Low energy effective action of the Gauss-Bonnet brane world

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Summary

- What is Gauss-Bonnet gravity ?
- The Gauss-Bonnet 5d brane world
- Low energy effective action from the 5d linearized equations of motion
  - Gauge choices
  - Bulk equations of motion
  - Boundary conditions on the branes
  - Effective scalar-tensor theory
- Conclusion

What is Gauss-Bonnet gravity ?

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{-g}[-2\Lambda + R$$

$$+\alpha(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})]$$

- Only R^2-order combination giving equations of motion with **no derivatives of higher order than two** and **divergence free**, like the Einstein tensor $G_{\mu\nu}$

  $\Rightarrow$ keeps the usual properties of Einstein's equations

- May appear as a consistent quantum gravity correction to General Relativity

- Why not so familiar ? In 4d, it is a **total derivative** $\Rightarrow$ does not contribute to the equations of motion

- In higher dimension $d > 4$ it is no longer a total derivative $\Rightarrow$ no reason to omit it !

- Why higher (space) dimensions ?

  Fundamentally motivated by string theory, predicting d=10 ;

  phenomenologically, allows natural hierarchies of gravitational origin, new scenarios of symmetry breaking...

The Gauss-Bonnet brane world

- Our universe = flat (3+1)d membrane at the border of an 5d anti de Sitter bulk where only gravity propagates :

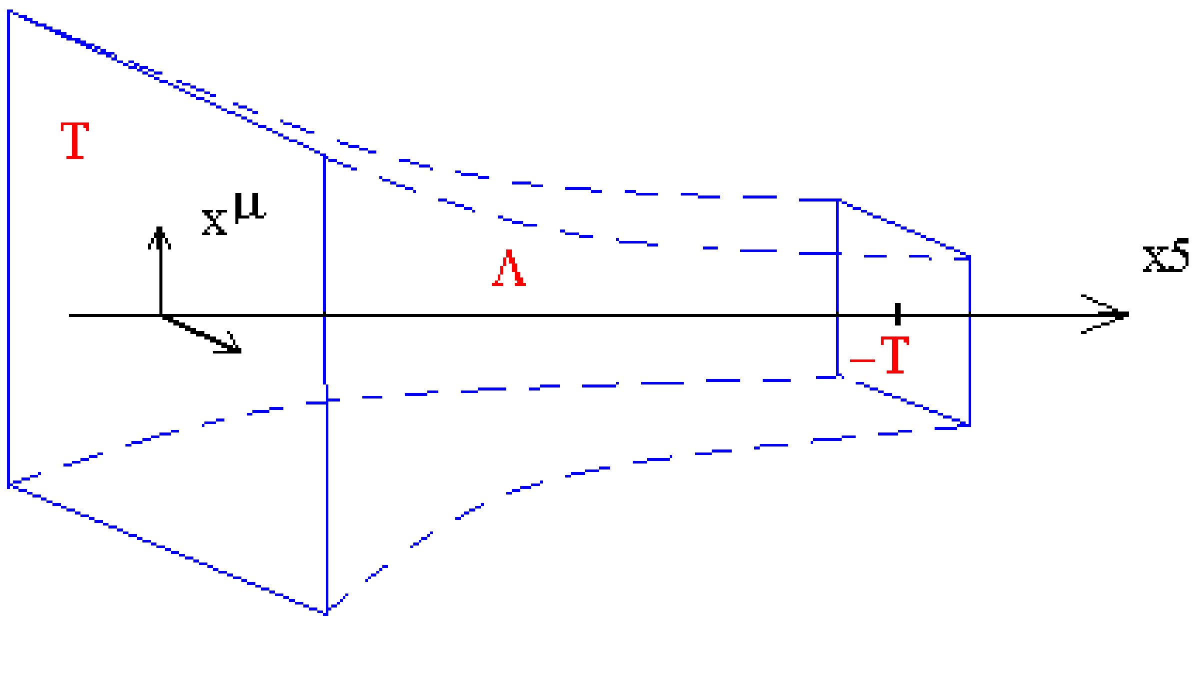
$$S = \frac{1}{\kappa_5^2}\int d^5x\sqrt{-g_5}[-\Lambda_5 + \frac{1}{2}R_5$$

$$+\frac{1}{2}\alpha(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd})]$$

$$+\sum_{i=1,2}\int_i d^4x\sqrt{-g}[-T_i + \frac{\beta_i}{2\kappa_5^2}R_i]$$

Background metric :

$$ds^2 = e^{-2kx^5}\eta_{\mu\nu}dx^\mu dx^\nu + (dx^5)^2$$



Linearized 5d equations of motion

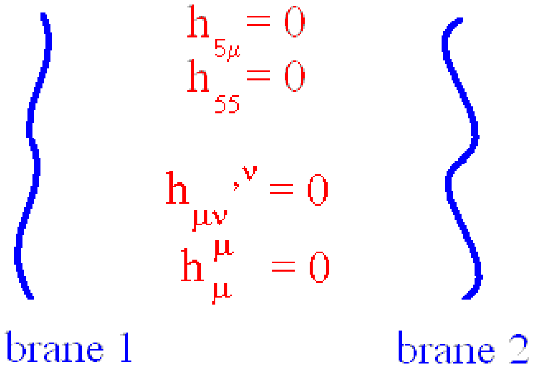
1) Gauge choices

Linear perturbations of the 5d metric : $g_{ab} = g_{ab}^{(0)} + h_{ab}$

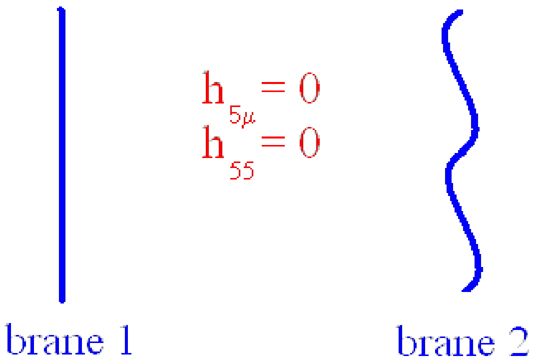
Possible gauge choices :

- "Axial gauge" (only $h_{\mu\nu}$ perturbations) + 4d transverse traceless

  $\Rightarrow$ bulk equations simpler but both branes are bent



- Gaussian normal coordinates with respect to one brane $\Rightarrow$ boundary conditions simpler (on that brane)



Linearized 5d equations of motion

2) Bulk equations of motion

[see e.g. S.C. Davis hep-th/0402151]

Linear perturbations : $ds^2 = e^{-2kx^5}(\eta_{\mu\nu} + \gamma_{\mu\nu})dx^\mu dx^\nu$

- Bulk equation of motion for $\gamma_{\mu\nu}$ in TT gauge :

  $(\partial_5^2 - 4k\partial_5 + e^{2kx^5}\Box_4)\gamma_{\mu\nu}(x^\lambda, x^5) = 0$

  No more than second order in derivatives !

- 5d graviton $\Leftrightarrow$ infinite tower of 4d mass eigenmodes (Kaluza-Klein modes) :

  $\gamma_{\mu\nu}(x^\lambda, x^5) = \sum_{n=0}^{\infty} f_n(x^5)\gamma_{\mu\nu}^{(n)}(x^\lambda)$

  $(\Box_4 + m_n^2)\gamma_{\mu\nu}^{(n)}(x^\lambda) = 0$

  $(\partial_5^2 - 4k\partial_5 - e^{2kx^5}m_n^2)f_n(x^5) = 0$

- At low energy, keep only the zero mass mode $m = 0$ and neglect massive modes. Legitimate only if there are no tachyonic modes $m^2 < 0$, that would make the vacuum unstable.

But in general for Gauss-Bonnet brane world with no induced gravity terms tachyonic gravitons always exist !

[C. Charmousis, JF Dufaux hep-th/0311267]

$\Rightarrow$ must have $(8\alpha k^2 + \beta_+ k)(8\alpha k^2 - \beta_- k) \leq 0$ to avoid tachyons.

Linearized 5d equations of motion

3) Boundary conditions

- On the branes $x^5 = 0, R$:

$$(\partial_5 + \tfrac{4\alpha k \pm \beta_\pm/2}{1-4\alpha k^2} e^{2kx^5} \Box_4)\gamma^\pm_{\mu\nu}$$

$$= \mp \tfrac{\kappa_5^2}{1-4\alpha k^2} e^{2kx^5} (T^\pm_{\mu\nu} - \tfrac{1}{3} T^\pm \eta_{\mu\nu} \pm 2\kappa_5^{-2}(1 \pm \beta_\pm k + 4\alpha k^2)\partial_\mu \partial_\nu \xi^\pm)$$

Both the brane energy momentum tensors and the scalars $\xi^\pm$ measuring the *brane bending* source the metric boundary conditions. The brane position fluctuations $\xi^\pm$ obey

$$\Box_4 \xi^\pm = \pm \tfrac{\kappa_5^2}{6(1 \pm \beta^\pm k + 4\alpha k^2)} T^\pm$$

- The interbrane distance fluctuation $(\xi^- - \xi^+)(x)$ can't be gauged away ; it is a physical 4d scalar mode, the **radion**, which contributes to the low-energy effective gravity.

Linearized 5d equations of motion

4) Resulting effective scalar-tensor gravity

- Low energy/large distance gravity theory looks 4-dimensional and contains the metric tensor + a massless scalar field (the radion).

  Standard Brans-Dicke form of a tensor-scalar action for observers on the positive tension brane :

  $S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} [\Phi R - \frac{\omega(\Phi)}{\Phi} (\partial\Phi)^2]$

  $+ S^{+}_{\text{matter}}(g_{\mu\nu}) + S^{-}_{\text{matter}}(A(\Phi) g_{\mu\nu})$

  with $\omega(\Phi) = \frac{3}{2} \frac{\Phi}{1-\Phi}$

  $A(\Phi) = \frac{1+\beta_+ k + 4\alpha k^2}{1-\beta_- k + 4\alpha k^2} (1-\Phi)$

  $\Phi = 1 - \frac{1-\beta_- k + 4\alpha k^2}{1+\beta_+ k + 4\alpha k^2} e^{-2kr}$

  $\kappa_4^2 = 2k\kappa_5^2 / (1 + \beta_+ k + 4\alpha k^2)$

- Solar system tests of G.R. constrain $\omega_{now} >$ some 10^3

  $\Rightarrow \frac{1+\beta_+ k + 4\alpha k^2}{1-\beta_- k + 4\alpha k^2} e^{2kr} >$ some 10^3

- Not a fully metric theory : matter on the other brane does not couple to the same metric.

- Brans-Dicke form for observers on the negative tension brane has $\omega < 0 \Rightarrow$ excluded in absence of a stabilization mechanism for the radion.

Conclusion

- The effective gravity of the Gauss-Bonnet brane-world is General Relativity + the radion scalar.

- 4d Minkowski vacuum can be made stable (no tachyonic graviton) by introducing appropriate induced gravity terms on the branes.

- G.R. solar system tests require large interbrane distance and observers on the positive tension brane, like in the pure Randall-Sundrum model (which has no Gauss-Bonnet, no induced gravity).

- Observationally, the effective gravity differs from the know pure Randall-Sundrum model only if

  $\beta_+ + \beta_- \neq 0$.

  In this case, the (observable) difference is in the ratio of the metrics minimally coupled to matter on each brane, and in the value of ω_{now}.

- Generalization to a full non-linear effective theory less obvious than in the pure RS model, partly because the "stacking technique" to find 5d solutions from 4d ones doesn't always work in the Gauss-Bonnet case.