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DESY 104

WHY?

$\dim(U) \stackrel{\text{today}}{=} 3$

BRANDENBERGER-VAFA MECHANISM

(Brandenberger and Vafa '89)

■ USING A THERMODYNAMICAL DESCRIPTION OF STRINGS AT LARGE ENERGIES (MICROCANONICAL ENSEMBLE) ONE CAN:

- ▶ Resolve the initial singularity of the Big Bang
- ▶ Potentially explain the dimensionality of the spacetime

■ STARTING SET-UP

- ▶ Weakly interacting string theory
(thermal equilibrium without back reaction)
- ▶ Adiabatic evolution of the Universe
(constant entropy $\Rightarrow T = T(R)$)
- ▶ Toroidal topology (let's say 9 dim)
(presence of winding degrees of freedom)
- ▶ Initially small Universe
($R \ll l_s$, cosmology)

STRINGS IN COMPACT SPACES (I)



■ STRING STATES

- ▶ Oscillatory Modes (stationary vibrating string)

$$\boxed{E \text{ independent of } R}$$

- ▶ Momentum Modes (moving strings)

$$\boxed{E = \frac{n}{R}}$$

- ▶ Winding Modes (strings winding one cycle)

$$\boxed{E = mR}$$

■ T-DUALITY

- ▶ Spectrum unchanged under the transformation

$$\left. \begin{array}{l} R \leftrightarrow 1/R \\ n \leftrightarrow m \end{array} \right\} \text{Physics}(R) \cong \text{Physics}(1/R)$$

- ▶ Distance may not be an invariant notion (in the sense of GR) for string theory at short length scales.

Quantum nature of measurement

- ▶ Existence of a minimal size

$$R_{\min} \sim l_s$$

DECOMPACTIFICATION MECHANISM (QUALITATIVE)

- WINDING MODES WORK AGAINST EXPANSION BECAUSE THEY CONTRIBUTE TO THE DYNAMICS WITH A NEGATIVE PRESSURE !!

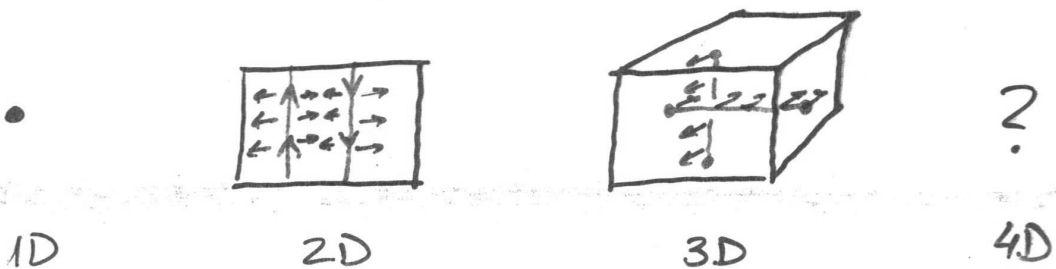
$$S = \text{const.} \rightarrow \left(p = - \frac{\partial E}{\partial V} \right) < 0 \quad (E \uparrow \text{ as } V \uparrow)$$

- EXPANSION CAN CONTINUE AS LONG AS THERMAL EQUILIBRIUM IS MAINTAINED (DECAY OF WINDING MODES) !!



$W + \bar{W} \leftrightarrow$ unwinding modes

- THERMAL EQUILIBRIUM IS ONLY POSSIBLE WITH THREE (OR LESS) LARGE EXPANDING DIMENSIONS !!! (TOPOLOGICAL ARGUMENT)



- IF THERE ARE MORE THAN THREE DIMENSIONS EXPANDING THE WINDING MODES CANNOT ANNIHILATE FALL OUT OF EQUILIBRIUM AND STOP THE EXPANSION (RECOLLAPSE) !!

BRANE GAS COSMOLOGY

- THE MECHANISM ALSO WORKS IF THE DYNAMICS OF D_p -BRANES IS TAKING INTO ACCOUNT
- FOR A GAS OF D_p -BRANES THERMAL EQUILIBRIUM IS MAINTAINED IF THERE ARE $D \leq 2p+1$!!!
LARGE EXPANDING DIMENSIONS
- THE FUNDAMENTAL STRING WINDING MODES (LIGHTER) DOMINATE THE DYNAMICS AT LATE TIMES AND THEN STILL 3 DIMENSIONS CAN GROW LARGE
- A HIERARCHY OF SMALL DIMENSIONS CAN BE CREATED

Type II SUGRA $T^9 \xrightarrow{p=2} T_L^5 \otimes T_S^4 \xrightarrow{p=1} T_{XL}^3 \otimes T_L^2 \otimes T_S^4$

- A BRANE PROBLEM EXISTS \rightarrow LOITERING PHASE APPEARS NATURALLY (No obstruction for the complete annihilation of winding modes)

$$\left[\begin{array}{l} \text{HUBBLE} \\ \text{RADIUS} \end{array} \gg \begin{array}{l} \text{SPATIAL} \\ \text{SIZE} \end{array} \right]$$

DYNAMICS (I)

(Easther et al. '03)

■ 11D-SUGRA (possible low energy limit of M-theory)

$$S = \frac{1}{2\kappa_{11}^2} \left[\int dx \sqrt{g} \left(R - \frac{1}{48} F_{[3]}^2 \right) + \frac{1}{6} \int A_{[3]} \wedge F_{[3]} \wedge F_{[3]} \right]$$

→ COUPLING

$$2\kappa_{11}^2 = (2\pi)^8 l_s^9$$

→ FIELD STRENGTH

$$F_{[3]} = dA_{[3]}$$

→ CHERN-SIMONS TERM

■ BACKGROUND METRIC

$$ds^2 = -dt^2 + \sum_{i=1}^{10} e^{2\lambda_i} dx_i^2$$

$$V = \prod_{i=1}^{10} e^{\lambda_i} \quad (l_s \sim 1)$$

$$0 \leq x_i \leq 1 \quad (\text{torus})$$

■ MATTER CONTENT

► SUGRA PARTICLES represented by a gas of massless particles with energy density ρ_s and pressure

$$P_s = \rho_s / 10$$

DYNAMICS (II)

■ MATTER CONTENT (cont.)

▶ A GAS OF M2-BRANES wrapping anisotropically different dimensions

• It can be characterized by a 10×10 MATRIX N_{ij}

• N_{ij} with $i < j$ ($i > j$) represent the number of branes (antibranes) in cycle (ij)

• Since $\#W = \#\bar{W}$ the matrix is symmetric

• CLASSIFICATION in terms of a wrapping type $m_1 - m_2 - m_3$

m_1 unwrapped

m_2 partially wrapped

m_3 fully wrapped

$$N_{ij} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A \\ 0 & A & B \end{pmatrix} \begin{array}{l} \updownarrow m_1 \\ \updownarrow m_2 \\ \updownarrow m_3 \\ \hline 10 \end{array}$$

• Averaging the EMT over transvers directions (non-relativistic branes)

$$\left(\begin{array}{l} P_B = \frac{1}{(2\pi)^2 V} \sum_{k \neq l} e^{i k x} e^{i l x} N_{kl} \\ P_B^i = -\frac{1}{(2\pi)^2 V} \sum_{k \neq l} e^{i k x} e^{i l x} (N_{kl} + N_{lk}) \leq 0 !!! \end{array} \right)$$

• (FREEZE OUT CONFIGURATION)

• (WHY NOT M1-BRANES)

DYNAMICS (II)

MATTER CONTENT (Cont.)

► For the gauge field we consider an ELEMENTARY (Electric) type solution (Friedland-Rubin '80)

- $F_{\alpha\beta}$ NON-ZERO only in a 3+1 submanifold

$$\left[\begin{aligned} F_{\mu\nu\alpha\beta} &= \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g_4}} F(t) & \mu, \nu, \alpha, \beta &= 0, 1, 2, 3 \\ F(t) &= f e^{-\lambda_4 r} \dots e^{-\lambda_{10}} \end{aligned} \right]$$

- ENERGY-MOMENTUM TENSOR

$$\left[\begin{aligned} p_G &= \frac{1}{\kappa^2} \left(\frac{F(t)}{2} \right)^2 \\ p_G^i &= \epsilon^i p_G \geq 0 \quad !!! \end{aligned} \right]$$

→ (-1, -1, -1, +1, ^(?), +1)

DYNAMICS (IV)

EQUATIONS OF MOTION

$$\sum_{k \neq l}^{10} \dot{\lambda}_k \dot{\lambda}_l = 2v_{H1}^2 (\rho_S + \rho_B + \rho_\sigma)$$

$$\ddot{\lambda}_i + 10 H \dot{\lambda}_i = v_{H1}^2 \left[\frac{1}{10} \rho_S + \frac{1}{3} \rho_B + \rho_B^i + (\epsilon^i - \frac{1}{3}) \rho_\sigma \right]$$

Mean Hubble parameter $H = \frac{1}{10} \sum_{i=1}^{10} \dot{\lambda}_i$

SCALING SOLUTION AT LATE TIMES (ATTRACTOR)!!

IND. INI. COND.

$$e^{\lambda_i} \sim \begin{cases} t^{\alpha_-} & \text{for } i=1,2,3 & \text{(unwrapped + gauge)} \\ t^{\alpha_+} & \text{for } i=7, -, m_1 & \text{(unwrapped)} \\ t^\beta & \text{for } i=m_1+1, -, m_1+m_2 & \text{(partially wrapped)} \\ t^\gamma & \text{for } i=m_1+m_2+1, -, 10 & \text{(fully wrapped)} \end{cases}$$

(Compos or)

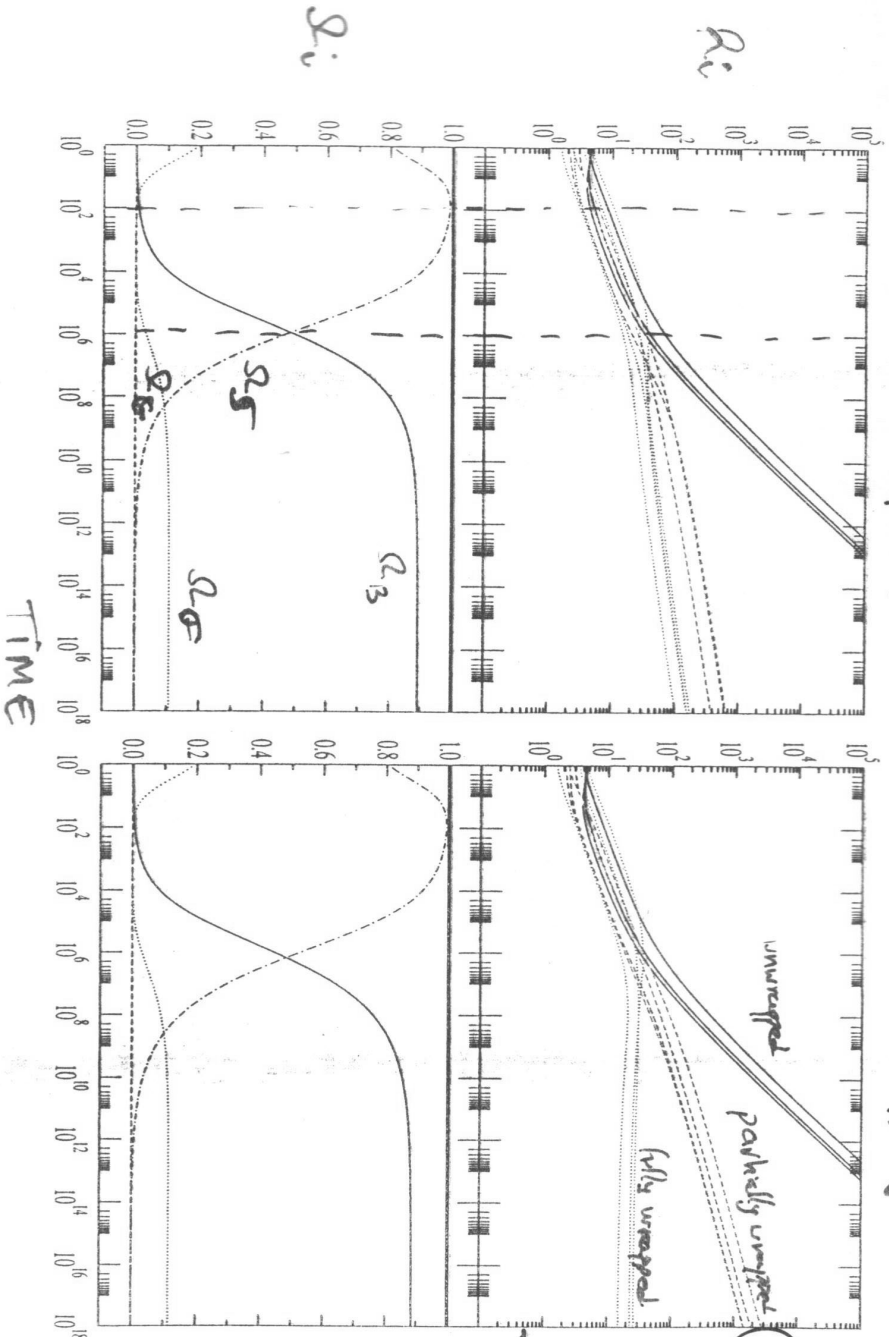
ANISOTROPIC PRESSURES \Leftrightarrow ASYMMETRY IN THE SIZE OF THE SPATIAL DIMENSIONS !!!

$$\left| \alpha = \frac{7}{13} \quad \beta = \gamma = \frac{1}{13} \right|$$

3-3-4 wrapping

$$\left| \alpha = \frac{8}{15} \quad \beta = \frac{2}{15} \quad \gamma = 0 \right|$$

3-4-3 wrapping



FLUXES
Σ NO

Ri

si

TIME

NOT VERY IMPRESSIVE RESULT

$m_2 = 3$ [YI?]

FINE-TUNNING

Figure 1: Cosmological evolution without fluxes. Graphs on the left are for a brane gas with a wrapping matrix of the type 3-3-4 and those on the right of the type 3-4-3. Plots on top represent the time evolution of the size (scale factor normalised by 2π) of all the spatial dimensions: solid curves are for unwrapped dimensions, dashed curves for partially wrapped dimensions, and dotted curves for fully wrapped dimensions. Plots on the bottom depict all the contributions to the expansion of the Universe as a function of cosmic time. The solid line represents Ω_B , the dashed line Ω_G , the dotted line Ω_σ , and the dotted-dashed line Ω_g . The thick solid line is the sum of all

(Easther et al '03)
Carpas '09

FLUXES

- GENERICALLY WITH $\rho_G \neq 0$ and $\rho_S = \rho_B = 0$ WE HAVE 7 EXPANDING AND 3 CONTRACTING DIMENSIONS

$$e^{d_i} \sim \begin{cases} t^{-2/7} & i=1,2,3 \\ t^{1/7} & i=4, \dots, 10 \end{cases}$$

- ALTHOUGH IT GIVES AN ASYMMETRY AMONG DIMENSIONS THE HIERARCHY IS NOT THE CORRECT ONE
- ANALITICALLY, ONE CAN EVEN SEE THAT A PHYSICAL SINGULARITY APPEARS AT FINITE TIME
- NOT VERY PROMISING PROPERTIES

$$\left[\alpha_- = \frac{3}{7} \quad \beta = \gamma = \frac{1}{7} \right]$$

$$\left[\alpha_- = \frac{46}{109} \quad \beta = \frac{22}{109} \quad \gamma = \frac{7}{109} \right]$$

FLUXES

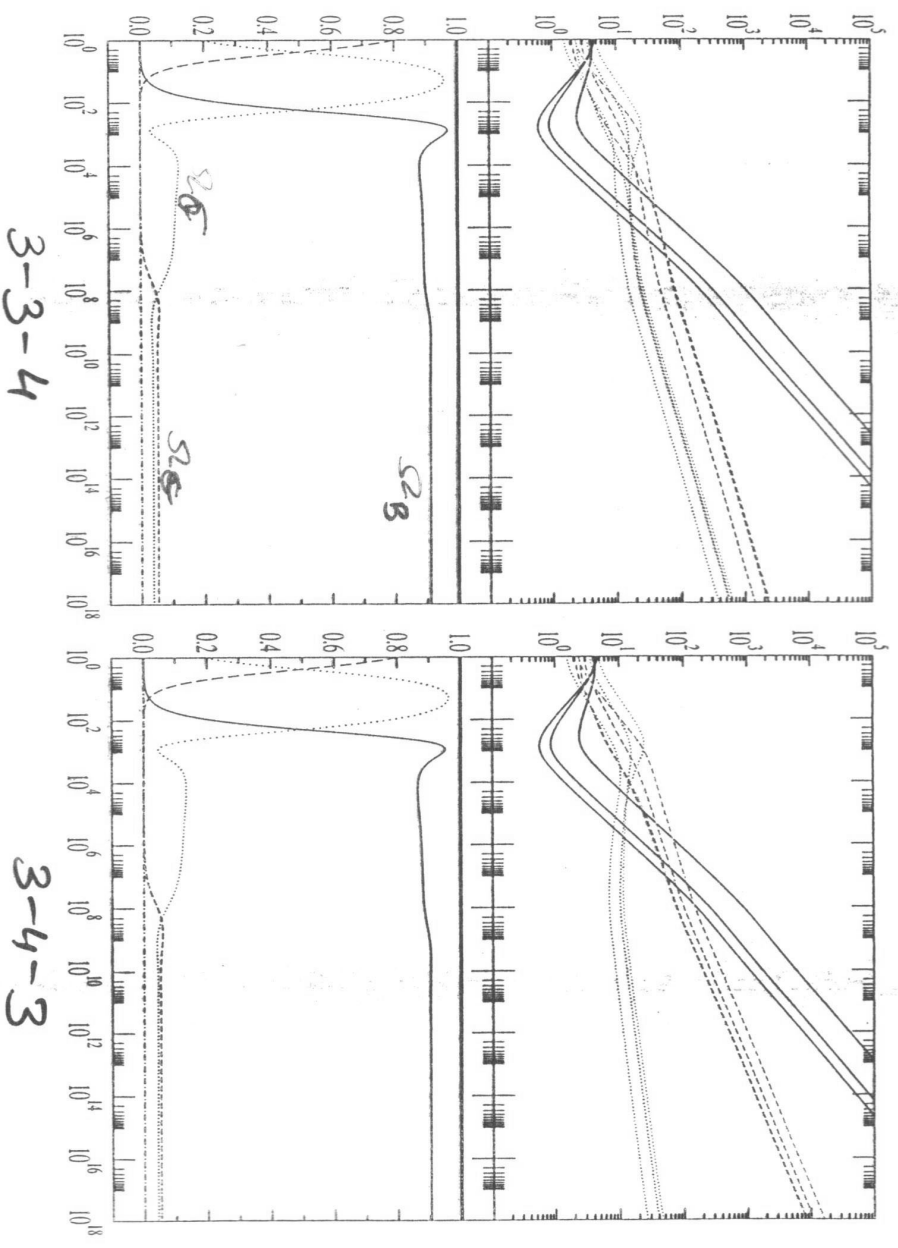


Figure 2: Cosmological dynamics of a brane gas of wrapping type 3-3-4 (left) and type 3-4-3 (right) with fluxes. The identification of plots and lines is the same as in Fig. 1.

$$\alpha_- = \frac{5w_2w_3 - 14}{5w_2w_3 + 49}$$

$$\beta = \frac{5w_3(w_2 - 3) + 7}{5w_2w_3 + 49}$$

$$\gamma = \frac{5w_2(w_3 - 3) + 7}{5w_2w_3 + 49}$$

$$\alpha = \frac{4}{11} \quad \rightarrow \quad \beta = \gamma = -\frac{1}{11}$$

$$\left\{ \begin{array}{l} \alpha_+ = \frac{5}{11} \\ \alpha_- = \frac{3}{11} \end{array} \right.$$

$$\beta = \gamma = -\frac{1}{11}$$

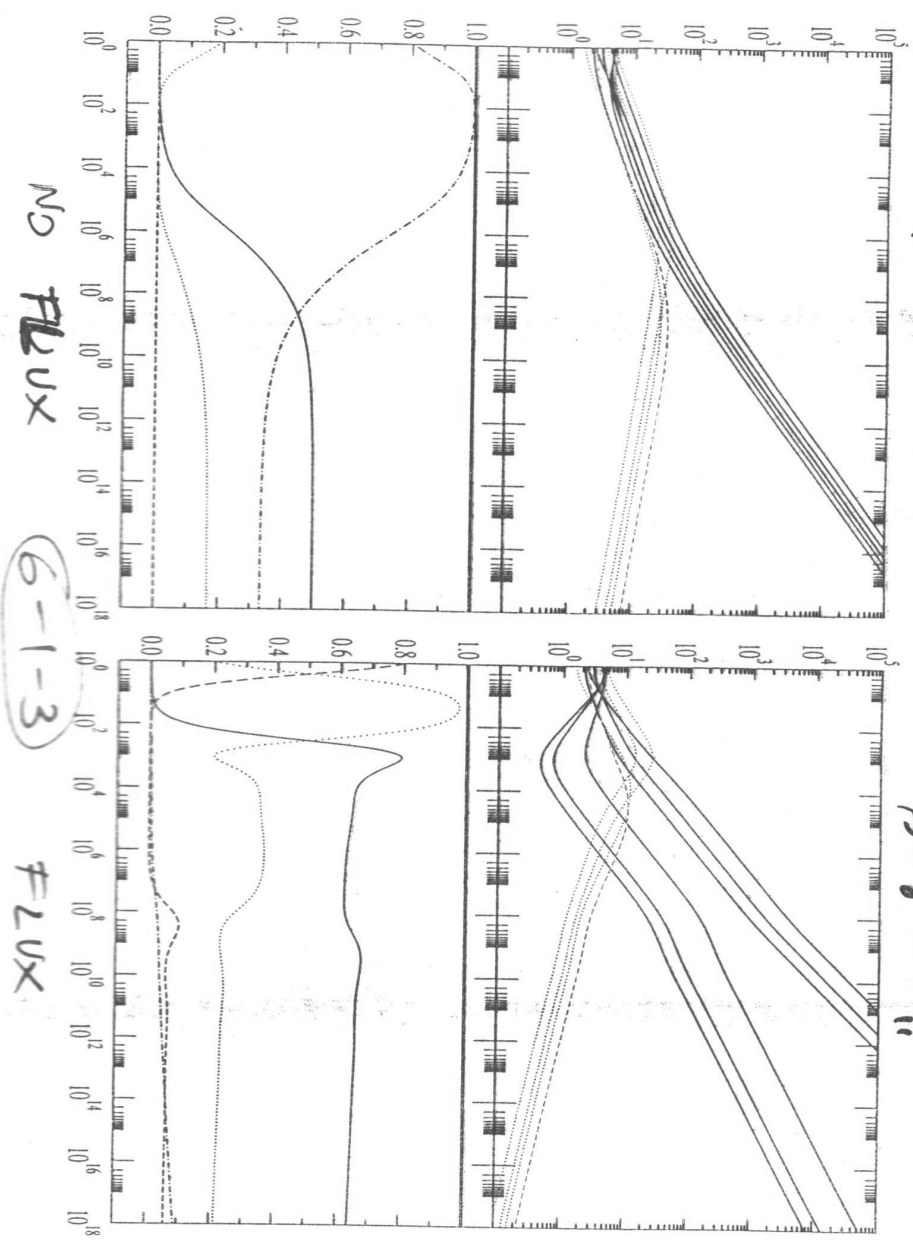


Figure 3: Cosmological dynamics of a brane gas of wrapping type 6-1-3 without (left) and with (right) fluxes. The identification of plots and lines is the same as in Fig. 1.

BRANE GASES WITH FLUXES

- EVEN THOUGH THE GAUGE FIELD CAN HAVE A SIGNIFICANT CONTRIBUTION TO THE EXPANSION IT DOES RESPECT THE HIERARCHIES OF THE SPATIAL DIMENSIONS CREATED BY THE GAS OF BRANES
- GAUGE FIELDS SUPPORT NEW SOLUTIONS WITH 3 GROWING DIMENSIONS FROM A CONFIGURATION WITH A LARGE # OF UNWRAPPED DIMENSIONS ($m_1 = 6$)
- IN FACT THERE ARE MORE SOLUTIONS WITH DIFFERENT VALUES OF $m_1 \Rightarrow$ LESS SEVERE FINE TUNING !!!
- ONE CAN EVEN MAKE A CONNECTION BETWEEN

DILATON \Leftrightarrow M-THEORY
GRAVITY

$$\left. \begin{array}{l} 0 - 9 - 1 \\ 0 - 3 - 1 \end{array} \right\} \rightarrow (11D \rightarrow 10D \text{ type II})$$
$$\left. \begin{array}{l} 0 - 9 - 1 \\ 0 - 3 - 1 \end{array} \right\} \rightarrow (\text{SOLITONIC})$$

- WHICH CONFIGURATION IS MORE PLAUSIBLE DEPENDS ON THE THERMODYNAMICS OF THE SYSTEM CLOSE TO THE HAGEDORN PHASE

TO CONCLUDE

- THE DYNAMICS OF FLUXES PLAYS A CRUCIAL ROLE IN THE LATE TIME BEHAVIOUR OF ANY IMPLEMENTATION OF THE BRANDENBERGER-VAFN MECHANISM FOR EXPLAINING THE DIMENSIONALITY OF THE SPACETIME
- A MUCH BETTER UNDERSTANDING OF THE THERMODYNAMICS OF BRANES CLOSE TO THE HAGEDORN TEMPERATURE IS NEEDED