Braneworld cosmology almost without branes

An introduction to patch inflation and noncommutative braneworlds

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The 5 W's

– **What?**

Braneworld early universe is a 3-brane in a 5D noncompact bulk; inflationary perturbations with a nonstandard effective Friedmann equation (4D, RS and GB scenarios).

– **Why?**

To look for cosmic signatures of high-energy, higher-derivative gravity models. This and next generation CMB experiments might be able to discriminate among them!

– Who and When?

- *4D essential:* Lidsey et al. 1997; Mukhanov, Feldman and Brandenberger 1992...

- *Cosmologies and gravities with extra dimensions:* Hořava and Witten, 1996a,b; Arkani-Hamed, Dimopoulos and Dvali 1998; Randall and Sundrum 1999a,b; Cline, Grojean and Servant 1999; Csáki et al. 1999; Binétruy, Deffayet and Langlois 2000; Binétruy et al. 2000; Chung and Freese 2000; Langlois, Maartens and Wands 2000; Charmousis and Dufaux 2002; Lidsey and Nunes 2003; Dufaux et al. 2004...

- *DBI (cosmological) tachyon:* Garousi 2000; Bergshoeff et al. 2000; Sen 2002a,b,c, 2003; Gibbons 2002; Fairbairn and Tytgat 2002; Mukohyama 2002; Feinstein 2002; Padmanabhan 2002; Frolov, Kofman and Starobinsky 2002; Kofman and Linde 2002...

- *Noncommutativity:* Brandenberger and Ho 2002; Huang and Li 203a,b,c; Tsujikawa, Maartens and Brandenberger 2003...

- *Patch approach:* hep-ph/0312246; hep-ph/0402126; hep-th/0406006; hep-ph/0406057; hep-th/0409088; G.C. and Tsujikawa, astro-ph/0407543.

hoW?

Radion (ultimate) stabilization

e.g. Goldberger and Wise 1999, 2000
5D massive scalar field with $V \sim O(\lambda)$

$\rho/\lambda \ll 1$

$\Downarrow$

standard effective Friedmann
equation (but bulk physics)

backreacting bulk

matter on the brane shifts
the minimum of V

$\rho/\lambda \gg 1$

$\Downarrow$

nonstandard cosmological
evolution

Example: Gauss–Bonnet gravity

$$S = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}} d^5x\sqrt{-g_5}\left[R - 2\Lambda_5 + \alpha\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right] + S_{\mathrm{matter}}$$

- Unique 5D action giving a 2nd order symmetric divergence-free tensor and field equations that are 2nd order in the metric (Lovelock 1970, 1971).

- Arises as α'-leading-order quantum corrections to gravity in the heterotic string effective action (Gross and Sloan 1987).

Friedmann equation:

$$H^2 = \frac{c_+ + c_- - 2}{8\alpha}$$

$$c_\pm = \left\{\left[(1 + 4\alpha\Lambda_5/3)^{3/2} + (\sigma/\sigma_0)^2\right]^{1/2} \pm \sigma/\sigma_0\right\}^{2/3}$$

$$\sigma_0^{-1} \equiv \sqrt{\alpha/2}\,\kappa_5^2\,, \qquad \sigma = \rho + \lambda$$

Three regimes:

$$H^2 = \left(\frac{\kappa_5^2}{16\alpha}\right)^{2/3} \rho^{2/3} \qquad (\sigma/\sigma_0 \gg 1)$$

$$H^2 = \frac{\kappa_4^2}{6\lambda}\rho^2 \qquad (\lambda/\sigma_0 \ll \sigma/\sigma_0 \ll 1)$$

$$H^2 = \frac{\kappa_4^2}{3}\rho \qquad (\rho/\sigma_0 \ll \sigma/\sigma_0 \ll 1)$$

Patch setup

Friedmann equation:

$$\boxed{H^2 = \beta_q^2 \rho^q}$$

$$\theta = 2(1 - q^{-1})$$

Regime	q	θ	β_q^2	$\zeta_q(h)$
dS	0	∞	H^2	$-$
GB	$2/3$	-1	$(\kappa_5^2/16\alpha)^{2/3}$	1
RS	2	1	$\kappa_4^2/6\lambda$	$2/3$
4D	1	0	$\kappa_4^2/3$	1

Assumptions

1. Confining mechanism ($\rho < M_5^4$).

2. No brane-bulk exchange.

3. Weyl tensor neglected.

4. Anisotropic stress neglected.

5. Large scale limit.

But

- Assumption 3. closes the system of equations on the brane without nonlocal contributions from the bulk.

- Assumptions 4. and 5. reduce the number of d.o.f. of gauge invariant scalar perturbations.

- Assumptions 3. and 5. nicely fit in the inflationary regime (dark radiation exponentially damped, long wavelength spectrum).

- Bulk physics mainly affects the small-scale/late-time cosmological structure (Gorbunov et al. 2001; Gordon and Maartens 2001; Ichiki et al. 2002; Leong et al. 2002; Koyama 2003; Koyama et al. 2004).

Who's the brane guy?

Perfect fluid:

$$p = w\rho$$

Continuity equation:

$$\dot{\rho} + 3H(\rho + p) = 0$$

Ordinary scalar field:

$$\rho = \dot{\phi}^2/2 + V(\phi) = p + 2V(\phi)$$

DBI tachyon:

$$\begin{aligned} \rho &= V(T)/c_S = -V(T)^2/p \\ c_S &= \sqrt{1 - \dot{T}^2} \end{aligned}$$

The DBI cosmological tachyon

Stringy p.o.v.:

- Condensation into the closed string vacuum (Carrollian limit).

- Near the minimum $g_s = O(1)$ and the perturbative description may fail down.

Cosmological p.o.v.:

- This is a toy model: like in the standard inflation, a reheating mechanism is required.

Slow-roll parameters

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv -\frac{\ddot{\psi}}{H\dot{\psi}} \qquad \cdots$$

$$\dot{\epsilon} = H\epsilon\left[\left(2-\widetilde{\theta}\right)\epsilon - 2\eta\right], \qquad \dot{\eta} = H\left(\epsilon\eta - \xi^2\right) \qquad \cdots$$

Inflation when $\epsilon < 1$:

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon)\,.$$

SR towers

$$\epsilon_{\phi V,0} \equiv \frac{q}{6\beta_q^2}\frac{V'^2}{V^{1+q}},$$

$$\epsilon_{\phi V,n} \equiv \frac{1}{3\beta_q^2}\left[\frac{V^{(n+1)}(V')^{n-1}}{V^{nq}}\right]^{1/n}, \qquad n \geq 1,$$

$$\epsilon_{TV,0} \equiv \frac{q}{6\beta_q^2}\frac{U'^2}{V^q},$$

$$\epsilon_{TV,n} \equiv \frac{1}{3\beta_q^2}\left[\frac{(U')^{n-1}}{V^{nq/2}}\left(\frac{U'}{V^{nq/2}}\right)^{(n)}\right]^{1/n}, \qquad n \geq 1.$$

Cosmological Perturbations – procedure

1. Write the linearly perturbed metric in terms of gauge-invariant scalar quantities.

2. Compute the effective action of the scalar field fluctuation and the associated equation of motion.

3. Write the perturbation amplitude as a function of an exact solution of the equation of motion with constant SR parameters.

4. Perturb this solution with small variations of the parameters.

Spectra and observables

$$
\begin{aligned}
A &= \frac{k}{5\pi z} \\
z(\phi) &= \frac{a\dot{\phi}}{H} \\
z(T) &= \frac{a\dot{T}}{c_S \beta_q^{1/q} H^{\theta/2}} \\
z(h) &= \frac{\sqrt{2}a}{\kappa_4 F_q} \\
F_q^2 &\equiv \frac{3q\beta_q^{2-\theta} H^{\theta}}{\zeta_q(h)\kappa_4^2}
\end{aligned}
$$

$$
\begin{aligned}
n_t &\equiv \frac{d \ln A_t^2}{d \ln k} \sim O(\epsilon) \\
n_s - 1 &\equiv \frac{d \ln A_s^2}{d \ln k} \sim O(\epsilon) \\
\alpha_t &\equiv \frac{d n_t}{d \ln k} \sim O(\epsilon^2) \\
\alpha_s &\equiv \frac{d n_s}{d \ln k} \sim O(\epsilon^2) \\
r &\equiv \frac{A_t^2}{A_s^2} = \epsilon/\zeta_q + O(\epsilon^2)
\end{aligned}
$$

Consistency equations – lowest SR order

$$
\begin{aligned}
\alpha_s(\phi) &\approx \zeta_q r[4(3+\theta)\zeta_q r + 5(n_s - 1)]\,,\\
\alpha_s(T) &\approx (3+\theta)\zeta_q r[(2+\theta)\zeta_q r + (n_s - 1)]\,,\\
n_t &\approx -(2+\theta)\zeta_q r + O(\epsilon^2)\,,\\
\alpha_t &= (2+\theta)\zeta_q r[(2+\theta)\zeta_q r + (n_s - 1)]\,.
\end{aligned}
$$

They are not degenerate!

CMB

WMAP 1st-year analysis: Bennett et al., Hinshaw et al., Kogut et al., Komatsu et al., Peiris et al., Spergel et al., 2003; Bridle et al. 2003.

$$r < 0.06\,, \qquad n_s \simeq 0.95\,.$$

Relative running:

$$\boxed{\alpha_s^{(\theta,\psi)} - \alpha_s^{(\theta',\psi')} \sim O(10^{-2}) \sim \textbf{WMAP estimate}}$$

$$\boxed{\textbf{Planck forecast: } \sim O(10^{-3})}$$

Noncommutative models

Brandenberger and Ho 2002; G.C. hep-th/0406006, hep-ph/0406057

Defining $\tau = \int a\, dt \approx a/H$, the SSUR on the brane is

$$\boxed{[\tau, x] = il_s^2}$$

*-product:

$$(f * g)(x,\tau) = e^{-(il_s^2/2)(\partial_x\partial_{\tau'} - \partial_\tau\partial_{x'})} f(x,\tau) g(x',\tau')\Big|_{\substack{x'=x \\ \tau'=\tau}}$$

Noncommutative observables

Noncommutative parameter

$$\boxed{\mu \approx (Hl_s)^4}$$

$$\begin{aligned} A(\mu,\, H,\, \psi) &= A^{(c)}(H,\, \psi)\,\Sigma(\mu) \\ \frac{d\ln\Sigma^2}{d\ln k} &= \sigma(\mu)\epsilon \\ n &= n^{(c)} + \sigma\epsilon \end{aligned}$$

UV and IR limits

$$\Sigma^2 \approx 1 - b_\Sigma \mu$$
$$\sigma \approx b_\sigma \mu$$

	UV		IR	
	b_Σ	b_σ	Σ^2	σ
BH1	4	16	$\mu^{-3/2}/2$	6
New1	3/2	6	$\mu^{-3/2}$	6
BH2	1	4	$\mu^{-1/2}$	2
New2	1/2	2	$\mu^{-1/2}$	2

Consistency relation

$$n_t = [\sigma - (2+\theta)]\zeta_q r$$

Consistency relation	n_t/r GB	RS	4D
Commutative UV ($\sigma = 0$)	-1	-2	-2
Class 1 IR ($\sigma = 6$)	5	2	4
Class 2 IR ($\sigma = 2$)	1	$-2/3$	0

$$\alpha_s^{(\theta,\psi)} - \alpha_s^{(\theta',\psi')} \sim O(10^{-1})$$

Likelihood analysis

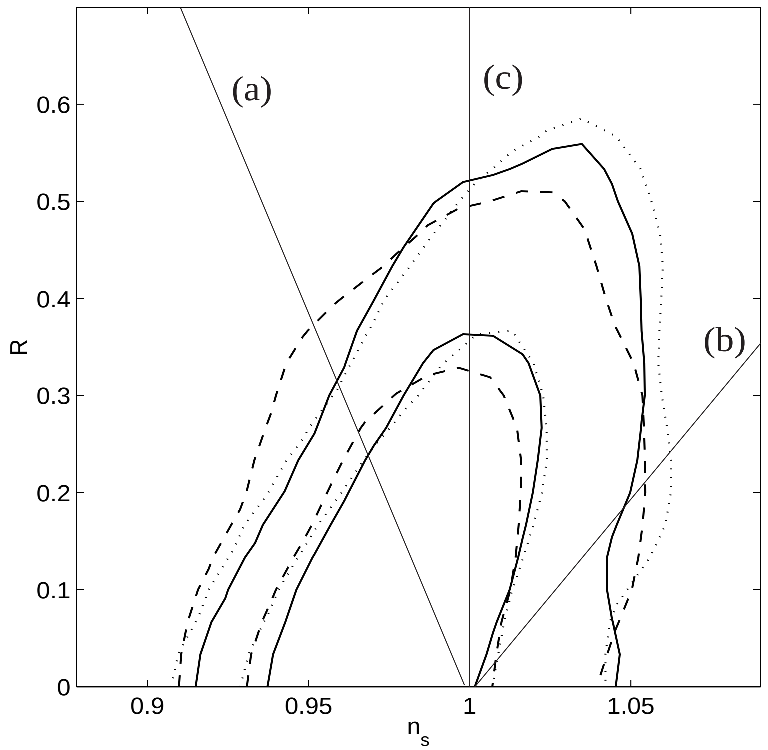
G.C. and Tsujikawa, astro-ph/0407543 (PRD to appear)

Cosmological Monte Carlo (CosmoMC) code with CAMB

Data set: 1st-year WMAP, 2dF, SDSS (+ CBI, VSA, ACBAR for small scales)

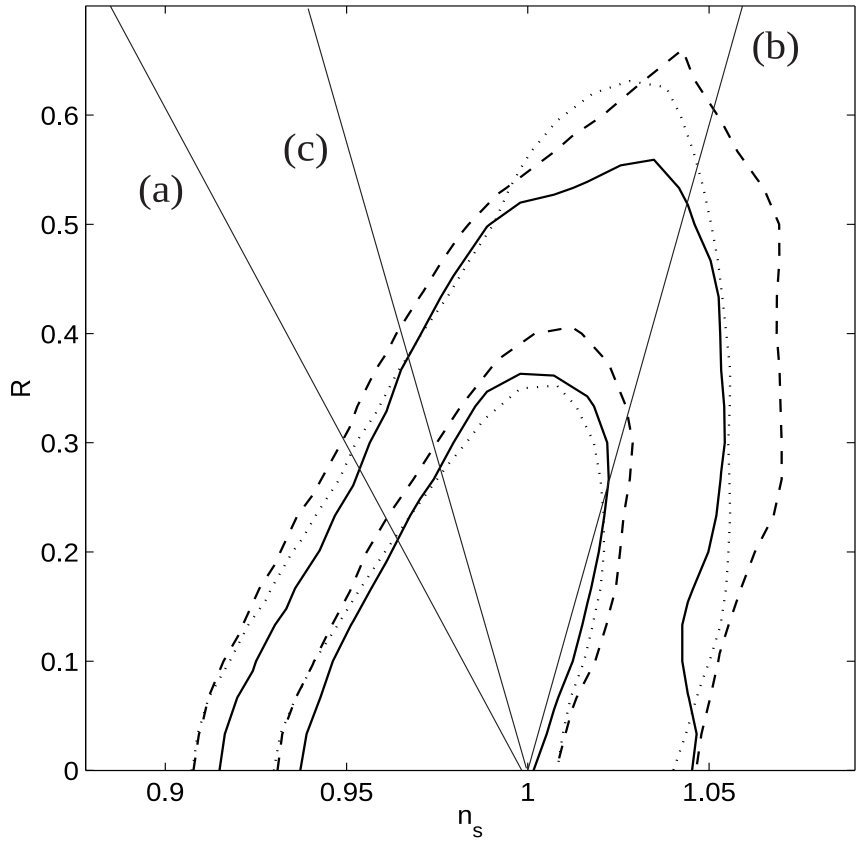
Parameter space: $\{A_s^2, R, n_s, n_t, \alpha_t, \alpha_s, \sigma\}$.

Likelihood analysis – 4D case



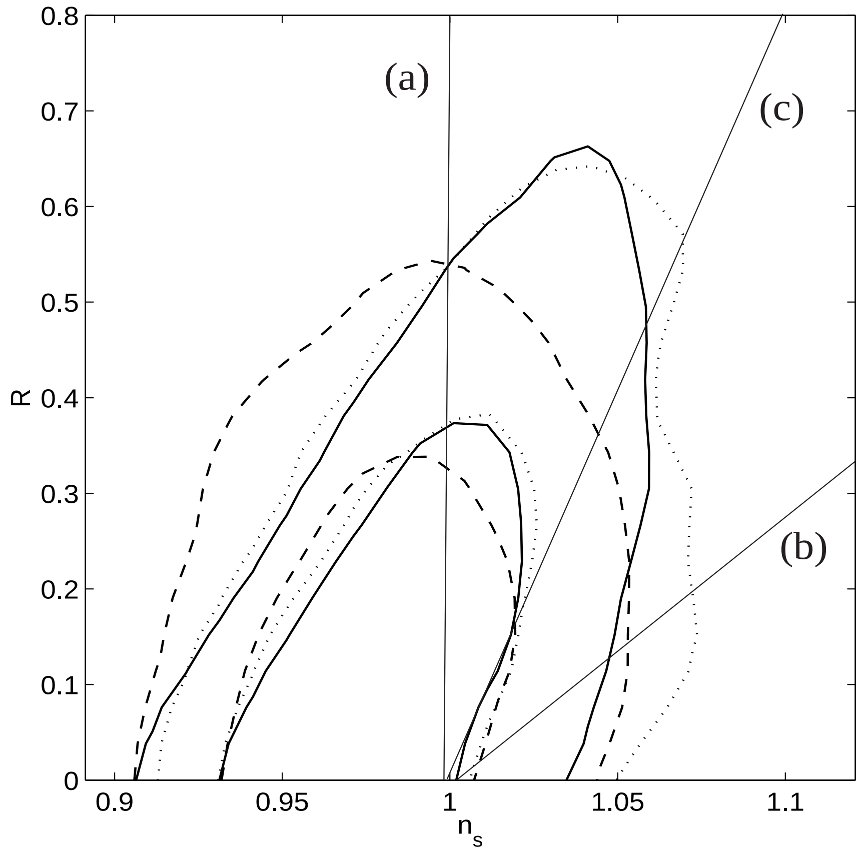
The 1σ and 2σ observational contour bounds for the 4D case. (a) $\sigma = 0$, solid; (b) $\sigma = 6$, dashed; (c) $\sigma = 2$, dotted. We also show the border of large-field (left) and hybrid (right) inflationary models.

Likelihood analysis – RS case



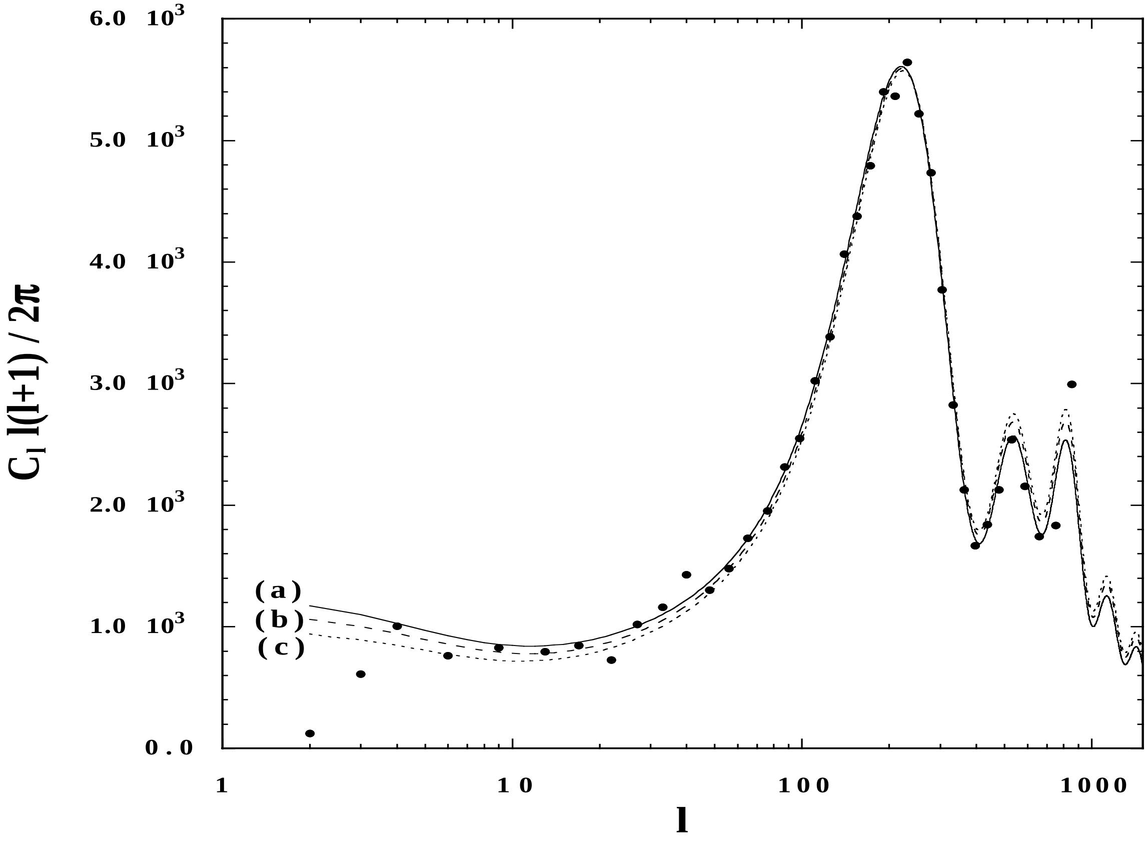
The 1σ and 2σ observational contour bounds for the RS case.

Likelihood analysis – GB case



The 1σ and 2σ observational contour bounds for the GB case.

CMB power spectrum – low multipoles suppression



(a): GR0 with $(n_s, R) = (0.967, 0.132)$ $(V = \phi^2)$; (b): GR1 with $(n_s, R) = (1.018, 0.144)$ $(V = \phi^2)$; (c): GR1 with $(n_s, R) = (1.049, 0.263)$, $(V = \phi^4)$.

Conclusions

- Not a full 5D calculation.
- Bulk physics should not dramatically improve large-scale results.
- Possibly detectable braneworld signatures within O(1) years.
- Extra physics can improve the results (e.g. noncommutativity).
- To M/string theorists: please find new cosmological scenarios with $\theta \neq 0, \pm 1$.