THE POWER SPECTRUM OF SUSY-CDM ON SUB-GALACTIC SCALES

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- Anne M. Green, S. H., Dominik J. Schwarz
  - to appear in MNRAS (astro-ph/0309621)
  - and JCAP in preparation
- S. H., Dominik J. Schwarz, Horst Stöcker, PRD64 083507 (2001)

MOTIVATION (Theory)

Standard folklore: CDM density perturbations

$$\Delta_{cdm}(k, z) = \underbrace{\Delta_{cdm}(k, z_i)}_{\text{initial perturbations}} \cdot \underbrace{T_\Delta^{1/2}(k, z)}_{\text{transfer-function}}$$

But: where is CDM microphysics?

CDM microphysics sets the typical scale for the first haloes in hierarchical structure formation:

$M_{mc} \approx 10^{-12}\, M_\odot$ for axion minicluster

(Kolb and Tkachev, 1996)

$M_{min} \approx 10^{-6}\, M_\odot$ for bino clouds

(Hofmann et al., 2001; Berezinsky et al., 2003; Green et al., 2003) + talks by Anne Green and Vyacheslav Dokuchaev

Small scale structure formation is sensitive on different CDM particle candidates

MOTIVATION (Experiments)

- Numerical:

Resolution of numerical simulations

$$M_{res} \approx 10^{+6} M_{\odot}$$

(Stoehr et al., 2003)

Substructure formation in simulations depends on numerical resolution (Moore et al., 1998)

Consistent and complete initial conditions?

- Real:

CDM searches, e.g.

$$\gamma\text{-ray flux} = \text{diffuse flux} + \underset{\Delta^2_{cdm}}{\underset{\approx}{\text{line contribution}}}$$

(Bergström et al., 2001; Ullio et al., 2002)

Knowledge of CDM substructures needed for robust predictions of expected signatures.

talk by Larry Widrow

TEMPERATURE SCALES

- Chemical decoupling : $\Gamma_{ann}(T_{cd}) \sim H(T_{cd})$

$$\frac{m}{T_{cd}} \approx \ln\left[10^{-4}\frac{M_{Pl}(M^4+m^4)\,m^3}{(M^2+m^2)^4}\right] \approx 25$$

$T > T_{cd}$: local chemical equilibrium

$T \sim T_{cd}$: relic abundance is fixed

$$\Omega_{cdm} = \frac{m\, n_{cd}\,(a_{cd}/a_0)^3}{3/8\pi\,(M_{Pl} H_0)^2} \approx 0.2\,\frac{m/T_{cd}/25}{\langle\sigma_{ann} v\rangle/1\,pb}$$

$T < T_{cd}$: chemical distinct systemes

- Kinetic decoupling : $\Gamma_{el}(T_{kd}) \sim H(T_{kd})\,N(T_{kd})$

$$T_{kd} \approx \left[10^{2}\,\frac{m^{\alpha}(M^2-m^2)}{M_{Pl}}\right]^{\frac{1}{3+\alpha}} \overset{\alpha \overset{!}{=} 1}{\approx} [10, 40]\ \mathrm{MeV}$$

$T > T_{kd}$: local thermal equilibrium

$T \sim T_{kd}$: spectrum of CDM density perturbation that survived collisional damping

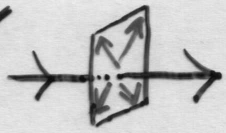
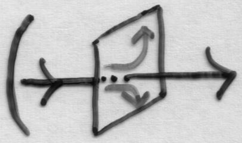
$$\Delta_{cdm}(k)\ ,\ k < k_{cd}$$

$T < T_{kd}$: free streaming

KINETIC DECOUPLING

At $T \sim T_{kd}$: CDM energy momentum tensor is given by

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + T^{\mu\nu}_{imperfect}$$

$T^{\mu\nu}_{imperfect} = \zeta \big($  $\big)^{\mu\nu} + \eta \big($  $\big)^{\mu\nu}$

bulk shear viscosity

$$\zeta = \frac{5}{3} n T \tau_{relax} \quad , \quad \eta = n T \tau_{relax}$$

CDM density perturbations $\widehat{=}$ oscillator with friction(t)

$$\Delta''_{cdm} + \underbrace{\frac{\zeta + \frac{4}{3}\eta}{\epsilon_{cdm}} \frac{k^2}{a}}_{\text{decay rate}} \Delta'_{cdm} + c^2_{cdm} k^2 \Delta_{cdm} = 0$$

Since Δ_{cdm} oscillates with complex frequency

$$D_d(k) \equiv \frac{\Delta_{cdm}(k, \eta_{kd})}{\Delta_{cdm}(k, \eta_i)} = \exp\left[-(k/k_d)^2\right]$$

with the characteristic damping scale

$$k_d \approx \frac{3{,}8 \cdot 10^7}{Mpc} \left(\frac{m}{100\,GeV}\right)^{1/2} \left(\frac{T_{kd}}{30\,MeV}\right)^{1/2}$$

FREE STREAMING

For $T_{ne} < T < T_{kd}$: CDM phase space distribution is given by

$$(p \cdot \nabla) F_{cdm}(x, p) = 0 \quad \text{with} \quad \tilde{F}_{cdm}\Big|_{kd} \propto D_d$$

Geodesic motion of CDM particles defuels sub-sf :

$$D_{fs}(k) \equiv \frac{\Delta_{cdm}(k, \eta)}{\Delta_{cdm}(k, \eta_{kd})} =$$

$$= \left[1 - \frac{2}{3}(k/k_{fs})^2\right] \exp\left[-(k/k_{fs})^2\right]$$

with the characteristic free streaming scale

$$k_{fs} \approx \frac{1{,}7 \cdot 10^6}{\mathrm{Mpc}} \frac{(m/100\,\mathrm{GeV})^{1/2} (T_{kd}/30\,\mathrm{MeV})^{1/2}}{1 + \ln(T_{kd}/30\,\mathrm{MeV})/19{,}2}$$

GRAVITATIONAL GROWTH

Solve for perturbation variables

$$\Delta_j = \frac{\delta\epsilon_j}{(\epsilon+P)_j} \quad , \quad \vec{V}_j = i\frac{\vec{k}}{k} V_j \quad (j \in \{\text{rad}, \text{mat}\}) \quad , \quad \phi$$

in two subhorizon regimes:

(I) $\epsilon_{rad} \gg \epsilon_{cdm}$

(II) $\epsilon_{cdm}\Delta_{cdm} \gg \epsilon_{rad}\Delta_{rad}$

overlapping!

Including baryons:

$z \gg z_{rec}$: baryons are tightly coupled to photons

$\Rightarrow$ baryonic small scale perturbations Δ_b are erased

$10^5 > z > z_{rec}$: tight coupling breaks down

$\Rightarrow$ Δ_b is growing, but still $\Delta_b \ll \Delta_{cdm}$

$z_{rec} > z \gtrsim z_b$: $\Delta_b \ll \Delta_{cdm}$ for $k > k_b \approx 10^3\,\text{Mpc}^{-1}$ until $z_b \approx 150$

Transferfunction:

$$T_\Delta(k,z) = (6c)^2\left[\ln\frac{k}{k_{eq}} + b\right]^2\left(\frac{1+z_{eq}}{1+z}\right)^\nu$$

with

	$\Omega_b/\Omega_m = 0$	$\Omega_b/\Omega_m = 0.16$
c	$3/2$	$1{,}37$
b	$-1{,}74$	$-1{,}57$
ν	2	$1{,}80$

SO FAR

We found

$$\Delta_{cdm}(k,z) = \Delta_{cdm}(k,z_i) \cdot T_\Delta^{1/2}(k,z) \cdot D(k)$$

initial density perturbations — transfer function — damping factor

$\uparrow$

CDM microphysics

$$T_\Delta(k,z) = (6c)^2 \left[\ln \frac{k}{k_{eq}} + b \right] \left(\frac{1+z_{eq}}{1+z} \right)^\nu$$

$$D(k) = \left[1 - \frac{2}{3}(k/k_{fs})^2 \right] \exp\left[-(k/k_{fs})^2 - (k/k_d)^2 \right]$$

$\tilde{B}$ino 'benchmark models':

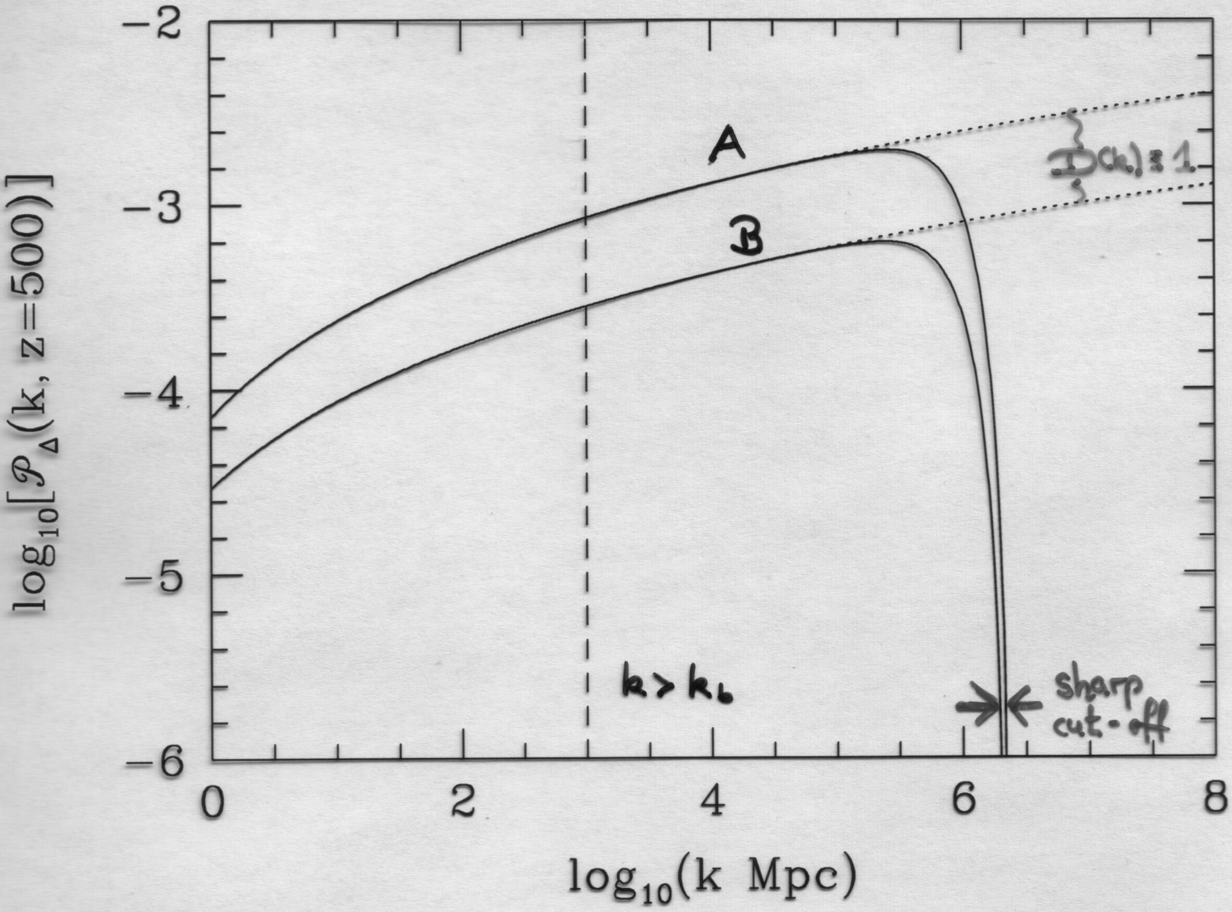
	m GeV	M GeV	T_{cd} GeV	$\Omega_{\tilde{B}}$	T_{kd} MeV
A	100	230	4,0	0,31	33
B	150	190	5,8	0,17	21

	$\lambda_{kd}(k_d)$ $1/H_{kd}$	M_d $10^{-11} M_\odot$	$\lambda_{eq}(k_{fs})$ $1/H_{eq}$	M_{fs} $10^{-7} M_\odot$
A	10^{-2}	9	10^{-8}	9
B		6		6

POWER SPECTRUM

We find for $k > k_b$ and $z_{eq} \gg z > z_b$

$$\frac{\mathcal{P}_\Delta(k,z)}{10^{-7}A} = 1{,}06\,c^2\left[\ln\frac{k}{k_{eq}} + b\right]^2 D^2(k)\left(\frac{1+z_{eq}}{1+z}\right)^{\nu}$$

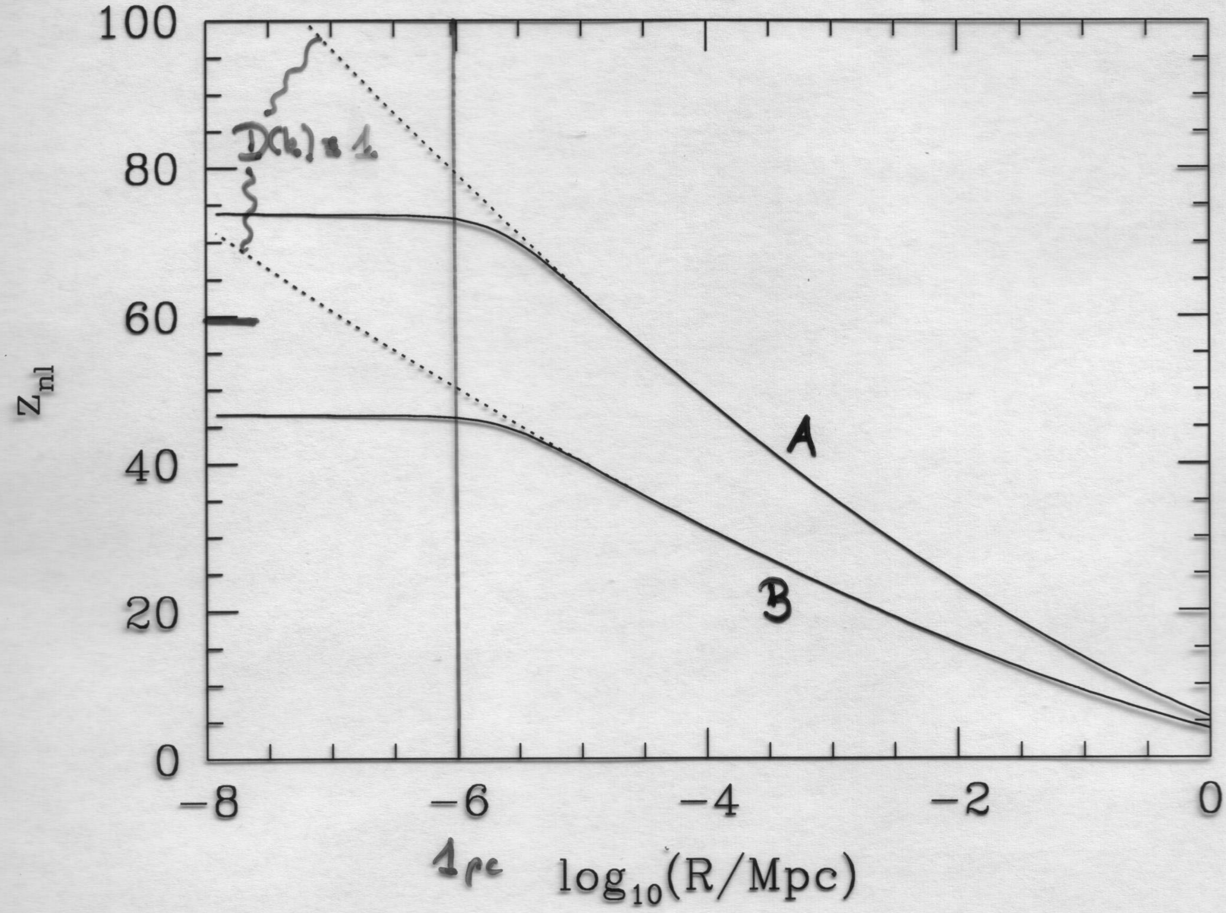


NONLINEAR REGIME

Typical fluctuations on comoving scale R go nonlinear

$$\sigma(R, z_{nl}) = 1$$

↑
mass variance



Spherical collapse model:

$$M(R) = 1{,}6 \cdot 10^{-7}\, M_\odot\, \frac{\omega_m}{0{,}14} \left(\frac{R}{pc}\right)^3$$

CDM overdensities that go nonlinear have $2M \approx M_\oplus$ and size

$$r = 1{,}05\, R / (1 + z_{nl}^{max}) \approx 0{,}02 \text{ pc}$$

$$\Rightarrow \Delta_{halo} = 7 (1 + z_{nl}^{max})^3 \approx 10^6$$

SUMMARY

- The power spectrum of CDM density perturbations has a sharp cut-off (physical) which is sensitive to the underlying CDM microphysics.
    - Allows for simulations of hierarchial structure formation using a complete initial spectrum that is consistent with CDM microphysics.
- The power spectrum has a maximum close to the cut-off which sets the typical scale for the 1st haloes in hierarchial structure formation.
    - Ab-initio calculation of sub-structure formation needed for CDM detection experiments (e.g. flux of annihilation products $\sim \Delta_{halo}^2$).

      CDM searches probe small scale structure.