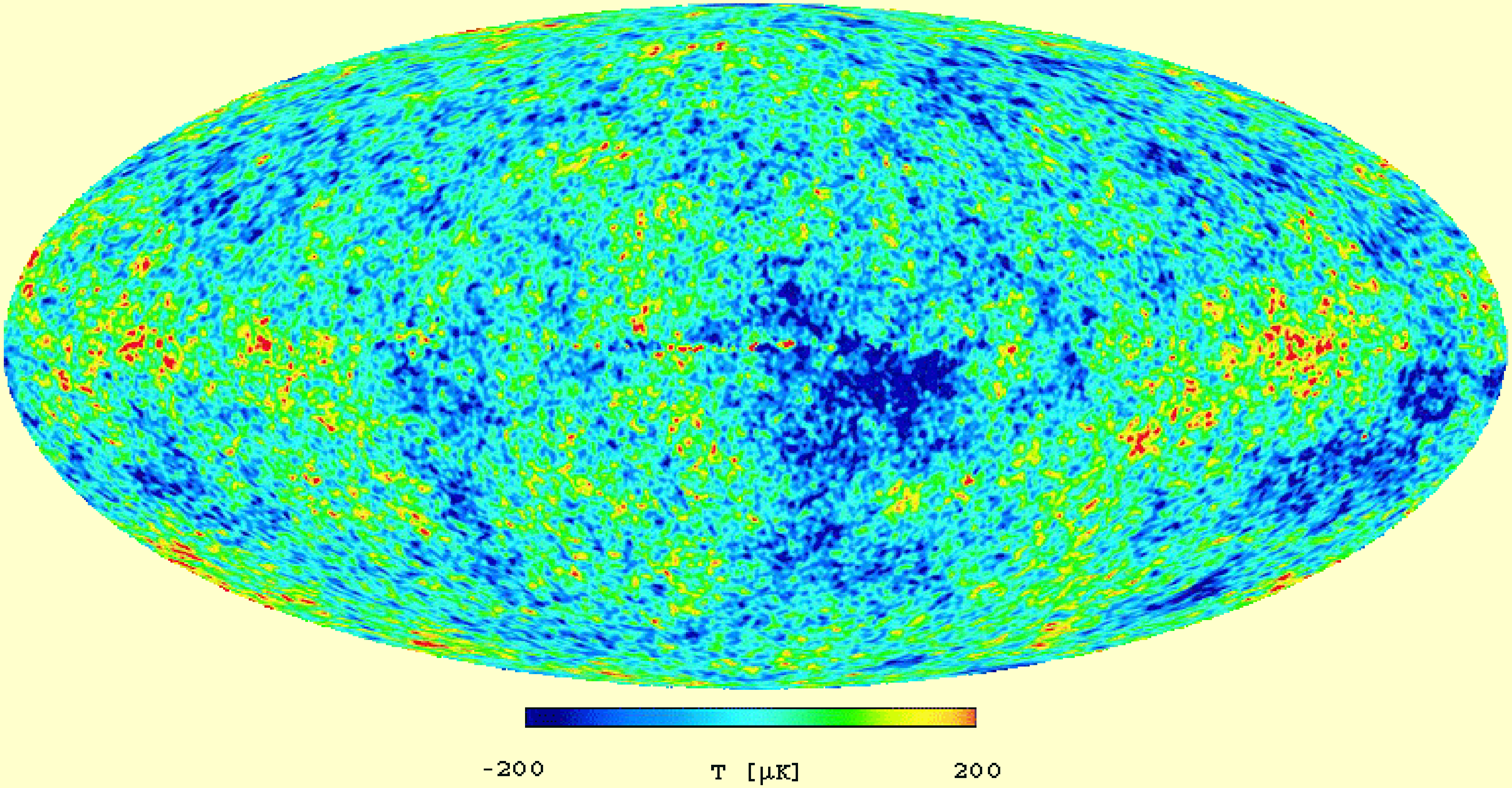
# *Inhomogeneities in the Universe*

DESY, 2004





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# Contents

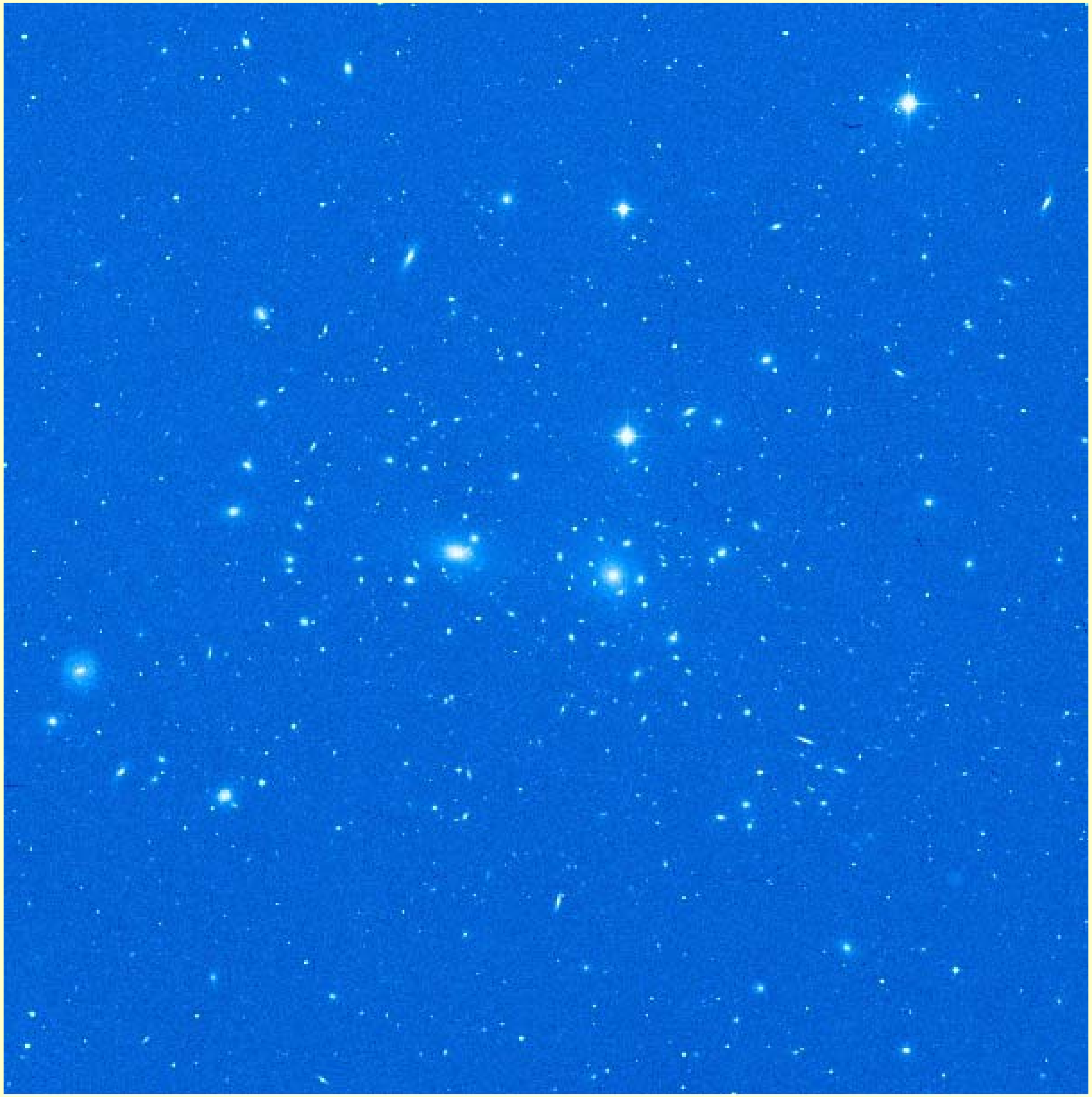
# *On 'sufficienty' small scales the universe is inhomogeneous*

- There are we
- the solar system...

- galaxies...

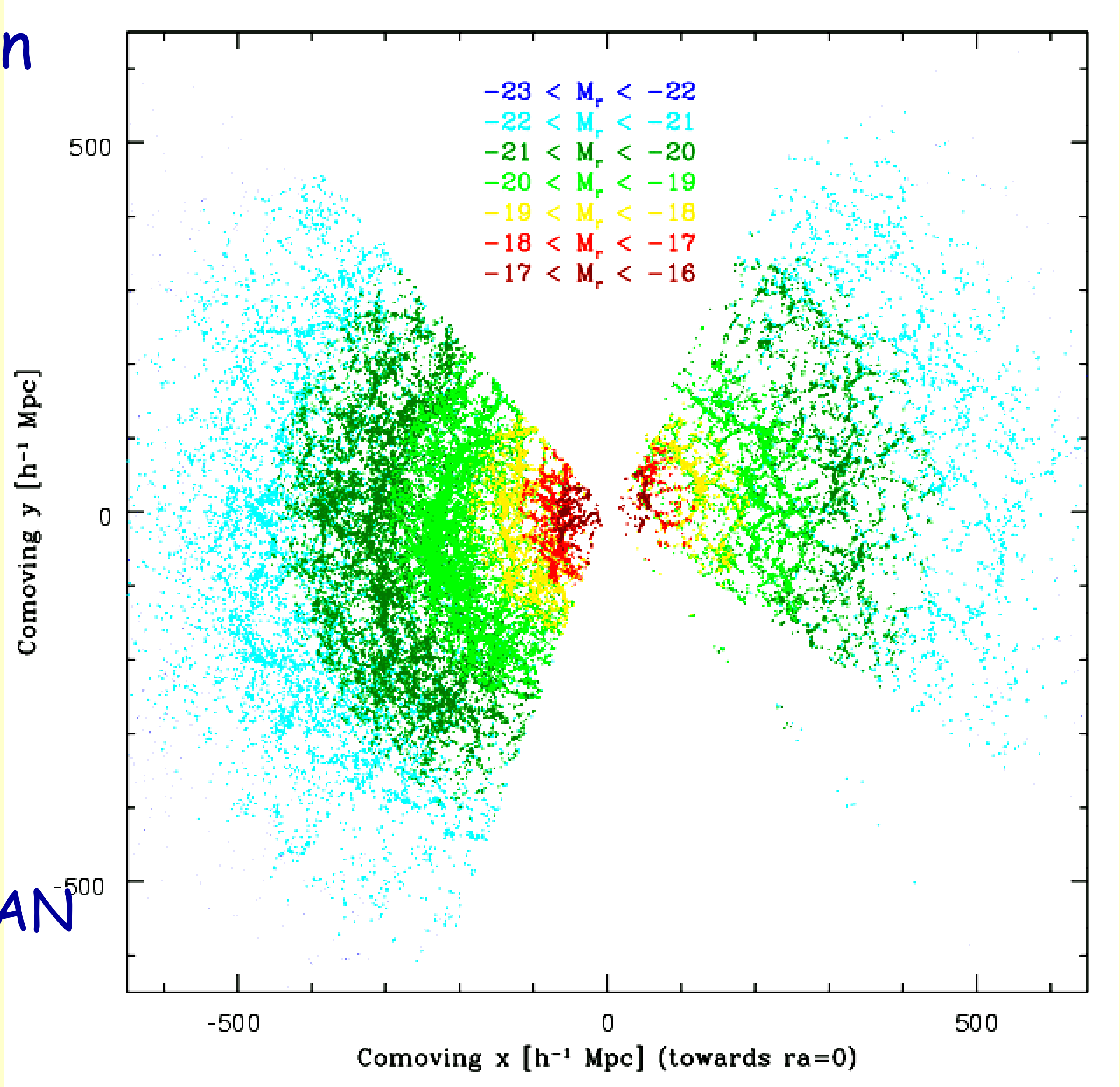
M100

- clusters of galaxies...



Coma cluster

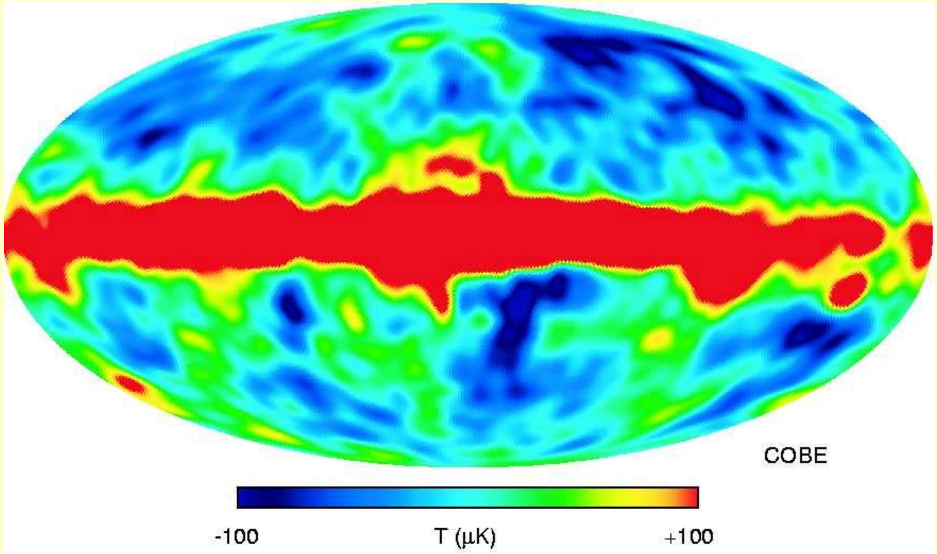
# Galaxies are arranged in sheets and filaments with voids in between



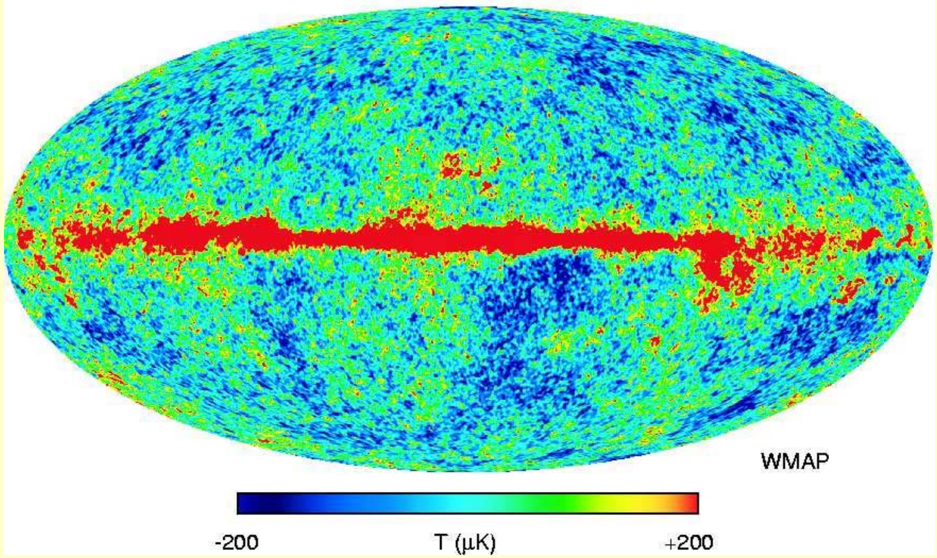

A slice of the SLOAN galaxy catalog

# *CMB anisotropies*

COBE (1992)




WMAP (2003)



The CMB has small fluctuations,

$$\Delta T/T \sim \text{a few} \times 10^{-5}.$$

As we shall see they reflect roughly the amplitude of the gravitational potential.

=> CMB anisotropies can be treated with linear perturbation theory.

The basic idea is, that structure grew out of small initial fluctuations by gravitational instability.

=> At least the beginning of their evolution can be treated with linear perturbation theory.

As we shall see, the gravitational potential does not grow within linear perturbation theory. Hence initial fluctuations with an amplitude of $\sim$ a few $\times 10^{-5}$ are needed. In N. Kaloper's talk you will hear about the main ideas how such fluctuations could emerge during an inflationary era of the universe.

# Linear cosmological perturbation theory

- **metric perturbations**

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2 h_{\mu\nu}$$

$$h_{\mu\nu} dx^\mu dx^\nu = -2A d\eta^2 - 2B_i d\eta dx^i + 2H_{ij} dx^i dx^j$$

- **Decomposition into scalar, vector and tensor components**

$$B_i = \nabla_i B^{(S)} + B_i^{(V)}$$

$$H_{ij} = H_L \gamma_{ij} + \left( \nabla_i \nabla_j - \frac{1}{3} \Delta \gamma_{ij} \right) H_T + \frac{1}{2} \left( H_{i|j}^{(V)} + H_{j|i}^{(V)} \right) + H_{ij}^{(T)}$$

$$\nabla_i B^{(V)i} = \nabla_i H^{(V)i} = \nabla_i H^{(T)ij} = 0$$

# Perturbations of the energy momentum tensor

**Density and velocity**

$$T^{\mu}_{\nu} u^{\nu} = -\rho u^{\mu}, \quad u^2 = -1$$

$$\rho = \bar{\rho}\,(1+\delta)\,, \quad u = u^0 \partial_t + u^i \partial_i$$

$$u^0 = \frac{1}{a}(1-A) \qquad u^i = \frac{1}{a} v^i$$

**stress tensor**

$$\tau^{\mu\nu} = P^{\mu}_{\alpha} P^{\nu}_{\beta} T^{\alpha\beta} \qquad P^{\mu}_{\nu} \equiv u^{\mu} u_{\nu} + \delta^{\mu}_{\nu}$$

$$\tau^{i}_{j} = \bar{p}\left[(1+\pi_L)\,\delta^{i}_{j} + \Pi^{i}_{j}\right]$$

# Gauge invariance

Linear perturbations change under linearized coordinate transformations, but physical effects are independent of them. It is thus useful to express the equations in terms of gauge-invariant combinations. These usually also have a simple physical meaning.

Gauge invariant metric fluctuations (the Bardeen potentials)

$$\Psi = A - \frac{\dot{a}}{a}(k^{-2}\dot{H}_T - k^{-1}B) - k^{-2}\ddot{H}_T + k^{-1}\dot{B}$$

$$\Phi = H_L + \frac{1}{3}H_T - \frac{\dot{a}}{a}(k^{-2}\dot{H}_T - k^{-1}B)$$

$\Psi$ is the analog of the Newtonian potential. In simple cases $\Phi=-\Psi$.

In longitudinal gauge, the metric perturbations are given by

$$h_{\mu\nu}^{(long)} = -2\Psi d\eta^2 + 2\Phi\gamma_{ij}dx^i dx^j$$

# Gauge invariant variables for perturbations of the energy momentum tensor

**The anisotropic stress potential** $\Pi$

**The entropy perturbation**

$$\Gamma = \pi_L - \frac{c_s^2}{w}\delta$$

$w = p/\rho$

$c_s^2 = p'/\rho'$

**Velocity and density perturbations**

$$V \equiv v - \frac{1}{k}\dot{H}_T = v^{(\text{long})}$$

$$D_g \equiv \delta + 3(1+w)\left(H_L + \frac{1}{3}H_T\right) = \delta^{(\text{long})} + 3(1+w)\Phi$$

$$D \equiv \delta^{(\text{long})} + 3(1+w)\left(\frac{\dot{a}}{a}\right)\frac{V}{k}$$

- **Einstein equations**

  **constraints**

$$4\pi G a^2 \rho D = (k^2 - 3\kappa)\Phi$$

$$4\pi G a^2 (\rho + p) V = k\left(\left(\frac{\dot a}{a}\right)\Psi - \dot\Phi\right)$$

  **dynamical**

$$-k^2\left(\Phi + \Psi\right) = 8\pi G a^2 p \Pi$$

- **Conservation equations**

$$\dot D_g + 3\left(c_s^2 - w\right)\left(\frac{\dot a}{a}\right) D_g + (1+w)kV + 3w\left(\frac{\dot a}{a}\right)\Gamma = 0$$

$$\dot V + \left(\frac{\dot a}{a}\right)\left(1 - 3c_s^2\right) V = k\left(\Psi - 3c_s^2\Phi\right) + \frac{c_s^2 k}{1+w} D_g + \frac{wk}{1+w}\left[\Gamma - \frac{2}{3}\left(1 - \frac{3\kappa}{k^2}\right)\Pi\right]$$

# Simple solutions and consequences

**matter**

$$D \propto a\ , \qquad V \propto \eta\ , \qquad \Psi = \text{const.}$$

**radiation**

$$D_g = D_2\left[\cos(x) - \frac{2}{x}\sin(x)\right] + D_1\left[\sin(x) + \frac{2}{x}\cos(x)\right]$$

$x = c_s k \eta$

$$V = -\frac{\sqrt{3}}{4}D_g' \qquad \Psi = -\frac{D_g + \frac{4}{\sqrt{3}x}V}{4+2x^2}$$

- The $D_1$-mode is singular, the $D_2$-mode is the adiabatic mode
- In a mixed matter/radiation model there is a second regular mode, the isocurvature mode
- On super horizon scales, $x<1$, $\Psi$ is constant
- On sub horizon scales, $D_g$ and $V$ oscillate while $\Psi$ oscillates and decays like $1/x^2$ in a radiation universe.

# lightlike geodesics

From the surface of last scattering into our antennas the CMB photons travel along geodesics. By integrating the geodesic equation, we obtain the change of energy in a given direction n:

$E_f/E_i = (n \cdot u)_f/(n \cdot u)_i = [T_f/T_i](1+ \Delta T_f/T_f - \Delta T_i/T_i)$

This corresponds to a temperature variation. In first order perturbation theory one finds for scalar perturbations

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)} n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))\, d\eta$$

acoustic oscillations

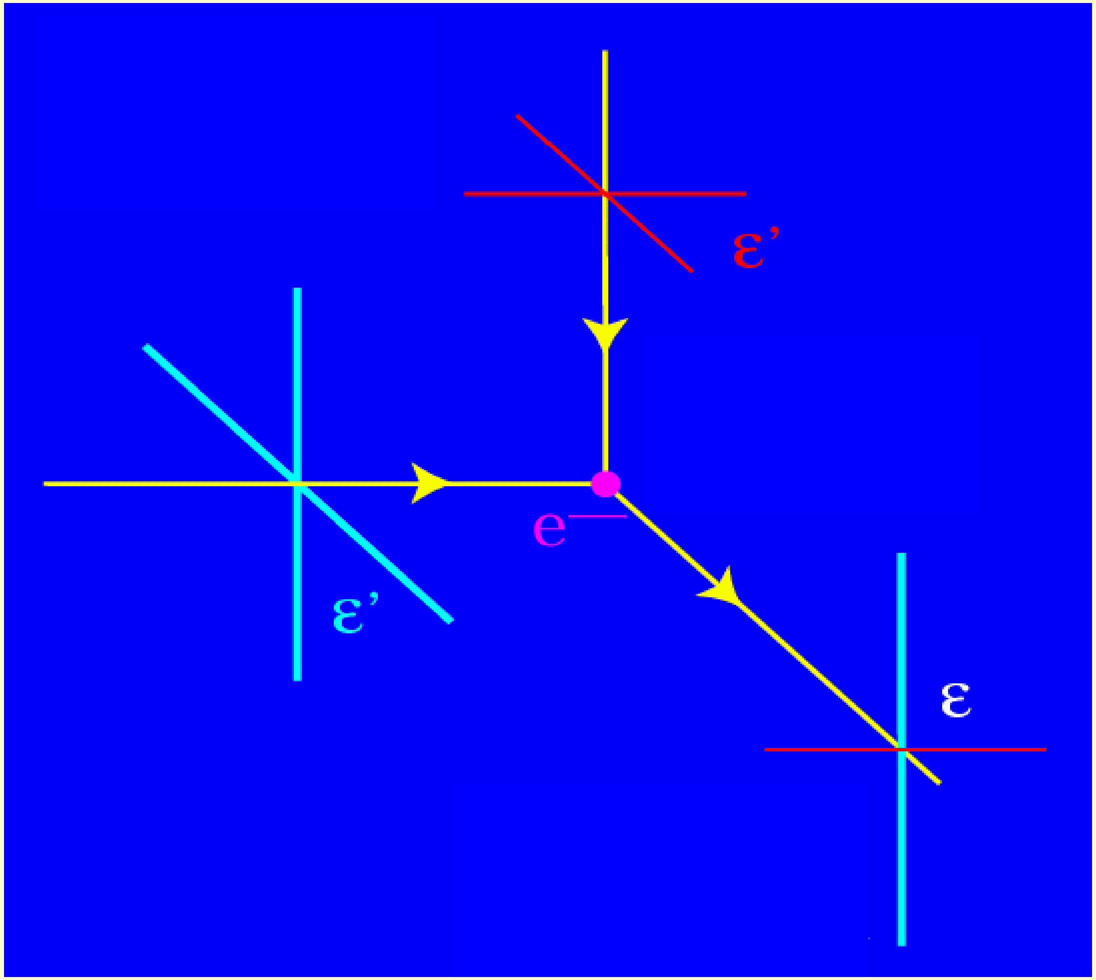
Doppler term

gravitat. potentiel (Sachs Wolfe)

integrated Sachs Wolfe ISW

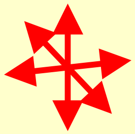
# Polarisation

- Thomson scattering depends on polarisation: a quadrupole anisotropy of the incoming wave generates linear polarisation of the outgoing wave.

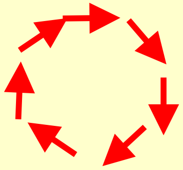


Polarisation can be described by the Stokes parameters, but they depend on the choice of the coordinate system. A better way is to split the polarisation field into a gradient- and a rotational part:

**E-polarisation**
(generated by scalar and tensor modes)



**B-polarisation**
(generated only by the tensor mode)



Due to their parity, T and B are not correlated while T and E are

An additional effect on CMB fluctuations is Silk damping: on small scales, of the order of the size of the mean free path of CMB photons, fluctuations are damped due to free streaming: photons stream out of over-densities into under-densities. To compute the effects of Silk damping and polarisation we have to solve the Boltzmann equation for the Stokes parameters of the CMB radiation. This is usually done with a standard, publicly available code like CMBfast, CAMBcode or CMBeasy.

# Reionization

The absence of the so called Gunn-Peterson trough in quasar spectra tells us that the universe is reionised since, at least, $z \sim 6$.
Reionisation leads to a certain degree of re-scattering of CMB photons. This induces additional damping of anisotropies and additional polarisation on large scales (up to the horizon scale at reionisation). It enters the CMB spectrum mainly through one parameter, the optical depth $\tau$ to the last scattering surface or the redshift of reionisation $z_{re}$ .

# *Matter power spectra*

The perturbations are random variables. We can only measure one realization of them, our observable Universe. However, for a given model of the Universe we can only reliably calculate expectation values, like $P(k)$, $\langle D(k)D^*(k')\rangle = \delta(k-k')P(k)$

where D(k) is the dark matter density fluctuation. Or

$P_v(k) = \langle V(k)V^*(k')\rangle = \delta(k-k')P_v(k)$

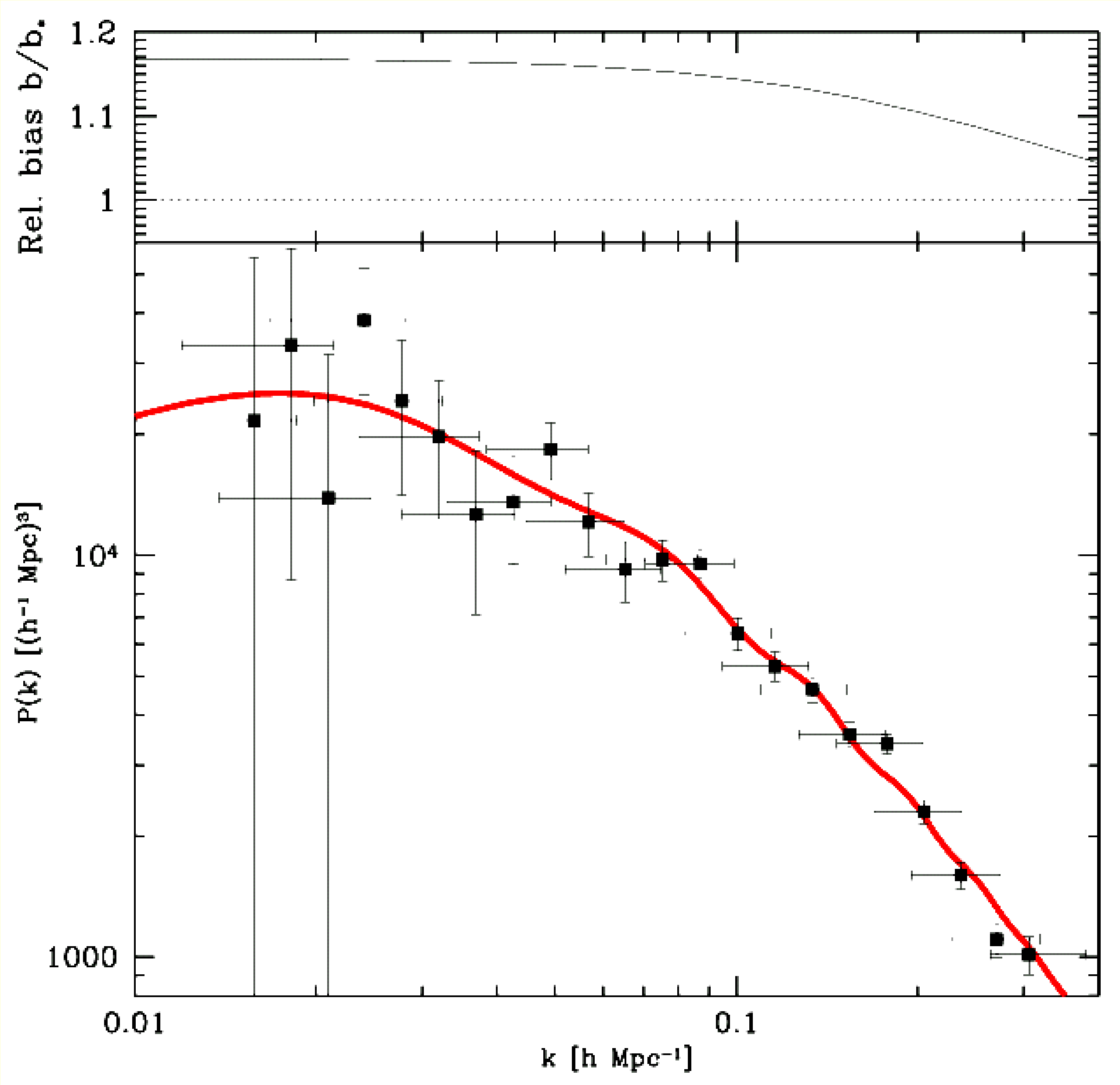
We then assume that these power spectra are independent of direction (isotropic random process), so that we can compare them with the Fourier transformed data averaged over directions in k-space.

If the random process describing the perturbations is Gaussian, these 2-point functions contain all the statistical information. Within linear perturbation theory these power spectra are related via the conservation equation,

$$P_V(k) \simeq \Omega^{0.6}(H/k)^2P(k)$$

# The dark matter power spectrum as inferred from SLOAN

There is an additional problem when comparing the calculated dark matter with the galaxy distribution: How are they related? The question of bias.



# The power spectrum of CMB fluctuations

$\Delta T(\mathbf{n})$ is a function on the sphere, we can expand it in spherical harmonics

$$\frac{\Delta T}{T}(\mathbf{x}_0, \mathbf{n}, \eta_0) = \sum_{\ell,m} a_{\ell m}(\mathbf{x}_0) Y_{\ell m}(\mathbf{n}) \qquad \langle a_{\ell m} \cdot a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

consequence of statistical isotropy

observed mean

$$\frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \equiv C_\ell^{\rm obs}$$

cosmic variance (if the $a_{lm}$'s are Gaussian)

$$\frac{\sqrt{\langle |(C_\ell^{obs})^2 - C_\ell^2| \rangle}}{C_\ell} = \sqrt{\frac{2}{2\ell+1}}$$

# The physics of CMB fluctuations

- Large scales : The gravitational potential on the surface of last scattering, time dependence of the gravitational potential $\Psi \sim 10^{-5}$ .

$$\theta > 1^o$$
$$\ell < 100$$

- Intermediate scales : Acoustic oscillations of the baryon/photon fluid before recombination.

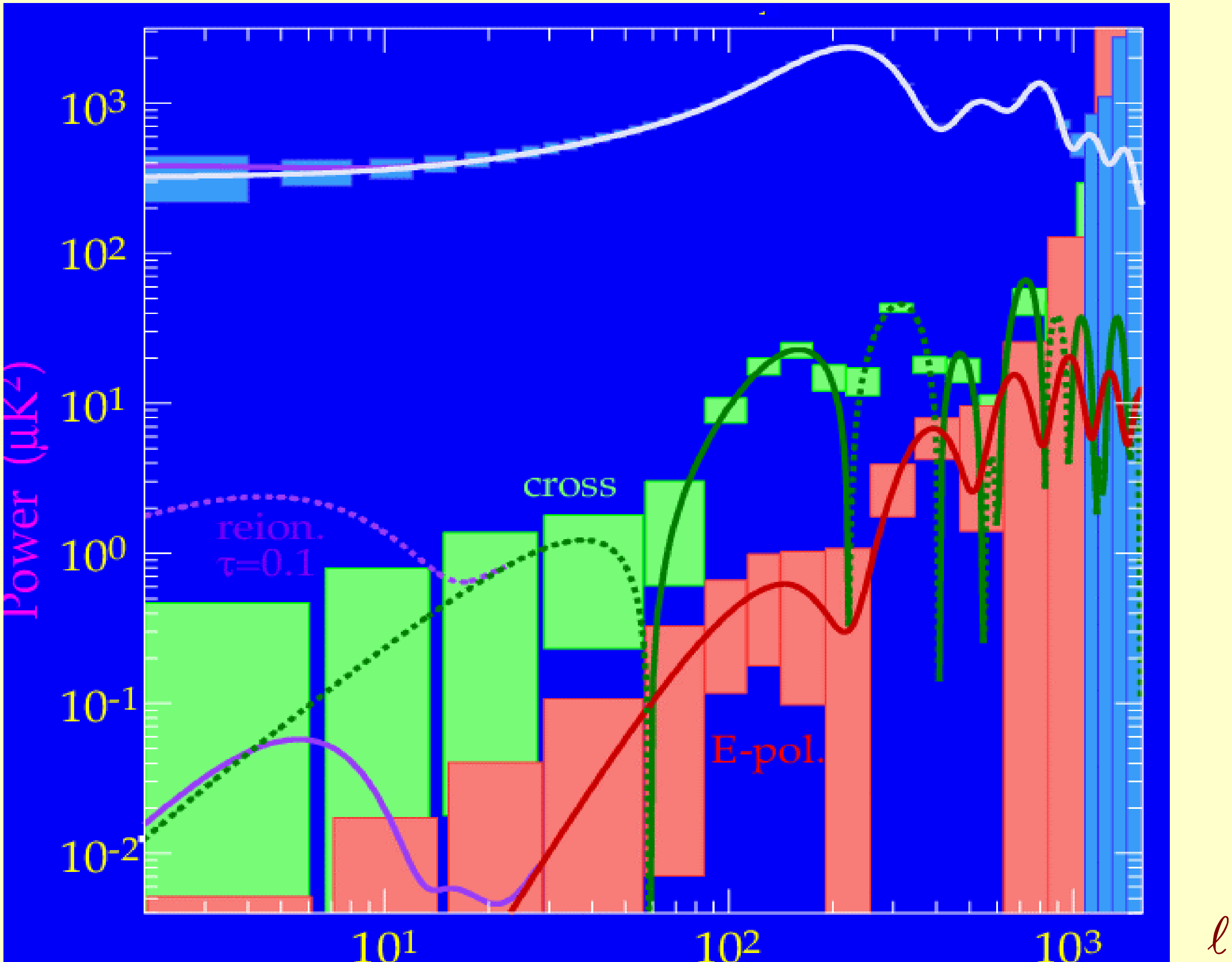
$$6' < \theta < 1^o$$
$$100 < \ell < 800$$

- Small scales : Damping of fluctuations due to the imperfect coupling of photons and electrons during recombination

  (Silk damping).

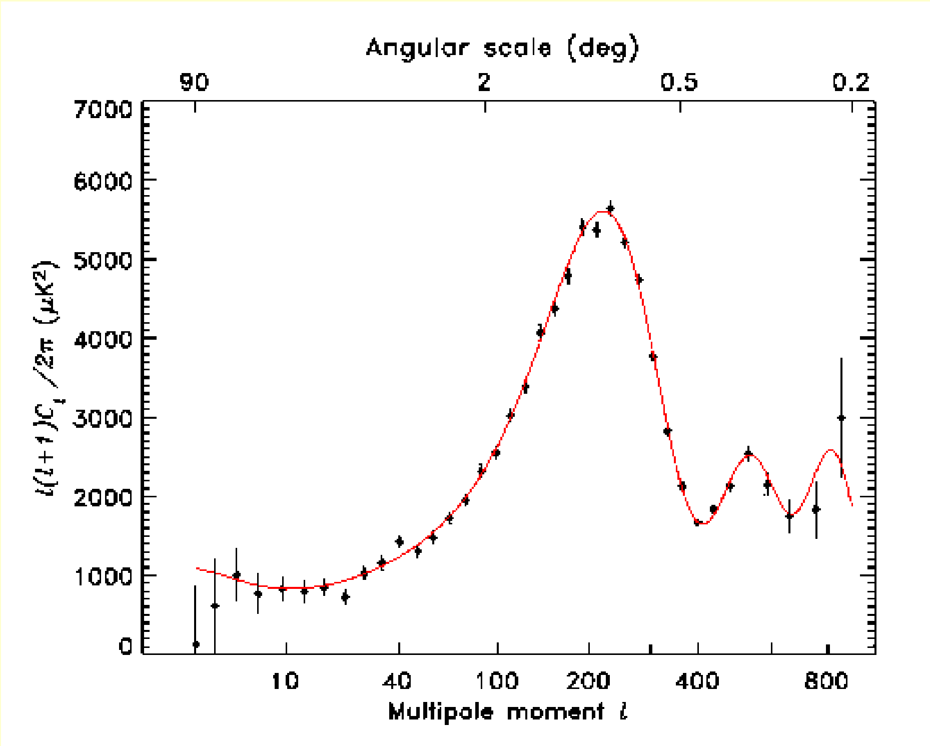
$$\theta < 6'$$
$$800 > \ell$$

# *Power spectra of scalar fluctuations*

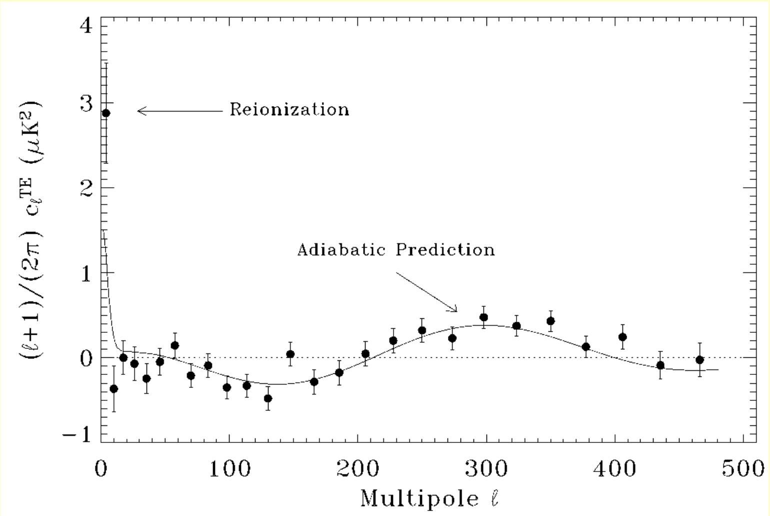


# WMAP data

Temperature (TT = $C_\ell$)



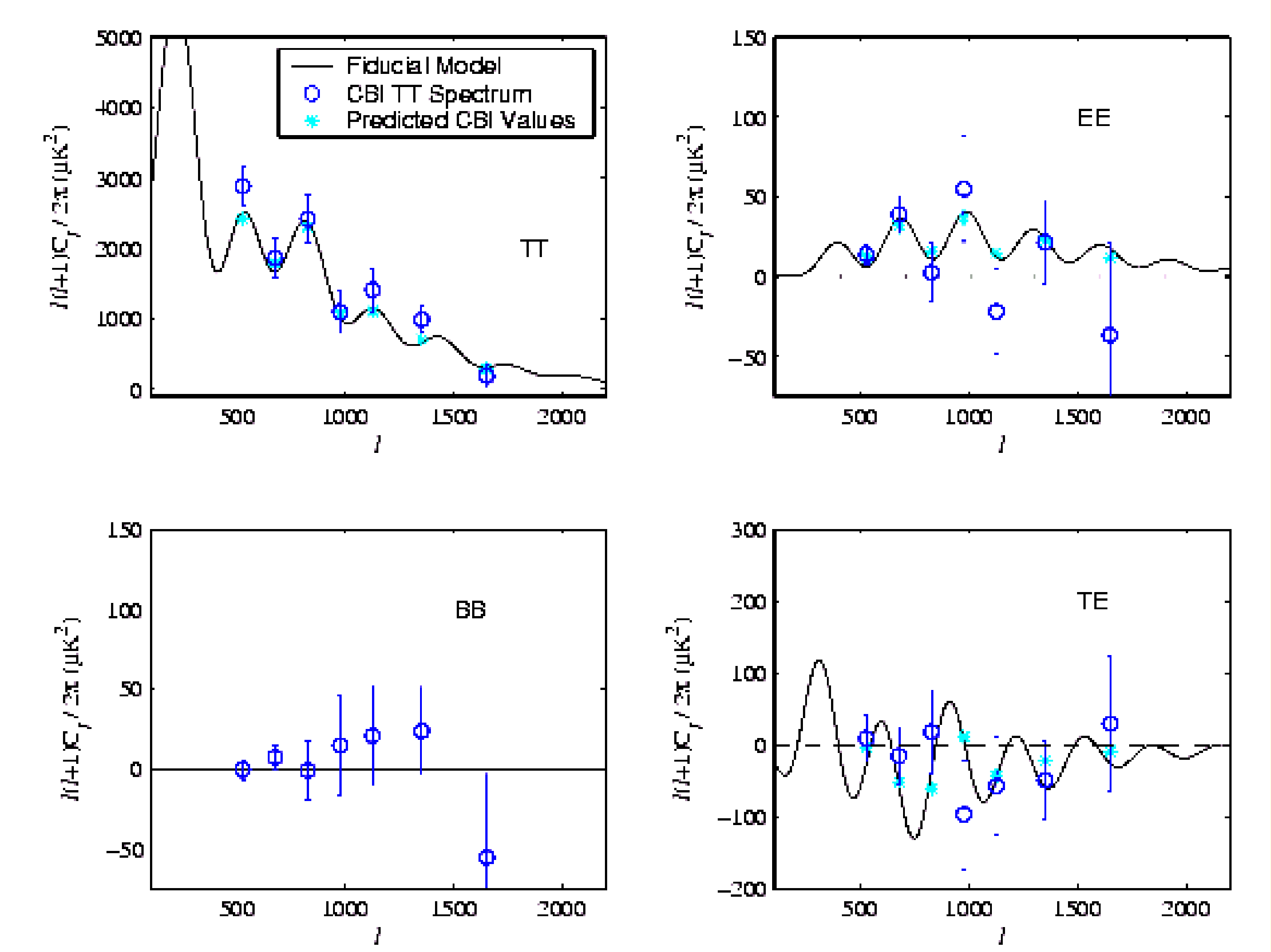

Polarisation (ET)




Spergel et al (2003)

# Newer data 1

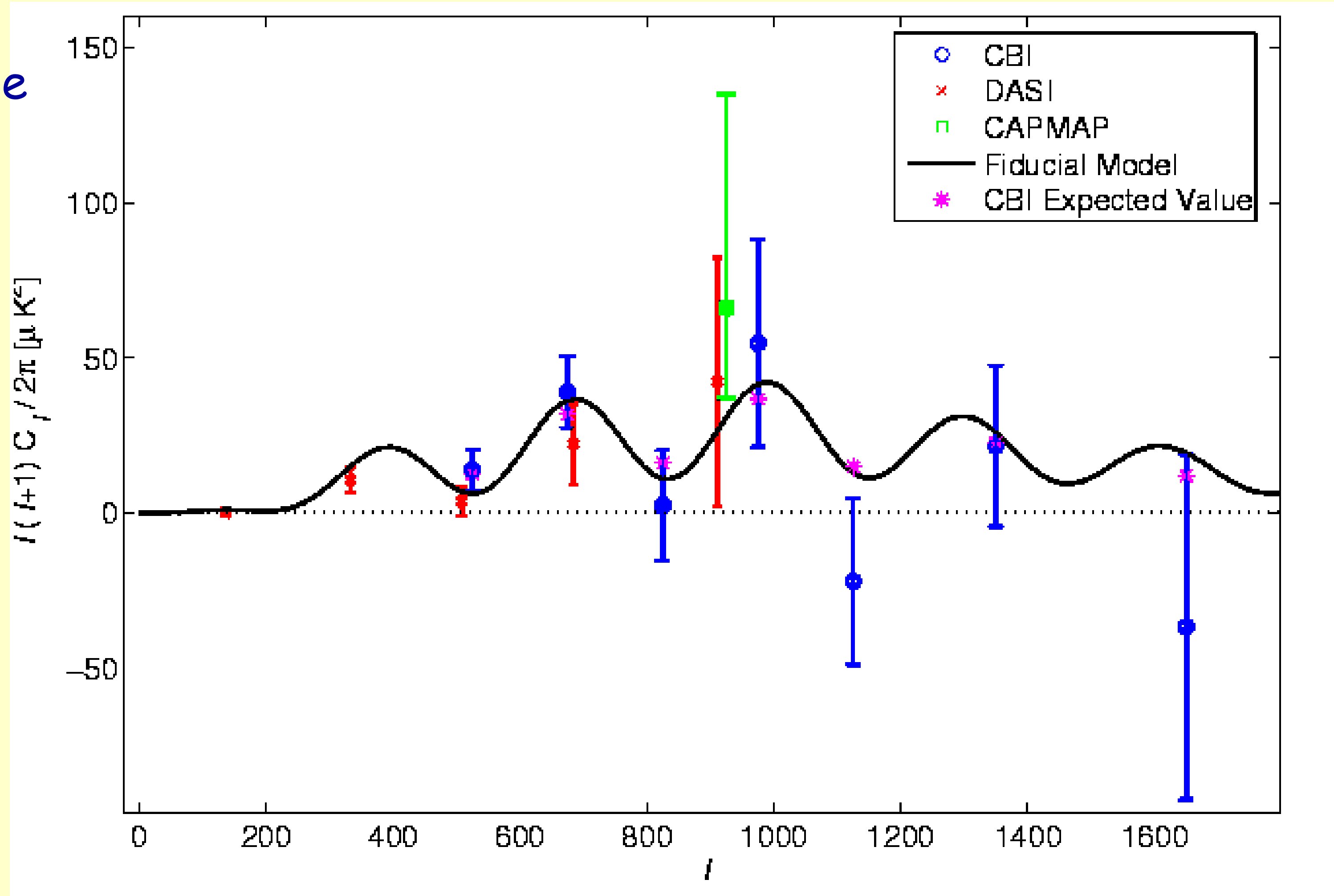
CBI




From Readhead et al. 2004

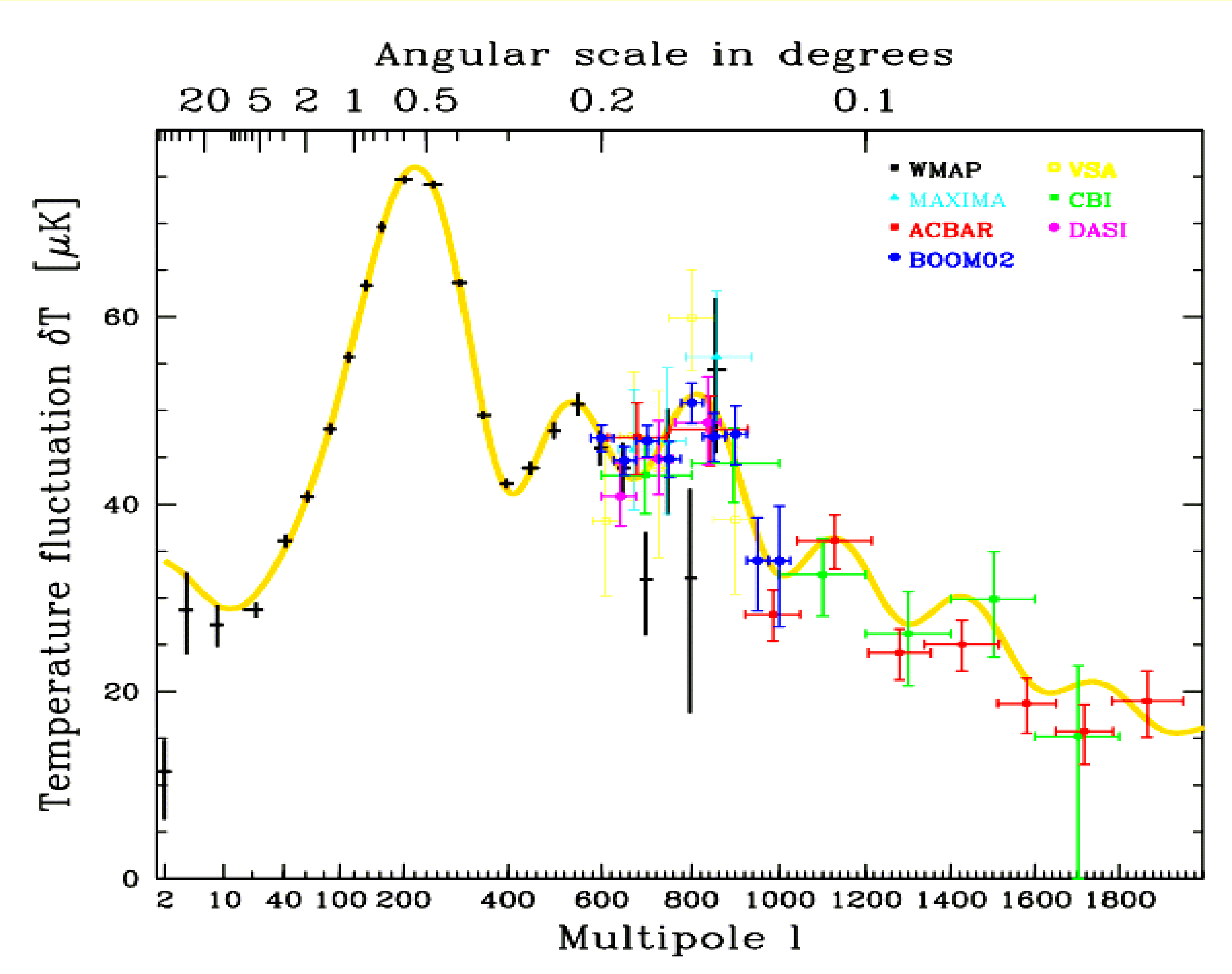
# *Newer data II*

Knowledge of the EE spectrum at present.




From Readhead et al. 2004

# *Observed spectrum of anisotropies*




Tegmark et al. 03

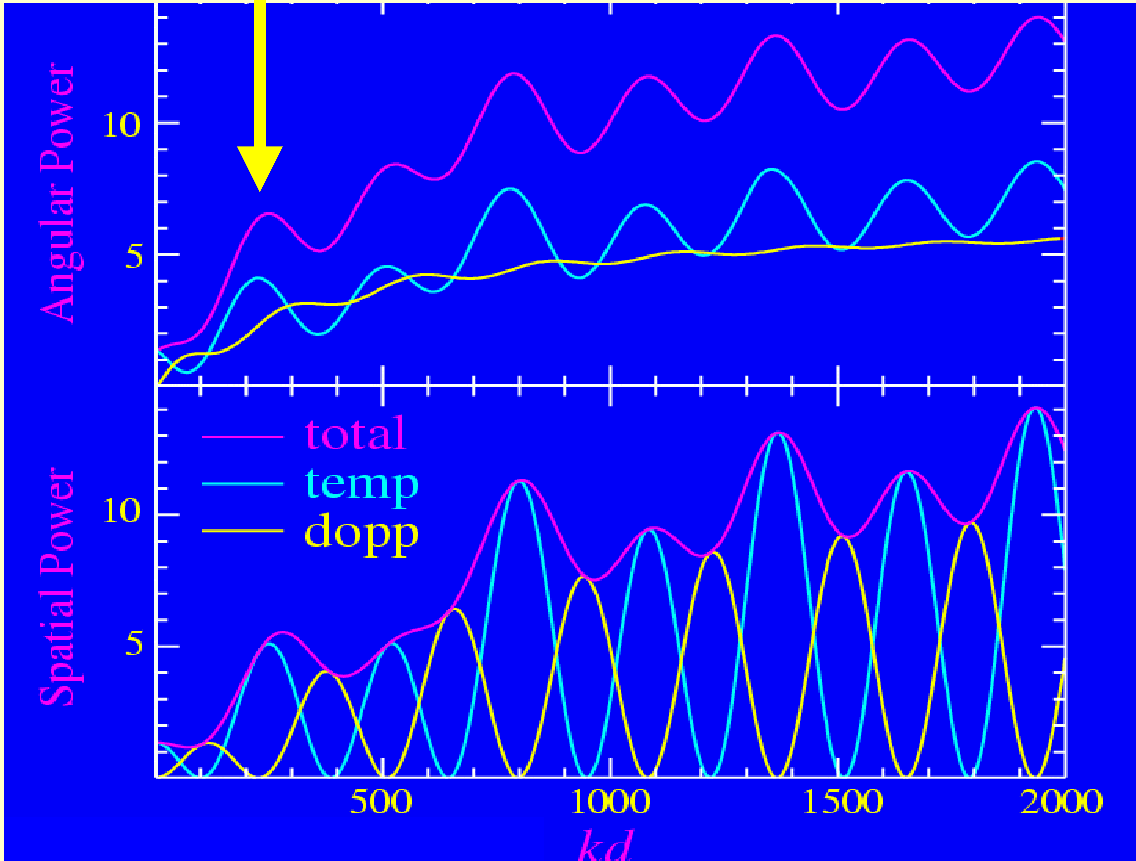
# Acoustic oscillations

Determine the angular distance to the last scattering surface, $z_1$

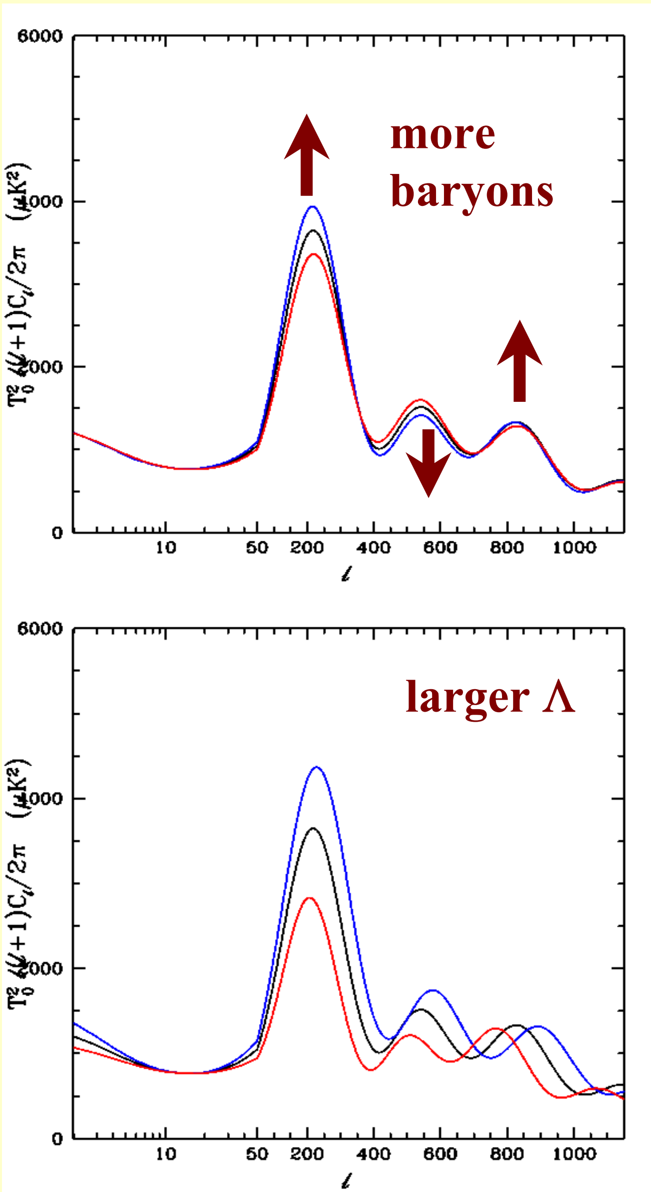
$$\eta_0 - \eta_1 = \frac{1}{H_0 a_0}\int_0^{z_1} \frac{dz}{[\Omega_{\rm rad}(z+1)^4 + \Omega_m (z+1)^3 + \Omega_\Lambda + \Omega_\kappa (z+1)^2]^{\frac{1}{2}}}$$

$$\eta_1 = \frac{1}{H_0 a_0}\int_{z_1}^{\infty} \frac{dz}{[\Omega_{\rm rad}(z+1)^4 + \Omega_m (z+1)^3 + \Omega_\Lambda + \Omega_\kappa (z+1)^2]^{\frac{1}{2}}}.$$

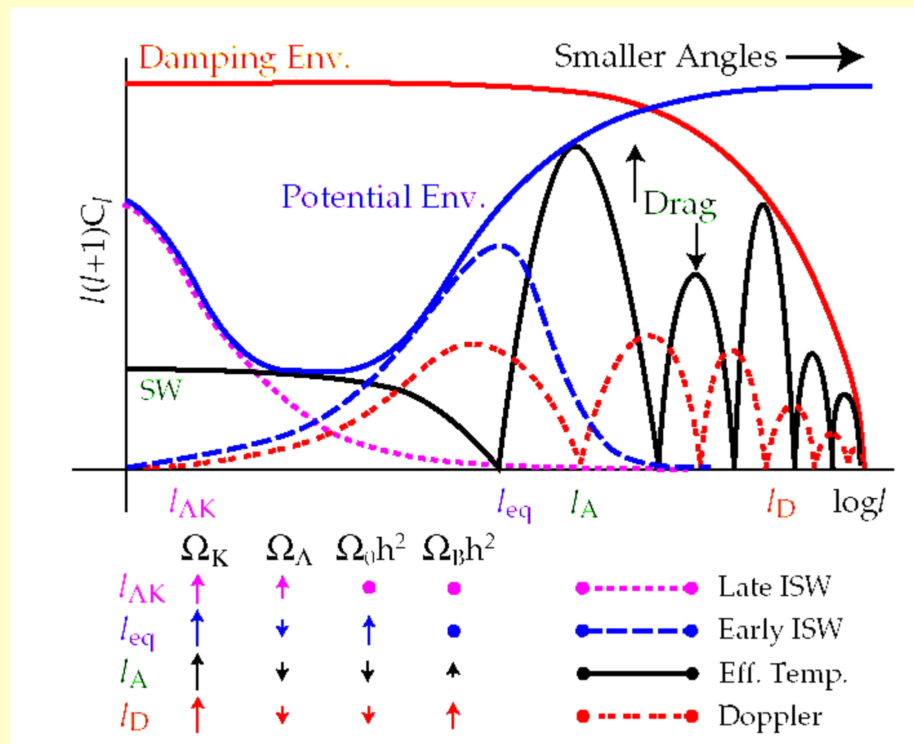
$$\vartheta_A = \frac{c_s \eta_1}{\chi(\eta_0 - \eta_1)}$$



# Dependance on cosmological parameters



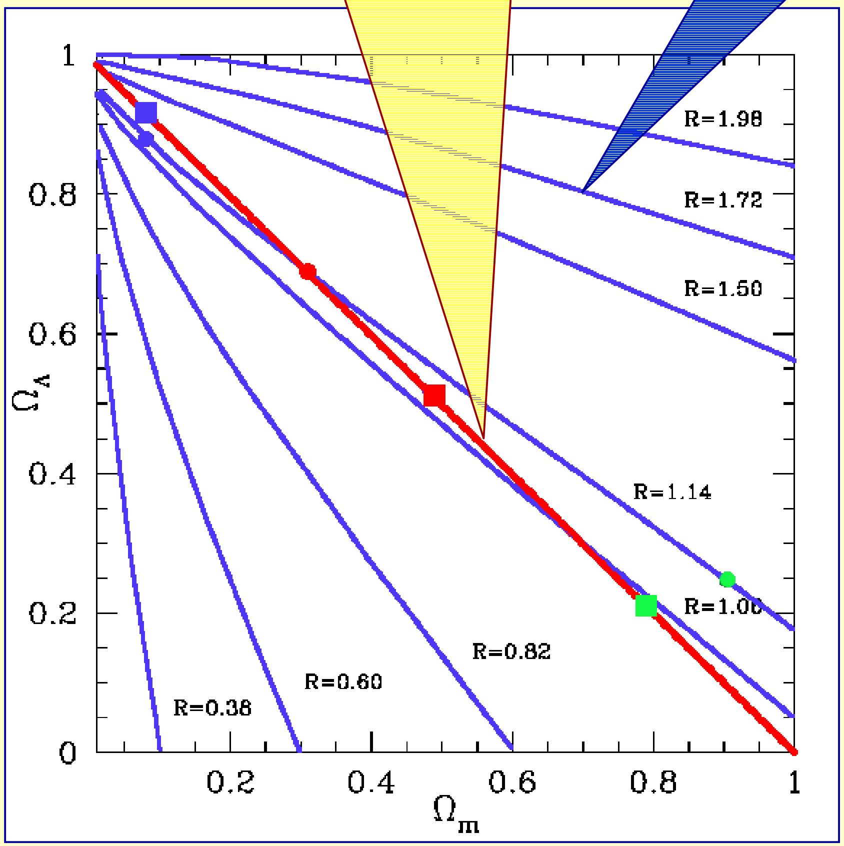

Most cosmological parameters have complicated effects on the CMB spectrum



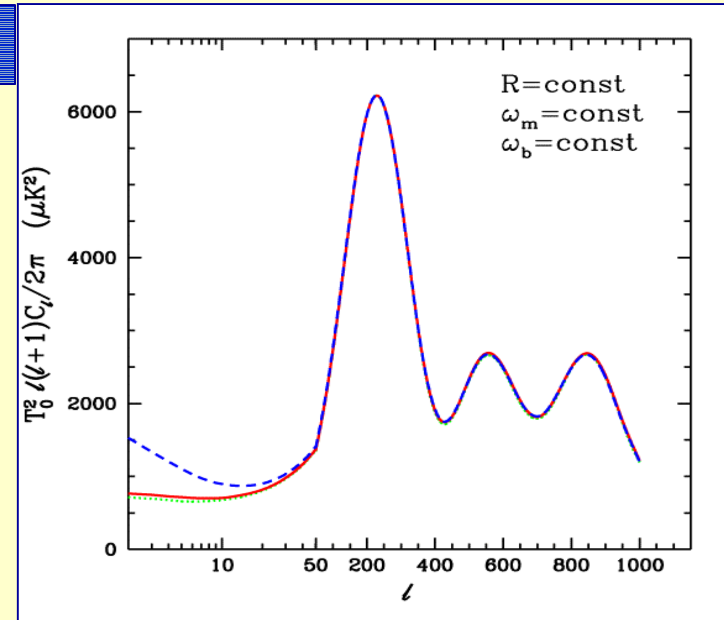
# *Geometrical degeneracy*

**Flat Universe (ligne of constant curvature $\Omega_K$=0 )**

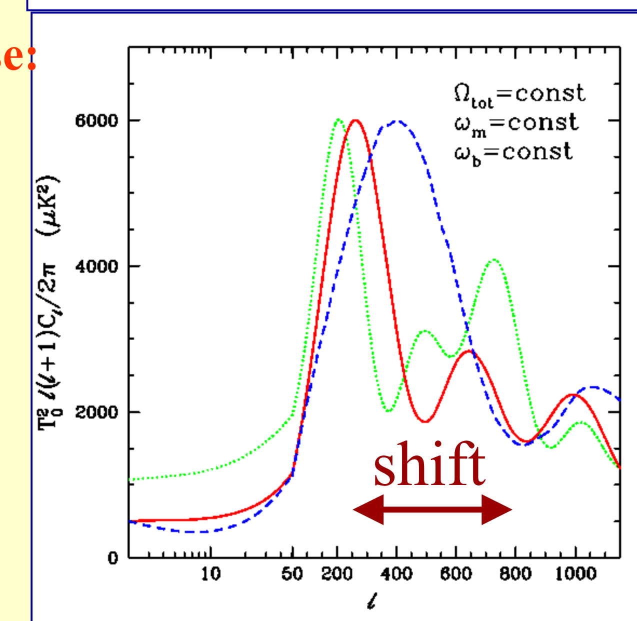
**degeneracy lines:**




**Degeneracy:**



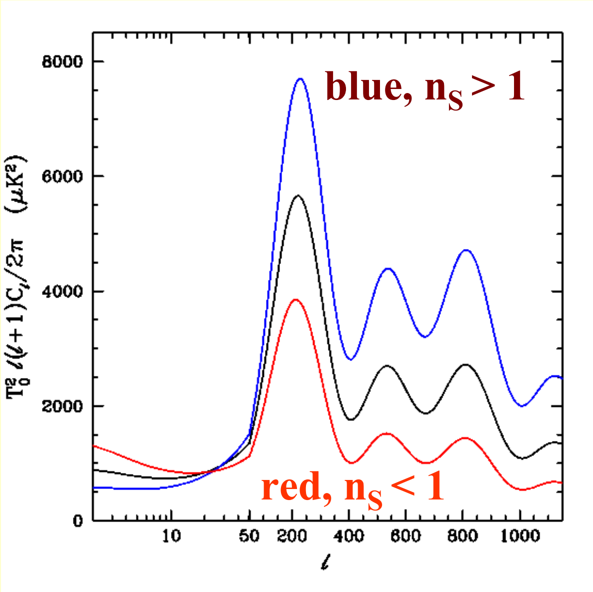

**Flat Universe:**




Shift parameter: $R = R(\Omega_\Lambda, \Omega_m)$

# *Primordial parameters*

Scalar spectum:



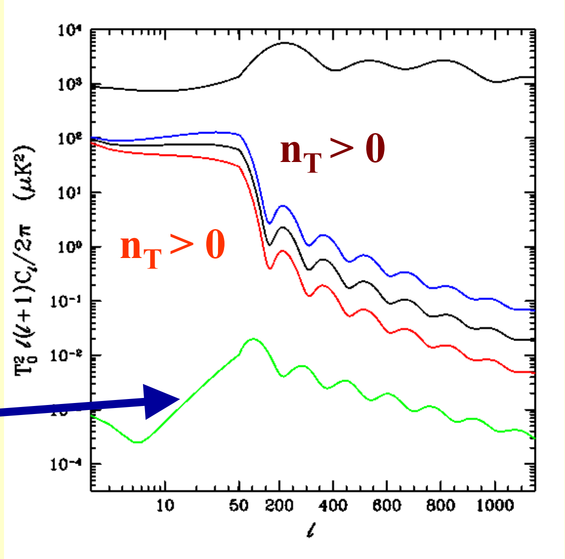

scalar spectral index $n_S$ and amplitude A

$$\langle \Psi^2 \rangle = A k^{n_S - 1}$$

$n_S = 1$ : scale invariant spectrum (Harrison-Zel'dovich)

Tensor spectum:
(gravity waves)

The 'smoking gun' of inflation, has not yet been detected: B modes of the polarisation (QUEST, 2005).



# *Mesured cosmological parameters*

(With CMB + flatness or CMB + Hubble)

Table 1. Power Law ΛCDM Model Parameters- *WMAP* Data Only

| Parameter | | Mean (68% confidence range) | Maximum Likelihood |
|---|---|---|---|
| Baryon Density | $\Omega_b h^2$ | $0.024 \pm 0.001$ | |
| Matter Density | $\Omega_m h^2$ | $0.14 \pm 0.02$ | |
| Hubble Constant | $h$ | $0.72 \pm 0.05$ | |
| Amplitude | $A$ | $0.9 \pm 0.1$ | 0.80 |
| Optical Depth | $\tau$ | $0.166^{+0.076}_{-0.071}$ | |
| Spectral Index | $n_s$ | $0.99 \pm 0.04$ | |
| | $\chi^2_{eff}/\nu$ | | 1431/1342 |

[a]Fit to *WMAP* data only

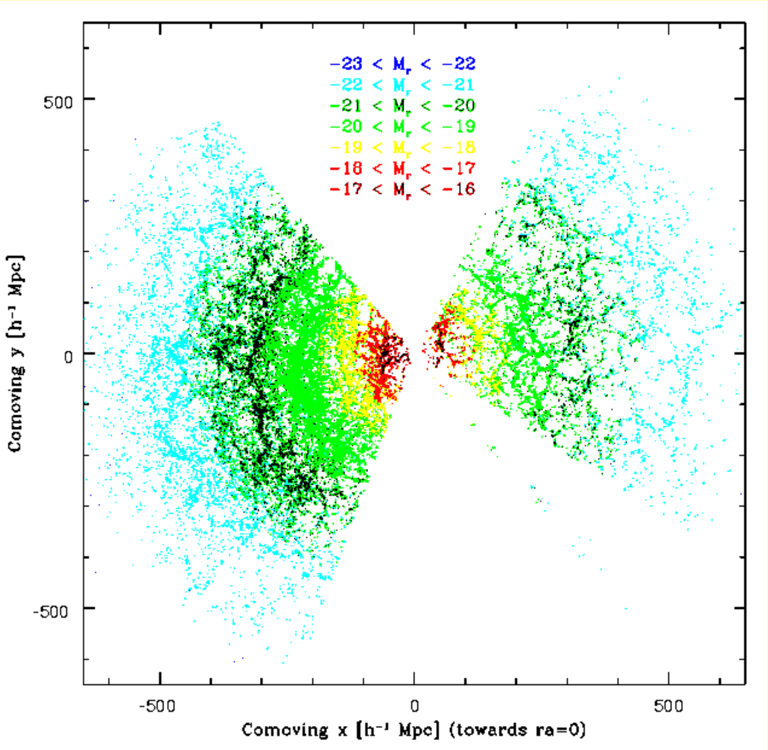
**a rigid constraint which is in slight tension with nucleosynthesis?**
**$\omega_{bar} = 0.02 \pm 0.002$**

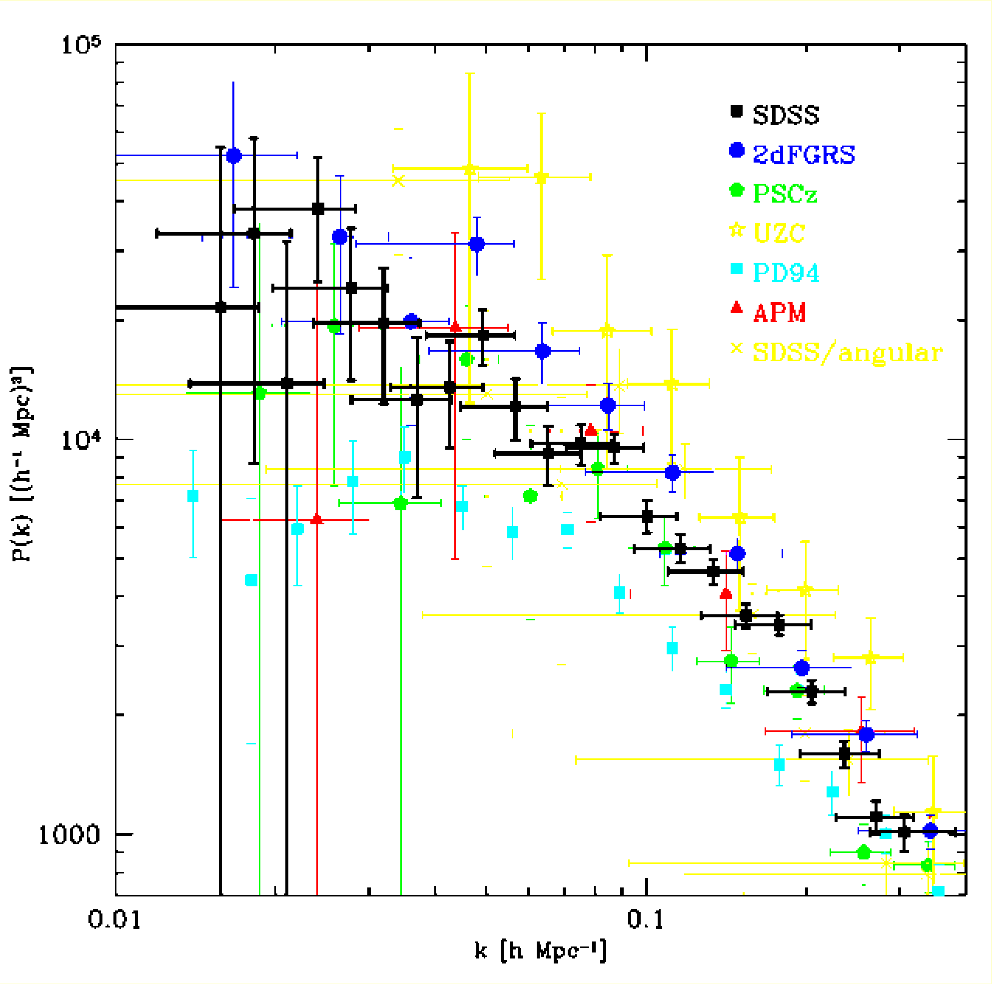
**$z_{reion} \sim 17$**
**unexpectedly early reionisation**

Attention: **FLATNESS** imposed!!!

On the other hand: $\Omega_{tot} = 1.02$ +/- 0.02 with the HST prior on *h*...

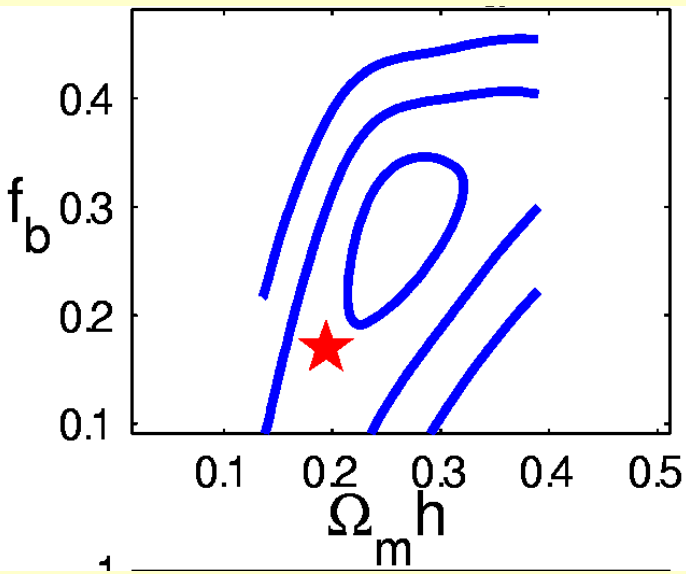
# *Galaxy distribution (CSS)*



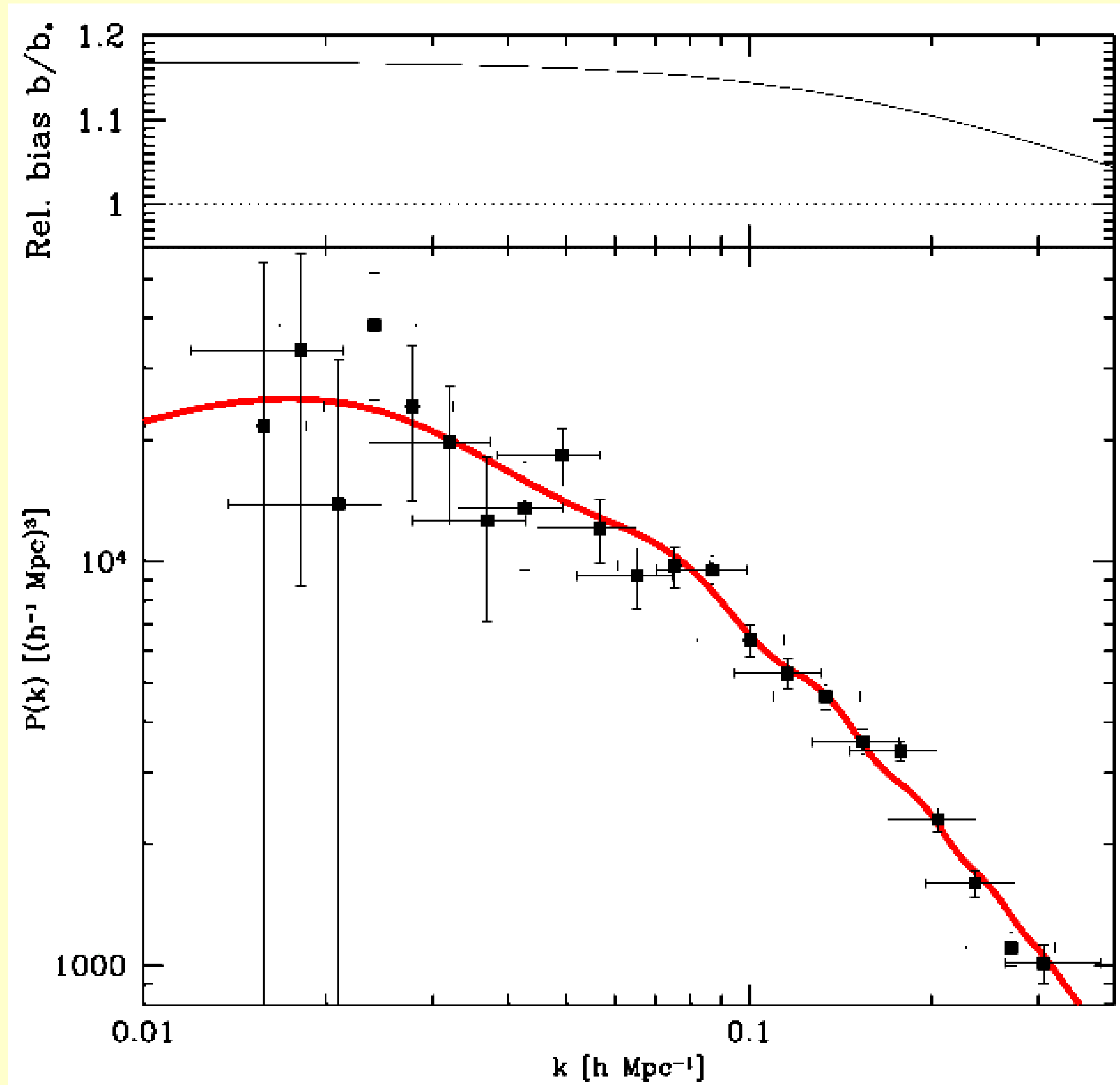



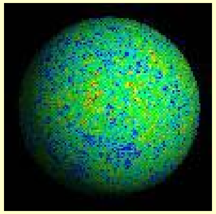

Tegmark et al. 2003

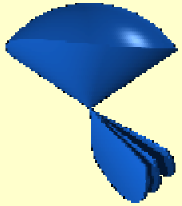
# *Parameters from SDSS* (Sloan Digital Sky Survey)




$$f_b = \frac{\Omega_b}{\Omega_m}$$







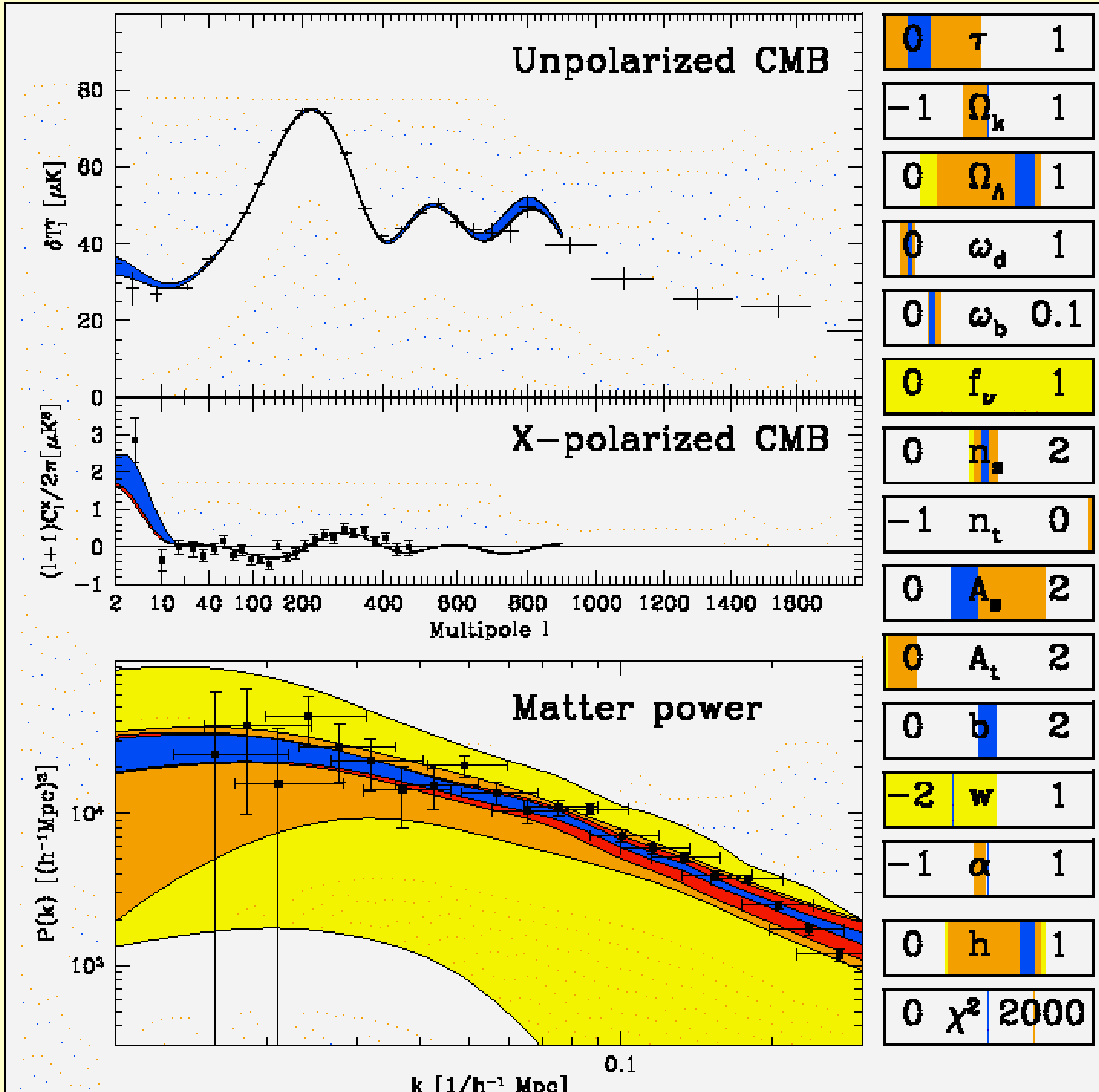
# CMB + LSS

CMB data

CMB data
+ $f_\nu = 0$,
$w = -1$

...
+ $\Omega_k = A_t = \alpha = 0$

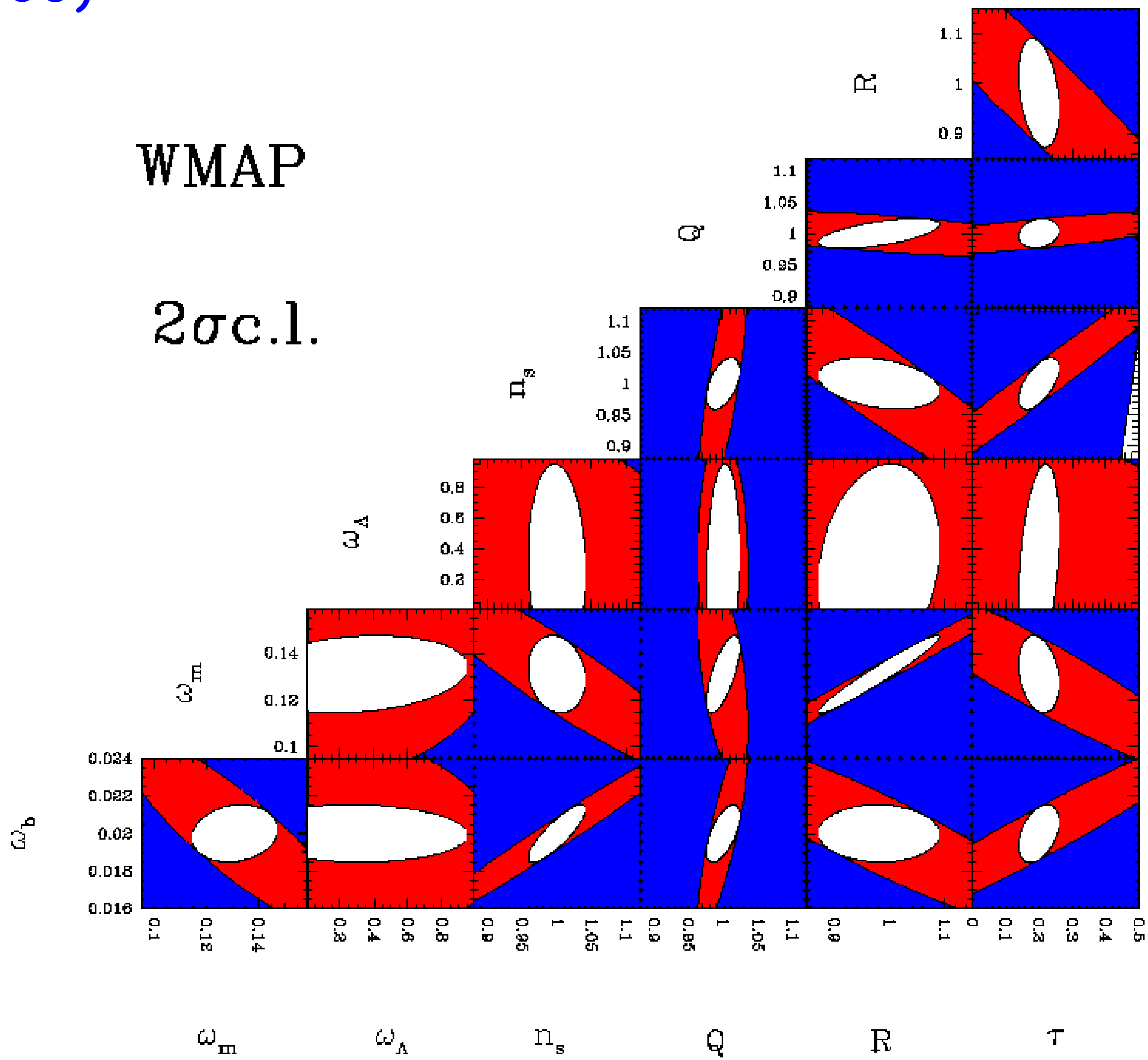
...
+ PS data



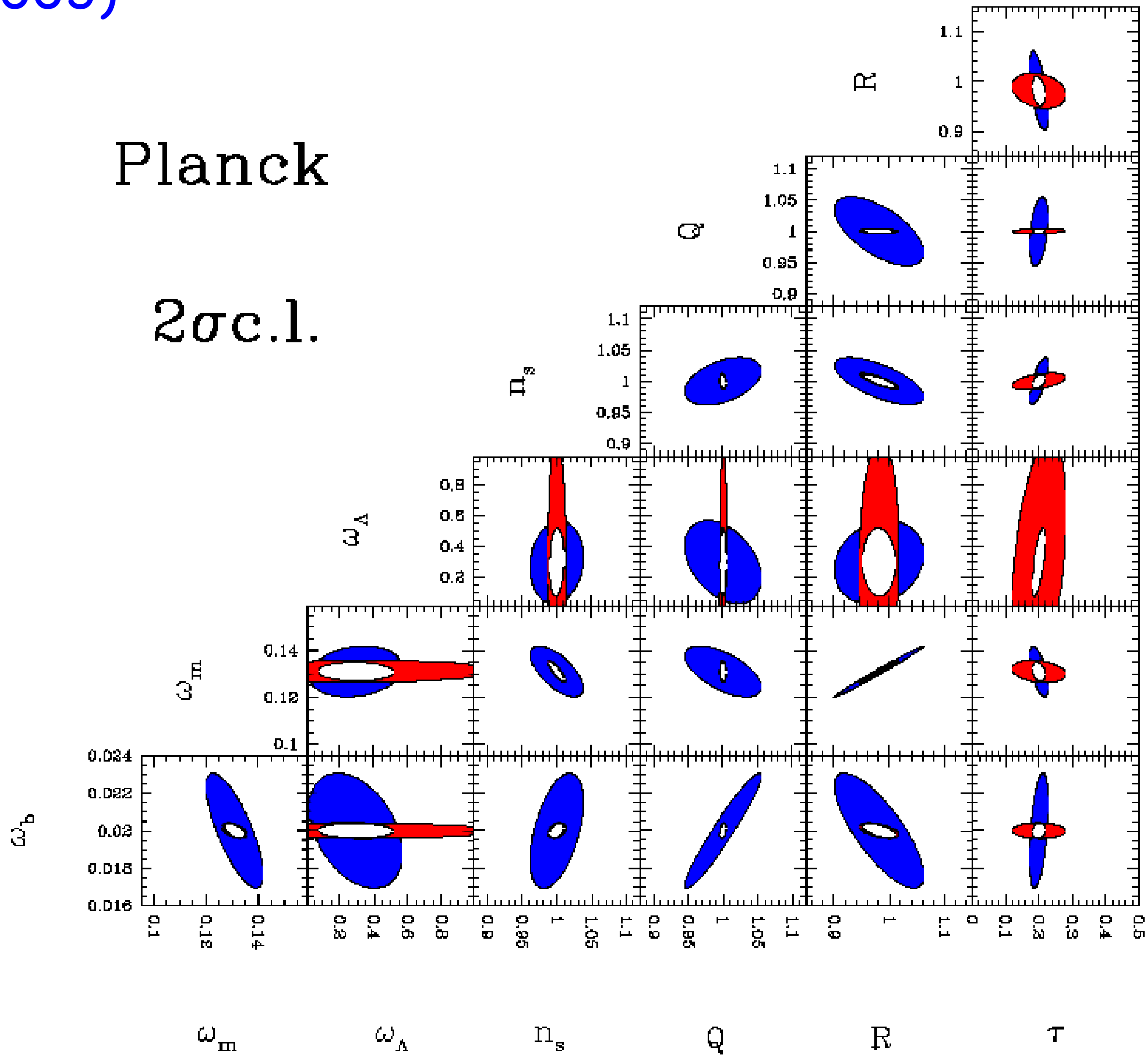

Tegmark et al. 03

# Forecast1: WMAP 2 year data
(Rocha et al. 2003)

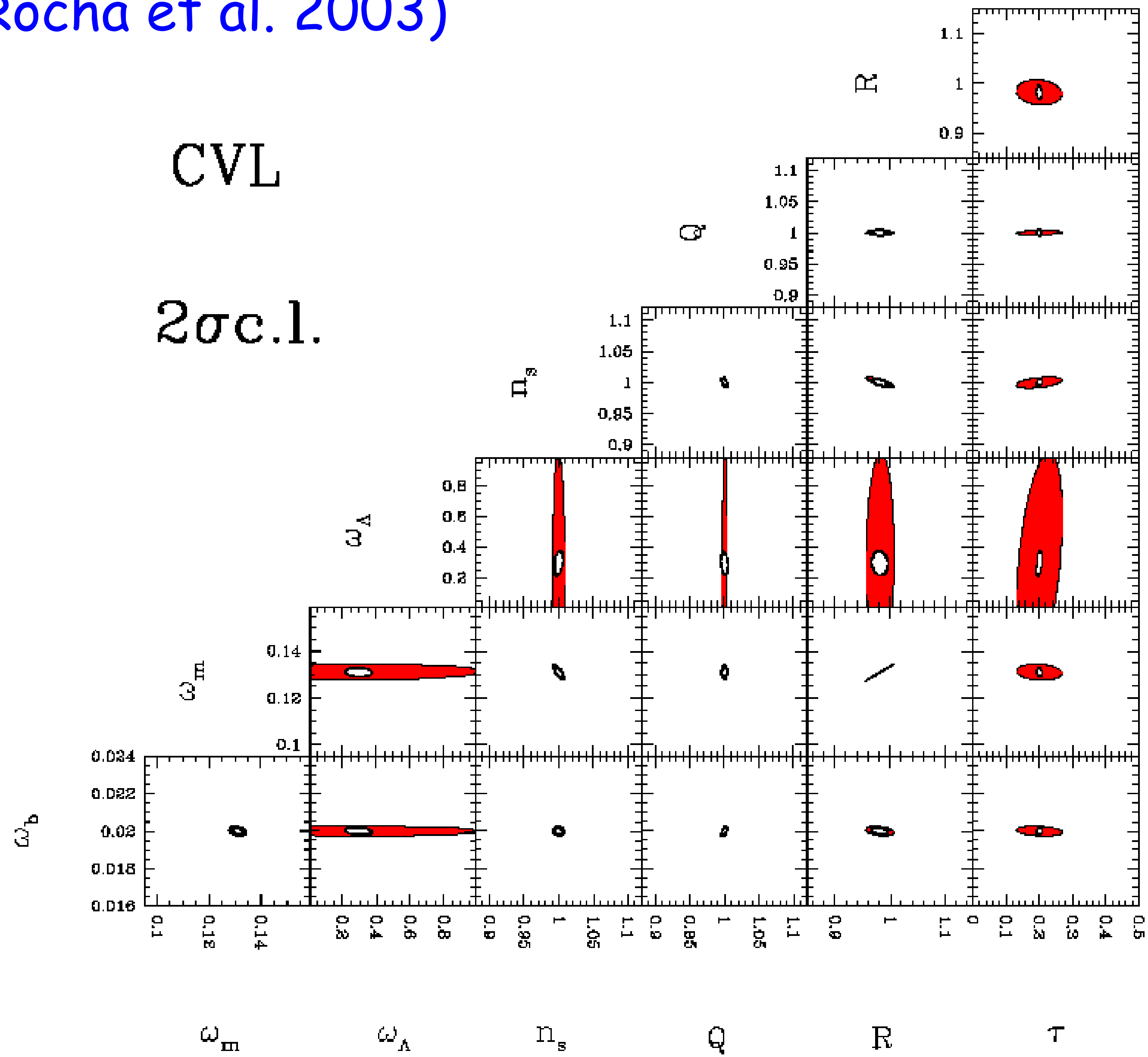
$\omega_b = \Omega_b h^2$
$\omega_m = \Omega_m h^2$
$\omega_\Lambda = \Omega_\Lambda h^2$
$n_s$ spectral index
Q quad. amplit.
R angular diam.
$\tau$ optical depth



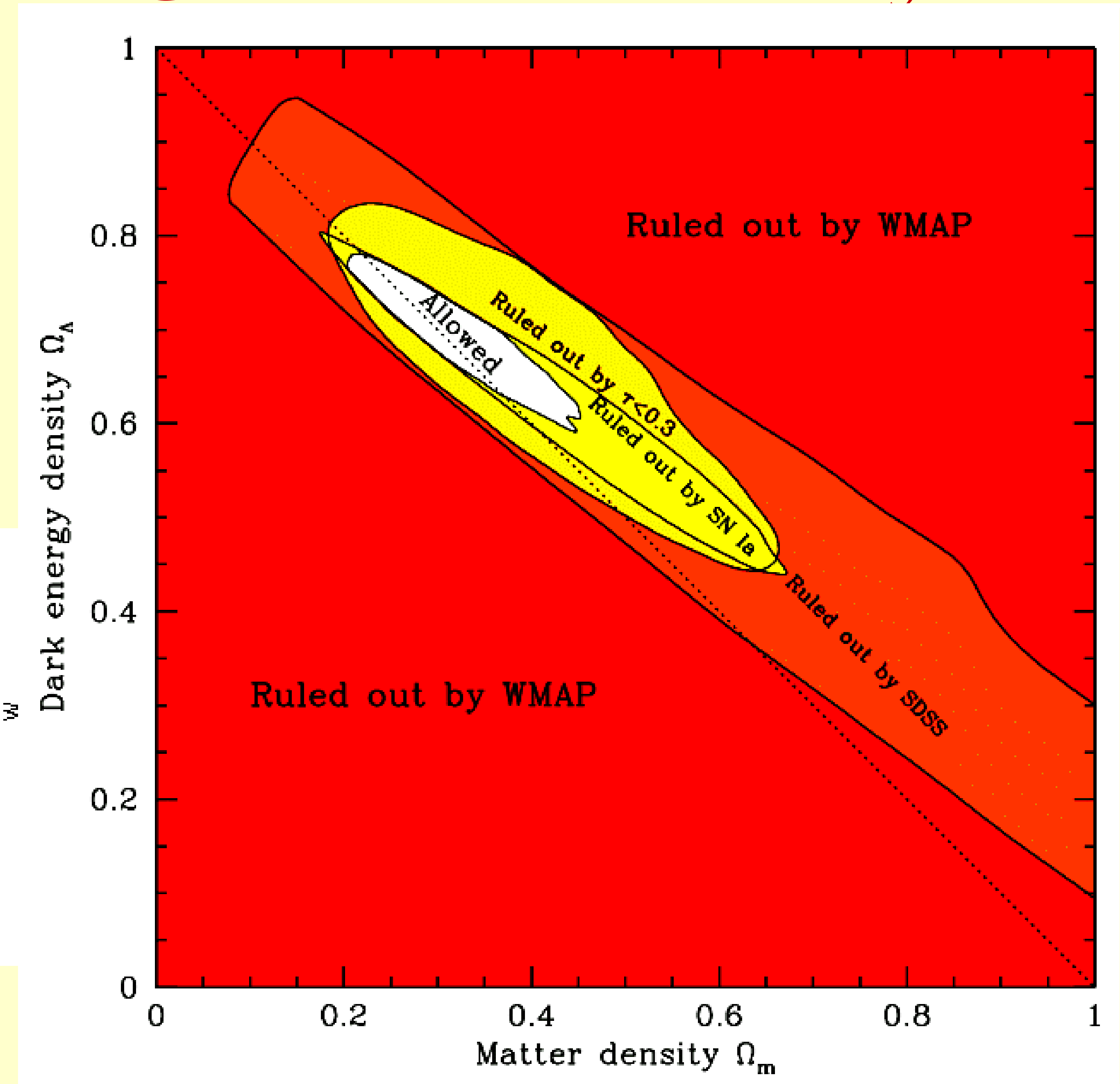
# Forecast2: Planck 2 year data
(Rocha et al. 2003)



# Forecast3: Cosmic variance limited data (Rocha et al. 2003)



# *Evidence for a cosmological constant*




Tegmark et al., 2003

)2

)02

# Conclusions

- We know the cosmological parameters with … sive precision which will still improve conside… uring the next years.
- We don't understand at all the biz… x' of cosmic components: $\Omega_b h^2 \sim 0.0$… $h^2 \sim 0.16$, $\Omega_\Lambda \sim 0.7$
- The simplest model of in… (scale invariant spectrum of scalar perturbations … g curvature) is a good fit to the data.
- What is d… atter?
- Wh… ark energy?
- … hat is the inflaton?

! We have not run out of problems in cosmology!