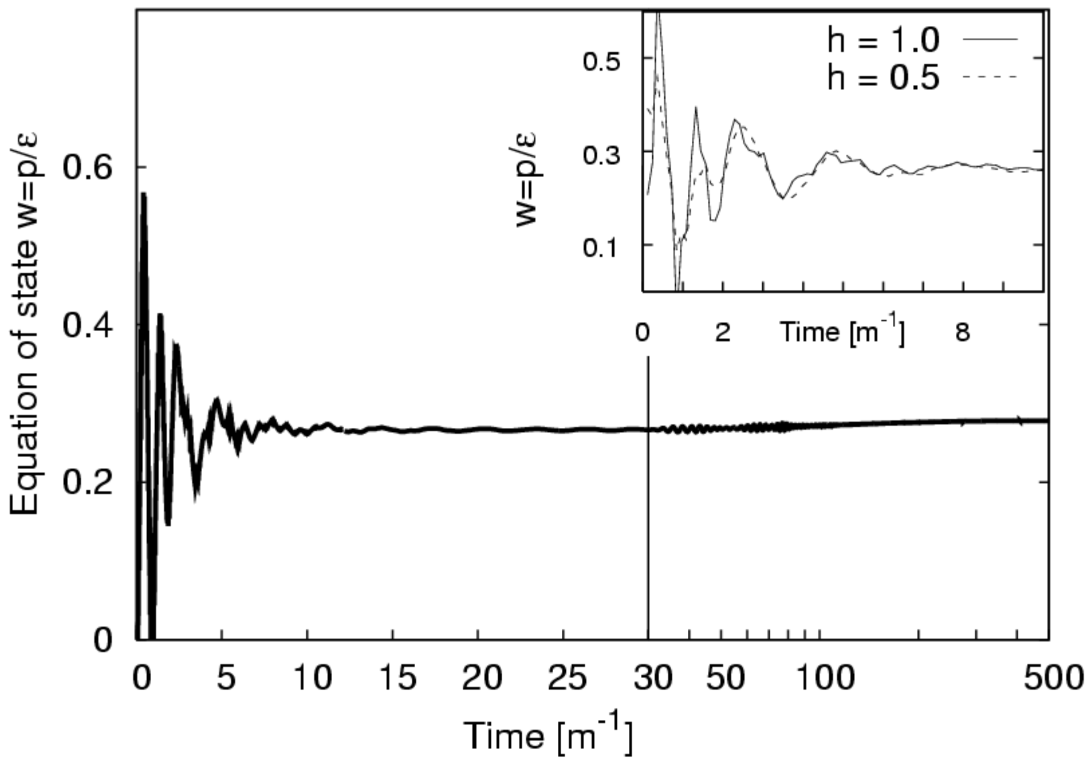
  $\left(n_p \sim \text{tr}\,\frac{p^i\gamma^i}{p}\langle[\psi,\bar{\psi}]\rangle_p\right)$



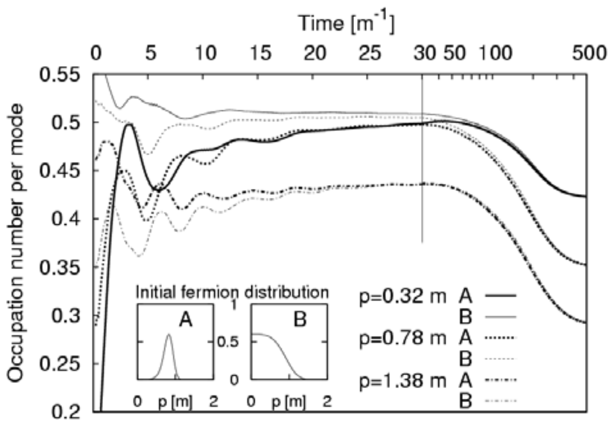
$$T_p^{(f)}(t) = T_p^{(s)}(t) = T_{\text{eq}}$$

Emergence of BE/FD at late times!

Equation of state



compare:



- Almost time-independent EOS builds up very early, even though distributions are far from equilibrium!
- Prethermalization-time independent of interaction details

$\rightsquigarrow$ $t_{\rm pt}$ is of the order of the characteristic inverse mass scale m^{-1}

- consequence of rapid loss of phase information ("dephasing")
- unrelated to the scattering-driven process of thermalization

$$\boxed{t_{\rm pt} \quad \ll \quad t_{\rm damp} \quad \ll \quad t_{\rm eq}}$$

Does a suitable global kinetic temperature $T_{\rm kin}$ *also exist at* $t_{\rm pt}$*?*

Practical definition:

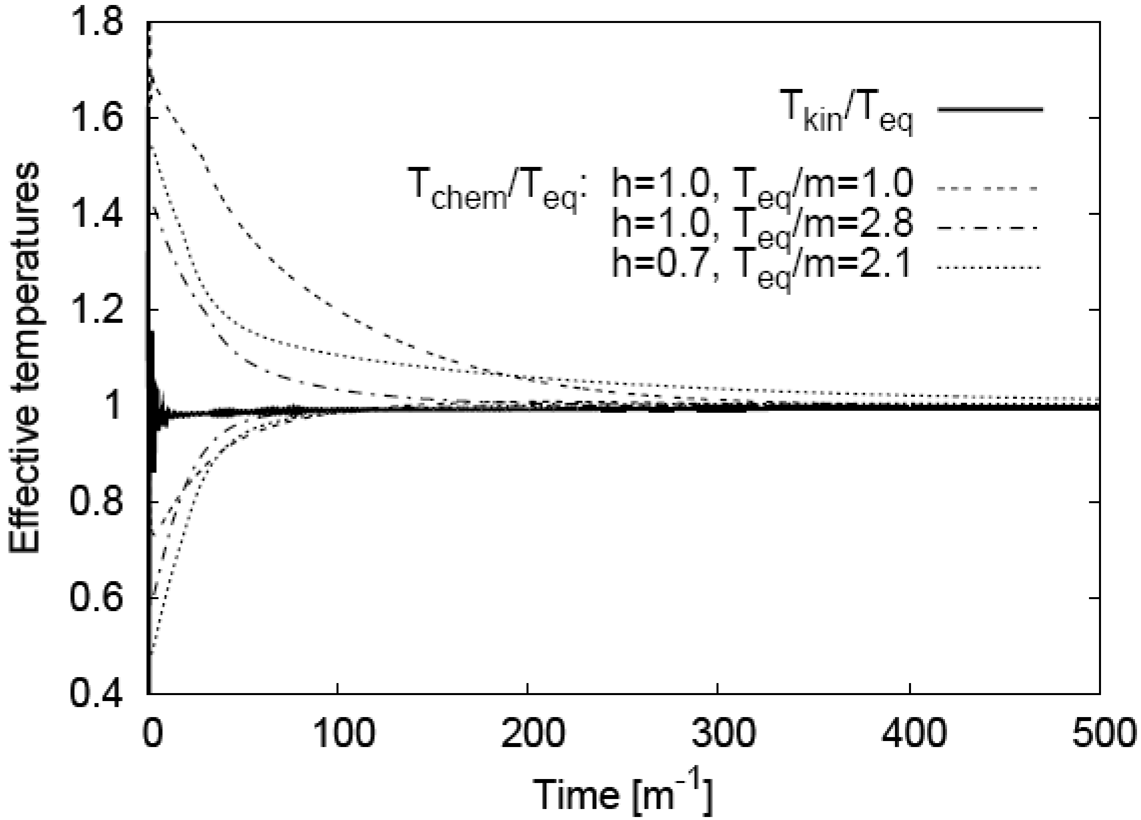
- association of temperature with average kinetic energy per d.o.f.

$$T_{\rm kin}(t) = E_{\rm kin}(t)/c_{\rm eq}$$

- $c_{\rm eq} = E_{\rm kin,eq}/T_{\rm eq}$ is given solely in terms of equilibrium quantities
  (E.g. relativistic plasma: $E_{\rm kin}/N = \epsilon/n = \alpha T$)

$$\boxed{\text{Kinetic equilibration:} \quad T_{\rm kin}(t) \; = \; T_{\rm eq}}$$

Kinetic prethermalization



upper curves: scalars

lower curves: fermions

$\rightsquigarrow$ $T_{\rm kin}(t)$ *prethermalizes on a very short time scale* $\sim m^{-1}$ *in contrast to chemical equilibration*

$\rightsquigarrow$ "quasi-thermal" description in a far-from-equilibrium situation!

Conclusions

2PI effective action techniques provide quantitative tool for

- far-from-equilibrium dynamics & thermalization in QFT
- analytic description of nonperturbatively large fluctuations

➨ Thermalization is a collective phenomenon with $t_{eq} >> t_{damp}$ (critical slowing down, `off-shell´ total particle number changing processes)

➨ Relevant prethermalization time t_{pt} for `bulk´ quantities can be dramatically shorter: $t_{pt} << t_{damp} << t_{eq}$